Math 335, Homework 3

Due Wednesday, February 17

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1. Let A, B, and C be sets, and let $f : A \to B$ and $g : B \to C$ be functions.

   (a) Prove that if f and g are injective, then $g \circ f$ is injective.

   (b) Prove that if f and g are surjective, then $g \circ f$ is surjective.

2. Prove that a function $f : A \to B$ is both injective and surjective if and only if it has an inverse function $f^{-1} : B \to A$.

3. Consider the following two elements of S_8, written in cycle notation:

$$f = (2, 3, 8, 4, 7)\ (5, 6)$$

$$g = (1, 2, 3, 4, 5)\ (6, 7, 8).$$

   (a) Write f in function notation. (That is, write $f(1) = \cdots, f(2) = \cdots$, et cetera.)

   (b) Compute f^{-1}. Express your answer in cycle notation.

   (c) Compute $f \circ g$. Express your answer in cycle notation, with each number only appearing once.

4. A *transposition* is defined as an element of S_n that swaps two numbers but sends every other number to itself; for example, $(2, 5)$ is a transposition in S_5 that swaps 2 and 5.

   (a) Express the element

$$f = (2, 4, 5)$$

   of S_5 as a composition of two transpositions.

   (b) Express the element

$$g = (1, 2, 3, 4)$$

   of S_4 as a composition of transpositions. (**Hint**: If you want to send 2 to 3, you can first swap 1 and 2 and then swap 1 and 3.)

   In fact, it's a theorem that every element of S_n can be expressed as a composition of transpositions. We won't cover this theorem in class, but you can find a proof in Gallian.

5. Explain in words how the group D_4 of symmetries of a square can be viewed as a subset of the symmetric group S_4. Are all elements of S_4 symmetries of a square?

Extra Credit Opportunity: Mathematical Writing

Turn in anytime before Wednesday, February 24

You'll be doing a lot of writing in this course, in the form of mathematical proofs and (at the end of the semester) a final project. Writing about mathematics is a hard skill that takes a lot of practice, and it also has many peculiar conventions that make it different from writing more generally.

For this extra credit opportunity, your assignment is the following:

1. Read the two articles "Elements of Style for Proofs" by D. C. Ernst and "Guidelines for Good Mathematical Writing" by Francis Su. (The titles in the previous sentence are clickable links to the articles.)

2. Write a paragraph summarizing your thoughts on the tips in these articles. For example, you might comment on whether you found any of them surprising or motivating.

3. Identify at least two specific things you learned from these articles that you want to work on this semester in your own mathematical writing.

Your responses can be hand-written or typed. They should be sent by Wednesday, February 24th (the day of our first exam) to me via e-mail, at eclader@sfsu.edu.

A thoughtful response to this assignment will earn you **5 points** extra credit.