Math 335, Homework 1

Due Wednesday, February 3

Mark Kim

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1. Write a brief (about two-paragraph) mathematical autobiography to help me get to know you. Some questions you might choose to answer in this autobiography are:

   - How did you become interested in mathematics? How long have you been studying it, and what courses or aspects of the subject have you particularly enjoyed?
   - What do you hope to do after you graduate? In what ways do you think your mathematics education will help you with those plans?
   - Based on our discussion on the first day of class, why do you think it might be useful—for you, personally—to learn abstract algebra?

   Answer

   I have had an interest in mathematics at a very young age. My parents' singular focus on academics exposed me to math concepts that were far beyond my grade level. I remember when I first encountered Algebra when I was in fourth grade. It was a revelation to me that unknown values could be depicted with letters. Nevertheless, family pressures left that curiousity and interest largely unexplored for most of my life: it was made clear to me that I was to be a Doctor. Throughout my academic studies, I eagerly absorbed whatever mathematical knowledge I could despite my focus on biological sciences. I am not completely sure exactly what draws me to mathematics, but I love how rewarding it is. Its difficulty requires hard work and reasoning, but when the pieces come together, the answers are elegant and awe inspiring. I don't really think I have encountered a course in math that I disliked, but perhaps the most enjoyable course I have taken was Math 301.

   After graduation, I intend to continue on to graduate school and particularly into a doctoral program. I want to research human behavior from a mathematical perspective. Specifically, I want to use data to understand hidden drivers for human behavior and life-defining decisions. I believe that gaining a substantial amount of mathematical maturity, I will be best equipped to pursue my interests. As for abstract algebra, I think it will be extremely useful to me. From our discussion on our first day of class, it seems that abstract algebra is a study of objects and operations. Since I am also a Computer Science major, I believe that understanding abstract algebra will give me a more intuitive understanding of abstractions within the context of programming. Outside of what we learned in class, I have heard that modern algebra is used quite extensively in cryptography (which I have a passing interest in as well).

2. In each of the following problems, when you are working modulo n, give an answer in the range $\{0, 1, 2, \ldots, n-1\}$.

   (a) Find a value of x such that $5x \equiv 1 \mod 11$.
   Answer: $x = 9$

(b) Is there a value of x such that $5x \equiv 1 \mod 10$? Carefully explain how you know.
Answer: Notice that $1 \mod 10$ is congruent to $1, 11, 21, 31, \cdots$. Starting at 1, we add or subtract 10 to reach conguence to $1 \mod 10$. However, $5x \neq 1 + 10y$ for all $x, y \in \mathbb{Z}$. Therefore, there is no value x such that $5x \equiv 1 \mod 10$.

3. The song "As" from Stevie Wonder's album *Songs in the Key of Life* mentions the equation

$$8 \times 8 \times 8 = 4.$$

Although this equation isn't true as stated, find all integers $n \geq 2$ such that the equation

$$8 \times 8 \times 8 \equiv 4 \mod n$$

is true.

Answer:

$$\begin{aligned} 8 \times 8 \times 8 &= 512 \\ 512 &\equiv 4 \mod 508 \\ &\equiv 4 \mod 2 \\ &\equiv 4 \mod 254 \\ &\equiv 4 \mod 4 \\ &\equiv 4 \mod 127 \end{aligned}$$

4. Is composition of symmetries of a square **commutative**—that is, if A and B are two symmetries of a square, is it true that $A \circ B = B \circ A$? Explain how you know.

Answer: Referring to the table we filled out in class, notice that $R_{90} \circ H = D$ and $H \circ R_{90} = D'$. Hence $R_{90} \circ H \neq H \circ R_{90}$, so the composition of symmetries of a square are **not** commutative.

5. Describe all of the symmetries of an equilateral triangle, in the same way that we did for symmetries of a square. In other words, list all of the symmetries and give them names, and write down a table showing the result of composing any two symmetries.

Answer:

- I: The identity (do nothing)
- R_{120}: Rotate 120° counterclockwise
- R_{240}: Rotate 240° counterclockwise
- V: Reflect along the vertical axis
- D: Reflect along the 210° diagonal
- D': Reflect along the 330° diagonal