AME40541/60541: Finite Element Methods

Course logistics

AME40541/60541: Finite Element Methods, 3 units
Lecture: MWF 10:30a-11:20a, 210 DeBartolo Hall
Recitation: W 11:30a-12:30p, 102 DeBartolo Hall

This course is being offered as a combined undergraduate (AME40541) and graduate (AME60541) course. The lectures for both listings will be co-located; however, the workload and level of rigor will be higher for those enrolled in AME60541. Undergraduate students can enroll in AME60541 for graduate credit with instructor approval.

Course description

The finite element method is the industry-standard for solving a range of thermal, structural, and fluid flow problems that commonly arise in engineering practice and research. Commercial software is well-suited for solving such problems; however, having a fundamental understanding of the underlying methods is crucial to effectively use the software, develop methods/code tailored to the particular problems they face, or undertake cutting-edge research. This course introduces the fundamental concepts of finite element methods with applications to structural analysis, heat flow, fluid mechanics, and multiphysics problems. It covers the basic topics of linear and nonlinear finite element technology including weak formulations and error analysis, domain discretization on structured and unstructured meshes, assembly of global equations, the isoparametric concept, essential and natural boundary conditions, numerical quadrature, variational crimes, and the structure of a finite element program. Throughout the course, students will build their own finite element code that will be used to investigate fundamental properties of finite element methods. In addition, the course makes use of commercial software to explore more advanced capabilities, validate their own code, and gain experience with software commonly used in engineering industry.

Learning goals

Upon successful completion of this course, you will be able to: (1) identify problems in engineering and science that can (and cannot) be solved with the finite element (FE) method, (2) apply the steps of the FE method to linear/nonlinear partial differential equations (PDEs) and engineering structures, (3) use commercial FE software to solve problems that arise in engineering and science, and (4) develop FE program from scratch that can be used/extended for research purposes.

Prerequisites

MATLAB programming, multivariable calculus, basic linear algebra

Instructor

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Office hours: MF 11:20a-1:00p, W 12:30p-1:30p, or by appointment

Teaching assistants

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Course material

Website: http://www.nd.edu/~mzahr/teach-nd-ame40541-spr20.html
Textbook: Claes Johnson. *Numerical Solution of Partial Differential Equations by the Finite Element Method*. Dover, 2009
Supplementary notes: provided by instructor on the website.

The primary website for the course will be used to distribute course materials such as handouts, lecture slides, code, homework and project prompts. Sakai will be used to distribute feedback, solutions to homeworks and the exam, and store grades.

Recitation session

An optional recitation session will be held every 1-2 weeks in 102 DeBartolo Hall on Wednesdays 11:30a-12:30p. These sessions will be used to dive deeper into material rarely covered in a traditional lecture: background on MATLAB coding, implementational aspects of FEM, interactive office hours, review sessions for examinations, final project assistance, etc.

Finite Element Software: COMSOL Multiphysics

We will use the finite element software COMSOL to explore more advanced properties and features of the finite element method and to validate code you write throughout the course. The School of Engineering has 11 COMSOL licenses installed on machines Bonzai00, Bonzai26-Bonzai35 in Fitzpatrick B19. To access COMSOL, you will need to use one of those machines. I have reserved the room every Friday 11:30a-1:00p and either the instructor or TA will be present to answer questions. To access Fitzpatrick B19 outside of office hours, make sure a class is currently not in session and use the code 26845#. You can check the schedule posted on the door to plan your visit.

Homework

There will 5-6 homeworks assigned roughly bi-weekly. They will be a combination of theory and programming exercises. Throughout the course, you will systematically build a finite element code in MATLAB, aided by the homework prompts and a minimal amount of starter code. Some homework assignments will also include a problem to solve using COMSOL.

Examinations

There will be two examinations. The first will be an in-class exam on 2/28 covering the material from chapters 1-4. The second exam will be a take-home exam, assigned 4/27 in-class and due 4/29 in-class, covering the material from chapters 5-10.

Project

There will be a project that will extend the finite element code you develop throughout the semester in the homework assignments to a general finite element code capable of handling unstructured meshes, nonhomogeneous natural boundary conditions, and nonlinear PDEs. You will use the code to solve a number of problems in heat flow, solid mechanics, and fluid mechanics.

Grading

The final score (g) will be calculated as a weighted combination of the homework (h) (30%), exams (e_1, e_2) (10% each), and project (p) (50%) scores. The raw scores will be used for the homework and project, i.e., not scaled or curved, with grades assigned based on rubric. Exam scores will be offset to account for the difficulty of the test. The offset value will be set by the instructor and based on data such as the highest raw score or performance of an independent test-taker. This offset factor will always be positive, i.e., only improve a student's performance on the exam. Thus, the final score will be computed as

$$g = 0.3h + 0.5p + 0.1(e_1 + o_1) + 0.1(e_2 + o_2) + \epsilon,$$

where $o_i \geqslant 0$ is the exam offset determined by the instructor and $\epsilon \in [0, 0.02]$ is determined from student's participation and engagement in class and office hours. Course grades are determined from the final score as follows:

Course score (g)	Course grade
$g \geqslant 0.94$	A
$0.94 > g \geqslant 0.90$	A-
$0.90 > g \geqslant 0.87$	B+
$0.87 > g \geqslant 0.84$	B
$0.84 > g \geqslant 0.80$	B-
$0.80 > g \geqslant 0.77$	C+
$0.77 > g \geqslant 0.74$	C
$0.74 > g \geqslant 0.70$	C-
$0.70 > g \geqslant 0.60$	D
$0.60 > g$	F

For example, a student with a 90% homework score ($h = 0.9$), 96% project score ($p = 0.96$), and 80% exam scores ($e_1 = e_2 = 0.8$) will receive a final score of

$$g = 0.3(0.9) + 0.5(0.96) + 0.1(0.8 + 0.1) + 0.1(0.8 + 0.1) + \epsilon = 0.93 + \epsilon,$$

where the exam offsets are $o_1 = o_2 = 0.1$, e.g., because the highest score was 90%. For a student with little-to-no participation in class and office hours ($\epsilon = 0$), this corresponds to a A-, while a student that actively participates in the course ($\epsilon = 0.02$) would receive a A.

Course schedule

MJZ travel: 2/12, 2/14, 3/18, 3/20

Lecture	**Date**	**Topics**	**Reading**
1	1/15	motivation, overview of FEM	PP 1 (on Sakai)
2-4	1/17, 1/20, 1/22	direct stiffness method	MJZ 1
5-8	1/24, 1/27, 1/29, 1/31	mathematical preliminaries	MJZ 2
9-12	2/3, 2/5, 2/7, 2/10	weighted residual method	MJZ 3
13-15	2/12, 2/14, 2/17	FEM, 1d	CJ 1.1-1.3, MJZ 4
16-18	2/19, 2/21, 2/24	variational formulation	CJ 2, MJZ 5
20	2/28	examination I (Ch 1-4)	-
19, 21	2/26, 3/2	FEM, formulation	CJ 1.4-1.7, 3, MJZ 6
22-25	3/4, 3/6, 3/16, 3/18	FEM, implementation	CJ 12, MJZ 7
26-27	3/20, 3/23	FEM, error analysis	CJ 4, MJZ 8
28-30	3/25, 3/27, 3/30	systems of PDEs	CJ 5, MJZ 9
31-33	4/1, 4/3, 4/6	nonlinear problems	CJ 13, MJZ 10
34-36	4/8, 4/15, 4/17	mixed methods	CJ 11, MJZ 11
37-39	4/20, 4/22, 4/24	time-dependent problems	CJ 8, MJZ 12
40-41	4/27, 4/29	hyperbolic problems	CJ 9, MJZ 13
-	4/27-4/29	examination II (Ch 5-10)	-

Recitation	**Date**	**Topics**	**Reading**
1	1/29	MATLAB programming	slides
2	2/5	Indicial notation	MJZ 2

Sakai

Technical support for Sakai is provided, not by me (instructor) or TAs, but by the OIT Help Desk and the Sakai Team. If you have a question or issue concerning the use of Sakai, please contact the Help Desk at oithelp@nd.edu or phone: 574-631-8111. You can also walk in; they're located in 115 DeBartolo Hall. Support Staff will contact me if they need to discuss the way I've set up our class. Please tell them the web address to our Sakai site, my NetID as your instructor and your NetID, and specifically name the assignment or task you need help with. Technical Tips Before Contacting the OIT Help Desk:

- Access Sakai with a newly opened fresh browser (not one thats been opened for a month and has 47 tabs running across the top).
- Before calling the Help Desk with issues, empty your browser history, close and re-open your browser. Often that's all it takes. (How-To Instructions Here).
- Never login to Sakai twice in two tabs in the same browser, especially when taking a test. You may lose answers.

Policies

Office hours: Students are strongly encouraged to utilize the office hours for assistance with the course material. To make the most out of office hours, students are encouraged to think through the problem on their own and avoid overly generic questions. For example, instead of "How do I do problem 3?", students should explain their thought process, what they have tried, and where they got stuck. If you cannot attend the scheduled office hours, contact the teaching staff directly to schedule an appointment.

Collaboration and honor code: Collaboration is permitted on the homework and project (not exams); however, each student must complete and submit their own assignment. Honor code violations will be handled through appropriate university channels (http://honorcode.nd.edu).

Disabilities: Any student who has a documented disability and is registered with Disability Services should speak with me as soon as possible regarding accommodations. More information can be found at http://disabilityservices.nd.edu.

Support for student mental health at Notre Dame

Care and Wellness Consultants provide support and resources to students who are experiencing stressful or difficult situations that may be interfering with academic progress. Through Care and Wellness Consultants, students can be referred to The University Counseling Center (for cost-free and confidential psychological and psychiatric services from licensed professionals), University Health Services (which provides primary care, psychiatric services, case management, and a pharmacy), and The McDonald Center for Student Well Being (for problems with sleep, stress, and substance use). Visit care.nd.edu.

Additional references

In addition to Johnson's book, there are a number of excellent FEM references, a few of which I have listed below. *These are not required for the course, but are solid addition to one's bookshelf, particularly if continuing with FEM.*

- N.S. Ottosen and H. Petersson. *Introduction to the Finite Element Method.* Prentice-Hall, 1992 (introductory)
- J.N. Reddy. *An Introduction to the Finite Element Method.* McGraw Hill, 3rd edition, 2006 (introductory)
- T.J. Hughes. *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis.* Dover, 1st edition, 2000 (introductory, more mathematical)
- Philippe G. Ciarlet. *The Finite Element Method for Elliptic Problems.* Society for Industrial and Applied Mathematics (SIAM), 2002 (advanced, finite element theory)
- Susanne C. Brenner and L. Ridgway Scott. *The Mathematical Theory of Finite Element Methods.* Springer Science & Business Media, 3rd edition, 2008 (advanced, finite element theory)
- Pavel Solin, Karel Segeth, and Ivo Dolezel. *Higher-Order Finite Element Methods.* Chapman & Hall/CRC, 2004 (advanced)
- T. Belytschko, W.K. Liu, and B. Moran. *Nonlinear Finite Elements for Continua and Structures.* John Wiley & Sons, 2013 (advanced)
- J. Hesthaven and T. Warburton. *Nodal Discontinuous Galerkin Methods: Algorithms, Analysis, and Applications.* Springer Science & Business Media, 2007 (introductory, specialized topic)