Exam 3 Review

- **Exam**: Wednesday 5/12, any 90 minutes, same format as Exams 1 + 2 (on iLearn, can use any non-human resource)

- **Office Hours**:
  - Today 10-11am
  - Tomorrow 10-11am
  - NOT on Wednesday
  - As usual next week (Mon 10-11am, Wed 2-3pm)

- **Classes**:
  - NO class Wednesday
  - Wrap-up class (with video) Friday

- **Final Project**:
  - Optional rough draft due Wednesday 5/12
  - Final project due Thursday 5/20, via submission link on iLearn

11:59pm

PRACTICE PROBLEM SOLUTIONS

① $G = \{1, g, g^2, \ldots, g^{14}\}$
$H = \langle g^5 \rangle = \{1, g^5, g^{10}\}$

(a) $G/H = \{1 \cdot H,\ g \cdot H,\ g^2 \cdot H,\ g^3 \cdot H,\ g^4 \cdot H\}$

(b)

	$1 \cdot H$	$g \cdot H$	$g^2 \cdot H$	$g^3 \cdot H$	$g^4 \cdot H$
$1 \cdot H$	$1 \cdot H$	$g \cdot H$	$g^2 \cdot H$	$g^3 \cdot H$	$g^4 \cdot H$
$g \cdot H$	$g \cdot H$	$g^2 \cdot H$	$g^3 \cdot H$	$g^4 \cdot H$	$1 \cdot H$
$g^2 \cdot H$	$g^2 \cdot H$	$g^3 \cdot H$	$g^4 \cdot H$	$1 \cdot H$	$g \cdot H$
$g^3 \cdot H$	$g^3 \cdot H$	$g^4 \cdot H$	$1 \cdot H$	$g \cdot H$	$g^2 \cdot H$
$g^4 \cdot H$	$g^4 \cdot H$	$1 \cdot H$	$g \cdot H$	$g^2 \cdot H$	$g^3 \cdot H$

② $G = S_3$
$H = \{e, (1,2)\}$

(a) E.g.

$$\underbrace{(1,3)}_{\in G} \circ \underbrace{(1,2)}_{\in H} \circ (1,3)^{-1}$$
$$= (1,3) \circ (1,2) \circ (1,3)$$
$$= (2,3) \notin H$$

(b) $eH = (1,2)H$ but $\underbrace{(e \circ (1,3))H}_{} \neq \underbrace{((1,2)\circ(1,3))H}_{}$

COMPOSITION OF COSETS IS NOT WELL-DEFINED!

$$= (1,3)H \qquad\qquad = (1,3,2)H$$
$$= \{(1,3)\circ e, (1,3)\circ(1,2)\} \qquad = \{(1,3,2)\circ e, (1,3,2)\circ(1,2)\}$$
$$= \{(1,3), (1,2,3)\} \qquad = \{(1,3,2), (2,3)\}$$

③ $\varphi : \mathbb{R}^* \longrightarrow \mathbb{R}^*$
$\varphi(x) = x^4$

(a) $\varphi(x \cdot y) = (x \cdot y)^4 = x^4 \cdot y^4 = \varphi(x) \cdot \varphi(y)$

(b)
$$\ker(\varphi) = \{x \in \mathbb{R}^* \mid \varphi(x) = 1\}$$
$$= \{x \in \mathbb{R}^* \mid x^4 = 1\}$$
$$= \{1, -1\}$$

(c)
$$\operatorname{im}(\varphi) = \{y \in \mathbb{R}^* \mid y = x^4 \text{ for some } x \in \mathbb{R}^*\}$$
$$= \mathbb{R}^+ \quad \text{(positive real \#s)}$$

(d) $\mathbb{R}^* / \{1,-1\} \cong \mathbb{R}^+$

4. In $\mathbb{Z}_3 \oplus \mathbb{Z}_6$:

(a) $\text{ord}((1,2)) = \text{lcm}\left(\underset{\text{in } \mathbb{Z}_3}{\text{ord}(1)}, \underset{\text{in } \mathbb{Z}_6}{\text{ord}(2)}\right)$

$= \text{lcm}(3, 3)$

$= \boxed{3}$

(b) $\langle (1,2) \rangle = \{(0,0), (1,2), (2,4)\}$

(c) $\mathbb{Z}_3 \oplus \mathbb{Z}_6 \cong \mathbb{Z}_3 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_3$

5. Abelian groups with 18 elements?

$18 = 2 \cdot 3 \cdot 3$

$= 2 \cdot 9$

$\boxed{\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_3 \quad \text{and} \quad \mathbb{Z}_2 \oplus \mathbb{Z}_9}$

6. (a) $\mathbb{Z}$ is an integral domain, not a field

$\mathbb{Z}_6$ is not an integral domain

(6) $\{0, 2, 4\} \subseteq \mathbb{Z}_6$ is an ideal because

- it's closed under $+$
  (the sum of even #s is even mod 6)
- it absorbs under $\cdot$
  (an even # times any # is even mod 6)