Math 335, Homework 9

Due Wednesday, April 21

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1. Complete the second exploration problem (EP2) for your final project. (Remember, these problems are listed at the end of the Final Project Guidelines.) Please write which project you're working on at the top of the page, for my reference.

2. Let
$$H = \{e,\ (1,2)(3,4)\} \subseteq S_4,$$
where e denotes the identity permutation.

   (a) Show that H is not a normal subgroup of S_4.

   (b) Consider the following three elements of S_4:
$$\alpha = (2,4,3),\quad \beta = (1,4,2),\quad \gamma = (1,3,2).$$
   Show that $\alpha H = \beta H$.

   (c) Show that $(\alpha \circ \gamma)H \neq (\beta \circ \gamma)H$.

   (d) What do parts (b) and (c) show?

3. Let $G = \mathrm{GL}(2,\mathbb{R})$ be the group of all 2×2 invertible matrices with coefficients in $\mathbb{R}$, and let
$$H = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} \,\middle|\, ad \neq 0 \right\} \subseteq G.$$
   Is H a normal subgroup of G? Prove your answer.

4. Let $\varphi : G \to H$ be a homomorphism of groups. Prove that $\varphi(g^{-1}) = \varphi(g)^{-1}$ for all $g \in G$.