Session 26: Gurobi Practice (Portfolio Optimization)

In this lab, you will practice your Gurobi coding skills by analyzing a large-scale portfolio optimization case.

1. Problem

Trojan investment is exploring new methods for updating its portfolio of US stocks based on mixed integer linear and quadratic optimization. In particular, it would like to optimize the trade-off between returns and risk, given the presence of transaction costs and managerial overhead. In particular, transaction cost implies that the new portfolio must not be too different from the current portfolio. Managerial overhead means that if the company invest in any stock, there should be a sufficiently large stake, and the number of stocks in the portfolio cannot be too large. The abstract formulation is given below.

Data:

- S: the set of stocks.
- w_i: the old weight of stock $i \in S$ before optimization. (The "weight" of a stock is % of total funds invested in the stock; weights of all stocks should add to one.)
- R_i: the expected annual return of stock $i \in S$.
- C_{ij}: the estimated covariance between stocks $i, j \in S$.
- σ_{target}: the maximum volatility of the final portfolio.
- Δ: the total movement allowed between the old weights and the new weights.
- k: the maximum # of stocks allowed in the portfolio.
- ϵ: the minimum non-zero weight allowed.

Decision variables:

- x_i: the new weight of stock i. (Continuous)

Objective and constraints: All summations are over the set S of stocks.

$$
\begin{array}{rll}
\text{Maximize:} & \sum_i R_i x_i & \text{(Average Return)} \\
\text{subject to:} & & \\
\text{(Valid weights)} & \sum_i x_i = 1 & \\
\text{(Risk tolerance)} & \sum_{i,j} C_{ij} x_i x_j \leq \sigma_{target}^2 & \\
\text{(Change in weights 1)} & x_i - w_i \leq \delta_i & \text{for each stock } i. \\
\text{(Change in weights 2)} & -(x_i - w_i) \leq \delta_i & \text{for each stock } i. \\
\text{(Change in weights 3)} & \frac{1}{2} \sum_i \delta_i \leq \Delta & \\
\text{(Non-negligible weights)} & \epsilon z_i \leq x_i \leq z_i & \text{for each stock } i. \\
\text{(Simplicity)} & \sum_i z_i \leq k & \\
\text{(Non-negativity)} & x_i \geq 0 &
\end{array}
$$

2. Data

The file "26-data.xlsx" (emailed to everyone and available on NBViewer along with other handouts and notes) contains two sheets. The sheet "s&p500" contains the stock prices of every stock on the S&P500 for 10 years. The sheet "oldPortfolio" contains the weights on the current portfolio. The following code can be used to load the data and calculate the returns R_i and covariances C_{ij}.

[1]:
```python
import pandas as pd
import numpy as np

oldPortfolio=pd.read_excel('26-data.xlsx',sheet_name='oldPortfolio'\
                           ,index_col=0)['Weight']
oldPortfolio
```

```
Stock
AMGN    0.306342
CNC     0.231379
FFIV    0.290586
FL      0.019480
LEG     0.152214
Name: Weight, dtype: float64
```

[2]:
```python
rawPrices=pd.read_excel('26-data.xlsx',sheet_name='s&p500'\
                        ,index_col=0).fillna(method='ffill')
logPrices=np.log(rawPrices)
priceChange=logPrices.diff(1).iloc[1:,:].fillna(0)
C=priceChange.cov()*252           # About 252 business days in a year
R=priceChange.mean()*252
```

[3]:
```python
R.head()
```

```
MMM     0.101382
AOS     0.252184
ABT     0.084367
ABBV    0.096193
ACN     0.141367
dtype: float64
```

[4]:
```python
C.iloc[:5,:5]
```

	MMM	AOS	ABT	ABBV	ACN
MMM	0.049054	0.042544	0.021191	0.008905	0.031119
AOS	0.042544	0.098905	0.025834	0.010012	0.039423
ABT	0.021191	0.025834	0.042142	0.012491	0.023052
ABBV	0.008905	0.010012	0.012491	0.039773	0.008844
ACN	0.031119	0.039423	0.023052	0.008844	0.063869

3. Optimizing for Given Parameters

Solve the optimization problem for the following parameters:

- σ_{target}: 0.25
- Δ: 0.3
- k: 20
- ϵ: 0.001

The code should save the result in an excel file "26-output.xlsx" with a single sheet, in the same format as the "oldPortfolio" sheet above.

[16]:

```
   Stock    Weight
5    CNC  0.231379
10  NFLX  0.214099
9   FFIV  0.153085
1   AMGN  0.115536
4   AVGO  0.114421
0   ALGN  0.096729
6   CHTR  0.037426
7    STZ  0.015109
3    BHF  0.007911
2   BKNG  0.007882
8   DLTR  0.006422
```

[6]:

```
Return: 0.28329526743055844
Risk: 0.2500199849510983
# stocks: 11.0
Change in portfolio: 0.5
```

4. Tradeoff between multiple objectives

The following example illustrates how to analyze problems with multiple objectives. It is based on Q1 from session 23, or DMD Example 8.1.

Decision variables: Let A, G, D denote the fraction of total investment to put in the assets Advent, GSS, and Digital.

Objective and constraints:

$$\begin{aligned}
\text{Maximize:} \quad & 11A + 14G + 7D \\
\text{subect to:} \quad & \\
\text{(Fractions)} \quad & A + G + D = 1 \\
\text{(Target risk)} \quad & \sqrt{16A^2 + 22G^2 + 10D^2 + 6AG + 2GD - 10AD} \leq \sigma \\
\text{(Nonnegativity)} \quad & A, G, D \geq 0
\end{aligned}$$

[7]:

```python
from gurobipy import Model, GRB
import numpy as np
mod2=Model()
```

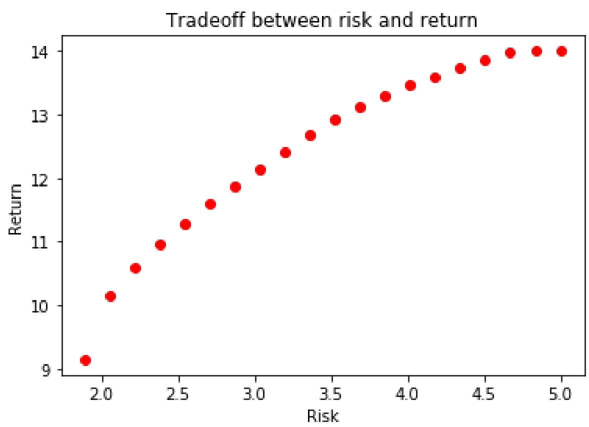
```
sigma=GRB.INFINITY
A=mod2.addVar()
G=mod2.addVar()
D=mod2.addVar()
ret=11*A+14*G+7*D
riskSquared=16*A*A+22*G*G+10*D*D+6*A*G+2*G*D-10*A*D
mod2.setObjective(riskSquared)
mod2.addConstr(A+G+D == 1)
mod2.setParam('outputflag',False)
mod2.optimize()
print('Minimum risk possible:',np.sqrt(riskSquared.getValue()))
```

```
Minimum risk possible: 1.8928303077552984
```

```
[8]: mod2.setObjective(ret,sense=GRB.MAXIMIZE)
     riskConstraint=mod2.addConstr(riskSquared<=GRB.INFINITY)
     mod2.setParam('outputflag',False)
     mod2.optimize()
     print('Maximum possible return:',ret.getValue())
     print('Corresponding sigma:',np.sqrt(riskSquared.getValue()))
```

```
Maximum possible return: 13.999999999968766
Corresponding sigma: 4.690415759786275
```

```
[14]: sigmaList=np.linspace(1.893,5,20)
      retList=[]
      for sigma in sigmaList:
          riskConstraint.QCRHS=sigma**2
          mod2.optimize()
          retList.append(ret.getValue())
      import matplotlib.pyplot as plt
      plt.plot(sigmaList,retList,'ro')
      plt.title('Tradeoff between risk and return')
      plt.xlabel('Risk')
      plt.ylabel('Return')
      plt.show()
```



(Optional) 4.1 Exercise

Analyze the tradeoff between return and risk (σ_{target}), as well as return and change in portfolio (Δ) in the problem for Trojan investment.

4.1.1 Tradeoff between return and risk

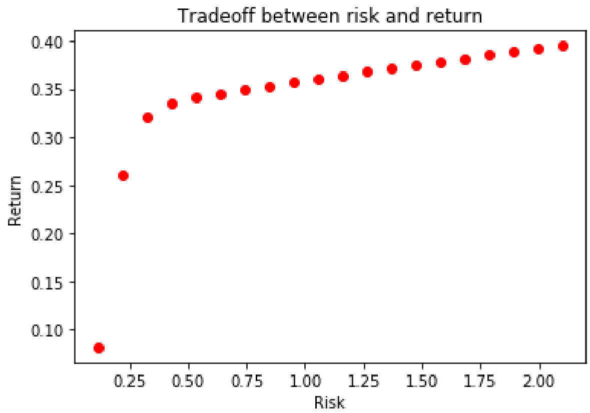
[10]:

```
Minimum total std: 0.11338791599055002
```

[11]:

```
Maximum return: 0.3949545858891008
Corresponding total std: 2.0745215455800414
```

[12]:



4.1.2 Tradeoff between return and transaction cost

[13]: