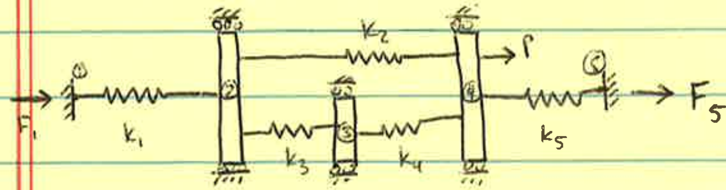
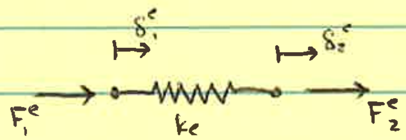
AME 50541, Lecture 2, 1/18



FBD of spring e

$$\text{1dof2gdof} = \begin{bmatrix} 1 & 2 & 2 & 3 & 4 \\ 2 & 4 & 3 & 4 & 5 \end{bmatrix}$$

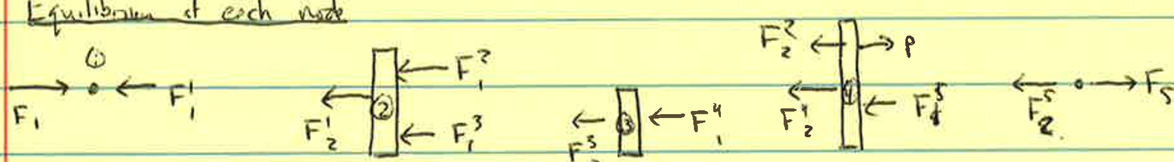


$$F_1^e = k_e(\delta_1^e - \delta_2^e)$$
$$F_2^e = k_e(\delta_2^e - \delta_1^e)$$

Compatibility

$$u_1 = \delta_1^1 \quad , \quad u_2 = \delta_2^1 = \delta_1^2 = \delta_1^3 \quad , \quad u_3 = \delta_2^3 = \delta_1^4 \quad , \quad u_4 = \delta_2^2 = \delta_2^4 = \delta_1^5 \quad , \quad u_5 = \delta_2^5$$

Equilibrium at each node



$$\cancel{F_1^1} \; F_1 = F_1^1 = k_1(\delta_1^1 - \delta_2^1)$$
$$0 = F_2^1 + F_1^2 + F_1^3 = k_1(\delta_2^1 - \delta_1^1) + k_2(\delta_1^2 - \delta_2^2) + k_3(\delta_1^3 - \delta_2^3)$$
$$0 = F_2^3 + F_1^4 = k_3(\delta_2^3 - \delta_1^3) + k_4(\delta_1^4 - \delta_2^4)$$
$$P = F_2^2 + F_2^4 + F_1^5 = k_2(\delta_2^2 - \delta_1^2) + k_4(\delta_2^4 - \delta_1^4) + k_5(\delta_1^5 - \delta_2^5)$$
$$F_5 = F_2^5 = k_5(\delta_2^5 - \delta_1^5)$$

- General rule:

compat

$$\begin{cases} F_1 = k_1(u_1 - u_2) \\ 0 = k_1(u_2 - u_1) + k_2(u_2 - u_4) + k_3(u_2 - u_3) \\ 0 = k_3(u_3 - u_2) + k_4(u_3 - u_4) \\ P = k_2(u_4 - u_2) + k_4(u_4 - u_3) + k_5(u_4 - u_5) \\ F_5 = k_5(u_5 - u_4) \end{cases}$$

apply bcs
$u_1 = u_5 = 0$

$$F_1 = k_1(-u_2)$$

$$0 = k_1(u_2) + k_2(u_2 - u_4) + k_3(u_2 - u_3) = (k_1 + k_2 + k_3)u_2 - k_3 u_3 - k_2 u_4$$

$$0 = (k_3 + k_4)u_3 - k_3 u_2 - k_4 u_4$$

$$P = (k_2 + k_4 + k_5)u_4 - k_2 u_2 - k_4 u_3 \cancel{- k_5 u_5}$$

$$F_5 = k_5(-u_4)$$

only relevant equations

$$\begin{bmatrix} k_1 + k_2 + k_3 & -k_3 & -k_2 \\ -k_3 & k_3 + k_4 & -k_4 \\ -k_2 & -k_4 & k_2 + k_4 + k_5 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ P \end{bmatrix}$$

$\rightarrow$ solve once k_e & P given!

~~★ Remainder of notes, ... MISZ 1~~

~~Let K be ndf x ndf stiffness matrix (before boundary conditions), then~~

~~$K(idx, idx)$ K_{ij} ...~~