Practice Problems for Exam 3

Math 335

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1. Let

$$G = \langle g \rangle = \{1, g, g^2, g^3, \ldots, g^{14}\},$$
$$H = \langle g^5 \rangle \subseteq G,$$

   where $\operatorname{ord}(g) = 15$.

   (a) List all of the elements in the quotient group G/H, being sure to list each one only once.

   (b) Make a table that shows how to multiply any element of G/H by any other to get a new element of G/H.

2. Let $G = S_3$, and let

$$H = \{e,\ (1,2)\} \subseteq G.$$

   (a) Show that H is **not** a normal subgroup of G.

   (b) Give an example of elements $a, a', b \in G$ such that

$$aH = a'H$$

   but

$$(a \circ b)H \neq (a' \circ b)H.$$

3. Let $\mathbb{R}^*$ be the set of all nonzero real numbers, which is a group under multiplication, and consider the function

$$\varphi : \mathbb{R}^* \to \mathbb{R}^*$$
$$\varphi(x) = x^4.$$

   (a) Prove that φ is a homomorphism.

   (b) What is the kernel of φ?

   (c) What is the image of φ?

   (d) What does the First Isomorphism Theorem say in this case?

4. This problem concerns the group $\mathbb{Z}_3 \oplus \mathbb{Z}_6$.

   (a) What is the order of the element $(1, 2)$ in this group?

   (b) List all of the elements of the subgroup generated by $(1, 2)$.

   (c) Is $\mathbb{Z}_3 \oplus \mathbb{Z}_6$ isomorphic to any other group built out of $\mathbb{Z}_n$'s by direct products?

5. List all abelian groups with 18 elements, making sure that no two groups on your list are isomorphic.

6. (a) Give an example of an integral domain that is not a field, and an example of a ring that is not an integral domain.

   (b) Is the subset

   $$\{0, 2, 4\} \subseteq \mathbb{Z}_6$$

   an ideal? Explain how you know.