Math 335, Homework 8

Due Wednesday, April 14

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1. Look at the Final Project Guidelines on the top of the course iLearn page, and choose which topic you'd like to pursue. Then, complete the first exploration problem (EP1) for your chosen topic; you'll find these problems listed at the end of the Final Project Guidelines.

2. Let $S = \mathbb{Z}$, and let $\sim$ be the equivalence relation defined by

$$a \sim b \quad \Leftrightarrow \quad a^2 = b^2.$$

   What is $[2]$? What is $[-3]$? What is $[0]$?

3. Let S be a set with an equivalence relation $\sim$. Let $a, b \in S$, and let $[a]$ and $[b]$ denote their equivalence classes under $\sim$. Prove that $a \sim b$ if and only if $[a] = [b]$.

4. Let $G = \mathbb{Z}_{12}$, which is a group under addition modulo 12, and let

$$H = \langle 3 \rangle \subseteq G.$$

   (a) Apply Lagrange's Theorem to compute the number of elements of G/H, without actually calculating those elements.

   (b) Now, list the elements of G/H. List each one only once, and for each element, identify it both by a name like $a + H$ and by writing the elements within $a + H$.

   (c) Make a table that shows how to add any two elements of the group G/H.

   (d) To which familiar group is G/H isomorphic? Write down an explicit isomorphism between G/H and this group.