Examples of Groups

- <u>Reminders</u>: · HW1 due today 5pm
  · Office Hours today 2-3pm

- <u>Announcement</u>: Mathematistas (regular meetings on Wednesdays at 3pm, Happy Hour this Friday at 3pm)

- <u>Video</u>: More examples of groups
  $(\mathbb{Z}_n, D_4, GL(n, \mathbb{R}))$ and the Cancellation Property

Worksheet 5: Examples and Non-Examples of Groups
Math 335

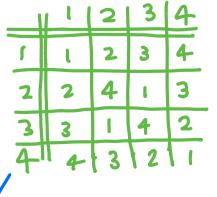**Recorder** (person whose first name comes alphabetically first): _____

Equity manager (person whose first name comes alphabetically second): _____

Reporter (person whose first name comes alphabetically third): _____

Get to know each other: Have you seen any good movies or TV shows recently?

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1. For each of the following, decide whether G is a group. If it is, try to prove that it's a group (assuming associativity). If it's not, try to figure out which of the group axioms fail.

   (a) $G = \{1, 2, 3\}$ under the operation of multiplication modulo 4

   Not a group

   - Closed? No: $2 \cdot 2 \equiv 0 \notin G$
     - $\in G$ $\in G$
   - Associative? ✓
   - Identity? $1 \in G$ ✓
   - Inverses? No: 2 has no inverse

	1	2	3
1			
2	2	0	2
3			

   (b) $G = \{1, 2, 3, 4\}$ under the operation of multiplication modulo 5

   Group

   - Closed? Yes (see table) ✓
   - Associative? ✓
   - Identity? $1 \in G$ ✓
   - Inverses? Yes (see table) ✓

	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

(c) $G = \{$odd integers$\}$ under the operation of addition

- **Closed?** No (e.g. $5 + 3 = 8 \notin G$)

- **Associative?** ✓

- **Identity?** No (0 isn't odd)

- **Inverses?** Yes (if a is odd, then $-a$ is odd) ✓

(d) $G = \{$even integers$\}$ under the operation of addition

- **Closed?** Yes (even + even = even) ✓

  Proof: $2k + 2\ell = 2(k + \ell)$

- **Associative?** ✓

- **Identity?** Yes (0 is even) ✓

  Proof: $0 = 2 \cdot 0$

- **Inverses?** Yes (if a even, then $-a$ even) ✓

  Proof: $a = 2k \implies -a = -2k = 2(-k)$