PDE-Constrained Optimization using Progressively-Constructed Reduced-Order Models

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Introduction

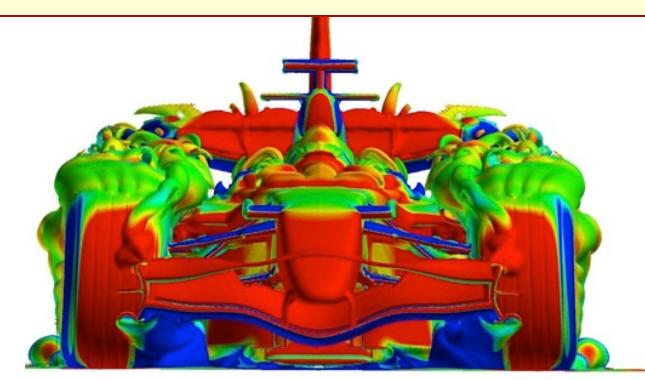
Rapidly solve a PDE-constrained optimization problem using a Reduced-Order Model (ROM) as a surrogate for the PDE

$$\begin{aligned} \underset{\mathbf{w}\in\mathbb{R}^N,\boldsymbol{\mu}\in\mathbb{R}^p}{\text{minimize}} \quad & f(\mathbf{w},\boldsymbol{\mu}) \\ \text{subject to} \quad & \mathbf{R}(\mathbf{w},\boldsymbol{\mu}) = 0 \end{aligned}$$

where the discretized PDE is

$$\mathbf{R}(\mathbf{w},\boldsymbol{\mu}) = 0$$

$\mathbf{w}$ is the state vector, $\boldsymbol{\mu}$ is the parameter vector, and N is very large



ROM-Constrained Optimization

Assume state vector lies in an r-dimensional subspace where $r \ll N$, defined by the Reduced Basis (RB), $\boldsymbol{\Phi}$

$$\mathbf{w} = \boldsymbol{\Phi}\mathbf{y}$$

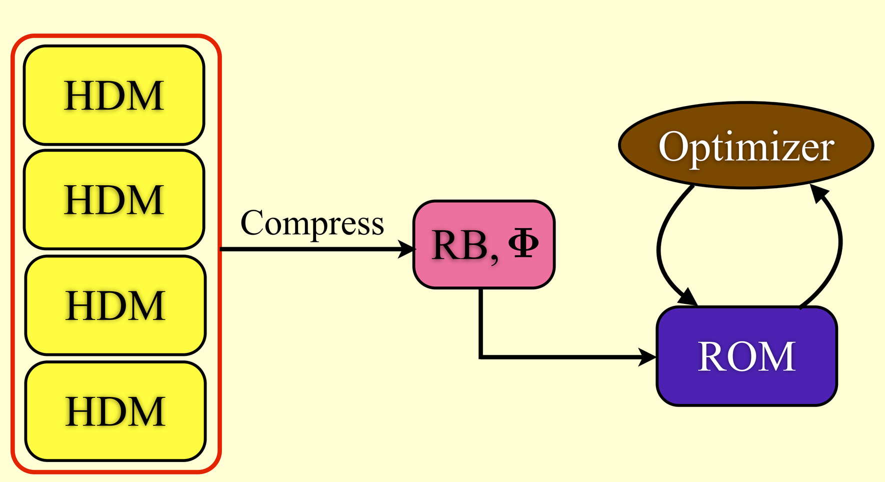
Project equations into another r-dimensional subspace

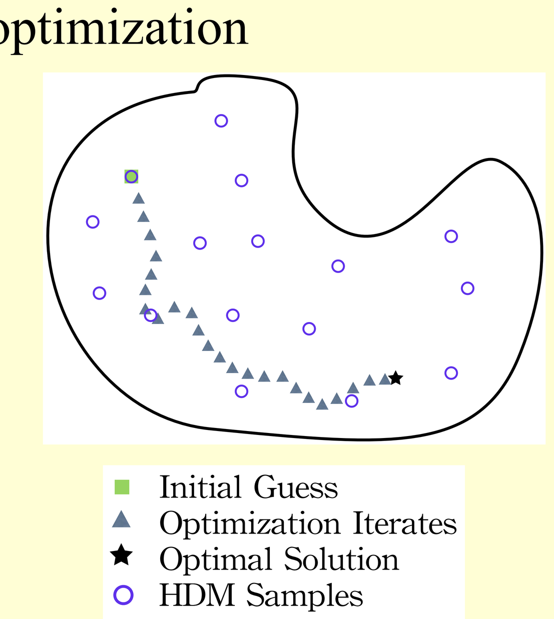
$$\boldsymbol{\Psi}^T\mathbf{R}(\boldsymbol{\Phi}\mathbf{y},\boldsymbol{\mu}) = 0$$

ROM-Constrained Optimization:

$$\begin{aligned} \underset{\mathbf{y}\in\mathbb{R}^r,\boldsymbol{\mu}\in\mathbb{R}^p}{\text{minimize}} \quad & f(\boldsymbol{\Phi}\mathbf{y},\boldsymbol{\mu}) \\ \text{subject to} \quad & \boldsymbol{\Psi}^T\mathbf{R}(\boldsymbol{\Phi}\mathbf{y},\boldsymbol{\mu}) = 0 \end{aligned}$$

Offline/Online approach to ROM-constrained optimization





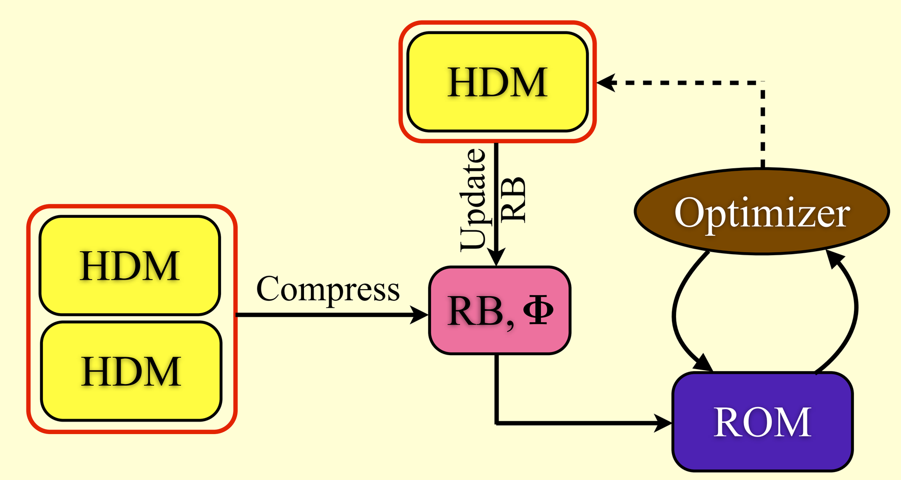
CPU effort breakdown for offline/online ROM-constrained optimization

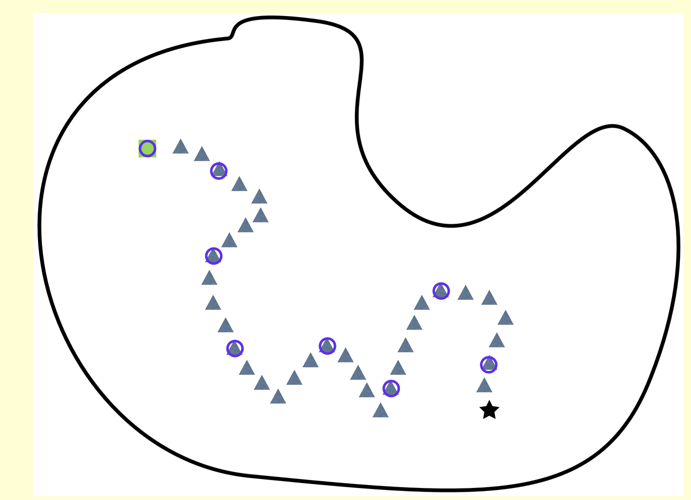
Progressively-Constructed ROMs for Optimization

Consider local version to ROM-constrained optimization

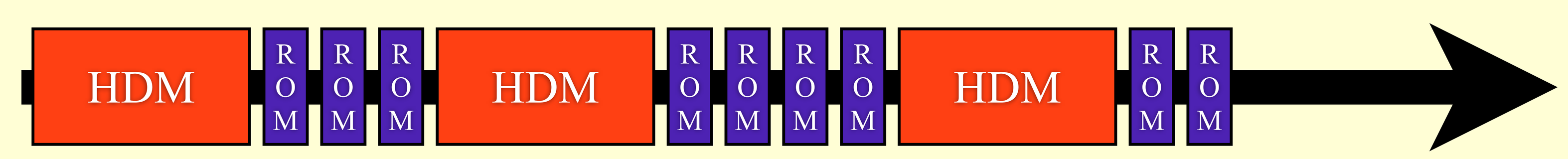
$$\begin{aligned} \underset{\mathbf{y}\in\mathbb{R}^r,\boldsymbol{\mu}\in\mathbb{R}^p}{\text{minimize}} \quad & f(\boldsymbol{\Phi}\mathbf{y},\boldsymbol{\mu}) \\ \text{subject to} \quad & \boldsymbol{\Psi}^T\mathbf{R}(\boldsymbol{\Phi}\mathbf{y},\boldsymbol{\mu}) = 0 \\ & ||\mathbf{R}(\boldsymbol{\Phi}\mathbf{y},\boldsymbol{\mu})|| \leq \epsilon \end{aligned}$$

Progressive approach to ROM-constrained optimization





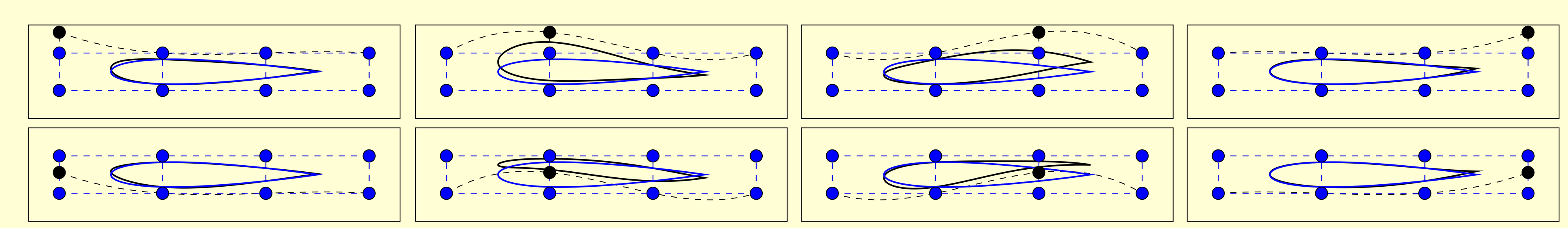
CPU effort breakdown for progressive ROM-constrained optimization

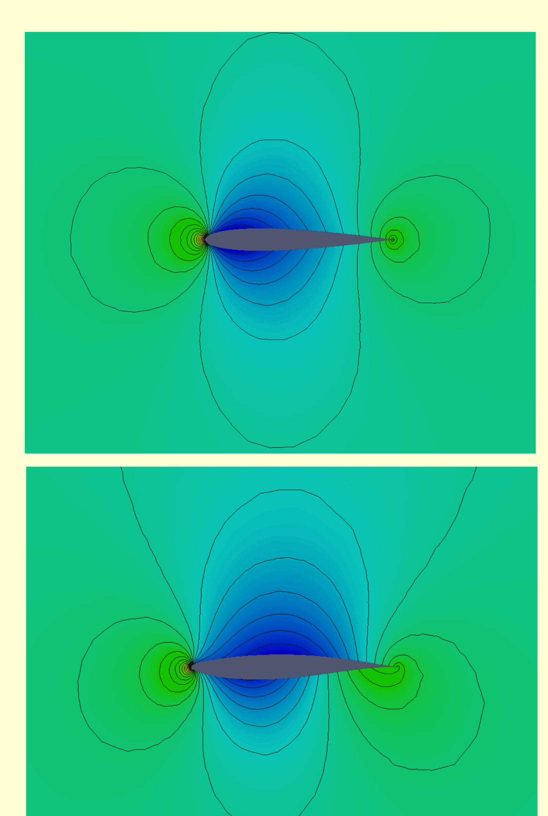


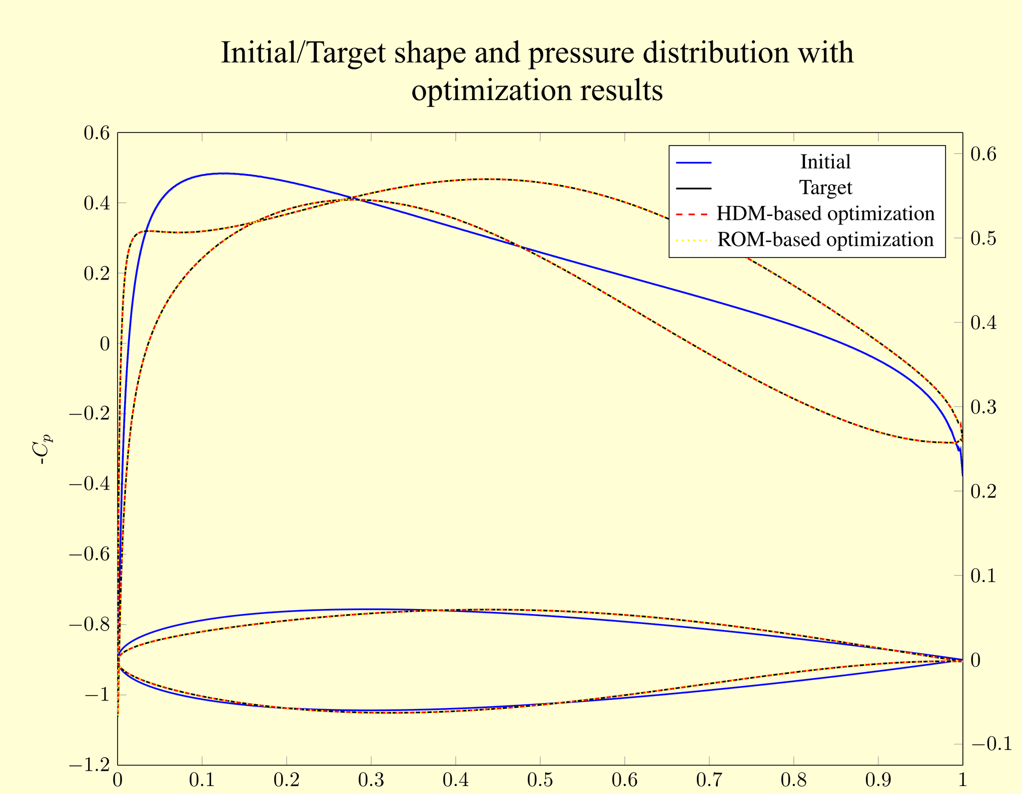
Aerodynamic Shape Optimization

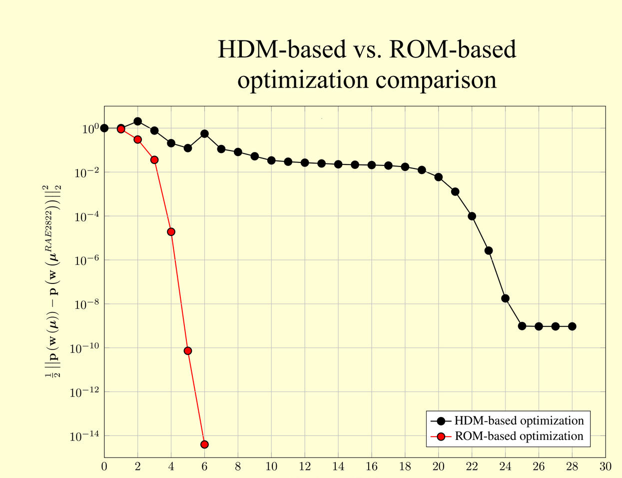
Parameter estimation shape optimization:

$$\begin{aligned} \underset{\mathbf{w}\in\mathbb{R}^N,\boldsymbol{\mu}\in\mathbb{R}^p}{\text{minimize}} \quad & \frac{1}{2}||\mathbf{p}(\mathbf{w}^*) - \mathbf{p}(\mathbf{w})||_2^2 \\ \text{subject to} \quad & \mathbf{R}(\mathbf{w},\boldsymbol{\mu}) = 0 \\ & \mathbf{c}(\mathbf{w},\boldsymbol{\mu}) \leq 0 \end{aligned}$$

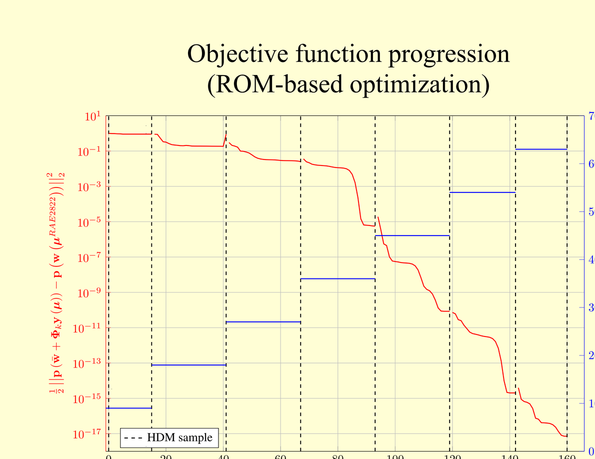


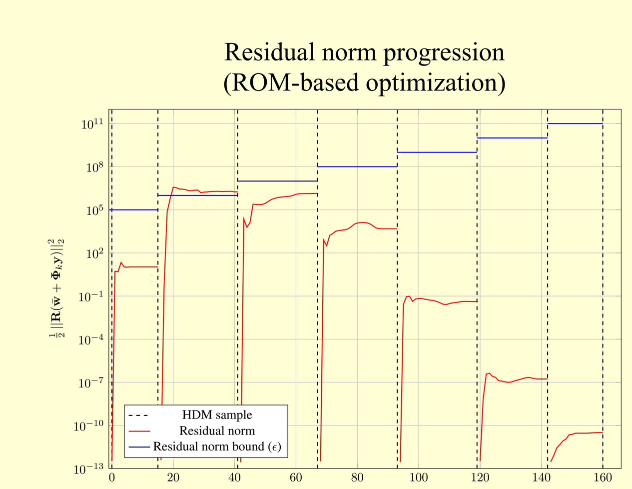






	HDM-based Optimization	ROM-based Optimization
# HDM Evaluations	29	7
# ROM Evaluations	-	346





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Further Information

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