Worksheet 15: The Subgroup Generated by an Element

Math 335

Reporter:

Recorder:

Equity Manager:

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1. In the group $G = S_3$, what is $\langle (1,3,2) \rangle$? Write out all the elements of this subgroup.

$$\langle (1,3,2) \rangle = \{e, (1,3,2), (1,2,3)\}$$

$$(1,3,2) \circ (1,3,2) = (1,2,3)$$

2. In the group $G = \mathbb{Z}_6$ (where the operation is addition modulo 6), what is $\langle 4 \rangle$? Write out all the elements of this subgroup.

$$\langle 4 \rangle = \{0, 4, 2\}$$

3. In the group $G = \mathbb{Z}$ (where the operation is addition), what is $\langle -2 \rangle$?

$$\langle -2 \rangle = \{0, -2, -4, -6, \ldots, 2, 4, 6, \ldots\}$$

$$= \{\text{even integers}\}$$

4. **Challenge 1**: Let G be a group and let $g \in G$. Do you see a relationship between the order of g and the subgroup $\langle g \rangle$?

ord(g) = # of elements in $\langle g \rangle$

(NOTE: This is even true if both of these are ∞)

5. **Challenge 2**: By definition, $\langle g \rangle$ contains not just the elements $e, g, g^2, g^3, \ldots$ but also the elements $g^{-1}, (g^{-1})^2, (g^{-1})^3, \ldots$. Sometimes, though, we get lucky and we can stop after just the positive powers of g, because the elements $g^{-1}, (g^{-1})^2, (g^{-1})^3, \ldots$ all already appear among those.

   (a) Look back at Problems 1–3. In which cases did $\langle g \rangle$ consist just of positive powers of g, and in which cases did we also need to separately include g^{-1} and its powers?

   - #1, #2 : $\langle g \rangle$ consists just of positive powers
   - #3: need to separately include g^{-1} and its powers

   (b) Do you have any guesses about a general rule for when $\langle g \rangle$ consists of just positive powers of g, and when positive powers alone aren't enough?

   $\langle g \rangle$ consists of just positive powers of g when ord(g) is finite, but positive powers alone aren't sufficient when ord$(g) = \infty$