

Math 335, Homework 6

Due Wednesday, March 17

1. Apply the Fundamental Theorem of Cyclic Groups to list all of the subgroups of \mathbb{Z}_{30} , which is a group under addition modulo 30.
2. Let $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$, which is a group under addition modulo 6, and let S_3 be the symmetric group, which is a group under composition. Consider the following function $\varphi : \mathbb{Z}_6 \rightarrow S_3$:

$$\begin{aligned}\varphi(0) &= e \\ \varphi(1) &= (1, 2) \\ \varphi(2) &= (1, 3) \\ \varphi(3) &= (2, 3) \\ \varphi(4) &= (1, 2, 3) \\ \varphi(5) &= (1, 3, 2).\end{aligned}$$

Is φ an isomorphism? Prove your answer.

3. Let G and H be two groups, and suppose that there exists an isomorphism $\varphi : G \rightarrow H$. Prove that G is abelian if and only if H is abelian.
4. Prove that the function $\varphi(x) = 10^x$ is an isomorphism from the group \mathbb{R} (under addition) to the group $\mathbb{R}^+ = \{\text{positive real numbers}\}$ (under multiplication).
5. In each of the following cases, decide whether G and H are isomorphic, and prove your answer. (**Hint:** In both cases, calculating orders of elements will be helpful.)
 - (a) $G = \mathbb{Z}_4$ (under addition modulo 4) and $H = \{1, a, b, c\}$, under the operation described by the following table:

	1	a	b	c
1	1	a	b	c
a	a	1	c	b
b	b	c	1	a
c	c	b	a	1

(The group H is called the *Klein four-group*.)

- (b) $G = S_4$ and $H = D_{12} = \{\text{symmetries of a regular 12-gon}\}$ (under composition)