

## Math 335, Homework 7

Due Friday, April 2 (note extended deadline!)

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1. Let

$$H = \{e, (1,2)(3,4), (1,3)(2,4), (1,4)(2,3)\} \subseteq S_4.$$

List all left cosets of  $H$  in  $S_4$ , being sure to list each one only once..

2. Let  $G = \langle g \rangle$  be a cyclic group with 30 elements. List all of the left cosets of  $\langle g^4 \rangle$  in  $G$ , being sure to list each one only once.

3. Let  $\mathbb{C}^*$  be the group of nonzero complex numbers (under multiplication). Recall that the *norm* of a complex numbers is defined as its distance from the origin in the complex plane; in other words, it's

$$|a + bi| := \sqrt{a^2 + b^2}.$$

Two facts about the norm, which you may assume for this problem, are

$$|z \cdot w| = |z| \cdot |w| \quad \text{and} \quad \left| \frac{1}{z} \right| = \frac{1}{|z|}.$$

Given this, let

$$H = \left\{ z \in \mathbb{C}^* \mid |z| = 1 \right\} \subseteq \mathbb{C}^*.$$

(The next page shows a picture of  $H$ , for your reference.)

- (a) Prove that, for  $v, w \in \mathbb{C}^*$ , we have  $vH = wH$  if and only if  $|v| = |w|$ .
- (b) Given part (a), draw a picture of the left coset  $5H$ , and of the left coset  $(1 + i)H$ .

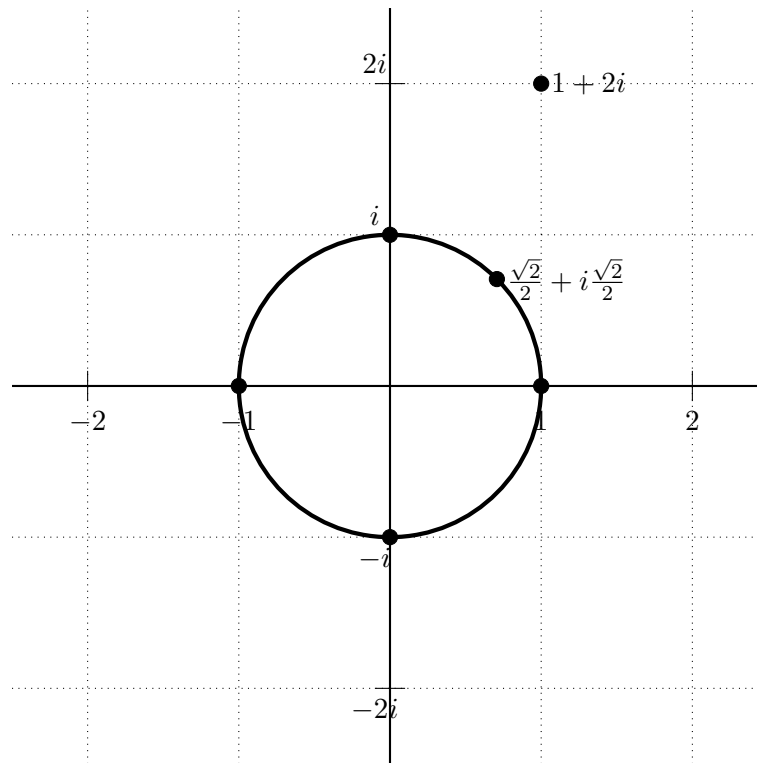
4. Let  $G$  be a group with 8 elements.

- (a) What are the possible orders of elements of  $G$ ?
- (b) Prove that  $G$  must have an element of order 2.

(**Hint:** Start by choosing a non-identity element of  $G$  at random. If it doesn't have order 2, try to cook up an element of order 2 out of it.)

5. Prove that a group with a prime number of elements must be cyclic.

To help you with Problem 3, here's a picture of  $\mathbb{C}$ , in which the set  $H$  appears as the unit circle.



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### Extra Credit Opportunity: Growth Mindset

Turn in anytime by Wednesday, April 7

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As we approach the second exam for this class, you may be feeling nervous about where you currently are in your understanding of modern algebra. To help put that in a bigger context of your ongoing mathematical journey, click on the following two links to watch two short videos:

- (~10 minutes) TED talk by Carol Dweck, ["The power of believing that you can improve"](#)
- (~4 minutes) YouTube video on [Growth Mindset](#).

For this extra credit opportunity, your assignment is to watch these videos and write a short reflection (one or two paragraphs) that addresses the following questions:

1. What are some of the key differences between growth mindset and fixed mindset, in terms of how they make you behave and how they make you feel?
2. What are one or two specific ways in which you could apply growth mindset to your experience in this class?

Your responses can be hand-written or typed. They should be sent by Wednesday, April 7 (the day of the second exam) to me via e-mail, at [eclader@sfsu.edu](mailto:eclader@sfsu.edu).

A thoughtful response to this assignment will earn you **5 points** extra credit.