Rings

- Finish discussion of Worksheet 37

- Video: A ring is a set with operations

  $+$ and $\cdot$,

  where

  1) $+$ forms an abelian group
  2) $\cdot$ is associative
  3) $+, \cdot$ satisfy distributive property

Worksheet 38: Rings

Math 335

Reporter:

Recorder:

Equity Manager:

$+$

Associative
Identity
Inverses
Commutative

$\cdot$

Associative
(Maybe identity)
(Maybe inverses)
(Maybe commutative)

1. In today's video, we learned about three special types of rings: commutative rings, rings with unity, and fields. Let's see if we can find an example and a non-example of each of these.

   (a) What is an example of a commutative ring? A ring that's not commutative?

   Commutative : $\mathbb{Z}$

   not commutative : $M_2(\mathbb{R})$

   (b) What is an example of a ring with unity? A ring without unity?

   with unity : $\mathbb{Z}$

   without unity : $2\mathbb{Z}$

   (c) What is an example of a field? A ring that's not a field?

   field : $\mathbb{R}$

   not a field : $\mathbb{Z}$, $M_2(\mathbb{R})$

2. In the remainder of this worksheet, we'll prove the following basic properties of rings:

   (i) For all $a \in R$,
   $$a \cdot 0 = 0 \quad \text{and} \quad 0 \cdot a = 0.$$

   (ii) For all $a, b \in R$,
   $$a \cdot (-b) = -(a \cdot b) \quad \text{and} \quad (-a) \cdot b = -(a \cdot b).$$

   (a) Use the distributive property to re-write
   $$a \cdot 0 + a \cdot 0$$
   in a different way. Can you use this to prove the first statement of property (i)? (The second statement of property (i) is proved analogously.)

$$a \cdot 0 + a \cdot 0 \overset{\text{distrib.}}{=} a \cdot (0 + 0) \overset{0 \text{ is identity for } +}{=} a \cdot 0$$

$$\Rightarrow \boxed{a \cdot 0 = 0} \quad \text{(cancellation)}$$

   (b) To prove the first statement of property (ii), try to use the distributive property to show
   $$a \cdot (-b) + a \cdot b = 0.$$
   (The second statement of property (ii) is proved analogously.)

$$a \cdot (-b) + a \cdot b \overset{\text{distrib.}}{=} a \cdot (-b + b) \overset{\text{inverses under } +}{=} a \cdot 0 \overset{\text{(i)}}{=} 0$$

$$\Rightarrow \boxed{a \cdot (-b) = -(a \cdot b)} \quad \text{(subtract } a \cdot b \text{ from both sides)}$$