Homework 9

Tuesday, April 20, 2021 6:56 PM

1. Topic #2 EP 2

a. For any face f of a cube, stab(f) has 4 elements

b. For any face g of a tetrahedron, stab(g) has 3 elements

2. $H = \{e, (1,2)(3,4)\} \subseteq S_4$ $\qquad ghg^{-1} \in H$

a. $\underset{g}{(1,3)} \circ \underbrace{(1,2)(3,4)}_{h} \circ \underset{g^{-1}}{(1,3)^{-1}} = (1,3) \circ (1,4,3,2)$

$= (1,4)(2,3) \notin H$

b. $\alpha H = \{(2,4,3), (2,4,3) \circ (1,2)(3,4)\} = \{(2,4,3), (1,4,2)\}$

$\beta H = \{(2,4,3), (1,4,2) \circ (1,2)(3,4)\} = \{(1,4,2), (2,4,3)\}$ SAME

c. $\alpha \circ \gamma = (2,4,3) \circ (1,3,2) = (1,2)(3,4)$

$(\alpha \circ \gamma)H = \{(1,2)(3,4), e\}$ not same

$\beta \circ \gamma = (1,4,2) \circ (1,3,2) = (1,3)(2,4)$

$(\beta \circ \gamma)H = \{(1,3)(2,4), (1,3)(2,4) \circ (1,2)(3,4)\} = \{(1,3)(2,4), (1,4)(2,3)\}$

d. Part b & c show that

$(\alpha H) \circ (\gamma H) = (\alpha \circ \gamma) H$ is not well-defined

Since $\alpha H = \beta H$, then $\alpha H \circ \gamma H = \beta H \circ \gamma H$. So if we apply the operation $(\alpha H) \circ (\gamma H) = (\alpha \circ \gamma) H$, we get

$(\alpha \circ \gamma) H = (\beta \circ \gamma) H$.

But as shown in c. $(\alpha \circ \gamma) H \neq (\beta \circ \gamma) H$.

3. Suppose $h = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \in H$ and $g = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ $\qquad g^{-1} = \begin{bmatrix} -1/3 & 2/3 \\ 2/3 & -1/3 \end{bmatrix}$

$\quad h \ {}^{-1} \ \begin{bmatrix} 1 & 2 \\ \end{bmatrix} \begin{bmatrix} 1 & 1 \\ \end{bmatrix} \begin{bmatrix} -1/3 & 2/3 \\ \end{bmatrix}$

$$gh g^{-1} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1/3 & 2/3 \\ 2/3 & -1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1/3 & 1/3 \\ 2/3 & -1/3 \end{bmatrix} = \begin{bmatrix} 5/3 & -1/3 \\ 4/3 & 1/3 \end{bmatrix} \notin H$$

can I row reduce this to make the bottom left zero?

4. Suppose $\varphi: G \to H$ is a homomorphism of groups. Let e_G be the identity of G and e_H be the identity of H.

Then $\varphi(e_G) = e_H$. By definition $gg^{-1} = e_G$, so we can say

$$\varphi(gg^{-1}) = e_H \quad \forall g \in G.$$

Since φ is a homomorphism, $\varphi(gg^{-1}) = \varphi(g)\varphi(g^{-1})$. and

$$\varphi(g)\varphi(g^{-1}) = e_H.$$

Notice that

$$\varphi(g)^{-1}\varphi(g)\varphi(g^{-1}) = \varphi(g)^{-1}e_H$$

$$e_H\,\varphi(g^{-1}) = \varphi(g)^{-1}e_H$$

$$\varphi(g^{-1}) = \varphi(g)^{-1} \qquad \square$$