Adding/Multiplying Cosets

- Final Project:

    - Choice of topics (see "Final Project Guidelines" at top of iLearn)
    - Each consists of three "Exploration Problems" on HW, a reading, and a final report (paper or video) due on 5/20
    - Reports will be individual, but for learning + brainstorming, work together!

- Note: HW8 on iLearn now, due Wed.

- Request: Send me your group-mate preferences if you have any!

- **Video:** If

$G =$ abelian group
$H \subseteq G$ subgroup,

then

$$\left\{ \begin{matrix} \text{left cosets of} \\ H \text{ in } G \end{matrix} \right\} = G/H \quad \text{"quotient group"}$$

forms a group under the operation

$$(aH) * (bH) = (a*b)H.$$

- **Ex:** $G = \mathbb{Z}$
$H = \{\text{multiples of } 3\}$

$$\Longrightarrow G/H = \{0 + H,\ 1 + H,\ 2 + H\}$$

- **Ex:** $G = \{1, g, g^2, g^3, g^4, g^5\}$ $(\text{ord}(g) = 6)$
$H = \{1, g^3\}$ ← operation is multiplication

$$(gH) \cdot (g^2 H) = g^3 H = \boxed{1 \cdot H}$$

$$g^3 H = \{g^3 \cdot 1, g^3 \cdot g^3\} = \{g^3, 1\} = 1 \cdot H$$

Worksheet 28: Adding/Multiplying Cosets

Math 335

Reporter:

Recorder:

Equity Manager:

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1. Let

$$G = \mathbb{Z}_9$$

and let

$$H = \{0, 3, 6\} \subseteq \mathbb{Z}_9.$$

$G/H = \{0+H, 1+H, 2+H\}$

(a) Apply Lagrange's Theorem to determine the number of distinct left cosets of H in G, and then make a list of those distinct left cosets.

# of left cosets = $|G|/|H| = 9/3 = 3$

List of cosets:
- $0+H = \{0,3,6\} = 3+H$
- $1+H = \{1,4,7\} = 4+H$
- $2+H = \{2,5,8\} = 5+H$

(b) Add the following two left cosets, and express your answer as one of the things from your list from part (a):

$$(2 + H) + (1 + H) = 3+H = \boxed{0+H}$$

same answer

(c) On the other hand, $2 + H = 5 + H$. (Make sure you believe me about this!) Try adding together

$$(5 + H) + (1 + H) = 6+H = \boxed{0+H}$$

and again expressing your answer as one of the things from your list from part (a). Did you get the same answer as you did in part (b)?

(d) What's another way you could express $2 + H$? Try adding that other expression to $1 + H$. Do you still get the same answer?

$$(8+H)+(1+H) = \boxed{0+H}$$

2. Let g be an element of order 15 in a group, and let

$$G = \langle g \rangle = \{1, g, g^2, g^3, \ldots, g^{14}\}$$
$$H = \langle g^5 \rangle = \{1, g^5, g^{10}\} \subseteq G.$$

(This is the same example from Worksheet 24.)

(a) By looking back at Worksheet 24 or re-calculating, make a list of the distinct left cosets of H in G.

$1H = \{1, g^5, g^{10}\}$
$gH = \{g, g^6, g^{11}\}$
$g^2H = \{g^2, g^7, g^{12}\}$
$g^3H = \{g^3, g^8, g^{13}\}$
$g^4H = \{g^4, g^9, g^{14}\}$

(b) Multiply the following two left cosets, and express your answer as one of the things from your list from part (a):

$$(g^3H) \cdot (g^4H) = g^7H = \boxed{g^2H}$$

(c) Check to make sure you believe me that $g^3H = g^8H$. Then try multiplying

$$(g^8H) \cdot (g^4H) = g^{12}H = \boxed{g^2H}$$

Did you get the same answer as you did in part (b)?

(d) What's another way you could express g^3H? Try multiplying that other expression by g^4H. Do you still get the same answer?

$$(g^{13}H) \cdot (g^4H) = g^{17}H = \boxed{g^2H}$$

NOTE: The results of this worksheet demonstrate that, in these two examples, the operation on cosets is well-defined.