Math 335, Midterm 1

October 4, 2019

Name:

Problem	Points Scored	Total Points Possible		
1		8		
2		8		
3		8		
4		8		
5		8		
Total		40		

Problem 1: You do not need to show any work or give justifications on this problem.

(a) Fill in the blank with a number in the range $\{0, 1, 2, 3, 4, 5\}$:

(b) We've seen that the set $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$ is a group under the operation of addition modulo 6. In this group, what is the inverse of 2?

(c) Give an example of a proper, nontrivial subgroup of \mathbb{Z}_6 . (Write out your subgroup explicitly, by listing all of its elements.)

to elements.)

$$\begin{cases}
0,3\end{cases}$$

b. Closed under addition

1. Associative (assumed)

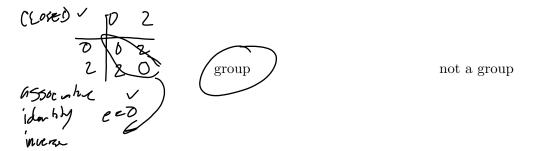
2. identity $e=0$

3. inverse $\frac{0.3}{0.3}$

(d) Give an example of a subset of \mathbb{Z}_6 that is *not* a subgroup.

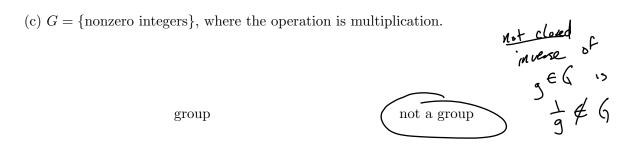
Problem 2: Which of the following are groups? In each case, circle "group" or "not a group"; you do not need to justify your answer.

(a) $G = \{0, 2\}$, where the operation is addition modulo 4.

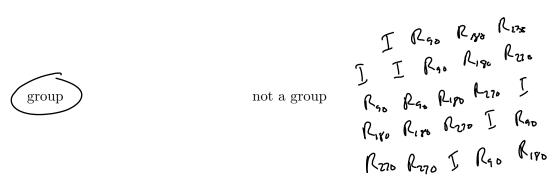


(b) $G = \{1, 2, 3\}$, where the operation is multiplication modulo 4.





(d) $G = \{\text{rotational symmetries of a square}\} = \{I, R_{90}, R_{180}, R_{270}\},$ where the operation is composition.



Problem 3: Suppose G is a group with four elements,

$$G = \{a, b, c, d\},\$$

and the group table for G

\vec{a} is the	e follow	ving:	Ĺ	The T		`
		a	b	c	d	
	a	d	c	a	b	
y }	b	c	d	b	a	
0	c	a	b	c	d	~ e
	d	b	a	d	c	

In other words, the entry in the top-right corner means that a * d = b.

(a) What is the identity element of G? Briefly explain how you know.

is the identity element of
$$G$$
? Briefly explain how you know.

 $e = c$. By definition $x * e = x$, and $e * x = x$

So notice that $c * x = x$ and $x * c = x$

the take

(b) What is the inverse of a? Briefly explain how you know.
$$a'' = b , \quad \text{By definition} \quad x * x'' = e \quad \text{and} \quad x'' + x = e$$

$$e = c, \quad \text{and} \quad a + b = c + b + a = c, \quad a'' = b$$

(c) Is the operation on G commutative—that is, is G abelian? Briefly explain how you know.

Problem 4: You do not need to show any work or give justifications on this problem.

(a) Carefully define what it means to say that a function $f: A \to B$ is **injective** (or **one-to-one**).

(b) In the symmetric group S_5 , let

$$\alpha = (1, 2, 3, 5), \quad \beta = (1, 3, 4).$$

What is $\alpha \circ \beta$? Express your answer in cycle notation, as a composition of disjoint cycles.

$$x^{3} = (1,5)(2,3,4)$$

(c) In the symmetric group S_6 , what is the order of

$$f = (1,3) (2,6,5,4)$$
?

(d) If f is the same permutation as in part (c), what is f^8 ?

$$f^{8} = (1)(2)(3)(4)(5)(4)$$

Problem 5:

(a) Carefully define what it means for a group G to be **cyclic**.

(b) Let

$$U_4 = \{1, -1, i, -i\} \subseteq \mathbb{C}^*,$$

which is a group under multiplication of complex numbers. Explain how you know that U_4 is cyclic.

(c) Let

$$\mathbb{Z}_5 = \{0, 1, 2, 3, 4\},\$$

which is a group under addition modulo 5. In this group, what is $\langle 3 \rangle$? List all of its elements.