Ideals

- Finish going over Worksheet 39.

- Punchline from last class:

field $\Rightarrow$ integral domain

- Video: An ideal is a subset $I \subseteq R$ that's
  1) closed under $+$
  2) absorbs under $\cdot$

something in I $+$ something in I $=$ something in I

something in I $\cdot$ anything in R $=$ something in I

Worksheet 40: Ideals and Quotient Rings

Math 335

Reporter:

Recorder:

Equity Manager:

E.g. $5 + 2x + 7x^2 \notin I$ (5 = constant term)

$2x + 7x^2 \in I$

Let $R = \mathbb{R}[x]$ and let

$$I = \{\text{polynomials with constant term } 0\}.$$

In other words, as we've seen, I consists of all polynomials of the form $x \cdot p(x)$ for all $p(x) \in \mathbb{R}[x]$.

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1. Thinking of R as a group under addition, why is I a subgroup?

- closed under +
- Contains 0
- contains negatives of all its elements

2. Why is I a normal subgroup?

Since R is abelian under +, every subgroup is normal.

Since I is a normal subgroup of R under addition, we can form the quotient group R/I. Elements of this are left cosets of I in R under addition, like

$$1 + I = \{1 + f(x) \mid f(x) \in I\}$$

or

$$(5x + 2) + I = \{5x + 2 + f(x) \mid f(x) \in I\}.$$

3. What are some of the elements in $1 + I$? Can you describe its elements in general?

$1 + I$ contains, e.g.

$1 + x$, $x \in I$; $1 + 2x + 5x^2$, $\in I$; $1 + 3x^2 - 7x^4$, $\in I$, ...

In general,

$1 + I = \{$polys w/ constant term $1\}$

4. What are some of the elements in $(5x + 2) + I$? Can you describe its elements in general?

$(5x+2) + I$ contains, e.g.

$5x + 2 + x$, $x \in I$, $2 + 6x$

$5x + 2 + 2x + 5x^2$, $\in I$, $2 + 7x + 5x^2$

$5x + 2 + 3x^2 - 7x^4$, $\in I$, $2 + 5x + 3x^2 - 7x^4$

In general,

$(5x+2) + I = \{$polys w/ constant term $2\}$

5. Using your answers to the previous two questions, what do you think the elements of a coset $a(x) + I$ look like in general?

$a(x) + I = \{$polys with same constant term as $a(x)\}$

Vague Challenge: What does your answer to Problem 5 tell you about the quotient R/I?

R/I has one element for each possible constant term (each real #), so

$R/I \cong \mathbb{R}$.