Orders in Cyclic Groups

- Recall: G is cyclic if G= <g>
 for some g=G

 ("every element of G is a

 power of one single element")
 - G<u>roal</u>: If G= < g>, then ord(gk) =efill this in!
 - Step 1: oralg) = # of elements in G
 - · Step a:

 "Lemma": < gk > = < ggcd(n,k) > n=# of elements
 in G

Worksheet 17: Orders of Elements in Cyclic Groups

Math 335

Reporter:

Recorder:

Equity Manager:

Suppose that g is a complex number such that

$$g^6 = 1$$

but no smaller positive power of g equals 1. (Trust me, there are complex numbers like this, though their expressions aren't so simple.) Let

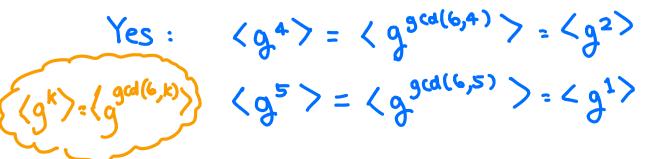
$$G=\{1,g,g^2,g^3,g^4,g^5\},$$

which forms a cyclic group under multiplication of complex numbers.

1. Write down all of the elements in the subgroup generated by each element of G. That is, fill in the following:

$$\langle 1 \rangle = \{1\}$$
 $\langle g \rangle = \{1, g, g^2, g^3, g^4, g^5\}$
 $\langle g^2 \rangle = \{1, g^2, g^4\}$
 $\langle g^3 \rangle = \{1, g^3\}$
 $\langle g^4 \rangle = \{1, g^4, g^2\}$
 $\langle g^4 \rangle = \{1, g^4, g^3, g^3, g^2, g^3\}$
 $\langle g^5 \rangle = \{1, g^5, g^4, g^3, g^2, g^3\}$
because $g^6 = g^6, g^4 = g^5$

2. Are your answers to Problem 1 consistent with the lemma from today's video?



3. Your answer to Problem 1 can be interpreted as a calculation of the order of every element in G. For example, since

$$\langle g^2\rangle=\{1,g^2,g^4\}$$

and we know that

$$\operatorname{ord}(g^2) = \# \text{ of elements in } \langle g^2 \rangle,$$

we see that $\operatorname{ord}(g^2) = 3$. Repeat this reasoning to calculate the order of each element in G.

4. Based on your calculations, can you guess a formula for the order of any element in G? Once you have a guess, check to make sure it agrees with your calculations in Problem 3.

$$\operatorname{ord}(g^{k}) = \frac{6}{\gcd(6,k)}$$

(E.g. ord(
$$g^2$$
) = $\frac{6}{9ca(6,2)} = \frac{6}{2} = 3$)

5. Vague Thought Question: This might feel weirdly similar to what you did on Worksheet 9, where you calculated orders of elements in \mathbb{Z}_6 . Can you see a relationship between orders of elements of the group G from this worksheet and orders of elements of the group \mathbb{Z}_6 ?