Projectile Motion

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May 30, 2024

Abstract

Projectile motion is the motion of an object as it is thrown into the air, with the influence of gravity. This paper explores the principles of projectile motion through an analysis and a live, interactive simulation. The mathematical equations that give the trajectory of a projectile are derived and explained. Additionally, to reinforce understanding, a web-based projectile motion simulator is presented, allowing users to adjust initial parameters like angle, velocity, and height to visualize the resulting projectile motion. By combining conceptual explanations with an interactive simulation, this paper aims to provide a comprehensive introduction to projectile motion.

1. Introduction

Projectile motion is a fundamental concept in physics, with applications in various fields, such as engineering, sports, and physics. You use projectile motion every time you kick a soccer ball, watch fireworks, or throw a pen for a friend to use.

Yet, despite its importance, projectile motion can be challenging to comprehend without visualization, especially for first-time students. The effects of initial velocity, constant acceleration, and so many other variables can be difficult to grasp with just static diagrams and equations.

To address this learning curve, this project creates an easy way for students to visualize and compare how different variables affect the curves of objects in projectile motion.

2. Computation and Math

This project is built with HTML, TailwindCSS, and Vanilla JavaScript. You can view the code here.

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Projectile motion involves the movement of an object along two axes: horizontal and vertical. Let's define some basic concepts:

Launch Angle: The angle the object is launched at, relative to the horizontal axis.
Initial Velocity: The velocity of the object when it is launched.
Trajectory: The projectile's path, which typically looks like a parabola.
Range: The total horizontal distance traveled by the projectile before hitting the ground.
Maximum Height: The maximum (vertex) the projectile reaches in the set motion.

Projectile motion is described with the following equations:

Horizontal Motion: $x = v_{0x}t$
Vertical Motion: $y = v_{0y}t - \frac{1}{2}gt^2$
Range: $R = \frac{v_0^2 \sin(2\theta)}{g}$
Maximum Height: $H = \frac{v_{0y}^2}{2g}$
Time of Flight: $T = \frac{2v_0 \sin(\theta)}{g}$
Velocity Components: $v_x = v_{0x}, v_y = v_{0y} - gt$

Where:
x and y are the horizontal and vertical displacements, respectively.
v_{0x} and v_{0y} are the initial horizontal and vertical velocities, respectively.
v_x and v_y are the horizontal and vertical velocities at time t.
g is the acceleration due to gravity (approximately 9.8 $\frac{m}{s^2}$ on Earth).
θ is the launch angle.
R is the range.
H is the maximum height.
T is the Time of Flight.

Several external factors can influence projectile motion:
Launch Angle: The angle affects its range and maximum height.
Initial Velocity: The velocity determines how far and high it will travel.
Gravity: Gravity affects the vertical motion of the projectile, accelerating it downward.
Air Resistance: Air resistance can affect the trajectory (not accounted for).
External Forces: Any external forces can alter its trajectory (not accounted for).

Now, let's use those formulas to help us find the trajectory. At a high level, the application needs to know the ball's position at time t from the initial velocity v_0 at an angle θ. We can easily do this with the horizontal and vertical motion.

While we figure out the trajectory, let's use an initial velocity of $v_0 = 20m/s$ at an angle of $\theta = 45°$ as an example.

We can decompose our initial velocity into horizontal and vertical components using trigonometry.

Horizontal Velocity: $v_{x0} = v_0 \cos(\theta) = 20 \cos(45°)$
Vertical Velocity: $_{y0} = v_0 \sin(\theta) = 20 \sin(45°)$

$$cos(45°) = \sin(45°) = \frac{\sqrt{2}}{2}$$

,

$$v_{x0} = v_{y0} = 20 \cdot \frac{\sqrt{2}}{2} = 10\sqrt{2}m/s$$

At time t, the x value can be calculated by integrating the velocity v_{x0} with t. In this example, t will be $1.5s$

$$x = v_{x0}t$$
$$x = 10\sqrt{2} \cdot 1.5$$
$$x \approx 21.213$$

We do something similar for y, but we must account for gravity. In this example, we're going to use Earth's gravity, $g = 9.8m/s^2$

$$y = v_{y0}t - \frac{1}{2}gt^2$$

$$y = 10\sqrt{2}(1.5) - \frac{1}{2}(9.8)(1.5)^2$$
$$y \approx 10.188$$

Thus, on every frame, the program simply finds the x and y coordinates of each ball t time after it's shot using the above steps.

We can quickly derive T and R.

The total time of flight T occurs when $y = 0$. Solving the vertical motion equation for t when $y = 0$ gives us:

$$0 = v_{y0}T - \frac{1}{2}gT^2$$

We can use the quadratic equation to solve for T. The only solution that makes physical sense (positive time) is:

$$T = \frac{2v_{y0}}{g} = \frac{2v_0 \sin(\theta)}{g}$$

We can use T to find R.

The range R is the horizontal distance traveled during the time of flight. Substitute the time of flight T into the horizontal motion equation:

$$R = v_{x0}T = v_0 \cos(\theta) \cdot \frac{2v_0 \sin(\theta)}{g}$$

We can simplify to find:

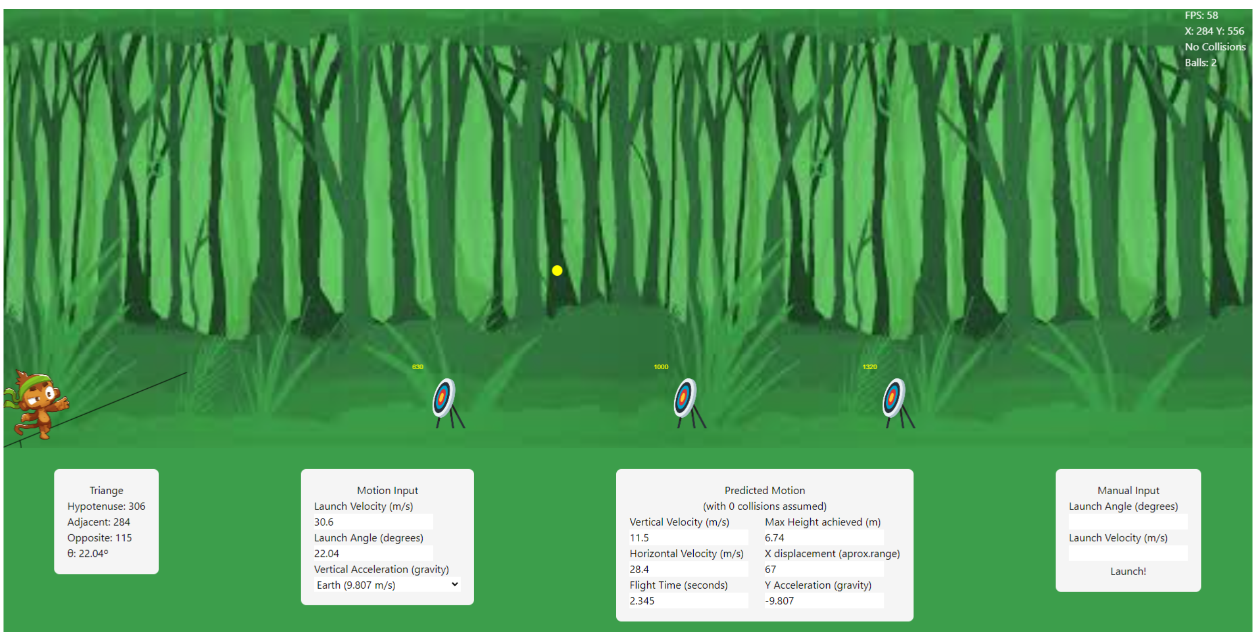
$$R = \frac{v_0^2 \sin(2\theta)}{g}$$

If you're curious, you can derive the provided equations to find the equation for the parabolic form of a projectile motion. The steps are left as an exercise to the reader, but the equation is:

$$y = x \tan(\theta) - \frac{gx^2}{2u^2 \cos^2(\theta)}$$

3. Demo

You can view a live demo of the app at https://apcalc.jyao.dev/.



Play around, edit variables, and have fun!

You can watch a video of the demo working here.

If you're interested, you can view the code as a GitHub Repository here.

4. Conclusion

Going into this project with no knowledge of projectile physics, learning the physics and math, and finally developing a simulation app was a really interesting journey that I would not have taken outside of this project. I learned a lot, and I was able to create a fully functional project while having fun.

References

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