UNIT-IV
TRANSFORMER

The transformer is a static device that transfers electrical energy from one electrical circuit to another electrical circuit through the medium of magnetic field & without a change in the frequency. The electric circuit which receives energy from the supply mains is called primary winding & the other circuit which delivers electric energy to the load is called the secondary winding.

The primary & secondary windings of a transformer are not connected electrically, but are coupled magnetically. This coupling magnetic field allows the transfer of energy in either direction, from high voltage to low voltage circuit or from low voltage to high voltage circuit.

If the transfer of energy occurs at the same voltage, the purpose of transformer is merely to isolate the two electric circuits. So the transformer called as isolated transformer. This is used in very rare in power application.

If the secondary winding ~~has~~ has more turns than the primary winding, then the secondary voltage is higher than the primary voltage and the transformer is called a step up transformer. In case secondary winding has less turns than primary, then the secondary voltage is lower than primary voltage & the transformer is called as step down transformer.

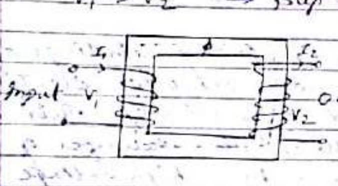
The same transformer can be used

as step up transformer as well as step down transformer.

$V_1 = V_2 \rightarrow$ Isolated Transformer.

$V_1 < V_2 \rightarrow$ Step up Transformer.

$V_1 > V_2 \rightarrow$ Step down Transformer.



A transformer is the most widely used device in both low & high current circuits. As such, transformers are built in an amazing ranges of sizes. In electronic circuit Transformer sizes may be so small that it weighs only a few tens of grams whereas in high voltage power circuits, it may weigh hundreds of tonnes.

Important task performed by the transformer

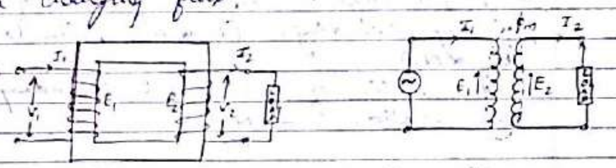
i) For decreasing & increasing voltage & current levels from one circuit to the another circuit.

ii) For matching the impedance of a source & its load for maximum power transfer in electronic & control circuits.

iii) For isolating one circuit from another.

Principle of Working of a Transformer

A transformer works on the principle of Electromagnetic Induction between two (or more) coupled circuits or coils. According to this principle, an emf is induced in a coil if it links a changing flux.



In the figure a two winding transformer is shown. The winding to which ac supply is connected is called primary winding & the winding to which load is connected is called secondary winding.

When ac supply of voltage V_1 is connected to primary, an alternating flux is set up in the core. This alternating flux links with the secondary winding & an emf is induced in it called mutually induced emf. The direction of this induced emf is opposite to the applied voltage According to Lenz's law.

The same flux also links with primary & produces self induced emf E_1. This emf E_1 also acts in opposite direction.

Although there is no electrical connection between primary & secondary winding, but electrical power is transferred from primary to secondary ckts through mutual flux.

The induced emf in the primary &

an secondary depends upon the rate of change of flux linkages (ie $N\frac{d\phi}{dt}$). The rate of change of flux ($\frac{d\phi}{dt}$) is same for both the windings.

Therefore, the induced emf in primary is proportional to number of turns of the primary windings ($E_1 \propto N_1$), Similarly, ($E_2 \propto N_2$)

The ratio of primary & secondary turns called turn ratio. $= \frac{N_1}{N_2}$

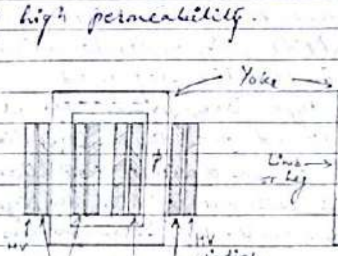
The ratio of secondary voltage to primary voltage is called Voltage transformation ratio of transformer. It is represented by K.

$$\boxed{K = \frac{E_2}{E_1} = \frac{N_2}{N_1}} \quad \left(\because E_2 \propto N_2 \ \& \ E_1 \propto N_1\right)$$

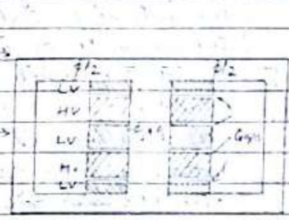
TRANSFORMER CONSTRUCTION

There are two types of transformers, the core type & the shell type. These two types differ from each other by the manner in which the windings are wound around the magnetic core.

The magnetic core is a stack of thin silicon steel laminations about 0.35 mm thick for 50 Hz transformer. In order to reduce the eddy current losses, these laminations are insulated from one another by thin layers of varnish, for reducing the core losses. Nearly all transformers have their magnetic core made from cold rolled grain oriented sheet steel (CRGO). This It has low core losses & high permeability.



Core type transformer



Shell type transformer.

In the core type, the windings surround a considerable part of steel core. In the shell type, the steel core surrounds a major part of the windings. For a given output & voltage rating, core type transformer require less iron but more conductor material as compared to a shell type transformer. The vertical portions of the core usually called limbs or legs & the top & bottom portions are called the yoke.

In iron core transformers, most of the flux is confined to high permeability core. There is however, some flux that leaks through the core legs and non-magnetic material surroundings the core. This flux is called leakage flux, links one winding & not the other. To reduce this leakage of flux in core type, we place half of the low voltage winding over one leg & other half over another leg & for HV winding also half of the winding is over one leg & other half over the second leg. LV winding is placed adjacent to the steel core & HV winding outside, in order to minimise the amount of insulation required.

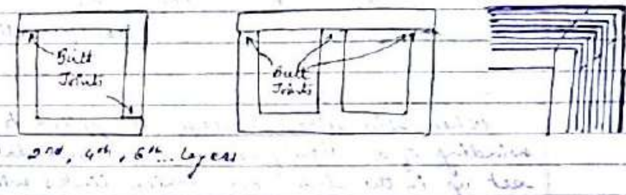
In the shell type transformer, the LV & HV windings are wound over the central limb and are interleaved or sandwiched. Note that the bottom and top LV coils are of half the size of other LV coils. Shell-type transformers are preferred for low-voltage, low power levels, whereas core type construction is used for high voltage, high power transformers.

In core type transformer, the flux has a single path around the legs or yokes. In the shell type transformer, the flux in the central limb divides equally and returns through the outer two legs.

The same type of laminations for the core and shell types of transformer is used here. In the figure the steel core is assembled in such a manner that the butt joints in adjacent layers are staggered as shown in figure. The staggering of the butt joints avoids continuous air gap. Therefore, the reluctance of the magnetic circuit is not increased.

Low power transformer are air cooled whereas large power transformers are ~~immersed~~ immersed in oil for better cooling. In oil cooled transformers, the oil serves as a coolant & also as an insulating medium.

1^{st}, 3^{rd}, 5^{th} ... layers



2^{nd}, 4^{th}, 6^{th} ... layers

Ideal Two Winding Transformer

For an ideal transformer, the various assumptions are as follows

1) Winding resistances are negligible.
2) All the flux set up by the primary, links the secondary windings i.e all the flux is confined to the magnetic core.
3) The core losses (hysteresis & eddy current losses) are negligible.
4) The core has constant permeability i.e the magnetization curve for the core is linear.

In actual practice, it is impossible to realize such a transformer (ideal).

In an ideal transformer there is no power loss. Therefore, output must be equal to the input.

$$\therefore \quad E_1 I_1 \cos\phi = E_2 I_2 \cos\phi$$

$$\text{or} \quad E_1 I_1 = E_2 I_2$$

or $\frac{E_1}{E_2} = \frac{I_2}{I_1}$

Since, $E_2 \propto N_2$ & $E_1 \propto N_1$

& $E_1 \cong V_1$ & $E_2 \cong V_2$

$\therefore \quad \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K$

(transformation ratio)

EMF EQUATION :-

When sinusoidal voltage is applied to the primary winding of a transformer, a sinusoidal flux is set up in the iron core which links with primary & secondary winding.

Let ϕ_m = Maximum value of flux

f = supply frequency

N_1 = No. of turns in primary

N_2 = No. of turns in secondary

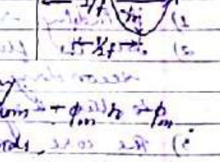


Fig shows the flux changes from $+\phi_m$ to $-\phi_m$ in half a cycle i.e. $\frac{1}{2f}$ seconds.

$\therefore$ Average rate of change of flux $= \frac{\phi_m - (-\phi_m)}{1/2f}$

$= 4f\phi_m$ wb/s.

Now, the rate of change of flux per turns is the average induced emf per turn in volts.

$\therefore$ Average emf induced/turn $= 4f\phi_m$ volts.

For sinusoidal wave, $\frac{\text{RMS Value}}{\text{Average Value}}$ = form factor = 1.11

$\therefore$ RMS value of emf induced/turns, $E = 1.11 \times 4f\phi_m$

$= 4.44 f\phi_m$ Volts.

Since primary & secondary have N_1 & N_2 turns resp.

$\therefore$ RMS value of emf induced in primary.

E_1 = (emf induced/turn) x No. of primary turns

$= 4.44\, N_1 f \phi_m$ Volts. --- (i)

Similarly, rms value of emf induced in secondary

$E_2 = 4.44\, N_2 f \phi_m$ Volts. --- (ii)

Again we can find the voltage ratio,

$\frac{E_2}{E_1} = \frac{4.44\, N_2 f \phi_m}{4.44\, N_1 f \phi_m} = \frac{N_2}{N_1} = K$ (transformation ratio)

Eqn (i) & (ii) can be rewritten as

$E_1 = 4.44\, N_1 f B_m A$ Volts

$E_2 = 4.44\, N_2 f B_m A$ Volts.

$[\because \phi_m = B_m A]$

Ques: A 25 KVA transformer has 500 turns on the primary & 40 turns on the secondary winding. The primary is connected to 3000 V, 50 Hz mains, calculate

(i) Primary and secondary currents in full load.

(ii) Secondary emf

(iii) The maximum flux in the core.

Neglect magnetic leakage, resistance of the wdg & the primary no load current in relation to the full load current.

Solⁿ:- (i) At full load, $I_1 = \frac{25 \times 10^3}{3000} = 8.33$ A

Now $\frac{I_1}{I_2} = \frac{E_2}{E_1} = \frac{N_2}{N_1}$

Secondary current, $I_2 = \frac{N_1 I_1}{N_2} = \frac{500 \times 8.33}{40}$

$$I_2 = 104.18\,A$$

ii) Secondary emf, $E_2 = \frac{N_2}{N_1} E_1$

$$= \frac{40}{500} \times 3000 = 240\,V$$

iii) Using relation, $E_1 = 4.44\, N_1 f \phi_m$

$$3000 = 4.44 \times 500 \times 50 \times \phi_m$$

$$\phi_m = 0.027\ Wb$$

Transformer on DC

A transformer cannot work on dc supply. If a rated dc voltage is applied across the primary, a flux of constant magnitude will be set up in the core. Hence, no self induced emf in the primary winding to oppose the applied voltage. The resistance of the primary winding is very low, so that primary current is quite high.

ie Primary current $= \frac{\text{dc applied voltage}}{\text{resistance of primary winding}}$

This current is much more than the rated full load current of primary winding. This it will produce the lot of heat (I^2R) loss & burns the insulation of the primary winding & the transformer will damaged.

That's why dc is never applied to transformers.

Transformer On No Load:-

The transformer on no load is ~~said~~ means the transformer having its secondary winding open circuited & the secondary current I_2 is zero. Neither the secondary winding has any effect on the magnetic flux in the core nor it has any effect on the primary current.

In actual transformer losses cannot be neglected. Therefore, if transformer is on no load, a small current I_e (usually 2 to 10% of the rated value) called exciting current is drawn by primary. This current has to supply ~~for~~ in iron losses (hysteresis losses & eddy current losses) in the core & very small amount of copper losses in the ~~core~~ primary winding. (the copper loss is too small as compared to iron loss, so we neglect it & copper loss is zero in secondary due to $I_2 = 0$.)

Therefore current I_e lags behind the voltage vector V_1 by an angle ϕ_e (called angle of hysteresis advance) which is less than 90°.

Angle of lag depends upon the losses in transformer.

The no load exciting current has two components.

i) One, I_w in phase with the applied voltage V_1, called active or working component. It supplies the iron losses & a small primary copper loss.

$$\boxed{I_w = I_e \cos\phi_0}$$

ii) The other, I_m in quadrature with the applied voltage V_1, called reactive or magnetising component. It produces flux in the core & does not consume any power.

$$\boxed{I_m = I_e \sin\phi_0}$$

Its function is to sustain the alternating flux in the core. It is wattless.

Now, I_e is the vector sum of I_w & I_m

$\therefore$ $I_e = \sqrt{I_w^2 + I_m^2}$

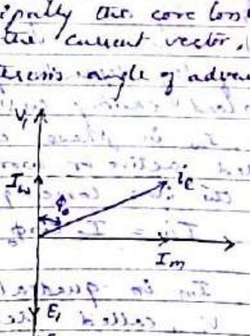
Important Points.

1) The no-load primary current I_e is very small as compared to the full load primary current. It is about 2% of the full load current.

2) The permeability of the core varies with the instantaneous value of the exciting current, the wave of ~~of~~ the exciting or magnetising current is not truly sinusoidal.

3) As I_e is very small, the no load primary Cu loss is negligibly small which means that no load primary input is practically equal to the iron loss in the transformer.

4) As it is principally the core loss which is responsible for shift in the current vector, angle ϕ_0 is known as hysteresis angle of advance.



Ques i) A 1-ϕ ~~ac~~ 2200/200 V transformer draws a no load primary current of 0.6A & absorbs 400 watts. Find the magnetising and iron loss currents.

ii) A 2200/250 V transformer takes 0.5A at a p.f. of 0.3 on o/c. Find magnetising & working component of no load primary current.

Sol^n (a) Iron-loss current $= \dfrac{\text{No-load input in watts}}{\text{Primary voltage}}$

$= \dfrac{400}{2200} = 0.182A$

Now, $I_e^2 = I_w^2 + I_m^2$

Magnetising component, $I_m = \sqrt{(0.6)^2 - (0.182)^2}$

$= 0.572A$

(b) $I_e = 0.5A$; $\cos\phi_0 = 0.3$

$I_w = I_e \cos\phi_0$

$= 0.5 \times 0.3$

$= 0.15A$

$I_m = \sqrt{(0.5)^2 - (0.15)^2} = 0.476A$

Ques A single phase transformer has 1000 turns on the primary and 200 turns on the secondary. The no load current is 3 amp. at a P.f. of 0.2 lagging. Calculate the primary current and P.f. when the secondary current is 280 amp. at a ~~p.f.~~ Power factor of 0.80 lagging.

Sol^n V_2 is taken as reference.

$\cos^{-1} 0.80 = 36.87°$

$I_2 = 280 \angle -36.87°$ A

turns ratio = $\frac{200}{1000} = \frac{1}{5}$

$$\therefore \frac{I_1}{I_2} = \frac{1}{5}$$

$$I_2' = \frac{I_2}{5}$$

$$I_2' = \frac{280}{5} \angle -36.87^\circ \text{ A}$$

$$= 56 \angle -36.87^\circ \text{ A}$$

$$\phi = \cos^{-1} 0.20$$
$$= 78.5^\circ$$
$$\sin\phi = 0.98$$

$$I_1 = I_e - I_2'$$
$$= 3(\cos\phi - j\sin\phi) + 56(\cos 36.87 - j\sin 36.87^\circ)$$
$$= 3[0.20 - j0.98] + 56[0.80 - j0.60]$$
$$= 0.6 - j2.94 + 44.8 - j33.6$$
$$= 45.4 - j2.94 + 44.8 - j33.6$$
$$= 45.4 - j36.54 = 58.3 \angle 38.86^\circ$$

Thus I lags behind the supply voltage by 38.86°.

Transformer On Load:

~~When the secondary is loaded, the sec. current I_2 is set up.~~

Transformer On Load :-

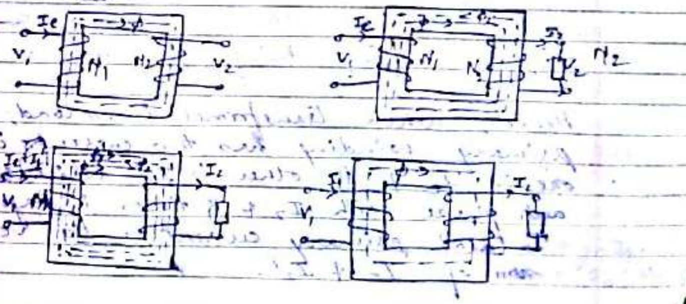
Firstly, the secondary is open circuited, so that current I_e (I_0) flows in the primary due to which flux ϕ is ~~flowing the~~ sets up in the core.

[Now, when secondary is loaded, the sec. current I_2 is set up. The magnitude and phase of I_2 with respect to V_2 is determined by the characteristic of load. Current I_2 is in phase with V_2 if load is resistive, it lags if load is inductive & leads if load is capacitive].

The secondary current sets up its own mmf ($= N_2 I_2$) & hence its own flux ϕ_2 which is in opposition to the main primary flux ϕ which is due to I_e. The sec. Amp turns $N_2 I_2$ are known as Demagnetising amp. turns or mmf.

[The opposing flux ϕ_2 weakens the primary flux ϕ momentarily, hence primary back emf E_1 tends to be reduced. For a moment V_1 gains the upper hand over E_1 & hence causes more current to flow in primary.

Let



Let the additional pri. current be I_2'. It is known as <u>load component of pri. current</u>. This current is antiphase with I_2. The additional primary mmf $N_1 I_2'$ sets up its own flux ϕ_2' which is in opposition to ϕ_2 (but in the same direction as ϕ) and is equal to it in magnitude. Hence, the two cancel out each other! So we find that the magnetic effects of secondary current I_2 are immediately neutralized by the additional primary current I_2' which is brought into existence exactly at the same instant as I_2.

Hence, whatever the load conditions, the net flux passing through the core is approximately the same as at no-load. Due to constancy of core flux at all loads the core loss is also practically the same under all load conditions.

As $\phi_2 = \phi_2'$

$$\therefore N_2 I_2 = N_1 I_2'$$

$$\therefore I_2' = \frac{N_2}{N_1} I_2$$

$$\boxed{I_2' = k I_2}$$

Hence, when transformer is on load, the primary winding has two currents in it, one is I_e & the other is I_2' which is anti phase with I_2 & K times in magnitude. Total primary current is the vector sum of I_e & I_2'.

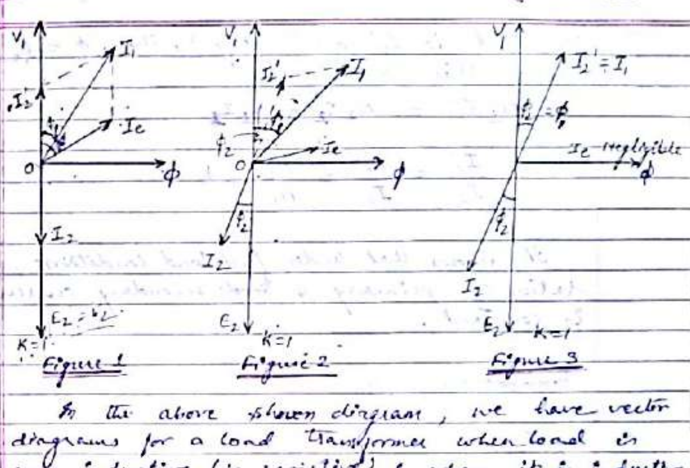


Figure 1 Figure 2 Figure 3

In the above shown diagram, we have vector diagrams for a load transformer when load is non inductive (ie resistive) & when it is inductive or capacitive (similar diagrams are drawn for inductive & capacitive load). The voltage transformation ratio is assumed to be unity so that primary vectors are equal to the secondary vectors.

In figure 1, I_2 is sec. current in phase with E_2 ($E_2 = V_2$). It causes primary current I_2' which is anti phase with it and equal to it in magnitude ($K=1$). Total primary current I_1 is the vector sum of I_e & I_2' and lags behind V_1 by an angle ϕ_1.

In fig 2 vectors are drawn for inductive load. Here I_2 lags E_2 by ϕ_2. Current I_2' is again antiphase with I_2 & equal to it in magnitude. Also I_1 is the vector ~~same~~ sum of I_2' & I_e and lags behind V_1 by ϕ_1.

It will be observed that ϕ_1 is slightly greater than ϕ_2. But if we neglect I_e as

EQUIVALENT CIRCUIT

In transformer, the problem of concerning voltage & currents can be solved by the use of phase diagrams. However, it is more convenient to represent the transformer by an equivalent circuit. ~~If~~ by an equivalent circuit calculation is ~~with~~ so easy & can be done by circuit theory. An equivalent circuit is merely a circuit interpretation of the equations which describes the behaviour of the device.

The resistances & leakage reactances of the primary & secondary are shown separately in the primary as well as in secondary. circuits are connected in parallel across the wdg. The effect of core loss is represented by a non inductive resistance R_0. ~~inductive reactance~~

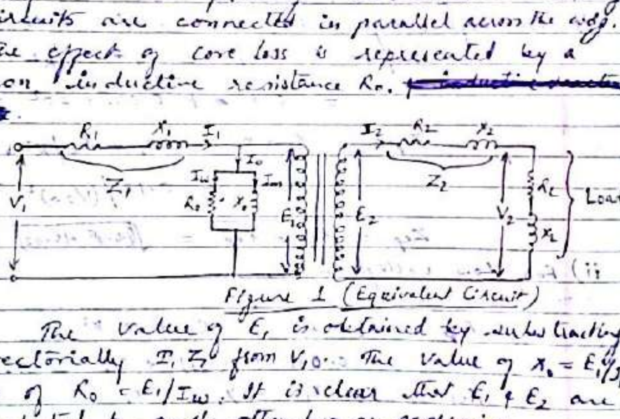


Figure 1 (Equivalent Circuit)

The value of E_1 is obtained by subtracting vectorially $I_1 Z_1$ from V_1. The value of $X_0 = E_1/I_m$ & of $R_0 = E_1/I_w$. It is clear that E_1 & E_2 are related to each other by an expression.

$$E_2/E_1 = N_2/N_1 = K$$

To make transformer calculations simpler, it is preferable to transfer voltage, current & impedance either to the primary or to the secondary. In that case we would have to work in only one winding which is more convenient.

The pri. eqvt. of the sec. induced voltage is $E_2' = E_2/K = E_1$

Similarly, The pri. —— " —— terminal or output voltage $V_2' = V_2/K$

Pri. eqvt of the sec. current is $I_2' = K I_2$

For transferring sec. impedance to pri. K^2 is used.

$$R_2' = R_2/K^2, \quad X_2' = X_2/K^2, \quad Z_2' = Z_2/K^2$$

The same relationship is used for shifting an external load impedance to the primary.

$$Z_L' = Z_L/K^2$$

When all the secondary impedances have been transferred to the primary side, the ~~only~~ sec. wdg need not be shown in the eqvt. ckt. The exact eqvt. ckt of the transformer with the impedances transferred to the primary is shown below.

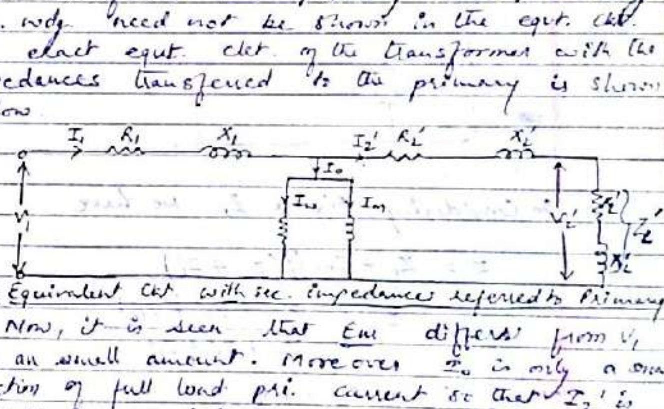
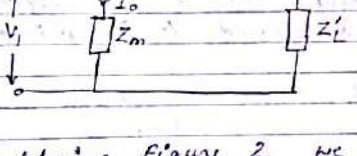


Figure 2 Equivalent Ckt with sec. impedances referred to Primary

Now, it is seen that E_1 differs from V_1 by an small amount. Moreover I_0 is only a small fraction of full load pri. current so that I_2' is practically equal to I_1. Consequently the equivalent circuit can be simplified by transferring the parallel branch consisting R_0 & X_0 to the left position of the circuit. This circuit is approximate equivalent circuit. Analysis with the approximate equivalent circuit gives almost the same result as the analysis with the exact equivalent ckt. However the analysis with the approximate eqvt. ckt. is simple because the resistances R_1 & R_2' and leakage reactances X_1 & X_2' can be combined as R_{ep} & X_{ep} respectively.

Figure 3 Approximate Eqvt. Ckt of Xmer



On Considering Figure 2, we have

$$Z = Z_1 + Z_m \,//\, (Z_2' + Z_L')$$

$$= Z_1 + \frac{Z_m (Z_2' + Z_L')}{Z_m + Z_2' + Z_L'}$$

where, $Z_2' = R_2' + jX_2'$

Z_m = impedance of the exciting circuit.

$$\therefore \boxed{V_1 = I_1 \left[Z_1 + \frac{Z_m (Z_2' + Z_L')}{Z_m + Z_2' + Z_L'} \right]}$$

On considering figure 3, we have

$$Z = (Z_1 + Z_2' + Z_L') \,//\, Z_m$$

$$= \frac{Z_m (Z_1 + Z_2' + Z_L')}{Z_m + Z_1 + Z_2' + Z_L'}$$

$$\therefore \boxed{V_1 = I_1 \left[\frac{Z_m (Z_1 + Z_2' + Z_L')}{Z_m + Z_1 + Z_2' + Z_L'} \right]}$$

PER UNIT SYSTEM

~~In this system~~ The per unit (PU) value of any quantity is defined as the ratio of the actual ~~value~~ value to in any unit & the base or reference value in the same unit.

While carrying out analysis of electrical machines or systems it is usual to express, voltage, current, VA & impedance in per unit (or percentage) of the base or reference value of these quantities. Such a method simplifies the calculations.

$$\text{Per unit Value} = \frac{\text{Actual Value in any unit}}{\text{Base or reference value in the same unit}}$$

While the base values can be selected ~~also~~ arbitrarily; but it is usual to assume the following base values:

Base Voltage (V_B) = Rated Voltage of Machine
Base Current (I_B) = — " — Current
Base impedance (Z_B) = ~~— " — Impedance — " —~~ Base Voltage / Base current
Base Power ($VA)_B$ = Base Voltage X Base current

Always Base Power & Base voltages are first selected & their choice automatically fixes the other base values.

As, $Z_B = \frac{V_B}{I_B} = \frac{V_B \times V_B}{I_B \times V_B} = \frac{V_B^2}{(VA)_B}$

$= \frac{(\text{Base Voltage})^2}{(\text{Base Power})}$

$Z_{PU} = \frac{Z_{actual}}{Z_{Base}}$ Then $\boxed{Z_{(PU)} = \frac{Z_{ac} \times (VA)_B}{V_B^2}}$ where, $Z_{ac} = Z_{actual}$

$\boxed{Z_{(PU)} = \frac{\text{Actual Impedance}}{\text{Base Impedance}}}$

In large devices & systems it is more practical to express the bases in kVA or MVA & kV.

Then, $Z_{(PU)} = \frac{Z_{actual} \times (kVA)_B}{1000 \quad (kV)_B^2}$

Advantages of Pu system

1) Calculations are simplified.
2) The characteristics of machines (generators, transformers, motors, etc) when described in per unit system are specified by almost the same no., regardless of the rating of machine.
3) For circuits connected by Xmers, per unit system is particularly suitable. By choosing suitable base kVs for the circuits the perunit resistances & reactance remains the same, referred to either sides of the transformer.
4) This method is useful to eliminate Ideal Xmer as circuit components.

Drawbacks

1) Some equations that hold in the unscaled cases are modified when scaled in Perunit. Factors such as $\sqrt{3}$ & 3 are removed or introduced by this method.
2) Eqvt. Ckts of the components are modified, making them somewhat more abstracts. Sometimes phase shifts that are clearly present in the unscaled ckt vanish in the Perunit system.

K) VOLTAGE REGULATION

At a constant supply voltage, the change in secondary terminal voltage from no load to full load / with respect to no load Voltage is called Voltage Regulation of the Xmer.

When a transformer is loaded, with a constant supply voltage, the terminal voltage changes depending upon the load & its power factor. The algebraic difference between the no-load & full load terminal voltage is measured in terms of voltage regulation.

Let,
E_2 = Secondary terminal voltage at no load
V_2 = —— // —— full load

Then, Voltage Regulation $= \frac{E_2 - V_2}{E_2}$ (Perunit) compared to I_2' as in fig 3, then $\phi_1 = \phi_2$. Under this assumption.

$$N_2 I_2' = N_2 I_2 = N_1 I_1$$

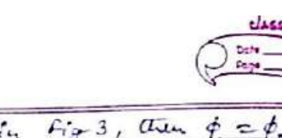
$$\therefore \boxed{\frac{I_2'}{I_2} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = k}$$

It shows that under full load conditions, the ratio of primary & ~~secd~~ secondary currents is constant.

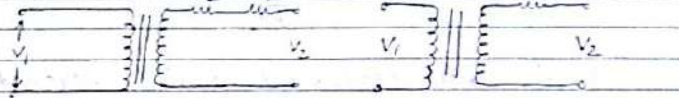
~~Transformer with~~

Equivalent Resistance :-

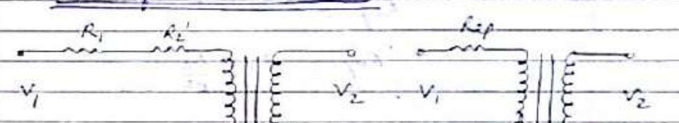
In the actual transformer the pri. & sec. windings have some resistances R_1 & R_2 respectively.



The resistances have been shown in the external windings in the given fig. Due to the ease of calculations, the resistances of the two windings can be transferred to either side. The resistances transferred from one side to other in such a manner that percentage voltage drop remains the same when represented on either side. Let the primary resistance R_1 be ~~referred referred~~ transferred to sec. side and the new value of this resistance be R_1' called equivalent resistance of pri. referred to sec. side as shown in fig. I_1 & I_2 be the full load pri. & sec. currents resp.



R_1 referred to sec. side.



R_2 referred to Pri. side

Then, $\frac{I_2 R_1'}{V_2} \times 100 = \frac{I_1 R_1}{V_1} \times 100$ [~~Stage~~

[Percentage voltage drops]

or $R_1' = \frac{I_1}{I_2} \times \frac{V_2}{V_1} \times R_1$

$= K^2 R_1$ $\left[\because \frac{I_1}{I_2} = \frac{V_2}{V_1} = k\right]$

Total equivalent resistance referred to sec.

$$R_{es} = R_2 + R_1'$$

$$\boxed{R_{es} = R_2 + K^2 R_1}$$

Similarly, R_2 referred to Pri, let its new value be R_2' called equivalent resistance of sec. referred to primary.

Then, $\frac{I_1 R_2'}{V_1} \times 100 = \frac{I_2 R_2}{V_2} \times 100$

$$\text{or}\quad R_2' = \frac{I_2}{I_1} \times \frac{V_1}{V_2} \times R_2$$

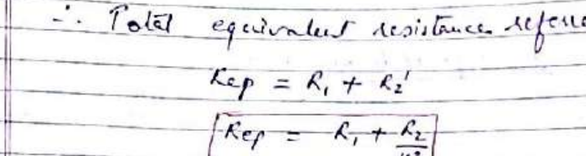
$$\boxed{R_2' = \frac{R_2}{K^2}}$$

∴ Total equivalent resistance referred to Pri.

$$R_{ep} = R_1 + R_2'$$

$$\boxed{R_{ep} = R_1 + \frac{R_2}{K^2}}$$

Equivalent Reactance

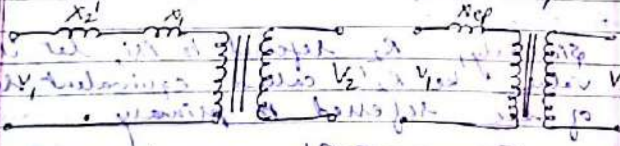


$$\frac{I_2 x_1'}{V_2} \times 100 = \frac{I_1 x_1}{V_1} \times 100 \quad (\%\ \text{voltage drop})$$

$$\text{or}\quad x_1' = \frac{I_1}{I_2} \times \frac{V_2}{V_1} \times x_1 = K^2 x_1$$

∴ Total equivalent reactance referred to secondary

$$x_{es} = x_2 + x_1' = x_2 + K^2 x_1$$



$$\frac{I_1 x_2'}{V_1} \times 100 = \frac{I_2 x_2}{V_2} \times 100$$

$$\therefore\ x_2' = \frac{I_2}{I_1} \times \frac{V_1}{V_2} \times x_2$$

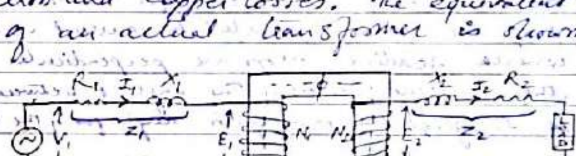
$$= \frac{1}{K^2} \times x_2$$

∴ Total Equivt. reactance referred to Primary.

$$x_{ep} = x_1 + x_2' = x_1 + \frac{1}{K^2} x_2 .$$

Actual Transformer

The transformer with resistances & leaky reactance is called an actual transformer. The actual transformer has pri & sec. resistances R_1 & R_2, pri. & sec reactances, iron and copper losses. The equivalent circuit of an actual transformer is shown in fig.



In the figure primary & secondary windings of a transformer with reactances taken out of the windings are shown. The primary impedance is given by

$$Z_1 = \sqrt{R_1^2 + X_1^2}$$

Similarly, secondary impedance is given by

$$Z_2 = \sqrt{R_2^2 + X_2^2}$$

The resistance & leakage reactance of each

winding is responsible for some voltage drop in each winding. In primary, the leakage reactance drop is $I_1 X_1$ (usually 1 or 2% of V_1).
Hence,

$$V_1 = (-E_1) + I_1(R_1 + jX_1)$$
$$= (E_1) + I_1 Z_1 \quad : V_1' + I_1 Z_1$$

Similarly, in secondary winding

$$E_2 = V_2 + I_2(R_2 + jX_2)$$
$$\underline{E_2} = V_2 + I_2 Z_2$$

The vector diagram for such a transformer for different kinds of load is shown in fig. In these diagrams, vectors for resistive drop are drawn parallel to current vectors whereas reactive drops are perpendicular to the current vectors. The angle ϕ_1 between V_1 & I_1 gives the power factor angle of the transformer.

Resistive **Inductive** **Capacitive**

It may be noted that leakage reactance can also be transferred from one winding to the other in the same way as resistance.

Ques) A 30 KVA, 2400/120 V, 50 Hz transformer has a high voltage winding resistance of 0.1 Ω & a leakage reactance of 0.22 Ω. The low voltage wdg resistance is 0.035 Ω & the leakage reactance is 0.012 Ω. Find the equivalent wdg resistance, reactance & impedance referred to the (i) high voltage side (ii) low voltage side.

Solution:- $K = 120/2400 = 1/20$

$R_1 = 0.1\,\Omega$, $X_1 = 0.22\,\Omega$

$R_2 = 0.035\,\Omega$, $X_2 = 0.012\,\Omega$

i) For high voltage side, it is a pri. side

$$R_{ep} = R_1 + R_2' = R_1 + R_2/K^2$$
$$= 0.1 + 0.035 \times 20^2 = 14.1\,\Omega$$

$$X_{ep} = X_1 + X_2' = X_1 + X_2/K^2$$
$$= 0.22 + 0.12/(1/20)^2 = 5.02\,\Omega$$

$$Z_{ep} = \sqrt{R_{ep}^2 + X_{ep}^2} = \sqrt{(14.1)^2 + (5.02)^2} = 15\,\Omega$$

ii) For Low voltage side,

$$R_{es} = R_2 + R_1' = R_2 + R_1K^2$$
$$= 0.035 + (1/20)^2 \times 0.1 = 0.03525\,\Omega$$

$$X_{es} = X_2 + X_1' = X_2 + K^2X_1$$
$$= 0.012 + (1/20)^2 \times 0.22 = 0.01255\,\Omega$$

$$Z_{es} = \sqrt{(R_{es})^2 + (X_{es})^2} = \sqrt{0.03525^2 + 0.01255^2} = 0.0375\,\Omega$$

$$\left[\text{or } Z_{es} = K^2 Z_{ep}\right]$$

Eddy current Loss :- Since the flux in a Xmer core is alternating, it links with the magnetic material of the core. This induced emf in the core & circulates eddy currents. Power is required to maintain these eddy currents. This power is dissipated in the form of heat & is known as eddy current loss. This can be minimised by making the core of the thin laminations.
Eddy current loss is represented by

$$P_e = K_e V f^2 t^2 B_m^2$$

ii) Copper Losses :- Copper losses occur in both the primary & secondary windings due to their ohmic resistance. If I_1, I_2 are the primary & sec. currents & R_1, R_2 are the pri & sec resistance

Then, Total copper losses $= I_1^2 R_1 + I_2^2 R_2$
$= I_1^2 R_{ep}$
$= I_2^2 R_{es}$

The currents in the pri & sec. wdgs vary according to the load, therefore, these losses vary according to the load & are known as variable losses. These losses vary as the square of load current.

EFFICIENCY OF A TRANSFORMER

The efficiency of a transformer is defined as the ratio of output power to the input power,

$\therefore$ Xmer Efficiency, $\boxed{\eta = \frac{\text{Output Power}}{\text{Input Power}}}$

$$= \frac{\text{Output Power}}{\text{Output Power} + \text{Losses}}$$

$$\boxed{\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + P_c}}$$

V_2 = sec. voltage
I_2 = sec current
$\cos\phi_2$ = Pf of the load.
P_i = Iron loss
P_c = Copper loss (at full load)

If x is the fraction of the full load, the efficiency of the transformer at this fraction is given by the relation.

$$\eta_x = \frac{x \times \text{output}}{x \times \text{output} + P_i + x^2 P_c}$$

$$\eta_x = \frac{x V_2 I_2 \cos\phi_2}{x V_2 I_2 \cos\phi_2 + P_i + x^2 I_2^2 R_{es}}$$

The copper losses vary as the square of the fraction of the load.

Condition for Maximum Efficiency :-

Efficiency of transformer

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{es}}$$

On dividing RHS by I_2, we get

$$\eta = \frac{V_2 \cos\phi_2}{V_2 \cos\phi_2 + P_i/I_2 + I_2 R_{es}} \quad \text{--- (1)}$$

The terminal voltage V_2 is approximately constant. Thus, for a given p.f. efficiency depends upon the load ~~cu~~ current I_2.

Now in eqn ①, the numerator is constant & the efficiency will be maximum if the value of denominator is minimum.

The maximum condition is obtained by differentiating the denominator w.r.t. the variable I_2 & equating that equation to zero.

$$\therefore \frac{d}{dI_2}\left[V_2 \cos\phi_2 + \frac{P_i}{I_2} + I_2 R_{es}\right] = 0$$

$$0 - \frac{P_i}{I_2^2} + R_{es} = 0$$

$$\boxed{P_i = I_2^2 R_{es}}$$

i.e. $\boxed{\text{Copper losses = iron losses}}$

Thus, we can say that the efficiency of a transformer will be maximum when ~~pro~~ copper losses are equal to the iron losses.

$$\therefore \boxed{\eta_{max} = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + 2P_i}} \quad - [\because P_i = P_c]$$

$$\because P_i = I_2^2 R_{es}$$

$$\therefore \boxed{I_2 = \sqrt{\frac{P_i}{R_{es}}}}$$

Ques ① - In a 25 kVA, 2000/200V power transformer the iron & full load copper losses are 350W & 400W resp. Calculate the efficiency at unit power factor at i) full load ii) half load.

Solⁿ:- $\cos\phi = 1$. $P_i = 350W$ $P_c = 400W$

$$\eta_x = \frac{x \text{ kVA} \times 1000 \times \cos\phi}{x \text{ kVA} \times 1000 \times \cos\phi + P_i + x^2 P_c}$$

i) At full load, $x = 1$

$$\eta = \frac{1 \times 25 \times 1000 \times 1}{1 \times 25 \times 1000 \times 1 + 350 + 1 \times 400} \times 100$$

$$= \frac{25 \times 1000}{25 \times 1000 + 750} \times 100$$

$$= 97.087\ \%$$

ii) At half load, $x = 0.5$

$$\eta_{(0.5)} = \frac{0.5 \times 25 \times 1000 \times 1}{0.5 \times 25 \times 1000 \times 1 + 350 + (0.5)^2 \times 400} \times 100$$

$$= 96.525\ \%$$

Ques In a 440/220V, 50Hz Xmer, the total iron loss is 2500 watts when the applied p.d. is 220V. At 25 Hz the corresponding loss is 850 watts. Calculate the eddy current loss at normal frequency and p.d.

Solⁿ:- Total iron loss is 2500 W at 220V, 50Hz

$$P_{i_1} = 2500\text{ W}$$

Total iron loss at 220V, 25Hz

$$P_{i_2} = 850\text{ W}$$

We know that, hysteresis loss

$$P_h = K_h V f B_m^{(1.6)}$$

or $P_h \propto f$ or $P_h = Af$ (A is constant)

Eddy current loss, $P_e = k_e V f^2 t^2 B_m^2$

or $P_e = B f^2$

or $P_e = f^2$

$\therefore P_{i_1} = A f_1 + B f_1^2$

or $\frac{P_{i_1}}{f_1} = A + B f_1$

or $\frac{2500}{50} = A + 50B$

$50 = A + 50B$ — (1)

$P_{i_2} = A f_2 + B f_2^2$

or $\frac{P_{i_2}}{f_2} = A + B f_2$

or $\frac{850}{25} = A + 25B$ or $34 = A + 25B$ — (2)

Substracting eqn (2) from (1) we get.

$25B = 16$

or $B = 0.64$

$\therefore$ Eddy current loss at normal frequency (50Hz) & potential difference (220V)

$P_e = B f^2$

$= 0.64 \times 50^2 = 1600\,W$

ALL DAY EFFICIENCY

The load on certain transformers fluctuate throughout the day. The distribution transformers are energised for 24 hours, but they deliver very light loads for major portion of the day. Thus, iron losses occur for whole day but copper losses ~~occ~~ occur only when the transformers are loaded.

Hence, the performance of such transformers cannot be judged by the ~~comm~~ commercial efficiency, but it can be judged by all day efficiency also known as operational efficiency or energy efficiency which is computed on the basis of energy consumed during a period of 24 hours.

The All day efficiency is defined as the ratio of output in kWh (or Wh) to the input in kWh (or Wh) of a transformer over 24 hours.

$\therefore$ All day efficiency, $\eta_{all\,day} = \frac{\text{Output in kWh (for 24 hours)}}{\text{Input in kWh}}$

To find ~~a~~ all day efficiency, we have to know the load cycle on the transformer.

Ques. A transformer has a maxi efficiency of 98% at 15 kVA at unit P.f. It is loaded as follows:- 12 hrs → 2 kW at p.f 0.5 ; 6 hrs : 12 kW at p.f 0.8 ; 6 hrs — 18 kW at P.f 0.9. Calculate all day efficiency of transformer.

Soln $\eta_{max} = \frac{kVA\cos\phi}{kVA\cos\phi + 2P_i}$

$\frac{98}{100} = \frac{15 \times 1}{15 \times 1 + 2P_i}$

$2P_i = \frac{15 \times 100}{98} - 15$

$P_i = ~~15.306~~ \; 0.153\,kW$

$P_c = 0.153\,kW$

During 24 hrs the Transformer is loaded as under:

Hrs	Load in kW	Pf	Load in kVA $= \frac{kW}{Pf}$	fraction of load $x = \frac{\text{given load in kVA}}{\text{full load in kVA}}$
12	2	0.5	$2/0.5 = 4$	$x_1 = \frac{4}{15} = 0.267$
6	12	0.8	$12/0.8 = 15$	$x = \frac{15}{15} = 1$
6	18	0.9	$18/0.9 = 20$	$x = \frac{20}{15} = 1.33$

kWh output in 24 hrs $= 2 \times 12 + 12 \times 6 + 18 \times 6$
$= 204$ kWh

Iron losses for hrs $= 0.153 \times 24$
$= 3.672$ kWh

Copper loss for 24 hrs $= (0.267)^2 \times 0.153 \times 12 + 1^2 \times 0.153 \times 6 + (1.33)^2 \times 0.153 \times 6$

$= 2.68$ kWh.

Input in 24 hrs $= 204 + 3.672 + 2.68$
$= 210.352$ kWh

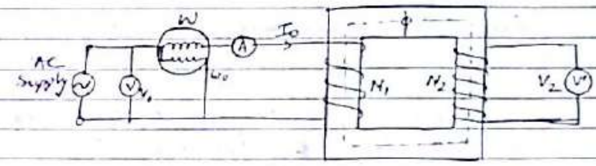
All day Efficiency, $\eta_{allday} = \frac{204}{210.352} \times 100$
$= 96.98\%$

Testing of Transformer

In order to determine the parameter of a transformer, voltage regulation & efficiency the following two tests are carried out.

i) Open Circuit or No-load test
ii) Short circuit test.

1) Open Circuit or No load Test :- This test is carried out to determined the no load loss or core loss or Iron loss and no load current I_0, which is helpful in finding the no load parameters R_0 & X_0 of the transformer.



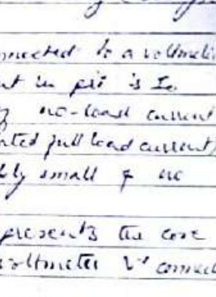
This test is carried out on the low voltage side of the transformer ie a wattmeter W, a voltmeter V' & an ammeter A are connected in low voltage winding (Primary). Supply is given as V_1 volts.

The 'sec' side is kept open or connected to a voltmeter V'. Since sec. is open, the current in pri is I_0 measured in Ammeter A. The value of no-load current I_0 is very small (2 to 6% of the rated full load current). Thus, the copper loss in pri are negligibly small & no copper loss in sec as it is open.

$\therefore$ Wattmeter reading W_0 only represents the core or Iron losses. V_2 is the reading of voltmeter V' connected in sec.

Let, the wattmeter reading $= W_0$.
Voltmeter reading $= V_1$
& Ammeter reading $= I_0$

Then, Iron losses of the Xmer $P_i = W_0$
& $V_1 I_0 \cos\phi_0 = W_0$
$\therefore$ No load P_f, $\cos\phi_0 = \frac{W_0}{V_1 I_0}$



Working components, $I_w = \frac{W_0}{V_1}$ $[\because I_w = I_0 \cos\phi_0]$

Magnetising component, $I_m = \sqrt{I_0^2 - I_w^2}$

No load parameters, ie;

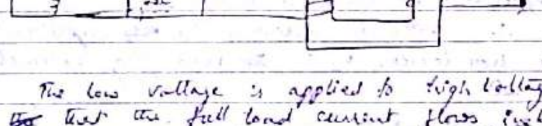
Equivalent exciting resistance, $R_0 = \frac{V_1}{I_w}$

Eqvt. Exciting Reactance, $X_0 = \frac{V_1}{I_m}$

2) Short Circuit Test :-

This test is carried out to determined copper loss at full load &, Equivalent impedance (Z_{es} or Z_{ep}), eqvt. resistance (R_{es} or R_{ep}) & leakage reactance (X_{es} or X_{ep}).

This test is usually carried out on the high voltage side of the transformer i.e. a wattmeter, voltmeter V & an ammeter A are connected in high voltage wdg (sec). The other wdg (Pri.) is shorted by a thick strip or by connecting an ammeter A' across the terminals



The low voltage is applied to high voltage wdg, so ~~the~~ that the full load current flows in both the wdgs. measured by ammeters A & A'.

Since, low voltage (usually 5 to 10% of normal rated voltage) is applied to the Xmer wdg. Therefore, the flux set up in the core is also small in amount $1/30^{th}$ to $1/8^{th}$ of normal flux. The iron losses are negligibly small due to low value of flux as these losses are approximately proportional to the square of the flux. Hence, wattmeter reading W_c only represent the copper losses in the Xmer wdgs. &

Let, the wattmeter reading $= W_c$

Voltmeter reading $= V_{2sc}$ (measured in V)

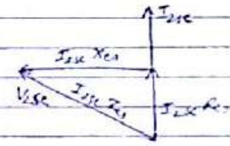
Ammeter reading $= I_{2sc}$ (measured in A)

Then, full load copper losses of the Xmer,

$$P_c = W_c = I_{2sc}^2 R_{es}$$

Eqvt resistance reffered to sec.

$$R_{es} = \frac{W_c}{I_{2sc}^2}$$



From phasor diagram

$$I_{2sc} Z_{es} = V_{2sc}$$

$\therefore$ Eqvt Impedance refered to sec.

$$Z_{es} = \frac{V_{2sc}}{I_{2sc}}$$

Eqvt. reactance refered to sec

$$X_{es} = \sqrt{Z_{es}^2 - R_{es}^2}$$

Auto Transformer :- An auto transformer is a transformer with only one winding wound on a laminated core. A part of this winding is common to both primary & secondary sides. On load, a part of the load current obtained directly from the supply & remaining part is obtained by transformer action. In an ordinary transformer the primary & secondary windings are electrically insulated from each other but connected magnetically. whereas, in an auto transformer the primary & secondary windings are