Order of an Element in a Group

- Note: HW3 includes an Extra Credit, added to "Extra Credit Opportunities" list on iLearn.

- Finish discussing Worksheet 8.

- Video: The order of $g \in G$ is the smallest (positive) power of g that equals e. If no positive power of g equals e, we say the order of g is ∞.

- Ex: $\text{ord}(e) = 1$

$e^1 = e$ ← identity, so $\text{ord}(e) = 1$

Worksheet 9: Orders of Elements in Groups

Math 335

Reporter:

Recorder:

Equity Manager:

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1. Consider the group

$$\mathbb{R}^* = \{\text{nonzero real numbers}\}$$

under the operation of multiplication.

(a) Calculate the order of -1 in $\mathbb{R}^*$.

$(-1)^1 = -1$

$(-1)^2 = 1$ ← identity, so $\boxed{\text{ord}(-1) = 2}$

(b) Calculate the order of $\frac{1}{2} \in \mathbb{R}^*$.

$(1/2)^1 = 1/2$

$(1/2)^2 = 1/4$

$(1/2)^3 = 1/8$

etc...

never = identity, so

$\boxed{\text{ord}\left(\frac{1}{2}\right) = \infty}$

(c) Can you make a general statement about the orders of all of the elements in $\mathbb{R}^*$?

$\text{ord}(1) = 1$

$\text{ord}(-1) = 2$

$\text{ord}(g) = \infty$ for all other $g \in \mathbb{R}^*$

2. Consider the group

$$\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$$

under the operation of addition modulo 6.

(a) Calculate the order of 2 in $\mathbb{Z}_6$. (Be careful: what do expressions like g^2 or g^3 mean when the operation is addition?)

$2 = 2$

$2+2 = 4$

$2+2+2 = 0$ ← identity, so $\boxed{\text{ord}(2)=3}$ ✓

(b) Calculate the order of 3 in $\mathbb{Z}_6$.

$3 = 3$

$3+3 = 0$ ← identity, so $\boxed{\text{ord}(3)=2}$

(c) **Challenge**: Can you guess a formula for the order of any element in $\mathbb{Z}_6$?

$$\text{ord}(g) = \frac{6}{\gcd(g,6)}$$

(We'll prove this later....)

E.g. $\text{ord}(2) = \frac{6}{\gcd(2,6)} = \frac{6}{2} = 3$

3. **Open-ended question**: Next time, we'll study the orders of elements in the symmetric group. As a warm-up for this, choose some elements in the group S_3, and try calculating their orders. What do you notice?

Next time!