Orders, continued

- Recall: The order of $g \in G$ is the smallest positive power of g that equals e.
- Video: $g^k = e \iff \text{ord}(g) \mid k$.

Worksheet 10: Orders of Elements in Groups, continued

Math 335

Reporter:

Recorder:

Equity Manager:

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1. Let

$$\alpha = (1, 4, 3) \in S_4.$$

Calculate α^2, α^3, and so on, until you find a power of α that equals e. (In other words, calculate the order of α.)

$\alpha = (1,4,3)$

$\alpha^2 = (1,4,3) \circ (1,4,3)$

$= (1,3,4)$

$\alpha^3 = (1,4,3) \circ (1,4,3) \circ (1,4,3)$

$= (1,3,4) \circ (1,4,3)$

$= (1)(2)(3)(4)$

$= e$

Identity element in S_4 is the function that sends 1 to 1, 2 to 2, ... I.e., $(1)(2)(3)(4)$

$\text{ord}(\alpha) = 3$

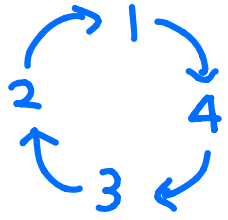
2. Do you have any guesses about the order of the element

$$(1, 4, 3, 2) \in S_4,$$

without doing any calculation?

order = 4

Idea:



Each power "turns the wheel once."

3. Calculating orders of elements in S_n that have more than one cycle is trickier, but it does follow a pattern. To explore this, have each person in the Breakout Room choose a different one of the following elements of S_6 and compute its order.

   - $f = (1,2)\,(4,5)$
   - $g = (1,2,3)\,(4,5,6)$
   - $h = (1,4)\,(2,3,5,6)$
   - $k = (1,2,3)\,(5,6)$

   (This may take a fair amount of computation! Take your time, and feel free to scroll to a different area of the Limnu board if you need more space.)

ord(f) = 2

ord(g) = 3

ord(h) = 4

ord(k) = 6

E.g.

$g = (1,2,3)(4,5,6)$

$g^2 = (1,2,3)(4,5,6)(1,2,3)(4,5,6)$

$= (1,2,3)(1,2,3)(4,5,6)(4,5,6)$

$= (1,3,2)(4,6,5)$

$g^3 = e$

4. Once each member of the Breakout Room has computed the order of an element of S_6, share your answers with one another. With all of this data, can you conjecture a formula for the order of any element of S_n? If so, try testing your conjecture on an element not in the above list.

order of an element in S_n

=

lcm of lengths of cycles