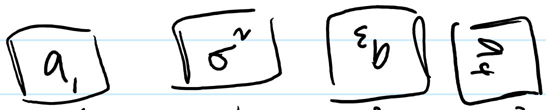
Homework 8

Wednesday, April 14, 2021 9:52 AM

1. Project Topic 2

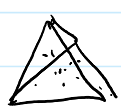
EP1. Considering a cube, the cube can have any one of 6 sides facing up. Each of these six sides, can be oriented 4 different ways and maintain symmetry. Therefore, there are 24 symmetries of a cube. Let us name the sides, a, b, c, d, e, f. and each position of each letter. Then for notation's sake, the position of the cube is marked by the side facing up. Then our symmetries



a^0 a^1 a^2 a^3

are as follows:

a^0, a^1, a^2, a^3
b^0, b^1, b^2, b^3
c^0, c^1, c^2, c^3
d^0, d^1, d^2, d^3
e^0, e^1, e^2, e^3
f^0, f^1, f^2, f^3



Similarly, consider the tetrahedron with for sides labeled a, b, c, d. Each side can be oriented 3 ways $\{e, R_{120}, R_{240}\}$. There can then be 12 symmetries as follows

$\{a^0, a^1, a^2, b^0, b^1, b^2, c^0, c^1, c^2, d^0, d^1, d^2\}$

2. $S = \mathbb{Z}$ Let $\sim$ be the equivalence relation

$a \sim b \iff a^2 = b^2$

What is $[2]$?

$$[2] = \{b \in \mathbb{Z} \mid 2 \sim b\}$$

$$= \{b \in \mathbb{Z} \mid 4 = b^2\} = \{b \in \mathbb{Z} \mid 2 = |b|\}$$

$$= \{2, -2\}$$

$$[-3] = \{b \in \mathbb{Z} \mid -3 \sim b\} = \{b \in \mathbb{Z} \mid (-3)^2 = b^2\}$$

$$= \{3, -3\}$$

$$[0] = \{b \in \mathbb{Z} \mid 0 \sim b\} = \{b \in \mathbb{Z} \mid 0 = b^2\} = \{0\}$$

3. Let $a, b \in S$. Suppose $a \sim b$. Let $x \in [a]$, which means $x \sim a$.
By definition, $a \sim b$ is transitive, so $x \sim b$. This means that $x \in [b]$ and $[a] \subseteq [b]$.

Again, by definition, $a \sim b$ is symmetric, and using the same argument as above, we find that $[b] \subseteq [a]$. Hence $[a] = [b]$.

Conversely suppose $[a] = [b]$. Suppose $y \in [a]$. Since $[a] = [b]$, $y \in [b]$, which shows $a \sim b$ $\forall a \in S$.

Suppose $y \in [b]$, then $y \in [a]$, which shows $b \sim a$ $\forall a \in S$, hence $a \sim b \Rightarrow b \sim a$. $\forall a, b \in S$

Suppose $[a] = [b]$ and $[b] = [c]$.

Then, $[a] = [c]$, If we let $y \in [a]$, $y \in [c]$ which shows $a \sim b$ and $b \sim c \Rightarrow a \sim c$.

Hence $a \sim b$.

4. Let $G = \mathbb{Z}_{12}$ (operation + mod 12)

$H = \langle 3 \rangle \leq G$

$G = \{0, 1, 2, 3, \ldots, 11\}$ 12 elements
$H = \{0, 3, 6, 9\}$ 4 elements

a. $\frac{|G|}{|H|} = \frac{12}{4} = 3$

b. $G/H = \{0+H, 1+H, 2+H\}$

$0+H = \{0, 3, 6, 9\}$
$1+H = \{1, 4, 7, 10\}$
$2+H = \{2, 5, 8, 11\}$

c.

	$0+H$	$1+H$	$2+H$
$0+H$	$0+H$	$1+H$	$2+H$
$1+H$	$1+H$	$2+H$	$0+H$
$2+H$	$2+H$	$0+H$	$1+H$

	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

d. $G/H \cong \mathbb{Z}_3$

$\varphi : G/H \to \mathbb{Z}_3$
$\varphi(0+H) = 0$
$\varphi(1+H) = 1$
$\varphi(2+H) = 2$