The Symmetric Group

- Finish discussing Worksheet 6.
- Video: The symmetric group S_n is

$$S_n = \{\text{bijections } \{1,2,\ldots,n\} \to \{1,2,\ldots,n\}\}.$$

E.g. one element of S_3 is

$$f : \{1,2,3\} \to \{1,2,3\}$$
$$f(1) = 3$$
$$f(2) = 1$$
$$f(3) = 2.$$

Worksheet 7: Introduction to the Symmetric Group

Math 335

Reporter:

Recorder:

Equity Manager:

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1. How many bijections from the set $\{1, 2, 3\}$ to itself can you think of? For example, we've seen one:

$$f(1) = 3$$
$$f(2) = 1$$
$$f(3) = 2.$$

Try to list as many others as you can.

$f(1) = 1$
$f(2) = 2$
$f(3) = 3$

$f(1) = 1$
$f(2) = 3$
$f(3) = 2$

$f(1) = 2$
$f(2) = 1$
$f(3) = 3$

$f(1) = 2$
$f(2) = 3$
$f(3) = 1$

$f(1) = 3$
$f(2) = 1$
$f(3) = 2$

$f(1) = 3$
$f(2) = 2$
$f(3) = 1$

2. How many different bijections do you think there are from the set $\{1, 2, 3\}$ to itself?

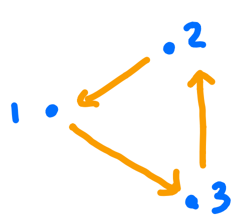
There are exactly 6 (the ones written above). In general, S_n has $n!$ elements.

3. It's cumbersome to have to write $f(1) = \cdots, f(2) = \cdots, f(3) = \cdots$ every time we want to specify a bijection. Brainstorm with your group members some possible short-hand notations for this.

Many possibilities! E.g. could write

$$f(1) = 3$$
$$f(2) = 2$$
$$f(3) = 1$$

as



[We'll use a short-hand called "cycle notation" in this class — next time....]

4. At the end of today's video, I mentioned the following theorem:

Theorem: The set

$$S_n = \Big\{\text{bijections from the set } \{1, 2, \ldots, n\} \text{ to itself}\Big\}$$

forms a group under the operation of composition.

What things do we need to check in order to prove this theorem? Some of those things are problems on Homework 3; do you see which ones?

0. Closure: The composition of two bijections is a bijection (HW3, #1).
1. Associativity: Assume, as always.
2. Identity: The identity function $f(1)=1, f(2)=2, \ldots, f(n)=n$ is a bijection.
3. Inverses: If f is a bijection, then it has an inverse function that's also a bijection (HW3, #2).