Math 335, Midterm 2

November 6, 2019

Name: ____

Problem	Points Scored	Total Points Possible
1		8
2		8
3		8
4		8
5		8
Total		40

Problem 1:

(a) Carefully define what it means for a group G to be **cyclic**.

(b) Let

$$U_4 = \{1, -1, i, -i\} \subseteq \mathbb{C}^*,$$

which is a group under multiplication of complex numbers. Explain how you know that U_4 is cyclic.

(c) Let

$$\mathbb{Z}_5 = \{0, 1, 2, 3, 4\},$$

which is a group under addition modulo 5. In this group, what is $\langle 3 \rangle$? List all of its elements.

Problem 2: You do not need to show your work or give justifications on this problem.

(a) Let

$$G = \langle a \rangle = \{1, a, a^2, a^3, \ldots, a^{19}\},$$

a group under multiplication in which a has order 20. List all the subgroups of G, being sure to list each one only once.

(b) Let G be the same group as in part (a). For which element $b \in G$ does $\langle b \rangle$ have exactly two elements?

(c) Give an example of a group G with no proper, nontrivial subgroups.

Problem 3: In each of the following cases, is the group G isomorphic to the group H? If so, write down an isomorphism $\phi : G \to H$. (You do **not** need to prove that it is an isomorphism.) If not, briefly explain how you know.

(a) $G = D_4$ (under composition) and $H = \mathbb{Z}_8$ (under addition modulo 8).

(b) $G = \mathbb{Z}_3$ (under addition modulo 3) and $H = \{0, 2, 4\} \subseteq \mathbb{Z}_6$ (under addition modulo 6) .

Problem 4: Let $G = S_3$ (under the operation of composition), and let $H \subseteq G$ be the subgroup generated by the element $(1, 2, 3)$.

(a) What is H? Write down all of its elements.

(b) What is the left coset $(1, 2)H$? Write down all of its elements.

(c) Apply Lagrange's Theorem to determine the number of left cosets of H in G.

(d) List all of the left cosets of H in G, being sure to list each one only once.

Problem 5:

(a) Let S be a set with an equivalence relation $\sim$, and let T be any set. Carefully define what it means to say that a function $f : S \to T$ is well-defined under $\sim$.

(b) Consider the equivalence relation on the nonzero real numbers $\mathbb{R}^*$ defined by

$$a \sim b \quad \Leftrightarrow \quad ab > 0.$$

What is $[1]$?

(c) Let $\sim$ be the same equivalence relation as in part (b). Give an example to show that

$$\begin{aligned} &f : \left\{ \begin{matrix} \text{equivalence classes} \\ \text{under } \sim \end{matrix} \right\} \to \mathbb{R} \\ &f([a]) = a + 1 \end{aligned}$$

is **not** well-defined.