Subgroups

- Reminder: Exam Wednesday
  (review Monday)

- Video: A subgroup of G is a subset

$$H \subseteq G$$

that's also a group, under the same operation as G.

Worksheet 12: Subgroups

Math 335

Reporter:

Recorder:

Equity Manager:

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1. Let $G = \mathbb{R}$, with the operation of addition. Which of the following are subgroups of G?

   (a) $H = \mathbb{Q}$

   subgroup

   closed: ✓
   associative: ✓
   identity: $0 = \frac{0}{1} \in \mathbb{Q}$
   inverses: ✓

   (b) $H = \{\text{irrational numbers}\}$

   not a subgroup

   not closed, e.g. $\underset{\in H}{\sqrt{2}} + \underset{\in H}{(1-\sqrt{2})} = \underset{\notin H}{1}$

   (c) $H = \{\text{even integers}\}$

   subgroup

   closed: ✓
   associative: ✓
   identity: $0 = 2 \cdot 0 \in H$
   inverses: ✓

2. What other subgroups of $\mathbb{R}$ (with the operation of addition) can you think of?

   $\mathbb{Z}$

   {multiples of 3}

   {half-integers} $= \{\ldots, -2, -\frac{3}{2}, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots\}$

   ... and many more

3. Now let $G = \mathbb{Z}_6$, with the operation of addition modulo 6. Which of the following are subgroups of H?

   (a) $H = \{0, 1, 2\}$

   not a subgroup (not closed, e.g. $\underset{\in H}{1} + \underset{\in H}{2} = \underset{\notin H}{3}$)

   (b) $H = \{0, 3\}$

   subgroup

+	0	3
0	0	3
3	3	0

   (closed: ✓
   associative: ✓
   identity: $0 \in H$ ✓
   inverses: ✓)

   (c) $H = \{3\}$

   not a subgroup (no identity $0 \notin H$)

4. What other subgroups of $\mathbb{Z}_6$ (with the operation of addition modulo 6) can you think of?

   $\{0, 2, 4\}$

   $\{0\}$

   $\{0, 1, 2, 3, 4, 5\}$

Challenge: Can you find *all* of the subgroups of $\mathbb{Z}_6$ (with the operation of addition modulo 6)?

See above. To argue we've found them all, try arguing in cases based on whether $1 \in H$, $2 \in H$, and so on.