Definition of a Group

- **Reminders:** • HW1 due Wednesday 5pm
  • Office hours today at 10am

- **Video:** A group is a set with a binary operation (e.g. + or •) that satisfies three axioms.

Worksheet 4: Definition of a Group

Math 335

Equity manager (person whose first name comes alphabetically last): ____________________

Reporter (person whose first name comes alphabetically second): ______________________

Get to know each other: Do you have any pets?

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1. Let's try to figure out whether $\mathbb{R}$ is a group under the operation of multiplication.

   0) **Closure**: Is $\mathbb{R}$ closed under multiplication?

   Yes: the product of two real numbers is a real number.

   1) **Associativity**: I'll spoil this one for you, since it's hard to check: multiplication of real numbers is associative.

   2) **Identity**: Does $\mathbb{R}$ have an identity element for multiplication? If so, what is it?

   Yes: the identity is 1.

   3) **Inverses**: Do all elements $a \in \mathbb{R}$ have inverses under multiplication? If so, what is the inverse of a?

   No! If $a \neq 0$, then the inverse of a is $1/a$, but 0 has no inverse.

2. Based on the answers to the above questions, is $\mathbb{R}$ a group under the operation of multiplication? If not, do you see a way you can modify it to make it a group?

   No, but

   $$\boxed{\mathbb{R}^* = \left\{\begin{matrix}\text{nonzero real} \\ \text{numbers}\end{matrix}\right\}}$$

   is a group under multiplication.

3. Can you think of any other sets that are groups under the operation of multiplication?

- $\mathbb{R}^+$ = {positive real numbers}
- $\mathbb{Q}^*$ = {nonzero rational numbers}
- $\mathbb{C}^*$ = {nonzero complex numbers}
- .... and many other examples!

NOTE: {nonzero integers} isn't a group under multiplication, because elements don't have inverses

Challenge: Repeat the steps of Problem 1 to try to figure out whether the set

$$G = \{2 \times 2 \text{ matrices with real numbers as entries}\}$$

is a group under the operation of matrix multiplication. If not, do you see a way to modify it to make it a group?

- closed? ✓
- associative? ✓
- identity? ✓ (Identity is $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$)
- inverses? ✗ (only if determinant is nonzero)

So G isn't a group, but {2x2 matrices with $\mathbb{R}$-entries + det $\neq 0$} is a group.