Listing Finite Abelian Groups

- Video: Zn & Zm = Znm (=> gcd(n,m)=1
- Goal: What are ALL the abelian
 groups with 4 elements (or
 some other # of elements)?
 Write a list so that ANY abelian
 group with 4 elements is isomorphic
 to something on your list.

Worksheet 36: Listing Finite Abelian Groups

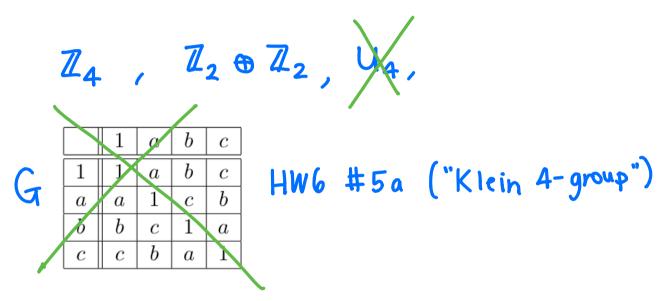
Math 335

Reporter:

Recorder:

Equity Manager:

1. How many abelian groups with four elements can you think of? List as many as you can. (There are hints at the bottom of this worksheet if you want them.)



2. Are any of the abelian groups on your list from Problem 1 isomorphic to each other? Try to make a new list, where you only list something if it's not isomorphic to something you've already written down.

$$U_4 \stackrel{?}{=} Z_4$$
So the new
$$G \stackrel{?}{=} Z_2 \oplus Z_2 \longleftarrow list has only$$

$$Z_4 and Z_2 \oplus Z_2$$

$$(9(1) = (0,0)$$

$$9(a) = (1,0)$$

$$9(b) = (0,1)$$

$$9(c) = (1,1)$$

3. How many abelian groups with twelve elements can you think of? (Again, see the hints if you want.)

$$\mathbb{Z}_{12}$$
 $\mathbb{Z}_{2} \oplus \mathbb{Z}_{6}$
 $\mathbb{Z}_{3} \oplus \mathbb{Z}_{4}$
 $\mathbb{Z}_{2} \oplus \mathbb{Z}_{3}$

4. Are any of the abelian groups on your list from Problem 3 isomorphic to each other? Try to make a new list, where you only list something if it's not isomorphic to something you've already written down.

$$Z_3 \oplus Z_4 \cong Z_{12}$$
 $Z_2 \oplus Z_2 \oplus Z_3 \cong Z_2 \oplus Z_6$

So the new list has only

 $Z_3 \oplus Z_4$ and $Z_2 \oplus Z_2 \oplus Z_3$

Hints:

- One group with four elements is \mathbb{Z}_4 , and another one appeared on Homework 6.
- Can you build a group with four elements by taking a direct product of \mathbb{Z}_n 's?
- In general, can you build a group with a certain number of elements by taking either \mathbb{Z}_n or a direct product of two or more \mathbb{Z}_n 's?