Lab Report: Revenue Management for Trojan Airlines

1. Executive Summary

Based on a simulation analysis, we find that the capacity rationing policy obtains the highest expected profit, with the best rationing policy obtaining an approximtaely 46% improvement over the best constant price policy. The optimal parameters for the three policies and the corresponding profits are as follows.

Policy	Expected Profit	Price 1	Price 2	Number of Seats Left
Constant Price	2670	255	-	-
Dynamic Price	3553	240	305	-
Capacity Rationing	3888	230	320	41

For grading, a correct answer needs to have the following:

- Capacity rationing policy is the best.
- Best constant price between 240 and 270, and profit between 2300 and 3000.
- Best dynamic p_1 between 225 and 255, best p_2 between 290 and 335, and best profit between 3200 and 3900.
- Best rationing p_1 between 215 and 245, best p_2 between 305 and 340, best l between 30 and 50, and best profit between 3500 and 4300.

2. Generating Customers

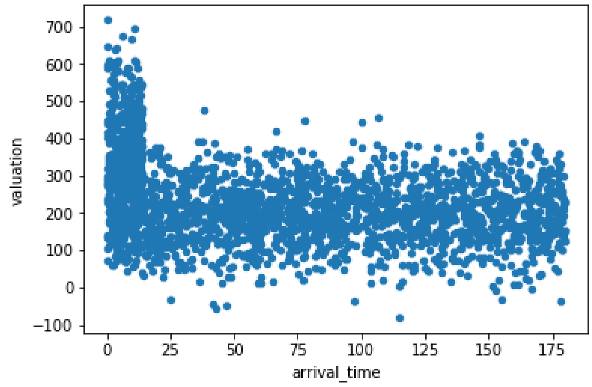
The following function creates customer data for one flight. It first generates the size of each segment, and then generate customer arrival time (time of purchase given as days before departure) and willingness to pay for each segment, according to the generated size. It stores the result in a DataFrame, which is sorted by the arrival time according to who came first.

[1]:
```
from scipy.stats import uniform, norm
import pandas as pd
def generateCustomers():
    leisureSize=max(0,int(round(norm(200,60).rvs())))
    businessSize=max(0,int(round(norm(60,20).rvs())))
    leisureTime=uniform(0,180)
    businessTime=uniform(0,14)
    leisureValue=norm(200,80)
    businessValue=norm(350,120)
    data=[]
    for i in range(leisureSize):
        data.append([leisureTime.rvs(),leisureValue.rvs()])
    for i in range(businessSize):
        data.append([businessTime.rvs(),businessValue.rvs()])
    customers=pd.DataFrame(data,columns=['arrival_time','valuation'])
    customers=customers.sort_values(by='arrival_time',ascending=False)
    customers.index=range(len(customers))
    return customers
```

[26]:
```
# Test code for grading
import numpy as np
np.random.seed(0)
```

```
pd.concat([generateCustomers() for i in range(10)])\
    .plot(x='arrival_time',y='valuation',kind='scatter')
```

```
<matplotlib.axes._subplots.AxesSubplot at 0x7fa747091518>
```



3. Simulating Pricing Policies

The following code cell defines three function for the three pricing policies. Each function calculates the total revenue given the customer information and the parameters of the pricing policy.

```
[52]: def simulateConstant(customers,seats,p):
          ''' Returns the revenue earned from charging a constant price of p,
          given a specified number of total seats.'''
          return min(seats,(customers['valuation']>=p).sum())*p

      def simulateDynamic(customers,seats,p1,p2):
          ''' Returns the revenue earned from charging price p1 up to 14 days
          before departure, and then changing the price to p2. The total number
          of seats is given as an input parameter, as well as the customer arrival
          times and valuations (in the customers DataFrame).'''
          revenue=0
          p=p1
          for i in customers.index:
              if seats==0:
                  break
              if customers.loc[i,'arrival_time']<=14:
                  p=p2
              if customers.loc[i,'valuation']>=p:
                  revenue+=p
                  seats-=1
          return revenue

      def simulateRationing(customers, seats, p1, p2, l):
          '''Returns the revenue earned from charging a price of p1 until
          only l tickets are left, and then charging a price of p2. The total number
          of seats is given as an input, as well as the customer arrival times and
          valuations (in the customers DataFrame)'''
          revenue=0
          p=p1
```

```
    for i in customers.index:
        if seats==0:
            break
        if seats<=1:
            p=p2
        if customers.loc[i,'valuation']>=p:
            revenue+=p
            seats-=1
    return revenue
```

```
[53]: # Test code for automatic grading
import pandas as pd
import numpy as np

def testConstant():
    data=[[170,50],[150,40],[13,95],[12,100]]
    customers=pd.DataFrame(data,columns=['arrival_time','valuation'])
    if not simulateConstant(customers,2,50)==100:
        return False
    if not simulateConstant(customers,2,100)==100:
        return False
    data=[[170,50],[150,40],[14,40],[13,95],[12,100]]
    customers=pd.DataFrame(data,columns=['arrival_time','valuation'])
    if not simulateConstant(customers,1,40)==40:
        return False
    if not simulateConstant(customers,3,50)==150:
        return False
    if not simulateConstant(customers,5,40)==200:
        return False
    return True

def testDynamic():
    data=[[170,50],[150,40],[13,95],[12,100]]
    customers=pd.DataFrame(data,columns=['arrival_time','valuation'])
    if not simulateDynamic(customers,3,40,95)==175:
        return False
    if not simulateDynamic(customers,2,40,95)==80:
        return False
    if not simulateDynamic(customers,1,50,100)==50:
        return False
    data=[[170,50],[150,40],[13,95],[13.5,40],[12,100]]
    customers=pd.DataFrame(data,columns=['arrival_time','valuation'])
    if not simulateDynamic(customers,2,40,100)==80:
        return False
    if not simulateDynamic(customers,4,40,95)==270:
        return False
    return True

def testRationing():
    data=[[170,50],[150,40],[13,95],[12,100]]
    customers=pd.DataFrame(data,columns=['arrival_time','valuation'])
    if not simulateRationing(customers,3,40,100,1)==180:
```

```
            return False
        if not simulateRationing(customers,2,40,100,1)==140:
            return False
        if not simulateRationing(customers,1,40,95,1)==95:
            return False
        data=[[170,50],[150,40],[120,50],[13,95],[12,100],[11,150]]
        customers=pd.DataFrame(data,columns=['arrival_time','valuation'])
        if not simulateRationing(customers,3,40,100,0)==120:
            return False
        if not simulateRationing(customers,3,40,100,1)==180:
            return False
        if not simulateRationing(customers,3,40,100,2)==240:
            return False
        if not simulateRationing(customers,3,40,151,2)==40:
            return False
        if not simulateRationing(customers,3,40,100,3)==200:
            return False
        return True

    def testAll():
        correct='1 (CORRECT)'
        wrong='0 (WRONG)'
        error='0 (RUNTIME ERROR, CHECK BY HAND USING BELOW CODE)'
        try:
            if testConstant():
                constant=correct
            else:
                constant=wrong
        except:
            constant=error
        try:
            if testDynamic():
                dynamic=correct
            else:
                dynamic=wrong
        except:
            dynamic=error
        try:
            if testRationing():
                rationing=correct
            else:
                rationing=wrong
        except:
            rationing=error
        print('Test results')
        print('\tsimulateConstant:\t',constant,'\n')
        print('\tsimulateDynamic:\t',dynamic,'\n')
        print('\tsimulateRationing:\t',rationing,'\n')

    testAll()
```

```
Test results
        simulateConstant:         1 (CORRECT)

        simulateDynamic:         1 (CORRECT)

        simulateRationing:         1 (CORRECT)
```

4. Optimizing Parameters

The following code first generates a list of 100 randomly generated DataFrames, each containing customer

4.1 Generating 100 Sets of Simulated Customer Data

```
[6]: # Generating a list of 1000 customer scenarios (like 1000 random flights)
     N=100
     import numpy as np
     np.random.seed(0)
     dataset=[generateCustomers() for i in range(N)]
```

4.2 Optimizing Constant Policy

```
[21]: import numpy as np
      seats=100
      fixedCost=20000
      resultsConstant=pd.DataFrame()
      i=0
      for p in range(0,750,5):
          revenue=np.average([simulateConstant(customers,seats,p) \
                                  for customers in dataset])
          profit=revenue-fixedCost
          resultsConstant.loc[i,'Price']=p
          resultsConstant.loc[i,'Profit']=profit
          i+=1
      resultsConstant.sort_values(by='Profit',ascending=False).head(1)
```

	Price	Profit
51	255.0	2669.5

4.2 Optimizing Dynamic Policy

```
[9]: # Take 1 with high jumps
     import numpy as np
     seats=100
     fixedCost=20000
     resultsDynamic=pd.DataFrame()
     i=0
     for p1 in range(0,750,50):
         for p2 in range(0,750,50):
             revenue=np.average([simulateDynamic(customers,seats,p1,p2)\
```

```
                            for customers in dataset])
            profit=revenue-fixedCost
            resultsDynamic.loc[i,'Price1']=p1
            resultsDynamic.loc[i,'Price2']=p2
            resultsDynamic.loc[i,'Profit']=profit
            i+=1
    resultsDynamic.sort_values(by='Profit',ascending=False).head(10).agg(['max','min'])
```

	Price1	Price2	Profit
max	250.0	400.0	3359.5
min	200.0	200.0	-266.0

```
[10]: # Take 2 with reduced bounds
      import numpy as np
      seats=100
      fixedCost=20000
      resultsDynamic2=pd.DataFrame()
      i=0
      for p1 in range(180,400,5):
          for p2 in range(180,400,5):
              revenue=np.average([simulateDynamic(customers,seats,p1,p2)\
                                  for customers in dataset])
              profit=revenue-fixedCost
              resultsDynamic2.loc[i,'Price1']=p1
              resultsDynamic2.loc[i,'Price2']=p2
              resultsDynamic2.loc[i,'Profit']=profit
              i+=1
      resultsDynamic2.sort_values(by='Profit',ascending=False)\
          .head(20).agg(['max','min'])
```

	Price1	Price2	Profit
max	245.0	330.0	3552.8
min	235.0	290.0	3447.5

```
[13]: resultsDynamic2.sort_values(by='Profit',ascending=False).head(1)
```

	Price1	Price2	Profit
553	240.0	305.0	3552.8

4.3 Optimizing Rationing Policy

```
[11]: # Take 1
      import numpy as np
      seats=100
      resultsRationing=pd.DataFrame()
      i=0
      for p1 in range(0,750,50):
          for p2 in range(0,750,50):
              for l in range(0,101,10):
                  revenue=np.average(\
                          [simulateRationing(customers,seats,p1,p2,l)\
                           for customers in dataset])
                  profit=revenue-fixedCost
```

```
                resultsRationing.loc[i,'Price1']=p1
                resultsRationing.loc[i,'Price2']=p2
                resultsRationing.loc[i,'Seats Left']=l
                resultsRationing.loc[i,'Profit']=profit
                i+=1
resultsRationing.sort_values(by='Profit',ascending=False)\
            .head(20).agg(['max','min'])
```

```
     Price1  Price2  Seats Left  Profit
max   250.0   400.0       100.0  3409.0
min     0.0     0.0         0.0  2620.0
```

```
[15]: resultsRationing.sort_values(by='Profit',ascending=False)\
                  .head(10).agg(['max','min'])
```

```
     Price1  Price2  Seats Left  Profit
max   250.0   400.0        60.0  3409.0
min   200.0   300.0        10.0  3012.0
```

```
[27]: # Take 2 with reduced bounds
      import numpy as np
      seats=100
      resultsRationing2=pd.DataFrame()
      i=0
      for p1 in range(190,270,5):
          for p2 in range(270,430,5):
              for l in range(0,101,1):
                  revenue=np.average([simulateRationing(customers,seats,p1,p2,l) \
                                      for customers in dataset])
                  profit=revenue-fixedCost
                  resultsRationing2.loc[i,'Price1']=p1
                  resultsRationing2.loc[i,'Price2']=p2
                  resultsRationing2.loc[i,'Seats Left']=l
                  resultsRationing2.loc[i,'Profit']=profit
                  i+=1
      resultsRationing2.sort_values(by='Profit',ascending=False)\
                      .head(10).agg(['max','min'])
```

```
     Price1  Price2  Seats Left  Profit
max   235.0   330.0        42.0  3887.7
min   230.0   315.0        35.0  3868.0
```

```
[30]: resultsRationing2.sort_values(by='Profit',ascending=False)\
                      .head(60).agg(['max','min'])
```

```
     Price1  Price2  Seats Left  Profit
max   235.0   340.0        45.0  3887.7
min   230.0   305.0        32.0  3842.6
```

```
[14]: resultsRationing2.sort_values(by='Profit',ascending=False).head(1)
```

```
      Price1  Price2  Seats Left  Profit
9131   230.0   320.0        41.0  3887.7
```

```
[17]: # Percentage improvement
      resultsRationing2['Profit'].max()/resultsConstant['Profit'].max()-1
```

```
0.4563401386027348
```