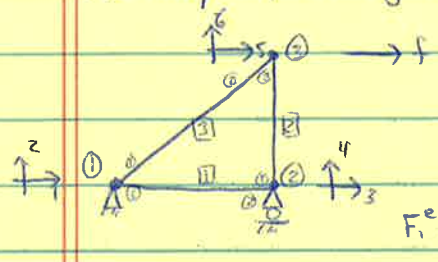
AME 50541, Lecture 5, 1/25

Assembly w/ idof2gdof



$$\text{idof2gdof} = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 4 & 2 \\ 3 & 5 & 5 \\ 4 & 6 & 6 \end{bmatrix}$$

Global equilibrium

$$R_i = \underbrace{\sum_{e=1}^{ne} \sum_{j=1}^{2} F_j^e \cdot \delta_{i,\text{idof2gdof}(j,e)}}_{\text{sum over all element local dofs corresponding to global dof } i} - \text{"+"} \sum_u \{\text{external}\} = 0$$

Verify:

$$R_1 = F_1^1 + F_1^3 - r_{1x} = 0$$
$$R_2 = F_2^1 + F_2^3 - r_{1y} = 0$$
$$R_3 = F_3^1 + F_1^2 = 0$$
$$R_4 = F_4^1 + F_2^3 - r_{2y} = 0$$
$$R_5 = F_3^3 + F_3^2 - f = 0$$
$$R_6 = F_4^3 + F_4^2 = 0$$

Compatibility

$u_j^e = u_i$ if $\{j,e\}$ corresponds to global dof i, i.e., $\text{idof2gdof}(j,e) = i$

$$= u_{\text{idof2gdof}(j,e)}$$
$$= \sum_{s=1}^{ndof} u_s \cdot \delta_{s,\text{idof2gdof}(j,e)}$$

Verify: $u_1 = u_1^1 = u_1^3$, $u_2 = u_2^1 = u_2^3$, $u_3 = u_3^1 = u_1^2$,

$u_4 = u_4^1 = u_2^2$, $u_5 = u_3^2 = u_3^3$, $u_6 = u_4^2 = u_4^3$

Element equations

$$\underline{F}^e = \underline{K}^e \cdot \underline{u}^e \quad , \quad F^e_i = \sum_{j=1}^{4} K^e_{ij} u^e_j$$

Assembly

$$R_i = \underbrace{\sum_e \sum_j F^e_j \cdot \delta_{i,\,\mathrm{idf2gdf}(j,e)}} - \sum\{\mathrm{external}\} = 0$$

$$\Leftrightarrow \sum_e \sum_j \sum_k K^e_{jk}\, u^e_k\, \delta_{i,\,\mathrm{idf2gdf}(j,e)} \sim \sum\{\mathrm{external}\} = 0$$

$$= \sum_e \sum_j \sum_k \sum_s K^e_{jk}\, \delta_{i,\,\mathrm{idf2gdf}(j,e)}\, u_s\, \delta_{s,\,\mathrm{idf2gdf}(k,e)}$$

$$= \sum_s \Big(\underbrace{\sum_e \sum_j \sum_k K^e_{jk}\, \delta_{i,\,\mathrm{idf2gdf}(j,e)}\, \delta_{s,\,\mathrm{idf2gdf}(k,e)}} \Big) u_s$$

$$K_{is}$$

$$\boxed{K_{is} = \sum_e \sum_j \sum_k K^e_{jk}\, \delta_{i,\,\mathrm{idf2gdf}(j,e)}\, \delta_{s,\,\mathrm{idf2gdf}(k,e)}}$$

Question: what to do with K^e_{jk}? $\xrightarrow{\text{add to}}$ $K_{\mathrm{idf2gdf}(j,e),\,\mathrm{idf2gdf}(k,e)}$

what to do with K^e? $\xrightarrow{\text{add to}}$ $K_{\mathrm{idf2gdf}(:,e),\,\mathrm{idf2gdf}(:,e)}$