Session 20: Optimization Modeling II (Abstract Formulation)

Example (Concrete formulation from last session's Q1)

Select which bestsellers to carry at an Amazon store's grand opening. The following table provides the list of Top 10 Bestsellers in Literature & Fiction, along with their genres.

Rank \ Genre	Literary	Sci-Fi	Romance	Thriller
1	$\surd$			
2		$\surd$		$\surd$
3			$\surd$	$\surd$
4	$\surd$		$\surd$	
5	$\surd$			
6			$\surd$	
7		$\surd$		
8				$\surd$
9	$\surd$	$\surd$		
10			$\surd$	

You wish to carry the minimum number of bestsellers, while ensuring that there are at least two bestsellers in each genre.

Concrete LP formulation from last session:

Decision variables: Let x_i be a binary decision variable denoting whether to carry book i, where $i \in \{1, 2, \cdots, 10\}$.

Objective and Constraints:

$$\begin{aligned}
\text{Minimize:} \quad & x_1 + x_2 + \cdots + x_{10} \\
\text{subject to:} \quad & \\
\text{(Literary)} \quad & x_1 + x_4 + x_5 + x_9 \geq 2 \\
\text{(Sci-Fi)} \quad & x_2 + x_7 + x_9 \geq 2 \\
\text{(Romance)} \quad & x_3 + x_4 + x_7 + x_{10} \geq 2 \\
\text{(Thriller)} \quad & x_2 + x_3 + x_8 \geq 2 \\
& x_i \in \{0, 1\} \text{ for every book } i \in \{1, 2, \cdots, 10\}.
\end{aligned}$$

Step 3a. Use variables to encode all input data

Data:

Step 3b. Formulate the LP/MIP in terms of only data and decision variables

Decision variables:

Objective and Constraints:

Q1 (Emergency Vehicle Location)

The city of Metropolis is divided into nine districts and is considering seven possible sites to place emergency vehicles. The table below shows the time (minutes) it takes for an emergency vehicle to travel from each district to each site. (The column labels are sites and row labels are districts.)

District \ Row	1	2	3	4	5	6	7
1	5	3	4	3	8	9	0
2	3	6	5	4	8	0	3
3	4	3	6	8	10	3	2
4	6	0	2	7	3	2	5
5	2	8	2	5	0	6	8
6	2	6	4	0	7	3	5
7	0	12	5	5	5	7	2
8	10	9	0	2	3	5	7
9	2	4	5	7	3	4	5

Formulate an optimization problem to find the minimum number of sites so that all districts are within three minutes of an emergency vehicle, then generalize this to be able to handle arbitrary data of a similar format.

Step 1. Identify the decision, objective, and constraints in English

Decision:

Objective:

Constraints:

Step 2. Formulate the optimization as linear expressions of decision variables

Decision variables:

Objective and constraints:

Step 3a. Use variables to encode all input data

Data:

Step 3b. Formulate the LP/MIP in terms of only data and decision variables

Decision variables:

Objective and constraints:

Q2 (Transportation Planning)

Your software company has launched a new Analytics product. As sales manager, you are planning to promote the product by sending salesforce to software conventions running concurrently in Los Angeles, Saint Louis, and Detroit.

You have 6 representatives available at each of your Little Rock and Urbana Branches. You would like to send at least 2 to the Los Angeles convention, 5 to the Saint Louis convention, and at least 4 to the Detroit convention.

Roundtrip airfare between the locations are as follows:

	Los Angeles	St. Louis	Detroit
Little Rock	250	150	200
Urbana	300	200	150

Formulate an optimization problem to allocate your sales force so as to minimize total airfare, then generalize the formulation to be able to handle arbitrary number of office branches, conventions cities, availability of representatives, requirement for conventions, and transportation cost.

Step 1. Identify the decision, objective, and constraints in English

Decision:

Objective:

Constraints:

Step 2. Formulate the optimization as linear expressions of decision variables

Decision variables:

Objective and constraints:

Step 3a. Use variables to encode all input data

Data:

Step 3b. Formulate the LP/MIP in terms of only data and decision variables

Decision variables:

Objective and constraints:

Q3 (Optimal Debt Payments)

Paris has come to you because she needs help paying off her credit card bills. Her statement at the beginning of month 1 shows the following balances:

Credit Card	Balance	Monthly Rate
Saks Fifth Avenue	$20,000	0.5%
Bloomingdale's	$50,000	1.0%
Macy's	$40,000	1.5%

Paris has agreed not to shop at any of these stores anymore, and she is willing to allocate up to 5,000 dollars per month to pay off these credit cards. All cards must be paid off within 36 months (meaning that her statement at the beginning of month 37 must be zero for all card). Paris' goal is to minimize the total of all her payments.

For this problem, assume that the interest for the month is applied after the payment for that month. For example, suppose Paris pays 5,000 on Saks for month 1. Then her Saks balance at the beginning of month 2 is $(1.005) \times (20000 - 5000) = 15075$.

Help Paris solve her problem by formulating it into a linear program, then generalize it to be able to handle arbitrary number of credit cards, balances, monthly rates, monthly budget, and time required for full payment.

Step 1. Identify the decision, objective, and constraints in English

Decision:

Objective:

Constraints:

Step 2. Formulate the optimization as linear expressions of decision variables

Decision variables:

Objective and constraints:

Step 3a. Use variables to encode all input data

Data:

Step 3b. Formulate the LP/MIP in terms of only data and decision variables

Decision variables:

Objective and constraints: