AME50541: Finite Element Methods

Course logistics

AME50541: Finite Element Methods, 3 units
Lecture: MWF, 10:30a-11:20a, 215 DeBartolo Hall

Course description

An introduction to the fundamental concepts of finite element methods with applications to structural analysis, heat flow, fluid mechanics, and coupled multiphysics problems. The course covers the basic topics of linear and nonlinear finite element technology including weak formulations and error analysis, domain discretization on structured and unstructured meshes, direct and integral approaches for assembly, the isoparametric concept, application of boundary conditions, numerical quadrature, variational crimes, the treatment of constraints, and the structure of a finite element program. Element technologies such as basic data structures, polynomial interpolation, and engineering elements (bars, beams, frames, and shells) will also be discussed. Throughout the course, students will build their own finite element code that will be used to investigate fundamental properties of finite element methods. In addition, the course makes use of commercial software to explore more advanced properties and capabilities and validate their own code.

Learning goals

Upon successful completion of this course, you will be able to: (1) identify problems in engineering and science that can (and cannot) be solved with the finite element (FE) method, (2) apply the steps of the FE method to linear/nonlinear partial differential equations (PDEs) and engineering structures (bars, beams, shells), (3) use commercial FE software to solve problems that arise in engineering and science, and (4) develop FE program from scratch that can be used/extended for research purposes.

Prerequisites

Basic MATLAB programming, multivariable calculus, basic linear algebra

Instructor

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Teaching assistant

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Course material

Website: `http://www.nd.edu/~mzahr/teach-nd-ame50541-spr19.html`
Text book: J.N. Reddy. *An Introduction to the Finite Element Method*. McGraw Hill, 3rd edition, 2006
Supplementary notes: provided by instructor on the website

The primary website for the course will be used to distribute course materials such as handouts, lecture slides, code, homework and project prompts. Sakai will be used to distribute feedback, solutions to homeworks and the exam, and store grades. Required reading tasks will be assigned from the text book, as well as portions of the homework assignments. A copy of the textbook has been placed on 2 hour reserve in the engineering library.

Finite Element Software: COMSOL Multiphysics

We will use the finite element software COMSOL to explore more advanced properties and features of the finite element method and to validate code you write throughout the course. The School of Engineering has 11 COMSOL licenses installed on machines Bonzai00, Bonzai26-Bonzai35 in Fitzpatrick B19. To access

COMSOL, you will need to use one of those machines. I have reserved the room every Friday from 11:30a-12:30p and either the instructor or TA will be present to answer questions. To access Fitzpatrick B19 outside of office hours, make sure a class is currently not in session and use the code 26845#. You can check the schedule posted on the door to plan your visit.

Homework

There will 8 homeworks assigned roughly bi-weekly. They will be a combination of theory and programming exercises. Throughout the course, you will systematically build a finite element code in MATLAB, aided by the homework prompts and a minimal amount of starter code. Some homework assignments will also include a problem to solve using COMSOL.

Examinations

There will be two examinations. The first will be on Friday, March 1 and will cover the material from Lectures 1-19. The second exam will be Wednesday, May 1 and cover the material for Lectures 21-41.

Project

There will be a project, assigned around Week 5, that will build on the finite element code you develop throughout the semester in the homework assignments. You will learn and implement an advanced topic in the finite element method; this year's topic is **topological optimization of structures**. This will involve reading a research paper on a simple but widely used topology optimization method, implementing it in your finite element code, verifying the results with COMSOL, and using your new tool to solve a number of interesting topology optimization problems.

Grading

The final score (g) will be calculated as a weighted combination of the homework (h) (30%), exams (e_1, e_2) (10% each), and project (p) (50%) scores. The raw scores will be used for the homework and project, i.e., not scaled or curved, with grades assigned based on rubric (see website). Exam scores will be offset to account for the difficulty of the test. The offset value will be set by the instructor and based on data such as the highest raw score and performance of an independent test-taker). This offset factor will always be positive, i.e., only improve a student's performance on the exam. Thus, the final score will be computed as

$$g = 0.3h + 0.5p + 0.1(e_1 + o_1) + 0.1(e_2 + o_2),$$

where $o_i \geqslant 0$ is the exam offset determined by the instructor. Course grades are determined from the final score as follows, where $\epsilon \in [0, 0.02]$ is determined from student's participation and engagement in class and office hours.

Course score (g)	Course grade
$g \geqslant 0.94 - \epsilon$	A
$0.94 - \epsilon > g \geqslant 0.90 - \epsilon$	A-
$0.90 - \epsilon > g \geqslant 0.87 - \epsilon$	B+
$0.87 - \epsilon > g \geqslant 0.84 - \epsilon$	B
$0.84 - \epsilon > g \geqslant 0.80 - \epsilon$	B-
$0.80 - \epsilon > g \geqslant 0.77 - \epsilon$	C+
$0.77 - \epsilon > g \geqslant 0.74 - \epsilon$	C
$0.74 - \epsilon > g \geqslant 0.70 - \epsilon$	C-
$0.70 - \epsilon > g \geqslant 0.60 - \epsilon$	D
$0.60 - \epsilon > g$	F

For example, a student with a 90% homework score ($h = 0.9$), 96% project score ($p = 0.96$), and 80% exam scores ($e_1 = e_2 = 0.8$) will receive a final score of

$$g = 0.3(0.9) + 0.5(0.96) + 0.1(0.8 + 0.1) + 0.1(0.8 + 0.1) = 93\%,$$

where the exam offsets are $o_1 = o_2 = 0.1$, e.g., either because the highest score was 90% or a semi-experienced, independent test-taker scored a 95%. For a student with little-to-no participation in class and office hours

($\epsilon = 0$), this corresponds to a A-, while a student that actively participates in the course ($\epsilon = 0.02$) would receive a A.

Sakai

Technical support for Sakai is provided, not by me (instructor) or TAs, but by the OIT Help Desk and the Sakai Team. If you have a question or issue concerning your use of Sakai, please contact the Help Desk at oithelp@nd.edu or phone: 574-631-8111. You can also walk in; they're located in 115 DeBartolo Hall. Support Staff will contact me if they need to discuss the way I've set up our class. Please tell them the web address to our Sakai site, my NetID as your instructor and your NetID, and specifically name the assignment or task you need help with. Technical Tips Before Contacting the OIT Help Desk:

- Access Sakai with a newly opened fresh browser (not one thats been opened for a month and has 47 tabs running across the top).
- Before calling the Help Desk with issues, empty your browser history, close and re-open your browser. Often that's all it takes. (How-To Instructions Here).
- Never login to Sakai twice in two tabs in the same browser, especially when taking a test. You may lose answers.

Policies

Office hours: Students are strongly encouraged to utilize the office hours of both the instructor and TA for additional assistance related to the course. To make the most out of office hours, students are encouraged to think through the problem on their own and avoid overly generic questions. For example, instead of "How do I do problem 3?", students should explain their thought process, what they have tried, and where they got stuck. If you cannot attend the scheduled office hours, contact the instructor or TA directly to schedule an appointment.

Collaboration and honor code: Collaboration is permitted on the homework and project (not exams); however, each student must complete and submit their own assignment. Honor code violations will be handled through appropriate university channels (http://honorcode.nd.edu).

Disabilities: Any student who has a documented disability and is registered with Disability Services should speak with me as soon as possible regarding accommodations. More information can be found at http://disabilityservices.nd.edu.

Additional references

In addition to Reddy's book, there are a number of excellent FEM references, a few of which I have listed below. *These are not required for the course, but are solid addition to one's bookshelf, particularly if continuing with FEM.*

- N.S. Ottosen and H. Petersson. *Introduction to the Finite Element Method.* Prentice-Hall, 1992 (introductory)
- T.J. Hughes. *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis.* Dover, 1st edition, 2000 (introductory, more mathematical)
- T. Belytschko, W.K. Liu, and B. Moran. *Nonlinear Finite Elements for Continua and Structures.* John Wiley & Sons, 2013 (advanced)
- J. Hesthaven and T. Warburton. *Nodal Discontinuous Galerkin Methods: Algorithms, Analysis, and Applications.* Springer Science & Business Media, 2007 (introductory, specialized topic)

Course schedule

Lecture	Date	Topics	Reading
1	1/16	motivation, overview of FEM	JNR 1
2-5	1/18, 1/21, 1/23, 1/25	direct stiffness method	JNR 4.1-4.2, MJZ 1
6-11	1/28, 1/30, 2/1, 2/4, 2/6, 2/8	mathematical preliminaries	JNR 2.1-2.5, MJZ 2-3
12-15	2/11, 2/13, 2/15, 2/18	finite element method	JNR 3, 8
16-18	2/20, 2/22, 2/25	continuum element technology	JNR 7.1, 9.1-9.4, MJZ 3
19	2/27	exam review	-
20	3/1	exam (lectures 1-19)	-
21	3/4	structure of FE code	JNR 7.2, MJZ 4
22-23	3/6, 3/8	linear elasticity	JNR 11
24-26	3/18, 3/20, 3/22	finite element analysis	MJZ 5
27-29	3/25, 3/27, 3/29	engineering elements: bars, beams, frames	JNR 5
30-31	4/1, 4/3	constraints	MJZ 6
32-35	4/5, 4/8, 4/10, 4/12	nonlinear and multiphysics problems	MJZ 7, 8
36-40	4/15, 4/17, 4/22, 4/24, 4/26	time-dependent problems	MJZ 9
41	4/29	exam review	-
42	5/1	exam (lectures 21-41)	-