Math 335, Homework 10

Due Wednesday, April 28

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1. Complete the third exploration problem (EP3) for your final project. (Remember, these problems are listed at the end of the Final Project Guidelines.) Please write which project you're working on at the top of the page, for my reference.

2. Let $\varphi : G \to H$ be a homomorphism. Prove that $\text{im}(\varphi)$ is a subgroup of H. (Recall, to do this, we need to check that $\text{im}(\varphi)$ is closed, that it contains the identity element, and that it contains inverses of all of its elements.)

3. Let $\varphi : G \to H$ be a homomorphism. Prove that $\ker(\varphi)$ is a *normal* subgroup of G.

4. This problem concerns the group $\mathbb{Z} \oplus \mathbb{Z}$, whose elements are pairs of integers (a, b), under the operation
$$(a, b) + (c, d) = (a + c, b + d).$$

   (a) What is the identity element of $\mathbb{Z} \oplus \mathbb{Z}$?

   (b) Let $\varphi : \mathbb{Z} \oplus \mathbb{Z} \to \mathbb{Z}$ be the homomorphism
   $$\varphi(a, b) = a - b.$$
   What is the kernel of φ? To what familiar group is $\ker(\varphi)$ isomorphic?

   (c) What is the image of φ?

   (d) What does the First Isomorphism Theorem say in this case?