

Design of Close Formation Controller for UAVs Based on Fuzzy PID Rule

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Abstract—The compact formation of UAVs is a formation in which the lateral distance between UAVs is twice the wingspan. Because it can effectively improve the aerodynamic performance of UAVs in formation, thereby reducing resistance, saving fuel, extending the duration, and also can confuse the enemy air defense system with the information of a super-large transport aircraft on the radar screen to achieve unexpected combat purposes, it has attracted much attention. In this paper, the aerodynamic coupling effect of UAV in close formation is studied, and the state space equation in three-dimensional space is established to describe the state of two UAVs. The fuzzy PID formation controller is designed and the effectiveness of the controller is verified by simulation.

Keywords—UAVs, close formation, aerodynamic coupling, fuzzy PID

I. INTRODUCTION

Unmanned systems have developed rapidly in recent years, attracting the attention of more researchers. UAVs have gradually entered the modern military and civilian fields due to their low cost, unmanned driving, and strong operability. UAV formations control, which can take full advantage of the performance of each UAV and use multiple UAVs' specific advantages such as cooperation of completing tasks, has become hot research objects. Compared with rotary wing UAVs, the control of fixed wing UAVs is more complicated, but it has irreplaceable advantages such as long-distance flight, heavy load and high speed. For the formation control of fixed-wing UAVs, some scholars have conducted some research on the aerodynamic coupling effects of UAVs, and established related UAV tight formation control models.

PID controller is the most widely used controller in the control field, including proportional controller, integral controller and differential controller. The control method is simple, and the control principle has nothing to do with the controlled object, only the input error. However, its shortcomings are also obvious. PID parameter tuning generally depends on engineering practice and control is often affected by the parameters. Once the parameters of traditional PID controllers are determined, they cannot be changed, which will cause the control effect in the entire control process. You cannot make adaptive adjustments based on the current system status.

Fuzzy control is a digital control technology created based on fuzzy mathematical theory. It is characterized in that different fuzzy rules can be applied to each special control system. During the control process, the corresponding usage rules are continuously adjusted and calculated according to the input. Output. Based on the above points, the UAV close formation controller is designed. The fuzzy control is applied

to the PID controller to optimize the control parameters in real time, and the traditional PID control results are compared to verify the effectiveness of the UAV close formation controller designed in this paper.

II. MODEL OF THE FORMATION FLIGHT SYSTEM

This paper mainly explores the relative position relationship between leadman and wingman in three-dimensional space. The mathematical model of UAV control and kinematics is as follows.

A. UAV Autopilot Model

According to the research of Proud [1], each UAV should carry an UAV autopilot, which can be divided into three parts: speed controller, heading controller and altitude controller. Among them, the mathematical expression of the speed controller and the heading controller is a first-order system, as altitude controller's is a second-order system:

$$\dot{V} = -\frac{1}{\tau_V}V + \frac{1}{\tau_V}V_c \quad (1)$$

$$\dot{\psi} = -\frac{1}{\tau_\psi}\psi + \frac{1}{\tau_\psi}\psi_c \quad (2)$$

$$\ddot{h} = -\left(\frac{1}{\tau_{h_a}} + \frac{1}{\tau_{h_b}}\right)\dot{h} - \frac{1}{\tau_{h_a}}\frac{1}{\tau_{h_b}}h + \frac{1}{\tau_{h_a}}\frac{1}{\tau_{h_b}}h_c \quad (3)$$

where V_c , ψ_c and h_c are the input control quantities, as τ_V , τ_ψ , τ_{h_a} and τ_{h_b} are the relevant time constants [3]. The structure of the equation solution shows that the UAV autopilot model can ensure that the speed, heading and altitude of the UAV gradually approach a given input control amount, and the convergence speed is related to the time constant.

B. UAV Formation Kinematics Model

The coordinate systems mainly used in this paper include [2]:

1. Ground coordinate system (inertial coordinate system) $i - O_i x_i y_i z_i$: The coordinate system fixed on the earth's surface, the origin O_i can usually be any point on the ground, $O_i z_i$ points to the center of the earth, $O_i x_i y_i$ takes the local horizontal plane, and the axis $O_i x_i$ and $O_i y_i$ are in the same horizontal plane, as the direction can be arbitrarily specified. If the UAV's flight distance and the flight time is short, the rotational angular velocity and the curvature of the earth can be ignored. Under this condition, the ground coordinate system is equal to the inertial coordinate system.

2. Body coordinate system $b - O_b x_b y_b z_b$ The origin O_b of the coordinate system fixed to the UAV is generally set to

the UAV's center of mass; the x-axis is consistent with the UAV's longitudinal axis and the head is positive; the z-axis is in the longitudinal symmetry plane of the UAV and is perpendicular to x-axis, pointing downwards; the y-axis is perpendicular to the longitudinal symmetry plane $O_b x_b z_b$ and points to the right-wing direction of the UAV.

3. Airflow coordinate system $a - O_a x_a y_a z_a$: The origin O_a of Airflow coordinate system, also called the wind axis system, is fixed to the center of mass of the UAV. The x-axis points in the direction of the UAV's speed vector and is forward. The axis-z, pointing directly below, is in the plane of symmetry of the UAV. The y-axis is perpendicular to the plane $O_a x_a z_a$, pointing to the right of the UAV.

The relative position of instantaneous plane between leadman and wingman is shown in Fig.1, where V_L and V_W represent the instantaneous speed vectors of aircrafts, ψ_L and ψ_W represent the instantaneous heading angle of vehicles, x represents the transverse space between lead and wing, and y represents the longitudinal space. According to Fig 1:

$$\frac{dx}{dt} = V_L \cos \psi_E + \dot{\psi}_W y - V_W \quad (4)$$

$$\frac{dy}{dt} = V_L \sin \psi_E + \dot{\psi}_W x \quad (5)$$

where $\psi_E = \psi_L - \psi_W$.

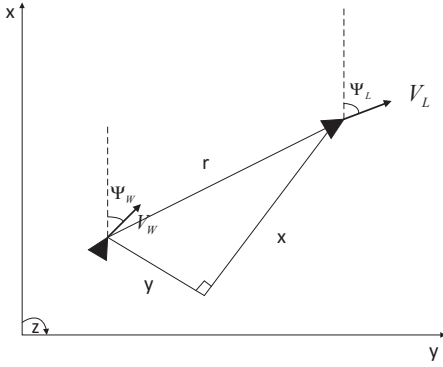


Fig.1 Wing Aircraft Rotating Reference Frame

III. ANALYSIS OF AERODYNAMIC COUPLING EFFECTS OF UAVS

For a fixed-wing UAV, its flight speed can reach tens of kilometers per hour. Such a high speed will produce a considerable air vortex at the wing tip of the wing, which will cause a strong disturbance to the following UAVs. Therefore, in order to design the UAV formation controller more accurately, it is necessary to give this aerodynamic coupling effect as a mathematical model and analyze it.

A. UAV Eddy Current Mathematical Model

According to relevant literature research, the eddy current model on the UAV wing is similar to two parallel eddy current zones [3], as shown in the following figure. Where b is the UAV wingspan and b' is the vorticity. According to the relevant theory of aerodynamics, it can be known that during the flight of the UAV, the airflow will flow through the wing and causes a pressure difference to cause the UAV to fly up. Later, the UAV caused a certain impact. Some relevant literatures show that the distance between the vortex vortices satisfies the following formula [4]:

$$b' = \frac{\pi}{4} b \quad (6)$$

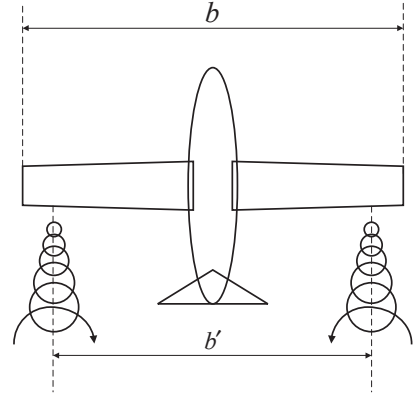


Fig.2 Vortex model of UAV

If the lateral distance between the UAVs is greater than or equal to twice the wingspan of the UAV, that is $x \geq 2b$ the length of the vortex can be considered to be infinite in real time [1]. At this time, according to the Biot-Safar law, velocity can be analogized to the magnetic field excited by the current element, so the additional speed generated by the leadman eddy current at a certain distance from the wingman can be rewritten as follows [1]:

$$\vec{W} = \frac{\hat{\Phi} \Gamma}{2\pi r_c} \quad (7)$$

According to the actual shape of the UAV, this speed can be orthogonally decomposed into an upside velocity and a lateral velocity, as shown in the following figure:

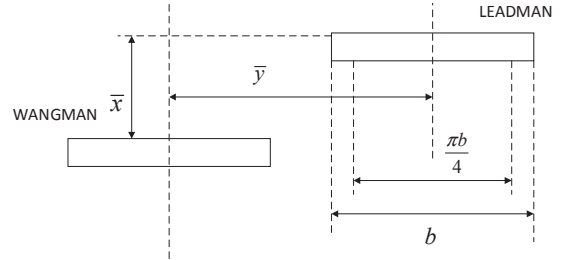


Fig.3 Top view of two aircraft formation

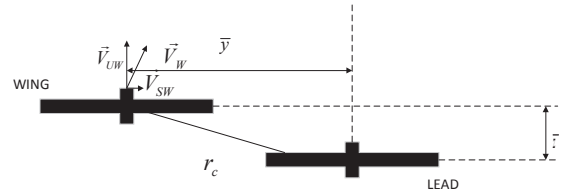


Fig.4 Main view of two aircraft formation

From the analysis of Figure 4, we can obtain the average value of upside velocity and lateral velocity:

$$\vec{V}_{UW_{avg}} = \frac{\Gamma_L}{4\pi b} \left[\ln \frac{y'^2 + z'^2 + \mu^2}{\left(y' - \frac{\pi}{4}\right)^2 + z'^2 + \mu^2} - \ln \frac{\left(y' + \frac{\pi}{4}\right)^2 + z'^2 + \mu^2}{y'^2 + z'^2 + \mu^2} \right] (-\hat{z}) \quad (8)$$

$$\vec{V}_{SW_{avg}} = \frac{\Gamma_L}{4\pi h_Z} \left[\ln \frac{\left(y' - \frac{\pi}{8}\right)^2 + z'^2 + \mu^2}{\left(y' - \frac{\pi}{8}\right)^2 + \left(z' + \frac{b}{h_Z}\right)^2 + \mu^2} - \ln \frac{\left(y' + \frac{\pi}{8}\right)^2 + z'^2 + \mu^2}{\left(y' + \frac{\pi}{8}\right)^2 + \left(z' - \frac{b}{h_Z}\right)^2 + \mu^2} \right] \hat{y} \quad (9)$$

It will bring the wingman changes in lift, resistance and lateral force, showing in Fig. 5:

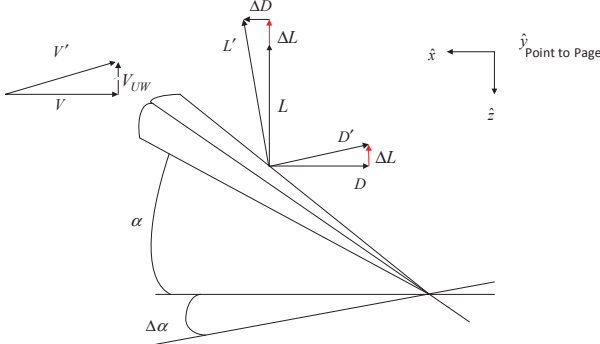


Fig.5 Sideview of Wingman's wing lift rotation

Meanwhile it creates some parameter, including [3]: lift coefficient ΔC_{LW} , resistance increment coefficient ΔC_{DW} , and lateral force increment coefficient ΔC_Y . We can calculate the dimensionless coefficients:

$$\sigma_{UW}(y', z') = \frac{2}{\pi^2} \left[\ln \frac{y'^2 + z'^2 + \mu^2}{\left(y' - \frac{\pi}{4}\right)^2 + z'^2 + \mu^2} - \ln \frac{\left(y' + \frac{\pi}{4}\right)^2 + z'^2 + \mu^2}{y'^2 + z'^2 + \mu^2} \right] (-\hat{z})$$

$$\sigma_{SW}(y', z') = \frac{2}{\pi} \left[\ln \frac{\left(y' - \frac{\pi}{8}\right)^2 + z'^2 + \mu^2}{\left(y' - \frac{\pi}{8}\right)^2 + \left(z' + \frac{h_Z}{b}\right)^2 + \mu^2} - \ln \frac{\left(y' + \frac{\pi}{8}\right)^2 + z'^2 + \mu^2}{\left(y' + \frac{\pi}{8}\right)^2 + \left(z' - \frac{h_Z}{b}\right)^2 + \mu^2} \right] (-\hat{z})$$

Then derive them in three directions, where $y' \equiv \frac{y}{b}$ and $z' \equiv \frac{z}{b}$. According to the data, we deduced that when the UAV satisfies $y = \frac{\pi}{4}b$ and the longitudinal distance is equal to 0, it can obtain the maximum lift factor, the smallest resistance increment factor, and a small lateral force increment factor. That is to say, at this time, the lift obtained by the UAV

reaches the maximum, the resistance is the smallest, and it is in the optimal formation.

B. Mathematical Model of UAV Close Formation with Aerodynamic Coupling Effect

According to the above analysis, the three-dimensional space state equation of the wingman in the tight formation of the UAV can be obtained [1], and specific parameters and calculation methods are shown in [5]:

$$\frac{d}{dt} \begin{bmatrix} x \\ V_W \\ y \\ \psi_W \\ z \\ \zeta \end{bmatrix} = A \begin{bmatrix} x \\ V_W \\ y \\ \psi_W \\ z \\ \zeta \end{bmatrix} + B \begin{bmatrix} V_{Wc} \\ \psi_{Wc} \\ h_{Wc} \end{bmatrix} + C \begin{bmatrix} V_L \\ \psi_L \\ h_{Lc} \end{bmatrix} \quad (10)$$

Among them, the distance between the wingman and the leadman in three directions, the wingman's forward speed, heading angle, and longitudinal speed are taken as the state vector $X = [x \ V_W \ y \ \psi_W \ z \ \zeta]^T$. The speed control amount, heading control amount, and altitude control amount output by the controller are used as control vectors $Y = [V_{Wc} \ \psi_{Wc} \ h_{Wc}]$. Then take the forward speed, heading angle, and altitude control of the leadman as the coupling amount $Z = [V_L \ \psi_L \ h_{Lc}]$.

IV. DESIGN OF FUZZY PID CONTROLLER

A. Principle of Traditional PID Controller

The mathematical model of a traditional PID controller is shown below:

$$u = K_P \cdot e + K_I \cdot \int e dt + K_D \cdot \frac{de}{dt} \quad (11)$$

For UAV formation systems, the main control variables include the speed, heading and distance between the three directions of the UAVs. The input control quantities in the UAV autopilot model are speed, heading, and altitude control [6], so the PID controller can be designed as follows:

$$\begin{aligned} V_c &= k_{x_p} e_x + k_{x_i} \int_0^t e_x dt + k_{x_d} \frac{de_x}{dt} \\ \psi_c &= k_{y_p} e_y + k_{y_i} \int_0^t e_y dt + k_{y_d} \frac{de_y}{dt} \\ h_c &= k_{z_p} e_z + k_{z_i} \int_0^t e_z dt + k_{z_d} \frac{de_z}{dt} \end{aligned} \quad (12)$$

where

$$\begin{aligned} e_x &= k_x x + k_{y_e} e_y \\ e_y &= k_y y + k_{\psi_e} e_{\psi} \\ e_z &= k_z z \end{aligned}$$

These factors represents the error signals in three directions of the UAVs, that is, the three distances between the formation of the UAVs, the linear combination of the speed error and the heading angle error between the main aircraft [1].

B. Design of Fuzzy PID Controller

Fuzzy control is a computer digital control technology based on fuzzy set theory, fuzzy linguistic variables and fuzzy logic reasoning [7]. The main control process is as follows:

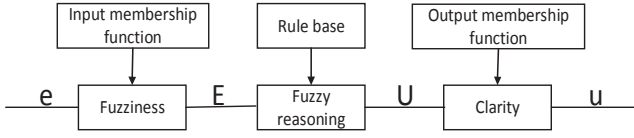


Fig.6 Fuzzy Controller Control Process

First of all, the input variables e need to be fuzzified, that is, the interval to which the variable belongs is determined according to the value range of the variable and the corresponding membership degree is obtained according to the membership function. Then the input variable e and the designed rule base are used to find the interval to which the output variable u belongs. The degree of membership of the input variable e is used to obtain an accurate output variable u , that is clarity. The more important step is to establish an accurate rule base[8]. For example, if the current error is positive and the error derivative is also positive, the control amount should be positive at this time. In addition, corresponding rules should be designed according to other situations. Generally speaking, rules are designed based on comprehensive expert experience and system models, and play a vital role in system control.

In this article, we used the fuzzy control principle to design a corresponding rule table to optimize the nine parameters $k_{x_p}, k_{y_p}, k_{z_p}, k_{x_I}, k_{y_I}, k_{z_I}, k_{x_D}, k_{y_D}$ and k_{z_D} in the PID controller. The input value are the error signals and their derivative in the three directions of the UAV. The output value are the parameter of PID controller. This article divides the variables into seven parts according to the value range: NB (negative large), NM (negative middle), NS (negative small), ZO (zero), PS (positive small), PM (positive middle), PB (Chia Tai). Common clarification methods generally include the center-of-gravity method, the maximum membership degree method, and the coefficient-weighted average method [9]. This article adopts the coefficient weighted average method and uses a two-dimensional fuzzy controller, that is, the input is the error and the first derivative of the error. The input-output relationship satisfies:

$$u = \frac{\sum k_i x_i}{\sum k_i} \quad (13)$$

where u is the output after deblurring; x_i is the fuzzy variable element; k_i is the membership degree of the corresponding element x_i [10]. The input and output membership functions are all triangular membership functions. The specific functions are shown in the following figure:

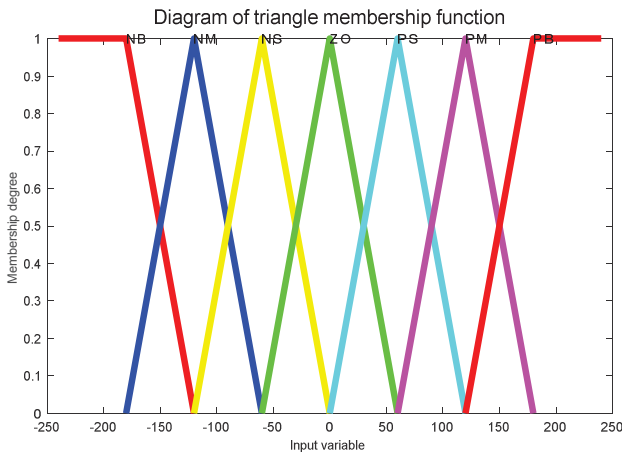


Fig.7 Triangle membership function graph

According to different parameters of PID controller, three rule bases are designed: proportional coefficient rule base, integral coefficient rule base and differential coefficient rule base [11]. In each iteration, the size of the nine parameters of the PID controller of this iteration is determined using the specific range of the input error and its error derivative.

C. UAV Tight Formation Control

Based on the above content, the UAV tight formation control system is designed:

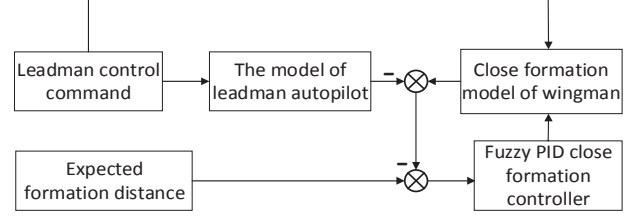


Fig.8 Fuzzy-PID Control System of UAVs formation

According to the figure above, the specific steps for tight formation control of UAVs using fuzzy PID controllers are as follows:

- Enter the leadman control instructions (speed control, heading control, altitude control) and the expected formation distance in three directions between aircrafts.
- Obtain the status of the leadman at the next moment according to the leadman autopilot model and control instruction
- Make a difference between the status of the leadman in the next time and the status of the wingman in the current time, and compare it with the expected formation distance. Use the linear combination of the differences as the input of the fuzzy PID controller to find the ideal control value of the wingman at the next moment.
- Treat the ideal control amount of the wingman at the next moment as an input, and use the wingman's close formation model to find the state of the wingman at the next moment
- Repeat steps 2-4 until the end of the set time

D. Simulation Comparison Test and Result Analysis

The traditional PID controller and the fuzzy PID controller are used to control the close formation model and the non close formation model of the UAV respectively, and the control effect of the two controllers is observed by changing the course control command of the long aircraft. The simulation time is set to 60 seconds and the step length is 0.02 seconds, and the simulation results are as follows. The initial values are shown in Table 1.

TABLE1 INITIAL STATE SETTING OF UAVS

States	Controls	Leadman	Wingman
Velocity (ft/s)	825	0	0
Heading ($^\circ$)	-30	0	0
Altitude (ft)	45000	0	0
Transverse space (ft)	60	-	60
Longitudinal space (ft)	23.5	-	23.5
Vertical spacing (ft)	0	-	0

In Fig.9 and Fig.10, the heading control command is uniformly reduced to 0 degree; in figure 11 to 13, the heading control command is uniformly reduced to -30 degree in the first 5 seconds. The black line is the stable value set at the beginning of the simulation. The red line indicates the state quantity change of the wingman, and the green line indicates the state quantity change of the host. The next two pictures show the changes of wingman status under close formation and non-close formation. It shows that the convergence speed of the wingman is slower, and the stable heading angle deviates to counteract the influence of the turbulence of the ebb and flow.

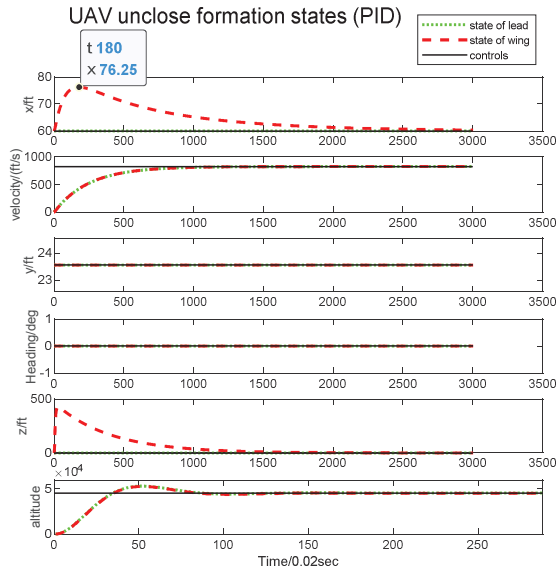


Fig.9 UAV unclosed formation states (PID)

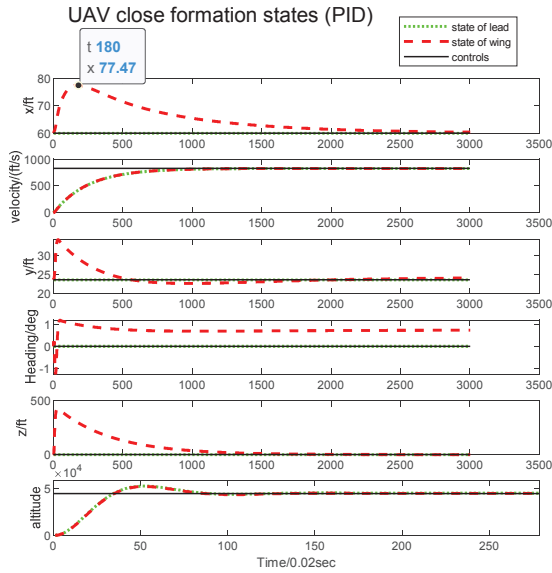


Fig.10 UAV closed formation states (PID)

It can be seen from the simulation of comparison tests that formation control with the aerodynamic coupling effect is more difficult, which is manifested in the slower convergence rate in the early stage of control, and the steady-state error in the later stage of control compared with the UAV formation control without aerodynamic coupling. The steady-state error is greater; compared to the traditional PID controller, the fuzzy PID controller responds faster to the UAV tight formation control, has better convergence performance, and is

more sensitive to heading changes in flight, but in the later stage of convergence, its stability is worse than that of the traditional PID controller. This is because its controlled parameter output will be affected by the input deviation in real time. It can obtain a better control effect in the early stage of control, but in the later stage of control, the deviation will produce a continuous small amplitude shocks can even cause the system to diverge in some cases. While the traditional PID controller has a slower convergence rate in the early stage and is prone to large-scale oscillations, it will be more stable than the fuzzy PID controller in the later stage due to the fixed parameters. Therefore, after analysis, the control method of using the fuzzy PID controller for the first 8 seconds and then converting to the PID controller can be adopted. The simulation result is shown in Fig.13. It can be seen that this method achieves the best control stability and convergence error of UAV under tight formation conditions.

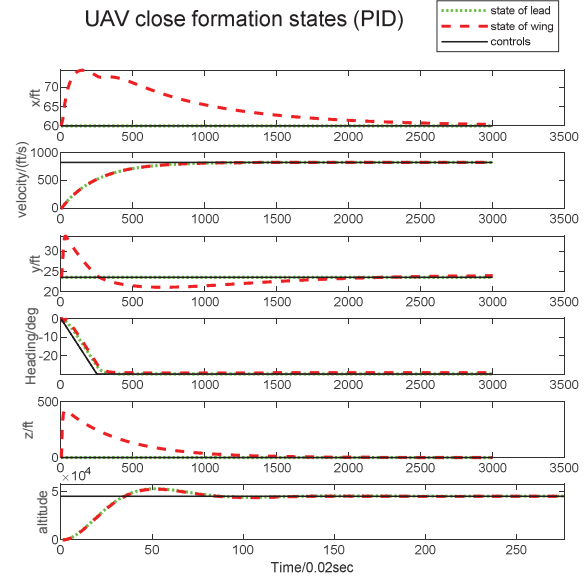


Fig.11 UAV closed formation states (PID)

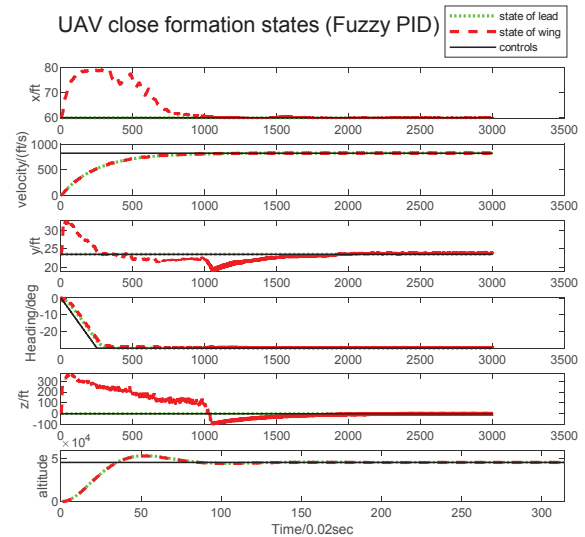


Fig.12 UAV closed formation states (Fuzzy PID)

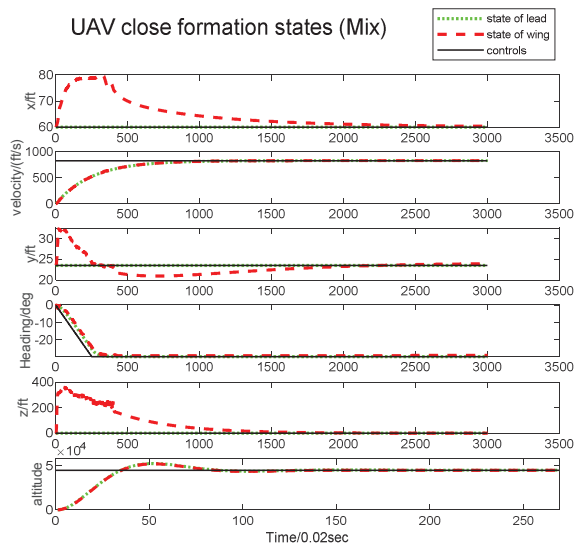


Fig.13 UAV close formation states (Mix)

V. CONCLUSION

This paper focuses on the formation control of UAV close formation, mathematically models the aerodynamic coupling effect of UAV close formation, and analyzes the theoretical optimal formation of UAV close formation. Based on this, a fuzzy PID controller is designed to optimize the control parameters of the traditional PID controller and improve its control performance. Comprehensive simulation comparison tests and analysis of their results can be drawn:

- UAV formation control under the influence of aerodynamic coupling will lead to more complicated wingman control, slower convergence speed and larger overshoot.
- The above analysis and simulation results show that the formation controller designed in this paper can

greatly improve the reliability of the tight formation of the UAV. The effectiveness of the controller for the tight formation control of the UAV is confirmed.

The above analysis and simulation results show that the formation controller designed in this paper can greatly improve the reliability of UAV close formation. The effectiveness of the controller for UAV close formation control is verified.

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