A Note on Optimal Foreign Reserve Management without Commitment[*]

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Abstract

This paper studies optimal policy for foreign exchange reserve management without commitment. The stylized linear-quadratic model incorporates three key ingredients: (i) fear of floating, (ii) forward-looking exchange rates, and (iii) cost of reserve management. The paper first studies two benchmark optimal policies with and without commitment: the Ramsey policy and the Markov-Perfect policy. It then provides a sustainable plan that resolves the time-inconsistency issue in reserve management. Following the plan, the government without commitment achieves the Ramsey outcome in equilibrium.

JEL Classification: E44, E52, E58, F31, F32, F41

Keywords: Foreign Exchange Reserves, Exchange Rates, Time-inconsistency, Ramsey policy, Markov-Perfect Policy, Sustainable Plan

1 Introduction

Emerging markets and developing countries (EMDEs) have been accumulating tremendous amount of foreign exchange (FX) reserves since the late 1990s. The dramatic increase in FX reserve holdings has prompted an active debate on optimal reserve management (International Monetary Fund 2011, 2013). What is the optimal policy for FX reserve management? This is a tricky question especially for EMDEs that have been suffering from the credibility problem in policy making.

This paper studies optimal FX reserve management without commitment. The paper builds on a stylized linear-quadratic small-open economy framework of Ghosh et al. (2016) and Basu et al. (2018). The model has three key ingredients: (i) fear of floating, (ii) forward-looking exchange rates, and (iii) cost of reserve management. First, as Calvo and Reinhart (2002) document, fear of floating is a prevalent motive in EMDEs. The exchange rate management is one of the key reasons for hoarding FX reserves. The

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model incorporates fear of floating as a convex cost of welfare, following Bianchi and Lorenzoni (2021). Second, the model incorporates the expectation channel of exchange rate determination as in Basu et al. (2018). Since the exchange rate is forward-looking, both the current and the future policies influence the current exchange rate determination. Lastly, the cost of reserve management is incorporated as a convex welfare cost in the model. The management cost is an important consideration for EMDEs that includes the foregone opportunity cost of investment and the quasi-fiscal sterilization cost.

In this framework, I first derive two optimal policies: the Ramsey policy and the Markov-Perfect policy. These are the benchmark policies for the government with and without commitment. The two policies differ in dealing with the trade-offs between fear of floating and reserve management cost. Comparing the equilibrium outcomes under the two policies, I show that the Ramsey policy attains higher welfare than the Markov-Perfect policy. This result highlights the importance of credibility in reserve management. Without commitment, however, the government may be tempted to deviate from the Ramsey policy. In other words, the Ramsey policy is time-inconsistent. I provide a way to attain the Ramsey outcome without commitment. In particular, I suggest a sustainable plan that incentivizes the government to follow the Ramsey policy, achieving the Ramsey outcome in equilibrium.

This paper is related to the literature on optimal FX reserve management. Jeanne and Rancière (2011) provide the measure for the optimal FX reserves in the economies susceptible to sudden stops. More recently, Arce et al. (2019) study macroprudential role of reserve accumulation that achieves the constrained-efficiency. Céspedes and Chang (2020) emphasize the interaction between ax-ante reserve accumulation and ax-post unconventional monetary policies. Despite its importance, the issue of commitment has been less explored in the literature. One exception is Basu et al. (2018) that studies FX intervention focusing on capital outflow episodes. In their work, time-inconsistency arises because of the zero lower bound on reserve holdings. In contrast, in the present paper, the cost of reserve management is the source of time-consistency. The government has an incentive to deviate to avoid the reserve management cost. The present study further provides a way to implement the Ramsey outcome, overcoming the credibility problem.

This paper also relates to the literature studying time-inconsistency of government policies (Calvo 1978; Kydland and Prescott 1980; Barro and Gordon 1983). Stokey (1989, 1991) and Chari and Kehoe (1990) propose the notion of a sustainable plan for the credible government policy.[1] The seminal work along this line includes Chang (1998) and Phelan and Stacchetti (2001). These studies provide complete characterizations of sustainable plans for monetary and fiscal policies using the technique of Abreu et al. (1990). The present study does not attempt to characterize the set of sustainable plans. Instead, it provides a simple sustainable plan, highlighting the issue of credibility in FX reserve management.

The paper is organized as follows. Section 2 describes the model. Section 3 studies the Ramsey and the Markov-Perfect policies. Section 4 provides a sustainable plan. Section 5 concludes.

[1] See also Evans and Sargent (2013) for the discussion on history-dependence of government policies.

2 Model

Time is discrete and denoted by $t \in \{0, \ldots, \infty\}$. Consider a small-open and perfect-foresight economy in which the equilibrium is characterized by three linear equations: capital flow, current account, and the balance of payment.

The capital flow is determined by

$$\Delta k_t = a(r_t - (r_t^* + e_{t+1} - e_t))$$

where Δk_t is the change in capital flows, r_t and r_t^* are the real interest rates in the domestic and the foreign economies, e_t is the real exchange rate, and $a > 0$.[2] This equation is a variant of the uncovered interest rate parity (UIP) condition that describes the private agents' behavior of absorbing excess returns in the financial market.[3] The capital flows out of the domestic economy, i.e., $\Delta k_t < 0$, whenever there is an excess return from holding foreign bonds after taking into account the expected exchange rate adjustment, i.e., $r_t < r_t^* + e_{t+1} - e_t$. The capital flows into the domestic economy whenever the opposite holds. Assume that the real interest rates are the same in the domestic and the foreign economies $r_t = r_t^*$, abstracting from policy rate considerations. The equation is then reduced to

$$\Delta k_t = a(e_t - e_{t+1}) \tag{1}$$

which shows that capital outflows ($\Delta k_t < 0$) are positively associated with the expected exchange rate depreciation ($e_t < e_{t+1}$).

The current account is normalized to be balanced when $e_t = 0$. In particular, the current account is

$$ca_t = ce_t \tag{2}$$

with $c > 0$, indicating that the current account surplus is associated with exchange rate depreciation.

Finally, from the balance of payment identity, the current account is equal to the change in the net foreign assets

$$ca_t = \Delta R_t - \Delta k_t \tag{3}$$

where ΔR_t is the change in the stock of FX reserves.

Definition 1. *A perfect-foresight equilibrium is composed of sequences of capital flow, exchange rate, and current account $\{k_t, e_t, ca_t\}_{t=0}^{\infty}$ satisfying (1), (2), and (3), given the stock of FX reserves $\{R_t\}_{t=0}^{\infty}$ chosen by the government, and the initial conditions k_{-1}, R_{-1}.*

Given $\{R_t\}_{t=0}^{\infty}$ chosen by the government and initial conditions k_{-1} and R_{-1}, the equilibrium condi-

[2] e_t increases when the exchange rate depreciates.

[3] The equation converges to the standard UIP condition as $a \to \infty$ where the capital flow is adjusted so that any expected excess return is absorbed in a frictionless market. A finite value of a stands for financial frictions that can be micro-founded by Gabaix and Maggiori (2015). See also Ghosh et al. (2016) and Basu et al. (2018) for the discussion on this specification.

tions are summarized as the following system of equations.

$$e_t = \frac{a}{a+c}e_{t+1} + \frac{1}{a+c}\Delta R_t \tag{4}$$

$$ca_t = ce_t \tag{5}$$

$$\Delta k_t = \Delta R_t - ce_t \tag{6}$$

Equation (4) shows that the exchange rate is forward-looking as it responds to the expected future exchange rate. The equation also shows that the exchange rate depreciates when reserves are accumulated, i.e., $\Delta R_t > 0$. This implies that the reserve management policy affects not only the exchange rate in the current period but also the exchange rates in all previous periods through the expectation.

Suppose that the per-period welfare of the economy is in quadratic form

$$-U(e_t, \Delta R_t) \equiv -\frac{(e_t - \bar{e})^2}{2} - \frac{b}{2}(\Delta R_t)^2 \tag{7}$$

The first term is fear of floating that incurs a convex cost at the exchange rate away from the bliss level $\bar{e} > 0$. The second term is the convex cost of reserve management.

How would the government optimally manage reserves? The welfare in equation (7) describes the trade-offs involved in reserve management. On the one hand, due to fear of floating, the government would like to achieve the bliss level of the exchange rate by adjusting the reserve stocks. On the other hand, due to the cost of reserve management, the government would try to avoid changing the reserve stocks. Let $f_t \equiv \Delta R_t$ be the variable for reserve management, i.e., positive (negative) f_t indicates reserve accumulation (decumulation). The government policy for reserve management is denoted by a sequence $\vec{f} = \{f_t\}_{t=0}^{\infty}$.

Definition 2. *The government policy for reserve management is $\vec{f}$ and the associated equilibrium outcome can be backed out from* (4) – (6)*.*

The next section studies two benchmark optimal policies for reserve management with and without commitment. With commitment, the government is able to follow the Ramsey policy achieving the first-best Ramsey outcome in equilibrium. The Markov-Perfect policy is a policy for the government without commitment.

3 Optimal policies for reserve management

This section derives the Ramsey policy and the Markov-Perfect policy, and then compares the equilibrium outcomes under the two policies.

3.1 Ramsey policy

Consider the Ramsey planning problem that maximizes the discount sum of the per-period welfare (7) subject to the equilibrium conditions (4) – (6). Define $x_t \equiv \begin{bmatrix} 1 \\ e_t \end{bmatrix}$ that contains the constant state and the forward-looking variable. The per-period welfare (7) can be rewritten as

$$-r(x_t, f_t) \equiv -x_t' \mathcal{R} x_t - \mathcal{Q} f_t^2$$

where $\mathcal{R} \equiv \begin{bmatrix} \frac{(\bar{e})^2}{2} & \frac{-\bar{e}}{2} \\ \frac{-\bar{e}}{2} & \frac{1}{2} \end{bmatrix}$ and $\mathcal{Q} \equiv \frac{b}{2}$. The equilibrium conditions (4) – (6) can be also rewritten in state-space form as

$$x_{t+1} = Ax_t + Bf_t \tag{8}$$

$$y_t = Cx_t + Df_t \tag{9}$$

where $A \equiv \begin{bmatrix} 1 & 0 \\ 0 & \frac{a+c}{a} \end{bmatrix}$, $B \equiv \begin{bmatrix} 0 \\ -1/a \end{bmatrix}$, $C \equiv \begin{bmatrix} 1 & 0 \\ 0 & c \\ 0 & -c \end{bmatrix}$, $D \equiv \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, and $y_t \equiv \begin{bmatrix} 1 \\ ca_t \\ \Delta k_t \end{bmatrix}$ contains the static variables.

The Ramsey problem in recursive form is composed of two subproblems.[4] In the first subproblem, the Ramsey planner solves

$$\max_{x_0} J(x_0)$$

where value function J solves the second subproblem of the Bellman equation

$$J(x) = \max_{f, x'} -r(x, f) + \beta J(x')$$

subject to

$$x' = Ax + Bf$$

The solution to this problem is called the Ramsey plan denoted by $\{\vec{x}^R, \vec{f}^R\}$, where $\vec{x}^R = \{x_t^R\}_{t=0}^{\infty}$ and $\vec{f}^R = \{f_t^R\}_{t=0}^{\infty}$. The associated equilibrium outcome can be backed out from (8) and (9). The details of the problem is given in Appendix A.

Definition 3. *The Ramsey policy for reserve management is $\vec{f}^R$ and the associated equilibrium outcome is called the Ramsey outcome.*

[4] See chapter 19 and 25 in Ljungqvist and Sargent (2018) and the associated QuantEcon lectures at https://quantecon.org/lectures for the recursive approach to the Ramsey problem.

3.2 Markov-Perfect policy

The Markov-Perfect policy depends only on the payoff-relevant current state. Since there is no state variable, the Markov-Perfect policy on reserve management $\vec{f}^{MP} = \{f_t^{MP}\}$ is constant in all periods, i.e., $f_t^{MP} = f^{MP}$ for some constant f^{MP}.

To derive f^{MP}, first iterate forward equation (4) and impose $f_{t+j+1} = f^{MP}$ for $j \geq 0$,

$$
\begin{aligned}
e_{t+1} &= \frac{1}{a+c} \sum_{j=0}^{\infty} \left(\frac{a}{a+c} \right)^j f^{MP} \\
&= \frac{1}{c} f^{MP}
\end{aligned}
$$

Hence, the exchange rate is expressed as

$$
e_t = \frac{a}{c(a+c)} f^{MP} + \frac{1}{a+c} f_t \tag{10}
$$

The choice f_t by the Markov-Perfect policy solves

$$
\max_{f_t} -U(e_t, f_t) + \sum_{s=t+1}^{\infty} \beta^{s-t} \left\{ -U \left(\frac{1}{c} f^{MP}, f^{MP} \right) \right\}
$$

subject to (10). The solution to this problem is f^{MP} and the Markov-Perfect policy is $\vec{f}^{MP} = \{f_t^{MP}\}$ such that $f_t^{MP} = f^{MP}$. The associated equilibrium outcome can be backed out from (5), (6) and (10). The details of this problem is given in Appendix B.

Definition 4. *The Markov-Perfect policy for reserve management is* $\vec{f}^{MP}$.

3.3 Comparison between Ramsey policy and Markov-Perfect policy

This section presents numerical results for the Ramsey and the Markov-Perfect policies. The parameter values are summarized in Table 1. Most of the parameters are taken from Basu et al. (2018). I set b to 5 and $\bar{e}$ to 0.5 in order to have strong enough welfare trade-offs. I normalize the initial values of k_{-1} and R_{-1} to zero.

Figure 1 describes the equilibrium outcomes under the Ramsey policy and the Markov-Perfect policy. Under the Markov-Perfect policy, reserves are accumulated at a constant rate. In each period, the government sets the same accumulation rate that balances out the trade-offs between fear of floating and the cost of reserve management. Accordingly, the exchange rate is constant over time, resulting in no changes in capital flows as shown by the constant current account. In contrast, under the Ramsey policy, the government resolves the trade-offs through the expectation channel of exchange rate. The government gradually increases the pace of reserve accumulation in the first 10 periods. In this acceleration phase, the exchange rate depreciates until it reaches the bliss point. The deprecation is not solely due to the contemporaneous reserve accumulation. The future expected accumulation also affects the exchange rates. The government

Parameter	Value	Description	Source
a	0.8	Responsiveness of capital flow to exchange rate	
c	0.15	Responsiveness of current account to exchange rate	Basu et al. (2018)
β	$a/(a+c)$	Discount factor	
b	5	Cost of FX reserve management	
$\bar{e}$	0.5	Bliss point of exchange rate	
T^S	4	Period parameter in $\vec{f}^S$	
$\bar{f}^S$	0.01	Accumulation parameter in $\vec{f}^S$	

Table 1: Parameter values

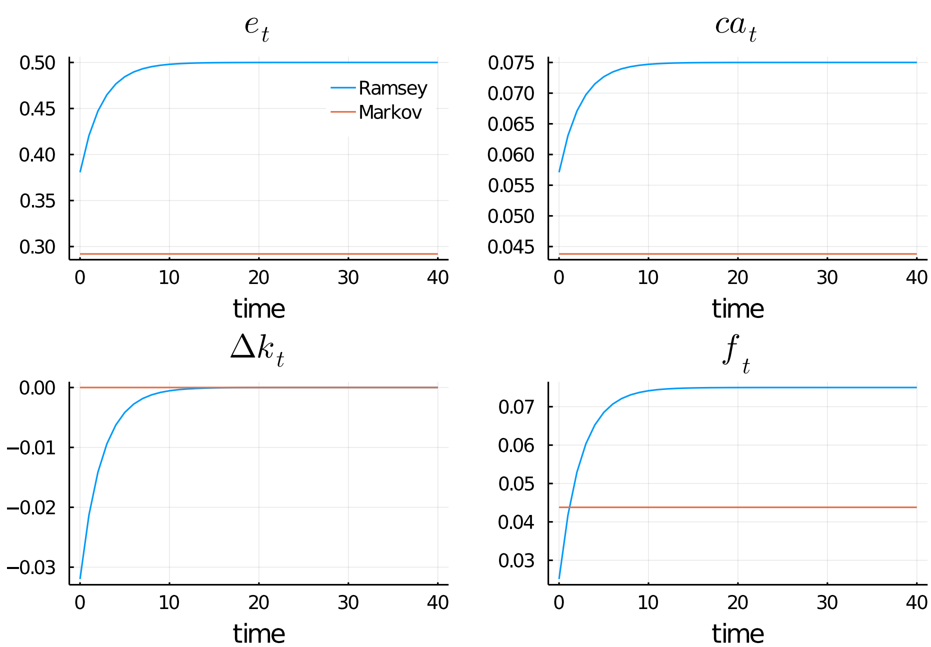

Figure 1: Equilibria under Ramsey and Markov-Perfect policies

credibly makes a promise about aggressive reserve accumulations in the future, letting the current exchange rate depreciate closer to the bliss level. After reaching the bliss level, the Ramsey policy keeps the accumulation at a constant rate to support this level of exchange rate.

Figure 2 describes the equilibrium levels of capital and reserve stocks under the two policies. Under the Markov-Perfect policy, the level of capital stays at the initial value. Under the Ramsey policy, the economy experiences capital outflows in the first 10 periods, resulting in lower level of capital stock. This is again due to the expected future exchange rate deprecations. Since the government gradually increases the pace of reserve accumulation under the Ramsey policy, the reserve stock is lower in the first few periods. Eventually, more reserves are accumulated under the Ramsey policy than the Markov-Perfect policy.

The Ramsey policy achieves higher welfare than the Markov-Perfect policy. Figure 3 shows that the overall welfare evaluated at time 0 is greater under the Ramsey policy (-0.06) than the Markov-Perfect policy (-0.167). The figure also illustrates that the welfare level is higher under the Ramsey policy along the

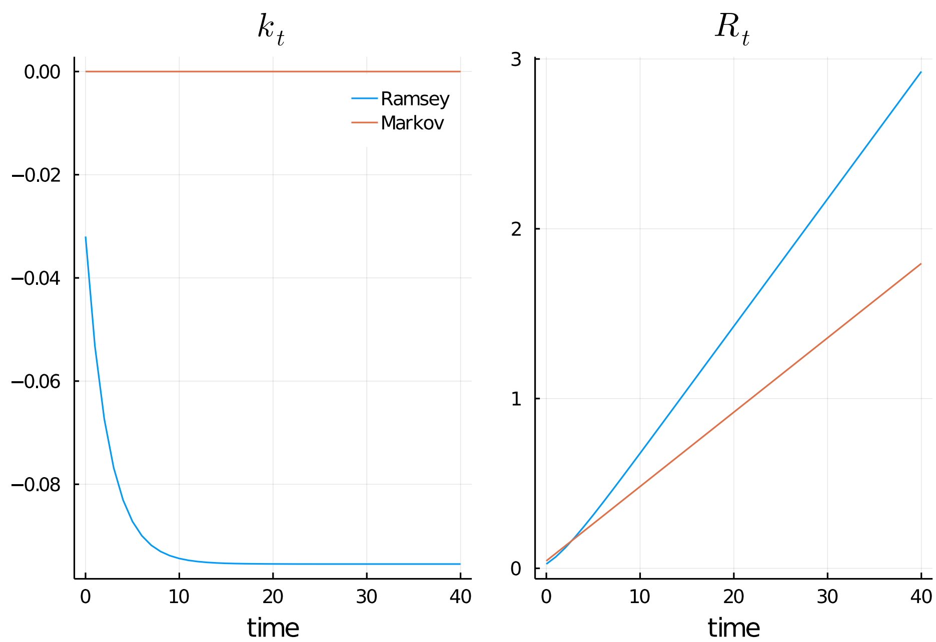

Figure 2: Capital flows and FX reserves under Ramsey and Markov-Perfect policies

time path. The overall welfare difference between the two policies is due to the availability of commitment device. The economy is better off if the government is able to make a credible promise about its reserve management.

Result 1. *The Ramsey policy $\vec{f}^R$ brings greater welfare than the Markov-Policy $\vec{f}^{MP}$.*

The above results leave several questions. What is the best equilibrium outcome for the government without commitment? Is it possible to achieve higher welfare than the one under the Markov-Perfect policy? If so, how? The next section provides a way to implement the Ramsey outcome for the government without commitment.

4 Sustainable plan for reserve accumulation

Without commitment, it is difficult for the government to implement the Ramsey policy. This is because the Ramsey policy is not time-consistent, and hence, the government faces deviation incentives from the policy. To see this, notice that there is a temporary gain from avoiding the cost of reserve management. The government gets higher one-period welfare

$$-r(x_t^R, 0) \geq -r(x_t^R, f_t^R)$$

if it sets $f_t = 0$ and deviates from the Ramsey policy f_t^R in period t. To achieve the Ramsey outcome, one must come up with a contingency plan that specifies not only the policy but also the punishment to deviations that might occur.

Definition 5. *A plan $\vec{f}$ is sustainable if it prevents any deviations from the plan.*

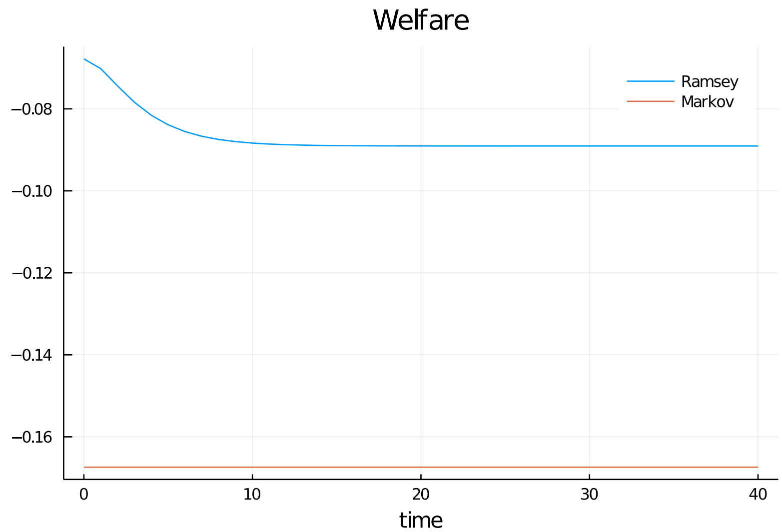

Figure 3: Welfare under Ramsey and Markov-Perfect policies

One way to construct the punishment scheme for a sustainable plan is to use a self-enforcing plan defined below.

Definition 6. *A plan $\vec{f}$ is self-enforcing if it deters any deviations by restarting the plan in period $j+1$ when there is a deviation in period $j \geq 0$.*

Consider a plan $\vec{f}^S$ that sets f_t^S at a low constant rate $\bar{f}^S$ for the first T^S periods and then switches to Ramsey policy $\vec{f}^R$ afterwards. The plan restarts itself if there is a deviation in any periods. Notice that this plan is self-enforcing and deters any deviations with $T^S = 4$ and $\bar{f}^S = 0.01$ as shown in Figure 4. Under $\vec{f}^S$, reserves are accumulated at a constant low rate $\bar{f}^S$ for the first four periods. After that, the government switches to $\vec{f}^R$. Compared to the Ramsey policy, the economy attains lower level of welfare (-0.12) evaluated at time 0. Importantly, unlike the Ramsey policy, this plan does not suffer from the time-inconsistency issue. The figure shows that the government cannot find any profitable deviations from the plan. Thus, the plan $\vec{f}^S$ successfully prevents deviation incentives by rewarding the government with high welfare associated with the Ramsey outcome only if it endures low welfare in the first four periods.

Result 2. *The plan $\vec{f}^S$ is self-enforcing.*

Now I construct a sustainable plan that delivers the Ramsey outcome in equilibrium using the self-enforcing plan $\vec{f}^S$ as a punishment. Specifically, the suggested plan, denoted by $\vec{f}^{RS}$, follows the Ramsey policy $\vec{f}^R$ until there is any deviation. When there is a deviation, the plan switches to $\vec{f}^S$ permanently. As long as this permanent punishment of $\vec{f}^S$ is strong, $\vec{f}^{RS}$ is sustainable. In other words, $\vec{f}^{RS}$ is sustainable if

$$v_j^R \geq -r(x_j^R, 0) + \beta v_0^S \tag{11}$$

for period $j \geq 0$. The left-hand side of (11) is the welfare associated with the Ramsey outcome in period j

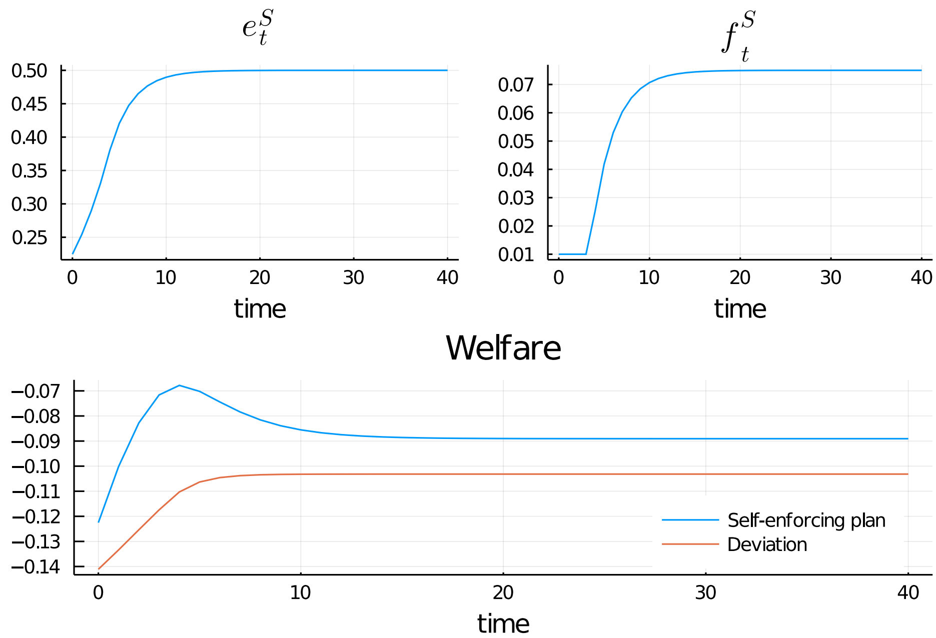

Figure 4: Self-enforcing plan

that the government can achieve following $\vec{f}^{RS}$

$$v_j^R \equiv \sum_{t=j}^{\infty} \beta^t \left\{-r(x_t^R, f_t^R)\right\}$$

The right-hand side of (11) is the sum of the temporary gain from deviating to $f_t = 0$ and the welfare from $\vec{f}^S$ that the government receives after the deviation

$$v_0^S \equiv \sum_{t=0}^{\infty} \beta^t \left\{-U(e_t^S, f_t^S)\right\}$$

Figure 5 shows that the suggested plan $\vec{f}^{RS}$ is sustainable. The welfare from $\vec{f}^{RS}$ is the same as the one from $\vec{f}^R$ shown in Figure 3. This is because $\vec{f}^{RS}$ achieves the Ramsey outcome on the equilibrium path without any deviations. Along the time path, the welfare from $\vec{f}^{RS}$ is greater than the one from deviations.[5] The permanent punishment with $\vec{f}^S$ is so strong that the government does not face any deviation temptations. Therefore, following plan $\vec{f}^{RS}$, the government without commitment achieves the Ramsey outcome in equilibrium.

Result 3. *The suggested plan $\vec{f}^{RS}$ is sustainable. Following this plan, the government can achieve the Ramsey outcome even without commitment.*

[5] The deviation is most profitable when the exchange rate is equal to the bliss level.

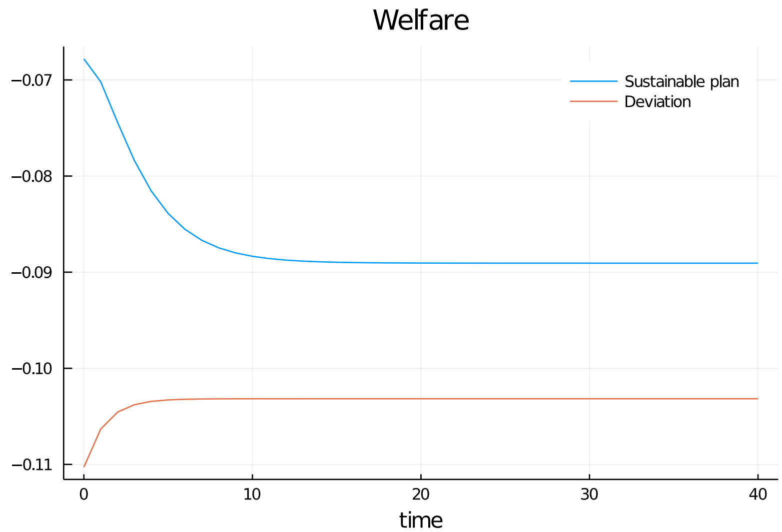

Figure 5: Welfare from the sustainable plan

5 Conclusion

FX reserve has been one of the important policy tools for the central banks in EMDEs. How to optimally manage the stock of FX reserves is still up for debate. Managing reserve is a challenging task, particularly for EMDEs with credibility issues in policy making.

This paper studies optimal policy for FX reserve management, emphasizing the role of commitment. It shows how the power of commitment can help address policy trade-offs arising from fear of floating and the management cost of reserves. With commitment, the government attains the first-best Ramsey outcome by making a credible promise about the future course of policy. The Ramsey policy gives rise to adjustments of exchange rates toward the desirable level through the expectation channel. Without commitment, however, the government policy is not credible. The Markov-Perfect policy only balances out the trade-offs period by period. The paper further provides a way to implement the Ramsey outcome in equilibrium for the government without commitment. A sustainable plan for reserve management is constructed with a strong punishment to deviations from the plan. Following this plan, the government implements the Ramsey outcome without suffering from time-inconsistency.

References

Abreu, Dilip, David Pearce, and Ennio Stacchetti (1990) "Toward a theory of discounted repeated games with imperfect monitoring," *Econometrica: Journal of the Econometric Society*, pp. 1041–1063.

Arce, Fernando, Julien Bengui, and Javier Bianchi (2019) "A Macroprudential Theory of Foreign Reserve Accumulation," Working paper.

Barro, Robert J and David B Gordon (1983) "Rules, discretion and reputation in a model of monetary policy," *Journal of monetary economics*, Vol. 12, No. 1, pp. 101–121.

Basu, Suman S, Atish R Ghosh, Jonathan D Ostry, and Pablo E Winant (2018) "Managing capital outflows with limited reserves," *IMF Economic Review*, Vol. 66, No. 2, pp. 333–374.

Bianchi, Javier and Guido Lorenzoni (2021) "The Prudential Use of Capital Controls and Foreign Currency Reserves,"Technical report, National Bureau of Economic Research.

Calvo, Guillermo A (1978) "On the time consistency of optimal policy in a monetary economy," *Econometrica: Journal of the Econometric Society*, pp. 1411–1428.

Calvo, Guillermo A and Carmen M Reinhart (2002) "Fear of floating," *The Quarterly journal of economics*, Vol. 117, No. 2, pp. 379–408.

Céspedes, Luis Felipe and Roberto Chang (2020) "Optimal foreign reserves and central bank policy under financial stress,"Technical report, National Bureau of Economic Research.

Chang, Roberto (1998) "Credible monetary policy in an infinite horizon model: Recursive approaches," *journal of economic theory*, Vol. 81, No. 2, pp. 431–461.

Chari, Varadarajan V and Patrick J Kehoe (1990) "Sustainable plans," *Journal of political economy*, Vol. 98, No. 4, pp. 783–802.

Evans, David and Thomas J Sargent (2013) "History-Dependent Public Policies," *Macroeconomics at the Service of Public Policy*, p. 120.

Gabaix, Xavier and Matteo Maggiori (2015) "International liquidity and exchange rate dynamics," *The Quarterly Journal of Economics*, Vol. 130, No. 3, pp. 1369–1420.

Ghosh, Atish R, Jonathan D Ostry, and Marcos Chamon (2016) "Two targets, two instruments: Monetary and exchange rate policies in emerging market economies," *Journal of International Money and Finance*, Vol. 60, pp. 172–196.

International Monetary Fund (2011) "Assessing Reserve Adequacy," IMF Policy Paper, (Washington: International Monetary Fund).

——— (2013) "Assessing Reserve Adequacy-Further Considerations," IMF Policy Paper, (Washington: International Monetary Fund).

Jeanne, Olivier and Romain Rancière (2011) "The Optimal Level of International Reserves For Emerging Market Countries: a New Formula and Some Applications," *The Economic Journal*, Vol. 121, No. 555, pp. 905–930.

Kydland, Finn E and Edward C Prescott (1980) "Dynamic optimal taxation, rational expectations and optimal control," *Journal of Economic Dynamics and control*, Vol. 2, pp. 79–91.

Ljungqvist, Lars and Thomas J Sargent (2018) *Recursive macroeconomic theory*: MIT press.

Phelan, Christopher and Ennio Stacchetti (2001) "Sequential equilibria in a Ramsey tax model," *Econometrica*, Vol. 69, No. 6, pp. 1491–1518.

Stokey, Nancy L (1989) "Reputation and time consistency," *The American Economic Review*, Vol. 79, No. 2, pp. 134–139.

——— (1991) "Credible public policy," *Journal of Economic dynamics and control*, Vol. 15, No. 4, pp. 627–656.

Appendix

A Ramsey problem

Since the Ramsey problem is linear-quadratic, guessing that $J(x) = -x'Px$ and substituting into the Bellman equation gives rise to the algebraic matrix Riccati equation

$$P = R + \beta A'PA - \beta^2 A'PB\left(Q + \beta B'PB\right)^{-1} B'PA$$

and the optimal decision rule

$$f_t = -Fx_t \tag{12}$$

where

$$F = \beta\left(Q + \beta B'PB\right)^{-1} B'PA$$

Hence, the value function $J(x_0)$ is

$$J\left(x_0\right) = -\begin{bmatrix} 1 & e_0 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 1 \\ e_0 \end{bmatrix} = -P_{11} - 2P_{21}e_0 - P_{22}e_0^2$$

Maximizing this with respect to e_0 yields the FOC:

$$-2P_{21} - 2P_{22}e_0 = 0$$

which implies

$$e_0^* = -\frac{P_{21}}{P_{22}}$$

Hence, the Ramsey plan $\{\vec{x}^R, \vec{f}^R\}$ is derived from (8) and (12) with $\vec{x_0}^R = [1 \quad e_0^*]'$.

B Markov-Perfect policy

The first-order condition from the Markov-Perfect government problem gives

$$\frac{-1}{a+c}\left(\frac{a}{c(a+c)}f^{MP} + \frac{1}{a+c}f_t - \bar{e}\right) - bf_t = 0$$

Imposing $f_t = f^{MP}$ and rearranging,

$$f^{MP} = \left(\frac{c}{1 + bc(a+c)}\right)\bar{e}$$

and the associated exchange rate is $e_t^{MP} = e^{MP}$ where

$$e^{MP} = \left(\frac{1}{1 + bc(a + c)}\right)\bar{e}$$