Math 335, Homework 4

Due Wednesday, March 3 (note extended deadline!)

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1. (a) Calculate the orders of each of the six elements in $\mathbb{Z}_6$, which is a group under the operation of addition modulo 6.

   (b) Calculate the orders of each of the five elements in $\mathbb{Z}_5$, which is a group under the operation of addition modulo 5.

2. (a) Let G be a group, and let $g \in G$ be an element with infinite order. Prove that, for any positive integers i and j,
$$g^i = g^j \Rightarrow i = j.$$

   (b) Give an example to show that the result of part (a) can fail if g has finite order.

3. (a) Can a finite group G have an element g with infinite order? If so, give an example. If not, prove your answer.

   (**Hint**: Consider the elements $g, g^2, g^3, g^4, \ldots$, and apply Problem 2.)

   (b) Can an infinite group G have an element g with finite order? If so, give an example. If not, prove your answer.

4. What is the order of the element
$$f = (1, 2, 3, 5)\ (2, 4, 5, 6, 7)$$
in S_7?

   (**Caution**: Our theorem about orders of elements of S_n doesn't immediately apply...)

5. What are the possible orders of elements in S_5? Explain your answer.