HW1 Example Solutions

(2b) Is there a value of x such that $5x \equiv 1 \mod 10$? Carefully explain how you know.

No integers x exist, since multiples of five can only ever be either 0 more than a multiple of ten, or 5 more.
Therefore,

$$5x \equiv \begin{cases} 0 & \text{if } x \text{ is even} \\ 5 & \text{if } x \text{ is odd} \end{cases}$$

and clearly it is not possible to have $5x \equiv 1 \mod 10$

b. Is there a value of x s.t. $5x \equiv 1 \mod 10$?
the range in this case would be $\{0,1,2,\ldots,9\}$
no; adding/subtracting 10 to a multiple of 5 will continuously give you another multiple of 5 which you cannot add or subtract 10 to to get 1.

b. $5x \equiv 1 \mod 10$ $\{0,\ldots,9\}$
$(a-b) \mid n$

$\frac{5x-1}{10}$

5x	5x - 1
5(9) = 45	45 - 1 = 44
5(8) = 40	40 - 1 = 39
5(7) = 35	35 - 1 = 34
5(6) = 30	30 - 1 = 29
5(5) = 25	25 - 1 = 24
5(4) = 20	20 - 1 = 19
5(3) = 15	15 - 1 = 14
5(2) = 10	10 - 1 = 9
5(1) = 5	5 - 1 = 4
5(0) = 0	0 - 1 = 0

There is no x value that works because no value of 5x - 1 is evenly divisible by 10.

④ Is composition of symmetries of a square commutative?

If we use the table of symmetries of square. We can see that $H \circ R_{90} = D$. However, $R_{90} \circ H = D'$. Therefore $H \circ R_{90} \neq R_{90} \circ H$. Thus, not commutative. $\square$

4) When it comes to symmetrirs of a square the commutative property works for some cases, but not all. If we look at the table for symmetries of a square, any 2 compositions of a rotation will get you the same result. For example $R_{270} \circ R_{180} = R_{90}$ and $R_{180} \circ R_{270} = R_{90}$. If we look at the reflections Section, compositions of the opposites will get you the same answer as well Such as $V \circ H = R_{180}$ and $H \circ V = R_{180}$. A special case that works with commutative property is any given reflection composed with R_{180} like $D \circ R_{180} = D'$ and $R_{180} \circ D = D'$. Therefore, the commutative property does not always work especially for compositions that limit which 2 symmetrirs can be used.

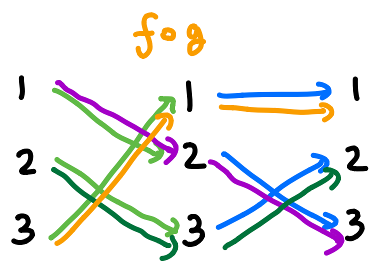
The Symmetric Group, continued

- Last time: $S_n = \left\{ \text{bijections } \{1,2,\dots,n\} \to \{1,2,\dots,n\} \right\}$

- Visual of composition:

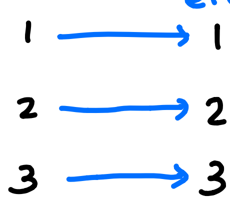
Ex:

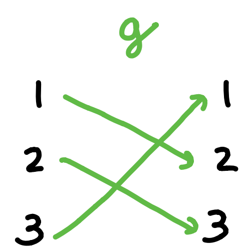
$\Longrightarrow$



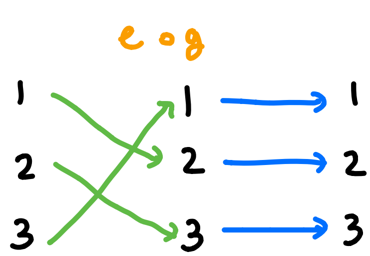
$(f \circ g)(1) = 3$
$(f \circ g)(2) = 2$
$(f \circ g)(3) = 1$

Ex: e = identity element in S_3





$\Longrightarrow$



$e \circ g = g$

Worksheet 8: Cycle Notation in the Symmetric Group

Math 335

Reporter:

Recorder:

Equity Manager:

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1. In S_6, write the permutation

$$(1,3)(2,5,6,4) = (2,5,6,4)(1,3) = (5,6,4,2)(1,3)$$

in function notation; that is, fill in:

$f(1) = $ 3
$f(2) = $ 5
$f(3) = $ 1
$f(4) = $ 2
$f(5) = $ 6
$f(6) = $ 4

2. In S_6, write the permutation

$$(1,3,2)(4,6)(5)$$

in function notation; that is, fill in:

$f(1) = $ 3
$f(2) = $ 1
$f(3) = $ 2
$f(4) = $ 6
$f(5) = $ 5
$f(6) = $ 4

3. In S_6, write the permutation

$f(1) = 6$
$f(2) = 2$
$f(3) = 5$
$f(4) = 4$
$f(5) = 3$
$f(6) = 1$

in cycle notation.

$(1,6)(2)(3,5)(4)$
$= (1,6)(3,5)$

4. In S_4, consider the two permutations

$$f = (1, 3, 2)$$

and

$$g = (3, 4).$$

(We're using the convention that numbers sent to themselves are omitted; for example, f sends 4 to itself.) What is the composition $f \circ g$? Express your answer in cycle notation.

$f \circ g = (1,3,4,2)$

5. For the same f and g as above, what is the composition $g \circ f$? Do f and g commute?

$g \circ f = (1,4,3,2)$

No, they don't commute: $f \circ g \neq g \circ f$.

6. In S_5, consider the two permutations

$$f = (1, 5, 2)$$

and

$$g = (3, 4).$$

Calculate both $f \circ g$ and $g \circ f$, in cycle notation. Do f and g commute now?

$f \circ g = (1,5,2)(3,4)$

$g \circ f = (1,5,2)(3,4)$

Yes, they do commute!

Theorem: If f and g are disjoint cycles (i.e., they have no numbers in common), then

$$f \circ g = g \circ f.$$