Midterm Review

- Logistics: Exam Wednesday, any 90 mins
 - · Use any non-human resource
 - · No class on Wednesday
 - · Office hours:
 - · Today 10-11am
 - · Tomorrow 1-2 pm
 - (Not on Wednesday)
- Topics: Modular arithmetic

 - · Symmetries of a square
 - · Definition/properties/examples of groups
 - · Sn (injective/surjective, cycle notation, composition, order)
 - · Orders of elements (definition/ properties/examples)
 - · Subgroups (definition/examples)
- Sample Exam: Ignore Problem 5

Math 335, Midterm 1

October 4, 2019

| Name: |
|-------|
|-------|

| Problem | Points Scored | Total Points Possible |
|---------|---------------|-----------------------|
| 1 | | 8 |
| 2 | | 8 |
| 3 | | 8 |
| 4 | | 8 |
| 5 | | 8 |
| Total | | 40 |

Problem 1: You do not need to show any work or give justifications on this problem.

(a) Fill in the blank with a number in the range $\{0, 1, 2, 3, 4, 5\}$:

$$-7 \equiv \underline{5} \mod 6.$$
 $\left(-7 \equiv -1 \equiv 5 \mod 6\right)$

(b) We've seen that the set $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$ is a group under the operation of addition modulo 6. In this group, what is the inverse of 2?

(because
$$2+4=0 \leftarrow identity$$
)

(c) Give an example of a proper montrivial subgroup of \mathbb{Z}_6 . (Write out your subgroup explicitly, by listing all of its elements.)

$$\{0,2,4\}$$
 or $\{0,3\}$

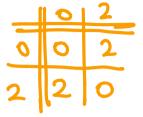
(d) Give an example of a subset of \mathbb{Z}_6 that is *not* a subgroup.

Problem 2: Which of the following are groups? In each case, circle "group" or "not a group"; you do not need to justify your answer.

(a) $G = \{0, 2\}$, where the operation is addition modulo 4.



not a group



(b) $G = \{1, 2, 3\}$, where the operation is multiplication modulo 4.

group



Not closed because 2eG but 2.2=0 isn't in G

(c) $G = \{\text{nonzero integers}\}\$, where the operation is multiplication.

group



Doesn't contain inverses, e.g. inv of $2 = \frac{1}{2} \notin G$

(d) $G = \{\text{rotational symmetries of a square}\} = \{I, R_{90}, R_{180}, R_{270}\},$ where the operation is composition.



| | I | Rao | Riso | R270 |
|------------------|------|------|------|------|
| not a group 📘 | I | R4. | Ruo | Reza |
| R ₁₀ | Rho | Riso | R27- | I |
| Rizo | Riso | Rzz. | I | Res |
| R ₂₇₀ | Res | I | R. | Riso |

Problem 3: Suppose G is a group with four elements,

$$G = \{a, b, c, d\},\$$

and the group table for G is the following:

| d | d | c | b | a | |
|---------------|---|---|----------|-----|---|
| a*d=b | b | a | c | d | a |
| a + b + d = a | a | b | d | c | b |
| (C*b = b | d | c | b | (a) | c |
| C# 4 = 0 | c | d | a | b | d |

In other words, the entry in the top-right corner means that a * d = b.

(a) What is the identity element of G? Briefly explain how you know.

(b) What is the inverse of a? Briefly explain how you know.

b, because
$$a * b = b * a = c$$
 and c is the identity

(c) Is the operation on G commutative—that is, is G abelian? Briefly explain how you know.

Problem 4: You do not need to show any work or give justifications on this problem.

(a) Carefully define what it means to say that a function $f: A \to B$ is **injective** (or **one-to-one**).

$$f: A \longrightarrow B$$
 is injective if $\forall a_1, a_2 \in A$,
 $f(a_1) = f(a_2) \Longrightarrow a_1 = a_2$.

(b) In the symmetric group S_5 , let

$$\alpha = (1, 2, 3, 5), \quad \beta = (1, 3, 4).$$

What is $\alpha \circ \beta$? Express your answer in cycle notation, as a composition of disjoint cycles.

$$(1,2,3,5) \circ (1,3,4) = (1,5)(2,3,4)$$

(c) In the symmetric group S_6 , what is the order of

$$f = (1,3) (2,6,5,4)$$
?

ora(f) =
$$lcm(2,4) = 4$$

(d) If f is the same permutation as in part (c), what is f^8 ?

$$f^8 = (f^4)^2 = e^2 = e$$

SKIP THIS PROBLEM— Problem 5: HAVEN'T COVERED THIS

(a) Carefully define what it means for a group G to be \mathbf{cyclic} .

MATERIAL YET

(b) Let

$$U_4 = \{1, -1, i, -i\} \subseteq \mathbb{C}^*,$$

which is a group under multiplication of complex numbers. Explain how you know that U_4 is cyclic.

(c) Let

$$\mathbb{Z}_5 = \{0, 1, 2, 3, 4\},\$$

which is a group under addition modulo 5. In this group, what is $\langle 3 \rangle$? List all of its elements.