Exam 2 Review
Math 335

The second exam for Math 335 will take place on **Wednesday, April 7**. You will be able to choose any 90-minute time window during that day (from 12:01am to 11:59pm) to complete the test. The questions will be posted on iLearn, and you'll submit your solutions by either hand-writing and scanning them or by writing them on a tablet and saving to a PDF. It will cover everything we learned up through the previous Friday's class, meaning:

1. Cyclic groups

   (a) Definition and examples of $\langle g \rangle$

   (b) Definition and examples/non-examples of cyclic groups

   (c) **Theorem**: $\text{ord}(g)$ = number of elements in $\langle g \rangle$

   (d) **Lemma**: If $G = \langle g \rangle$ has n elements, then $\langle g^k \rangle = \langle g^{\gcd(n,k)} \rangle$

   (e) **Theorem**: If $G = \langle g \rangle$ has n elements, then $\text{ord}(g^k) = \dfrac{n}{\gcd(n, k)}$

   (f) **Fundamental Theorem of Cyclic Groups**: If G is a cyclic group with n elements, then G has exactly one subgroup of size d for each divisor d of n—namely, that subgroup is $\langle g^{n/d} \rangle$.

   (g) Listing the subgroups of a cyclic group, either in additive notation like $\mathbb{Z}_{12}$ or in multiplicative notation like $\{1, g, g^2, g^3, \ldots, g^{11}\}$.

2. Isomorphisms

   (a) Definition of isomorphism

   (b) Examples/non-examples of isomorphisms

   (c) Definition of two groups being isomorphic

   (d) Proving that two groups are or are not isomorphic

3. Cosets

   (a) Definition and examples of the left coset aH

   (b) All left cosets of H in G have the same size (equal to $|H|$), and they're disjoint if they're not equal

   (c) **Lagrange's Theorem**: The number of distinct left cosets of H in G is equal to $\dfrac{|G|}{|H|}$

   (d) The size of any subgroup of G (and the order of any element of G) divides $|G|$

4. Equivalence relations

   (a) Definition and examples of equivalence relations

   (b) Definition and examples of equivalence classes

   (c) If $a \sim b$ is defined to mean $a^{-1}b \in H$, then $[a] = aH$

   (d) Definition and examples/non-examples of a function being well-defined

You will be able to use your notes or books, but you will not be allowed to get help from any other person. Stylistically, you can expect a mixture of

- example problems ("give an example of ...."),

- short computations (e.g. "compute the composition of these two elements of S_3"), and

- short proofs (e.g. "prove that the following is a group").

The proofs will not be as involved as the sorts of things you do on your homework; think more along the lines of what you do on your in-class worksheets. For a sample of the sort of things I've asked in the past, see the "Sample Midterm," which is the actual midterm from my Fall 2019 Math 335 class (slightly modified to remove problems on topics we haven't yet covered).

One very good way to prepare for the exam is to review all of the definitions from our class videos (specifically, those that were highlighted as <u>Definition</u>: ...), ideally with flashcards or some other self-testing mechanism. When doing so, test yourself both on the precise definition and on whether you can give both an example and a non-example of the concept. The in-class worksheets are a good source of these examples, and reviewing them is another good way to prepare.

I also encourage you to use each other as a resource! Please let me know if you're organizing a study session over Zoom or some other platform, and (with your permission) I'll advertise it to the class.