Q1) Derive Heisenberg's Uncertainity Principle, Show that electron cannot reside inside the nucleus.

Sol Heisenberg's uncertainity principle states that:
"It is impossible to determine the exact position (Δx) and momentum (Δp) of a particle simultaneoubly with accuracy".

i.e i) $\Delta x \cdot \Delta p \simeq h$ or $\frac{\hbar}{2}$

ii) $\Delta E \cdot \Delta t \simeq h$ or $\hbar/2$

1) Position and Momentum

Let us consider two Louis debroglie waves y_1 and y_2 moving with angular frequency ω_1 and ω_2 and progation constants k_1 and k_2 resp.

$$\therefore \quad y_1 = A \sin(\omega_1 t - k_1 x)$$
$$y_2 = A \sin(\omega_2 t - k_2 x)$$

By principle of superposition,
The resultant wave is given by $Y = y_1 + y_2$

$$Y = A\left[\sin(\omega_1 t - k_1 x) + \sin(\omega_2 t - k_2 x)\right]$$

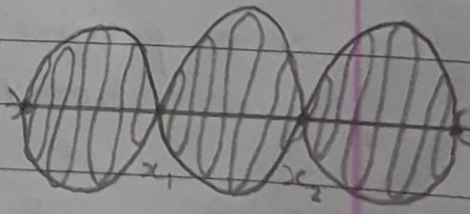
$$Y = A\left[2 \sin\left(\frac{\omega_1 t - k_1 x + \omega_2 t - k_2 x}{2}\right) \cos\left(\frac{\omega_1 t - k_1 x - \omega_2 t + k_2 x}{2}\right)\right]$$

$$Y = A\left[2 \sin\left(\frac{(\omega_1 + \omega_2)t - (k_1 + k_2)x}{2}\right) \cos\left(\frac{(\omega_1 - \omega_2)t - (k_1 - k_2)x}{2}\right)\right]$$

$$\left[\begin{array}{lll} \text{Let} & \frac{\omega_1 + \omega_2}{2} \simeq \omega & \frac{k_1 + k_2}{2} \simeq k \\ \text{and} & \omega_1 - \omega_2 \simeq \Delta\omega & k_1 - k_2 \simeq \Delta k \end{array}\right]$$

$$Y = A\left[2 \sin(\omega t - kx) \cos\left(\frac{\Delta\omega t}{2} - \frac{\Delta k x}{2}\right)\right]$$

The particle is found between two consecutive nodes and nodes are formed when the amplitude is 0.



Since, amplitude is given by the cos component i.e $\cos\left(\frac{\Delta\omega t}{2} - \frac{\Delta k x}{2}\right)$

$\therefore$ nodes are formed when $\cos\left(\frac{\Delta\omega t}{2} - \frac{\Delta k x}{2}\right) = 0$

i.e $\frac{\Delta\omega t}{2} - \frac{\Delta k x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \cdots = (2n+1)\frac{\pi}{2}$

Let the position of nodes be $x_1, x_2, x_3, \ldots$

$\therefore$ at $n=0$, at x_2, $\frac{\Delta\omega t}{2} - \frac{\Delta k x_2}{2} = (2n+3)\frac{\pi}{2}$

at x_1, $\frac{\Delta\omega t}{2} - \frac{\Delta k x_1}{2} = (2n+1)\frac{\pi}{2}$

(Subtracting them) $\frac{\Delta k x_1}{2} - \frac{\Delta k x_2}{2} = \frac{\pi}{2}[2n+3-2n-1]$

$$\frac{\Delta k}{2}(x_1 - x_2) = \frac{\pi}{2}(2)$$

$$\frac{\Delta x\, \Delta k}{2} = \pi \qquad \left[\Delta k = \frac{2\pi}{\Delta\lambda}\right]$$

$$\frac{\Delta x \times \cancel{2}\cancel{\pi}}{\Delta\lambda \times \cancel{2}} = \cancel{\pi} \qquad \left[\Delta\lambda = \frac{h}{\Delta p}\right]$$

$$\Delta x = \Delta\lambda$$

$$\Delta x = \frac{h}{\Delta p}$$

$$\boxed{\Delta x \cdot \Delta p = h}$$

for system of more than two waves, $\boxed{\Delta x \cdot \Delta p \geq \frac{\hbar}{2}}$

2) Energy and time

Let us consider a Louis debroglie particle moving with velocity 'v' in positive direction of x axis.

i.e $v = \frac{dx}{dt}$

The energy of particle is given by, $E = \frac{1}{2}mv^2$

Differentiating partially w.r.t v

$$\frac{\Delta E}{\Delta v} = \frac{1}{2} m \times 2v$$

$$\frac{\Delta E}{\Delta v} = m \times \frac{\Delta x}{\Delta t}$$

$$\Delta E \cdot \Delta t = \Delta x \times \underbrace{m \cdot \Delta v}$$

$$\Delta E \cdot \Delta t = \underbrace{\Delta x \cdot \Delta p}$$

$$\boxed{\Delta E \cdot \Delta t = h}$$

for system of more than two particles, $\boxed{\Delta E \cdot \Delta t \geq \frac{\hbar}{2}}$

Now, ~~Let us consider~~ To determine the position of electron,
Let us consider an electron inside the nucleus,
As we know, radius of nucleus $= 10^{-14}$ m. $\therefore$ diameter $= 2 \times 10^{-14}$ m

$$\therefore \Delta p = \frac{\hbar}{2\Delta x} = \frac{5.272 \times 10^{-35}}{2 \times 10^{-14}} = 2.636 \times 10^{-21} \text{ kg} \frac{m}{sec}$$

But theoritically,
energy of electron $E = \sqrt{p^2c^2 + m_0^2c^4} \sim 10$ MeV

But from β-decay experiment, $E \sim 4$ MeV
which is contradictory
Therefore, our assumption is wrong that electron resides inside in nucleus.
Thus, electron cannot reside inside the nucleus.

Q2> Define phase velocity, group velocity and particle velocity. Derive their expression, show that in a non-dispersive medium, the group velocity equal to the phase velocity.

Ans Phase velocity → The velocity of component wave in a wave packet is called phase velocity. It is given by

Phase velocity i.e $V_p = \frac{\omega}{k}$ where ω = Angular frequency
k = prop. const.

group velocity → The velocity with which a wavefront, formed due to Superpositioning of waves, travels is called group velocity

i.e Group velocity $V_g = \frac{d\omega}{dk}$

Particle velocity → The change in position of a particle w.r.t. change in time is called particle velocity.

i.e $v = \frac{dx}{dt}$

Let us consider a louis debroglie particle, moving with angular freq. ω and propogation constant k.

∴ Its phase velocity i.e $V_p = \frac{\omega}{k}$

$$\boxed{\omega = V_p k}$$

Its group velocity i.e $V_g = \frac{d\omega}{dk}$

$$k = \frac{2\pi}{\lambda}$$

$$V_g = \frac{d(V_p \cdot k)}{dk}$$

$$\frac{dk}{d\lambda} = 2\pi \times \frac{-1}{\lambda^2}$$

$$= V_p \frac{dk}{dk} + k \frac{dV_p}{dk}$$

$$\boxed{dk = \frac{-2\pi}{\lambda^2} d\lambda}$$

$$= V_p + \frac{\cancel{2\pi}}{\cancel{\lambda}} \frac{dV_p}{\frac{-\cancel{2\pi}}{\lambda^{\cancel{2}}} d\lambda}$$

$$\boxed{V_g = V_p - \lambda \frac{dV_p}{d\lambda}}$$

In non-dispersive medium, phase velocity does not depend on ~~group velocity~~ wavelength i.e $\frac{dV_p}{d\lambda} = 0$ ∴ $V_g = V_p - \lambda(0)$

$$\boxed{V_g = V_p}$$

Thus, in Non-dispersive medium, group velocity is equal to phase velocity

Q3> Derive a moving matter particle is equal to wave packet. ($v_g = v$ relativity relation).

Sol Let us consider a relativistic particle with mass $m^* = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

Its phase velocity i.e $V_p = \frac{\omega}{k} = \frac{2\pi\nu}{2\pi/\lambda} = \frac{E}{h} \times \frac{h}{m^* v} = \frac{m^* c^2}{m^* v}$

$$\boxed{V_p = \frac{c^2}{v}}$$

Its ~~gr~~ angular freq, $\omega = 2\pi\nu = 2\pi\frac{E}{h} = \frac{2\pi m^* c^2}{h} = \frac{2\pi c^2}{h}\frac{m^\circ}{\sqrt{1-\frac{v^2}{c^2}}}$ — ①

Its ~~angular~~ propagation const, $k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} = \frac{2\pi m^* v}{h} = \frac{2\pi v}{h}\frac{m^\circ}{\sqrt{1-\frac{v^2}{c^2}}}$ — ②

Differentiating ω w.r.t v

$$\frac{d\omega}{dv} = \frac{d}{dv}\left(\frac{2\pi c^2 m^\circ}{h} \times \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$= \frac{\cancel{2}\pi \cancel{c^2} m^\circ}{h}\left[-\frac{1}{\cancel{2}}\left(1-\frac{v^2}{c^2}\right)^{-3/2} \times \frac{-2v}{\cancel{c^2}}\right]$$

$$\boxed{\frac{d\omega}{dv} = \frac{2\pi m^\circ v}{h}\left(1-\frac{v^2}{c^2}\right)^{-3/2}} \text{ — (A)}$$

Differentiating k w.r.t v

$$\frac{dk}{dv} = \frac{2\pi m^\circ}{h}\frac{d}{dv}\left(v \times \left(1-\frac{v^2}{c^2}\right)^{-1/2}\right)$$

$$= \frac{2\pi m^\circ}{h}\left[v\left(\frac{-1}{\cancel{2}}\left(1-\frac{v^2}{c^2}\right)^{-3/2} \times \frac{-\cancel{2}v}{c^2}\right) + \left(1-\frac{v^2}{c^2}\right)^{-1/2}\right]$$

$$= \frac{2\pi m^\circ}{h}\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left[\cancel{\frac{v^2}{c^2}} + 1 - \cancel{\frac{v^2}{c^2}}\right]$$

$$\boxed{\frac{dk}{dv} = \frac{2\pi m^\circ}{h}\left(1-\frac{v^2}{c^2}\right)^{-3/2}} \text{ — (B)}$$

Group velocity i.e $V_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv}$

$$= \frac{\frac{2\pi m_0}{h} v \left(1-\frac{v^2}{c^2}\right)^{-3/2}}{\frac{2\pi m_0}{h} \left(1-\frac{v^2}{c^2}\right)^{-3/2}}$$

$$\boxed{V_g = V}$$

This is the celativity ~~read~~ relation i.e a maving particle is equal to wave packet.

Q10) Explain debroglie wavelength. Calculate de-broglie wavelength associated with a proton moving with a velocity equal to 1/20 velocity of light.

Sol According to debroglie, a moving matter particle is surrounded by a wave whose wavelength depends on the mass and velocity of the particle. These waves associated with moving particles are called matter waves and their wavelength is called debroglie wavelength.

Given, velocity of proton i.e $v = \frac{1}{20} \times$ velocity of light $= \frac{1}{20} \times 3 \times 10^8 \frac{m}{sec}$

Mass of protom i.e $m_p = 1.67 \times 10^{-27}$ kg

wavelength i.e $\lambda = ?$

$$\lambda = \frac{h}{p} = \frac{h}{m_p V} = \frac{6.62 \times 10^{-34}}{1.67 \times 10^{-27} \times \frac{1}{20} \times 3 \times 10^8} = \frac{662 \times 20 \times 10^{-7}}{167 \times 3 \times 10^8}$$

$$= \frac{13240}{501} \times 10^{-15} = 26.427 \times 10^{-15}$$

debroglie Wavelength i.e $\boxed{\lambda = 2.64 \times 10^{-14}\ m}$

Q11> An electron has a speed of 3.5×10^7 m/sec within the accuracy of 0.0098%. Calculate the uncertainty in position of electron.

Sol Speed of electron $= 3.5 \times 10^7 \ m\,sec^{-1}$

mass of electron $m_e = 9.1 \times 10^{-31}$ kg

$$\Delta v = \frac{98}{10000 \times 100} \times 3.5 \times 10^7 = 3430$$

$$\Delta p = m_e \Delta v = 9.1 \times 10^{-31} \times 3430$$
$$= 3.1213 \times 10^{-27}$$

$$\Delta x = \frac{\hbar}{2} \times \frac{1}{\Delta p} = \frac{5.27 \times 10^{-35}}{3.12 \times 10^{-27}} = 1.68 \times 10^{-8}$$

Q4> What is wave function. Explain normalization of wave function.

Ans Wave function is a function that expresses the probability of the particle to be at the position (x, y, z) at time t.

It is given by ψ and it should satisfy following conditions:

i> ψ should be finite everywhere

ii> ψ should be continuous everywhere.

iii> ψ and its Ist derivative $\left(\frac{d\psi}{dx}, \frac{d\psi}{dy}, \frac{d\psi}{dz}\right)$ should be continuous

If we assume that the displacement of any point in space is given by ψ then the particle density at that point will be directly proportional to the square of displacement i.e ψ^2

$\therefore$ ψ^2 is used to express the particle density

But since, in some cases, the value of ψ can be zero then the question arises that what is the position of particle.

So Max-Born suggested that the quantity $|\psi|^2$ represents the probability of ~~finding~~ particle to be in state ψ and so the quantity $|\psi|^2$ is the measure of probabity density

$$\psi = \psi_0 e^{-Et/\hbar}$$

$$\psi^* = \psi_0 e^{Et/\hbar} \qquad [\psi^* \text{ is complex conjugate}]$$

$$\therefore \psi \psi^* = \psi_0 e^{-Et/\hbar} \cdot \psi_0 e^{Et/\hbar} = \psi_0^2 \cdot 1$$

$$\psi \psi^* = |\psi|^2 = \text{real value}$$

Then the quantity $\psi \psi^* dV$ or $|\psi|^2 dV$ tells us the probability of finding the particle in infinite volume V.

Now, if the particle is in limited region under some forces (such as electron in atom or a particle in a box with impenetrable walls, then the probability of finding the particle in that finite space is unity i.e $\iiint_V |\psi|^2 dV = 1$ or $\iiint_V \psi_{(r,t)} \psi^*_{(r,t)} = 1$

where, $\iiint_V$ represent the integration of the entire volume.

The above condition is called condition of normalization and the wave functions which satisfy the above condition are called normalized wave function.

~~But if the particle~~ But the probability of finding the particle at infinite distance, while it is still bound inside the limited region, is zero i.e $\psi_{(r,t)} \psi^*_{(r,t)} = 0$ when $r \to \infty$

Q 3> Derive Energy operator
Q 4> Derive Momentum operator
Q 5> Derive time dependent Schrodinger wave equation.
Q 6> Derive time independent Schrodinger wave equation.

Ans Let us consider a Louis debroglie material particle which shows dual nature. Its mathetical wave equation or wave function is given by,

$$\psi = A e^{-i[\omega t - Kx]}$$

$$\psi = A e^{-i[2\pi\nu t - \frac{2\pi}{\lambda}x]}$$

$$\psi = A e^{-i2\pi[\frac{E}{h}t - \frac{x}{h}p]}$$

$$\psi = A e^{-i\left(\frac{2\pi}{h}\right)[Et - Px]}$$

$$\boxed{\psi = A e^{-i/\hbar [Et - Px]}} \quad - (1)$$

Energy - Momentum operators

→ Differentiating partially eq (1) w.r.t t

$$\frac{\partial \psi}{\partial t} = A \frac{\partial}{\partial t} e^{-i/\hbar [Et - Px]} = \underline{A e^{-i/\hbar [Et - Px]}} \left(-\frac{i}{\hbar} E\right)$$

$$\frac{\partial \psi}{\partial t} = \psi \times \frac{-i^2}{\hbar i} E$$

$$E\psi = i\hbar \frac{\partial \psi}{\partial t}$$

where, $\boxed{E = i\hbar \frac{\partial}{\partial t}}$ is the Energy Operator

(Q8)

→ Differentiating partially eq (1) w.r.t x

$$\frac{\partial \psi}{\partial x} = A \frac{\partial}{\partial x} e^{-i/\hbar [Et - Px]} = \underline{A e^{-i/\hbar [Et - Px]}} \left(-\frac{i}{\hbar}(-P)\right)$$

$$\frac{\partial \psi}{\partial x} = \psi \frac{i}{\hbar} P$$

$$P\psi = \frac{\hbar}{i} \frac{\partial \psi}{\partial x}$$

where, $\boxed{P = \frac{\hbar}{i} \frac{\partial}{\partial x}}$ is Momentum operator

(Q7)

The total energy of system of particle is given by,

$$E = K.E. + P.E$$

$$E = \frac{1}{2} m v^2 + V$$

$$E = \frac{p^2}{2m} + V$$

Multiplying both sides by ψ

$$E\psi = \frac{p^2\psi}{2m} + V\psi$$

$$i\hbar\frac{\partial\psi}{\partial t} = \left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)^2\psi \times \frac{1}{2m} + V\psi$$

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{\hbar^2}{i^2}\frac{\partial^2\psi}{\partial x^2} \times \frac{1}{2m} + V\psi$$

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\cdot\frac{\partial^2\psi}{\partial x^2} + V\psi} \quad \text{— (A)}$$

(Q5) This is the Time Dependent Schrodinger Wave equation

eq (1), $\psi = A e^{-i/\hbar\,[Et - Px]}$

$$\psi = A e^{-i/\hbar\, Et} e^{i/\hbar\, Px} \qquad \left[\text{Let } \psi_x = A e^{i/\hbar\, Px}\right]$$

$$\boxed{\psi = \psi_x e^{-i/\hbar\, Et}} \quad \text{— (B)}$$

Differentiating partially, eq (B) w.r.t. t

$$\frac{\partial\psi}{\partial t} = \psi_x \frac{\partial}{\partial t} e^{-i/\hbar\, Et}$$

$$\boxed{\frac{\partial\psi}{\partial t} = \psi_x e^{-i/\hbar\, Et}\left[\frac{-iE}{\hbar}\right]} \quad \text{— (2)}$$

Differentiating partially, eq (B) w.r.t x

$$\frac{\partial\psi}{\partial x} = e^{-i/\hbar\, Et}\frac{\partial\psi_x}{\partial x}$$

$$\frac{\partial}{\partial x}\frac{\partial\psi}{\partial x} = e^{-i/\hbar\, Et}\frac{\partial}{\partial x}\frac{\partial\psi_x}{\partial x}$$

$$\boxed{\frac{\partial^2\psi}{\partial x^2} = e^{-i/\hbar\, Et}\frac{\partial^2\psi_x}{\partial x^2}} \quad \text{— (3)}$$

Substituting eqn (B), (2) and (3) in eq (A)

$$i\hbar\, \psi_x\, e^{-i/\hbar\, Et}\left(\frac{-i}{\hbar}E\right) = \frac{-\hbar^2}{2m} e^{-\frac{i}{\hbar}Et} \frac{\partial^2 \psi_x}{\partial x^2} + V \psi_x e^{-i/\hbar\, Et}$$

$$\frac{-i^2 \cancel{\hbar} E\psi_x}{\cancel{\hbar}} \cancel{e^{-i/\hbar\, Et}} = \cancel{e^{-i/\hbar\, Et}}\left[\frac{-\hbar^2}{2m}\frac{\partial^2 \psi_x}{\partial x^2} + V\psi_x\right]$$

$$\boxed{E\psi_x = \frac{-\hbar^2}{2m}\frac{\partial^2 \psi_x}{\partial x^2} + V\psi_x}$$

where $E_x = H_x$ i.e Hamiltonian operator $\boxed{H_x = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V}$

This is the Time Independent Schrodinger Wave equation

Q19> Explain Schrodinger wave equation for a particle in a three-dimentional box. Solve it to obtain Eigen Function and show that Eigen Energy are discrete

Ans Schrodinger time independent wave equation for particle in one-dimention is,

$$E\psi_x = \frac{-\hbar^2}{2m}\frac{\partial^2 \psi_x}{\partial x^2} + V\psi_x$$

$$(E-V)\frac{2m}{\hbar^2}\psi + \frac{\partial^2 \psi}{\partial x^2} = 0 \quad \text{— (A)}$$

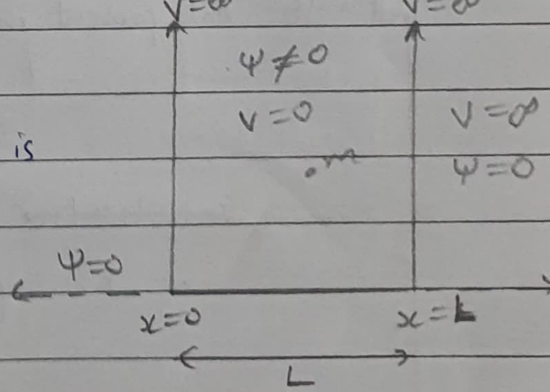
For three dimensional box,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ i.e Laplacian Operator

Let us consider a particle of mass 'm' inside of box of width 'L'. The particle can move inside the box as the potential (V) is zero. But the particle cannot move outside the box as the potential is very high



$\therefore$ Boundary conditions are : $V = 0$ at $0 < x < L$

$V = \infty$ at $x \leq 0$ and $x \geq L$

By using Schrodinger's time independent wave equation (eq (A)) and the boundary conditions, the energy of the particle can be calculated

Inside the box, $V = 0$ $\therefore$ eq (A) becomes,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} E \psi = 0$$

$$\left[\text{let } K^2 = \frac{2m}{\hbar^2} E \right] \text{ (B)}$$

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} + K^2 \psi = 0}$$

The solution of IInd order derivative is given by.

$$\psi = A \sin k \cancel{x} + B \cos kx \quad \text{— (1)}$$

$\psi = 0$, at $x = 0$

$$\therefore \quad 0 = A \sin k(0) + B \cos k(0) = B$$

$$\boxed{B = 0}$$

$\psi = 0$ at $x = L$

$$\therefore \quad 0 = A \sin kL + B \cos k(L) = A \sin kL + 0(\cos kL)$$

$$0 = A \sin kL$$

$A \neq 0$

because at $A = 0$, $\psi = 0$

(particle not present in box)

$$\sin kL = 0$$

$$kL = 0, \pi, 2\pi \cdots$$

$$kL = \pm n\pi$$

$$\boxed{k = \frac{\pm n\pi}{L}} \text{ — (2)}$$

~~Substitution~~ Equating eq (B) and (2)

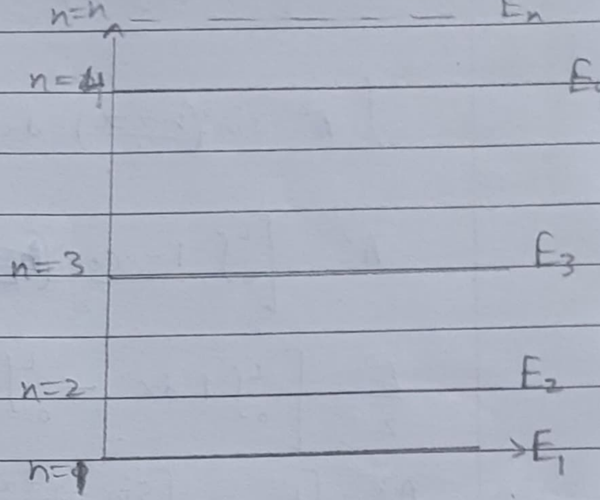
$$K^2 = \left(\frac{\pm n\pi}{L} \right)^2$$

$$\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{L^2}$$

$$E = \frac{n^2 \pi^2 \hbar^2}{2mL^2} = \frac{n^2\pi^2}{2mL^2} \times \frac{h^2}{4\pi^2}$$

$$\boxed{E = \frac{n^2 h^2}{8mL^2}}$$

i.e Energy values where $n = 1, 2, 3 ---$



For $n=1$, $E_1 = \frac{h^2}{8mL^2}$

For $n=2$, $E_2 = \frac{4h^2}{8mL^2} = 4E_1$

For $n=3$, $E_3 = \frac{9h^2}{8mL^2} = 9E_1$

For $n=4$ $E_4 = \frac{16h^2}{8mL^2} = 16E_1$

Since, Energy is not continuous (or not equispaced)
Thus, Discontinuous form of energy is obtained
$\therefore$ Eigen energy is Discrete

eq ①, $\psi = A \sin kx + B\cos kx$ $\quad \left[\because B = 0, \ k = \frac{\pm n\pi}{L}\right]$

$\psi = A\sin kx + 0(\cos kx)$

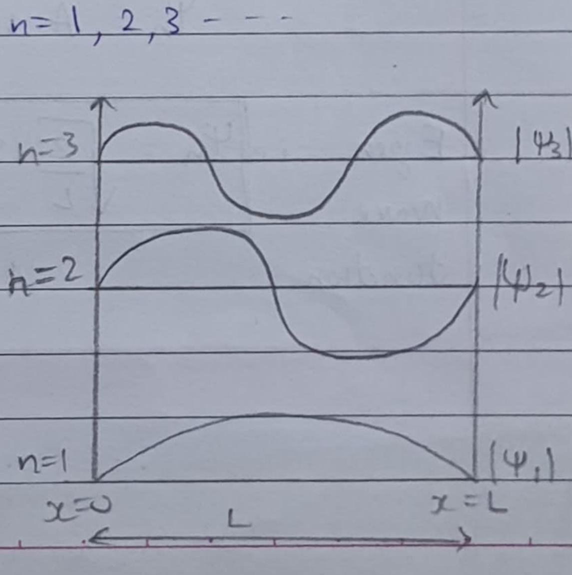
$$\boxed{\psi_n = A\sin\left(\frac{n\pi}{L}x\right)} \quad — ©$$

This is the Eigen Function where $n = 1, 2, 3 ---$

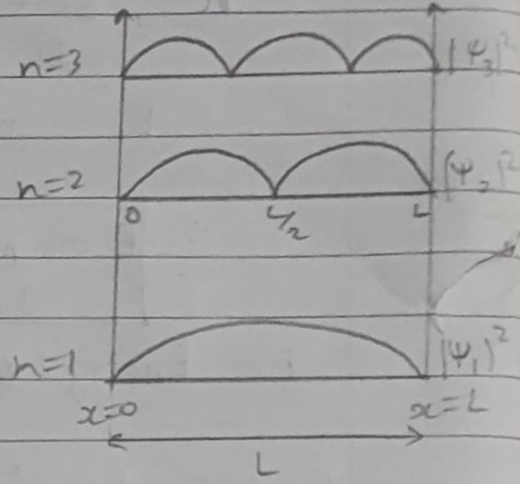
For $n=1$, $\psi_1 = A\sin\left(\frac{\pi}{L}x\right)$

For $n=2$, $\psi_2 = A\sin\left(\frac{2\pi}{L}x\right)$

For $n=3$, $\psi_3 = A\sin\left(\frac{3\pi}{L}x\right)$



Here, $|\psi|^2$ represents the probability at finding the particle at that instant



By using normalisation ~~eq~~ condition,

$$\int_0^L |\psi_n|^2 \, dx = 1$$

$$\int_0^L \left| A \sin\left(\frac{n\pi x}{L}\right)\right|^2 dx = 1$$

$$\int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

$$\left[1-\cos 2\theta = 2\sin^2\theta \qquad \sin^2\theta = \frac{1-\cos 2\theta}{2} \right]$$

$$A^2 \left[\int_0^L \frac{1-\cos 2\left(\frac{n\pi x}{L}\right)}{2} dx \right] = 1$$

$$\frac{A^2}{2}\left[\int_0^L 1\, dx - \int_0^L \cos\left(\frac{2n\pi x}{L}\right) dx\right] = 1$$

$$\frac{A^2}{2}\left[L - \left[\frac{\sin \frac{2n\pi x}{L}}{\frac{2n\pi}{L}}\right]_0^L \right] = 1 \qquad [\because \sin n\pi = 0]$$

$$\frac{A^2}{2}(L-0) = 1$$

$$A^2 = \frac{2}{L}$$

$$\boxed{A = \sqrt{\frac{2}{L}}}$$

eq ©,

$$\psi_n = A \sin\left(\frac{n\pi x}{L}\right)$$

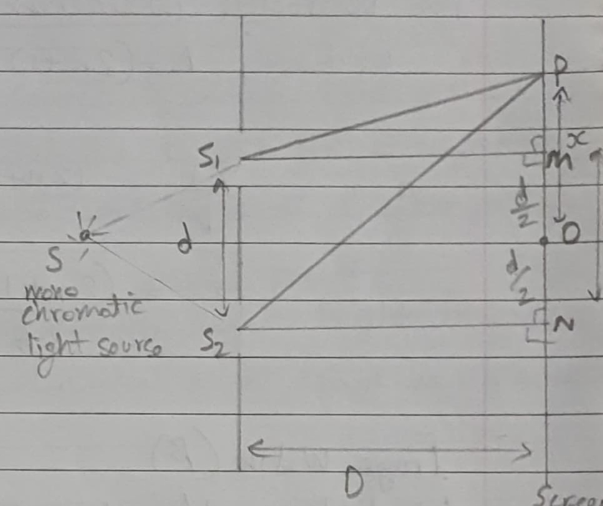
Eigen wave function i.e. $\boxed{\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)}$

(intro)
Thomas young in 1801 demostrated interference of light and division of waveforms.

Q1) Explain Young's double slit experiment. Derive fringe width.

Sol Let us consider a monochromatic light source (S) in front of which there are two narrow slits (S_1) and (S_2) at (d) distance from each other. Let the distance b/w the slits & screen be (D). We need to find the conditions of bright & dark fringes at point (P) on the screen. The distance between (P) and O is (x).
Let us draw S_1M, S_2N perpendiculars on screen.

Path diff. b/w S_1 & S_2 $\boxed{\Delta = S_2P - S_1P}$ —(I)



By pythagoras,

In ΔS_1PM

$$(S_1P)^2 = (PM)^2 + (S_1M)^2$$
$$= \left(x - \frac{d}{2}\right)^2 + D^2$$

$$\boxed{(S_1P)^2 = x^2 + \frac{d^2}{4} - xd + D^2} \text{ —(1)}$$

In ΔS_2PN

$$(S_2P)^2 = (PN)^2 + (S_2N)^2$$
$$= \left(x + \frac{d}{2}\right)^2 + D^2$$

$$\boxed{(S_2P)^2 = x^2 + \frac{d^2}{4} + xd + D^2} \text{ —(2)}$$

Subtracting eq (2) from (1)

$$(S_2P)^2 - (S_1P)^2 = \cancel{x^2} + \cancel{\frac{d^2}{4}} + xd + \cancel{D^2} - \cancel{x^2} - \cancel{\frac{d^2}{4}} + xd - \cancel{D^2}$$

$$(S_2P - S_1P)(S_2P + S_1P) = 2xd$$

$$S_2P - S_1P = \frac{2xd}{S_1P + S_2P} = \frac{2xd}{D + D} \qquad [S_1P \sim S_2P \sim D]$$

$$\Delta = S_2P - S_1P = \frac{2xd}{2D}$$

$$\boxed{\Delta = \frac{xd}{D}} \text{ —(A)}$$

For Constructive interference (Bright fringes)

we know, $\Delta = n\lambda$ where $n = 1, 2, 3 \ldots$

$$\frac{xd}{D} = n\lambda$$

$$\boxed{x_n = \frac{n\lambda D}{d}}$$

For Destructive interference (dark fringes)

we know, $\Delta = (2n+1)\frac{\lambda}{2}$ where, $n = 0, 1, 2, 3 \ldots$

$$\frac{xd}{D} = (2n+1)\frac{\lambda}{2}$$

$$\boxed{x_n = \frac{(2n+1)\lambda D}{2d}}$$

Fringe Width (β)

→ Let distance b/w two consecutive Bright fringes be x_n, x_{n+1}

$$x_n = \frac{n\lambda D}{d} \qquad x_{n+1} = \frac{(n+1)\lambda D}{d}$$

$$\beta = x_{n+1} - x_n = \frac{\lambda D}{d}(n+1-n)$$

Fringe width $= \beta = \frac{\lambda D}{d}$ for bright fringes.

→ Let dist b/w any two consecutive dark fringes be x_n and x_{n+1}

$$\beta = x_{n+1} - x_n$$

$$= \frac{(2(n+1)+1)\lambda D}{2d} - \frac{(2n+1)\lambda D}{2d} = \frac{\lambda D}{2d}(2n+3-2n-1)$$

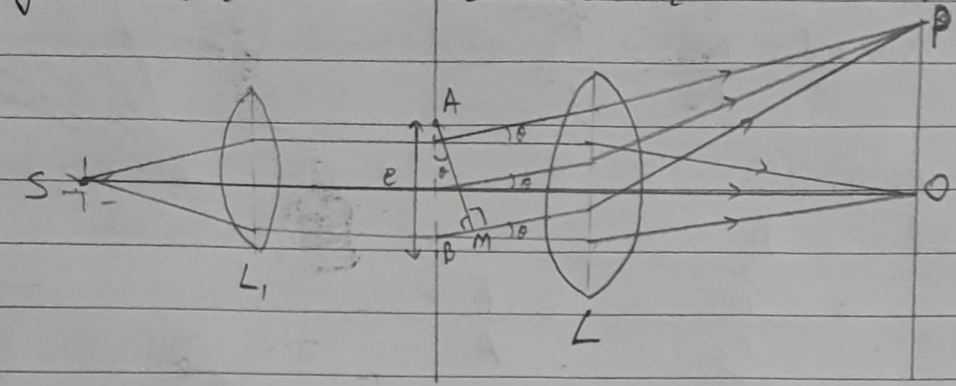
$$\beta = \frac{2\lambda D}{2d} \Rightarrow \boxed{\beta = \frac{\lambda D}{d}}$$ for dark fringes

Conclusion → $\beta \propto \lambda$, $\beta \propto D$, $\beta \propto \frac{1}{d}$ where D = dist b/w slits & screen

d = dist b/w two slits

Q6> Discuss the phenomena of Fraunhofer diffraction at a single slit and show that the relative intensities of successive maximum are nearly $1 : 4/9\pi^2 : 4/25\pi^2 : 4/49\pi^2$ - - - -

Sol



Let us consider a monochromatic light source S whose light passes through convex lens L_1 then gets diffracted due to slit AB of length (e) and forms principal maxima at O and ~~and~~ fringe at P after passing through lens L.

A pattern of central bright fringe surrounded by alternate dark & weak bright fringes of decreasing intensity on either side of central bright fringe is observed on the screen.

Path difference $\Delta = BM$

In Δ MAB, $\sin\theta = \frac{BM}{AB} = \frac{BM}{e}$

$$\boxed{\Delta = BM = e\sin\theta}$$

Phase difference $\delta = \frac{2\pi}{\lambda} \times \Delta$

$$\boxed{\delta = \frac{2\pi e\sin\theta}{\lambda}}$$

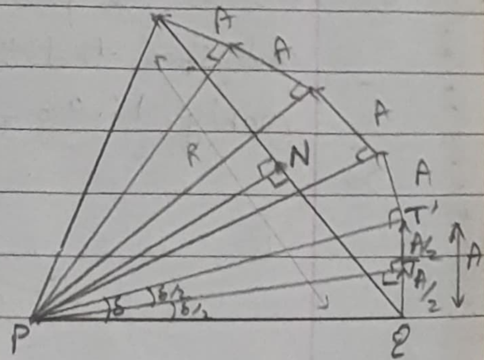
If the slit is divided in n-equal parts then the phase difference of wave reaching the screen becomes,

$$\boxed{\delta = \frac{1}{n}\frac{2\pi}{\lambda} e\sin\theta}$$

and amplitude of each phase reaching the screen is A'

To obtain Resultant amplitude, let phase δ be substended at point A.

In ΔPQT, $\sin\frac{\delta}{2} = \frac{QT}{PQ} = \boxed{\frac{A/2}{PQ}}$ — ①



In ΔPNQ, $\sin\frac{n\delta}{2} = \frac{NQ}{PQ} = \boxed{\frac{R/2}{PQ}}$ — ②

Dividing eq ② & ①

$$\frac{\sin\frac{n\delta}{2}}{\sin\frac{\delta}{2}} = \frac{\frac{R}{\cancel{2}}\big/\cancel{PQ}}{\frac{A}{\cancel{2}}\big/\cancel{PQ}} = \frac{R}{A}$$

$$R = \frac{A\sin\frac{n\delta}{2}}{\sin\frac{\delta}{2}} = \frac{A\sin\frac{1}{2}\left(\cancel{n}\times\frac{1}{\cancel{n}}\frac{2\pi}{\lambda}e\sin\theta\right)}{\sin\frac{1}{\cancel{2}}\left(\frac{1}{n}\frac{\cancel{2}\pi}{\lambda}e\sin\theta\right)}$$

$\left[\text{Let } \frac{\pi}{\lambda}e\sin\theta = \alpha\right]$

$$R = \frac{A\sin\alpha}{\sin\frac{\alpha}{n}} \qquad \left[\because n >>> \alpha \;\; \therefore \sin\frac{\alpha}{n} \sim \frac{\alpha}{n}\right]$$

$$R = \frac{A\sin\alpha}{\alpha/n}$$

$$\boxed{R = \frac{nA\sin\alpha}{\alpha}}$$

Intensity i.e $I \propto (R)^2$

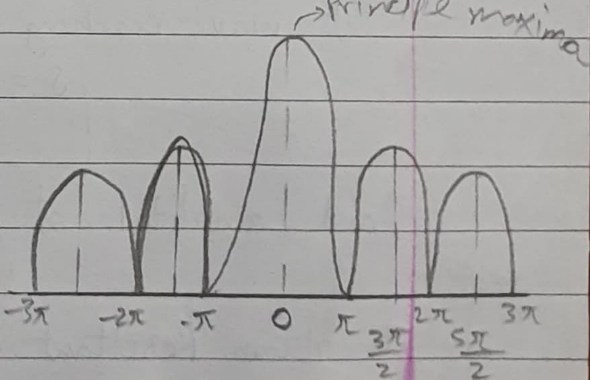
$$I = k(nA)^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \left[\text{where, } kn^2A^2 = I_0\right]$$

$$\boxed{I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2}$$

① Principle Maxima

$\alpha = 0$

$\frac{\pi}{\lambda}e\sin\theta = 0$



$\sin\theta = 0$

at $\therefore \boxed{\theta = 0^\circ}$ we get principal maxima

② Minima

$$\frac{\sin\alpha}{\alpha}=0$$

$$\alpha = 0, \pi, 2\pi, ---$$

$$\alpha = \pm n\pi$$

$$\frac{\pi}{\lambda} e\sin\theta = \pm n\pi$$

$$\boxed{e\sin\theta = \pm n\lambda}$$

This is the condition for minima where $n=1,2,3---$
$n\neq 0$ because at $n=0$, we get principal maxima.

③ Secondary maxima.

differenting I w.r.t α

$$0 = \frac{d}{d\alpha} I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$0 = I_0\left[\cancel{\cos\alpha}\ 2\left(\frac{\sin\alpha}{\alpha}\right)\left[\frac{\cos\alpha}{\alpha} - \frac{\sin\alpha}{\alpha^2}\right]\right]$$

$\therefore$ $$2I_0\frac{\sin\alpha}{\alpha}=0$$

$$\sin\alpha=0$$

This gives the condition for minima

or $$\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}=0$$

$$\alpha\cos\alpha = \sin\alpha$$

$\alpha = \tan\alpha$ which is similar to eq^n of st. line

$$\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} ---$$

$$\alpha = (2n+1)\frac{\pi}{2}$$

$$\frac{\pi}{\lambda} e\sin\theta = (2n+1)\frac{\pi}{2}$$

$$\boxed{e\sin\theta = (2n+1)\frac{\lambda}{2}}$$

This is the condition for secondary maxima where $n=1,2,3--$

If $\alpha = \frac{3\pi}{2}$, $I_1 = I_0 \frac{(\sin \frac{3\pi}{2})^2}{(\frac{3\pi}{2})^2} = \frac{I_0 (-1)^2}{\frac{9\pi^2}{4}} = \boxed{\frac{4I_0}{9\pi^2}}$

If $\alpha = \frac{5\pi}{2}$, $I_2 = I_0 \frac{(\sin \frac{5\pi}{2})^2}{(\frac{5\pi}{2})^2} = \frac{I_0 (1)^2}{\frac{25\pi^2}{4}} = \boxed{\frac{4I_0}{25\pi^2}}$

If $\alpha = \frac{7\pi}{2}$, $I_3 = I_0 \frac{(\sin 7\pi/2)^2}{(\frac{7\pi}{2})^2} = \frac{I_0(-1)^2}{\frac{49\pi^2}{4}} = \boxed{\frac{4I_0}{49\pi^2}}$

∴ Relative intensities of successive maximum are

$I_0 : \frac{4I_0}{9\pi^2} : \frac{4I_0}{25\pi^2} : \frac{4I_0}{49\pi^2}$

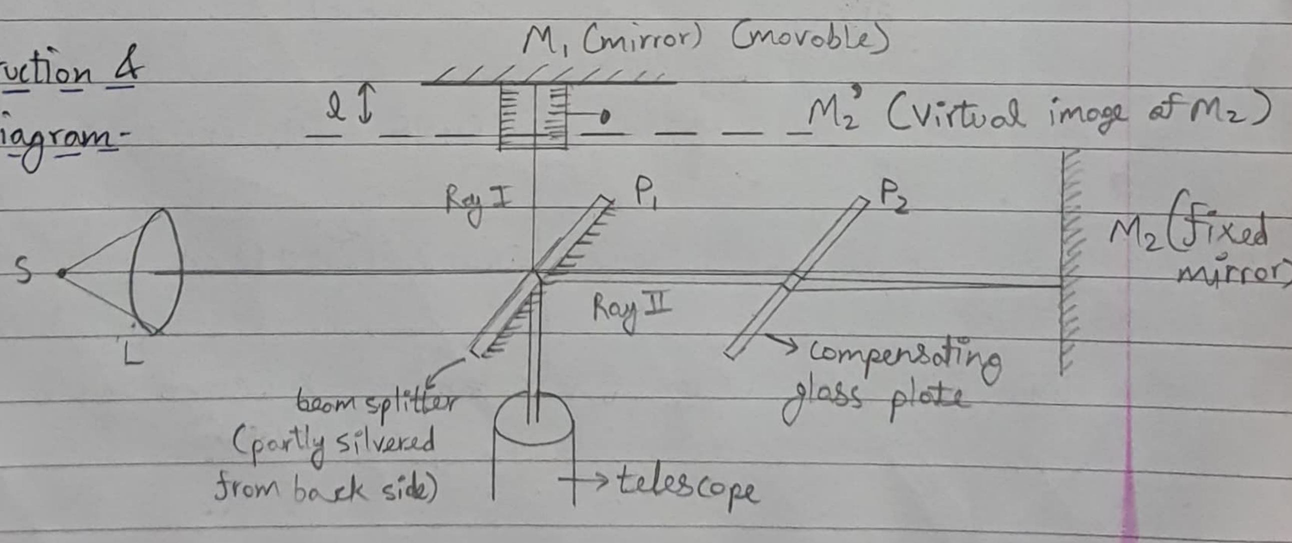
i.e $\boxed{1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2}}$

Q5> Describe Michelson's interferometer and explain the formation of fringes in it. How this can be used to measure the wavelength of monochromatic light?

Ans. Michelson's Interferometer

Introduction & Principle - In Michelson's interferometer, light wave from an external source is divided in two coherent light beams by partial reflection at lightly silvered surface of glass plate and then made to reunite at same plate after travelling different optical path.

Construction & Diagram-



Working ~~*~~ the monochromatic source of light is kept on the focus of lens L so ~~as~~ to get a parallel ~~emegent~~ emergent ray of light.

* The ray of light is incident on plate P_1 (beam splitter) which is inclined at an angle of 45° and is partly silvered on back/lower side, so that the incident light is equally reflected and transmitted by it.

* Half of the incident ray is reflected by P_1 towards the movable Mirror M_1 as Ray I and the other half is transmitted through P_1 towards fixed Mirror M_2 as Ray II.

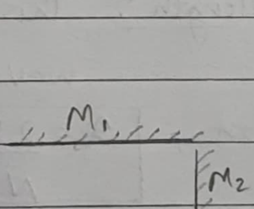
* Both rays, Ray I & Ray II, incident normally on the mirrors and hence on reflection, they retrace their path.

* On returning to glass plate P_1, a part of the amplitude of Ray I is ~~transmitt~~ transmitted towards the telescope whereas a part of the Ray II is reflected towards the telescope.

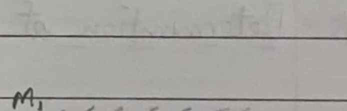
* Since there are two beams towards the telescope and they are derived from the same incident wave, thus they are coherent.

* These two waves form interference fringes at the field of view of telescope such that their path difference $\Delta = 2\mu t \cos r + \lambda/2$, Condition for ~~maxima~~ constructive interference is $\Delta = n\lambda$ and for destructive interference $\Delta = (2n+1)\frac{\lambda}{2}$

Types of Fringes :

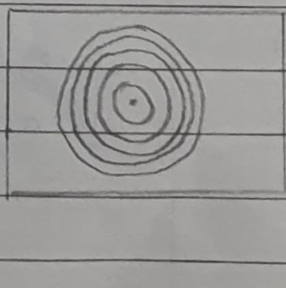


or



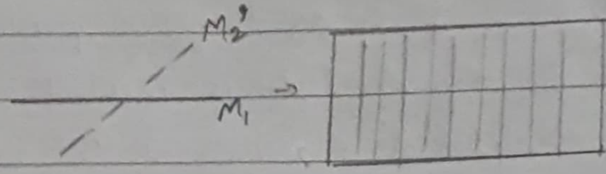
① Circular fringes

When mirror M_1 and mirror M_2 are exactly perpendicular or mirror M_1 and virtual image M_2' are parallel to each other, then we get Circular fringes

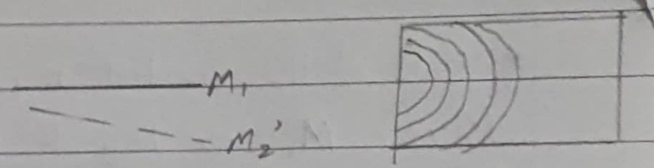


(2) Wedge or Localized fringes

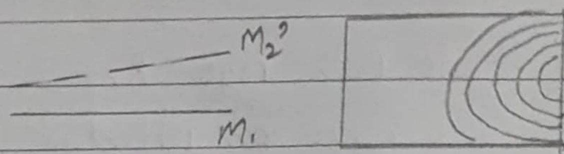
i) When M_2' is inclined over M_1 then straight line fringes are obtained



ii) When M_2' is inclined below M_1 then semicircular fringes, on left side of field of view of telescope, are obtained.



iii) When M_2' is inclined above M_1, then semicircular ~~fring~~ fringes, on right side of field of view of telescope, are obtained



* Function of compensating glass plate :-

→ To equalize the path of waves, a glass plate P_2, of same thickness and same material as glass plate P_1, is introduced in the path of Ray II towards mirror M_2.

Thus, glass plate P_2 is called compensating glass plate.

→ ~~If~~ In the absence of P_2, the path of Ray I & Ray II are not equal as Ray I travels through P_1 twice whereas Ray II does not travel through P_2 twice.

* Determination of wavelength :- This can be used to determine the wavelength of monochromatic light by using the equation, $\boxed{\lambda = \frac{2l}{N}}$

where, (N) is the number of fringes ~~in~~ that move across the field of view of telescope when mirror is displaced through a distance (l).

Q9) Explain ~~princi~~ application of Michelson's interferometer.

Sol The application of Michelson's interferometer are as follows

(1) Determination of thickness of thin transparent sheet :-

By using the formula, thickness i.e $\boxed{t = \frac{\lambda N}{2(\mu - 1)}}$

where, μ is the refractive index of the material of thin transparent sheet and (N) is the number of fringes moving across the field of view when mirror is displaced through a distance (l).

(2) Determination of two spectral line or for the resolution of two closed spaced spectral line :-

By using the relation, $\boxed{\Delta\lambda = \frac{\lambda_{avg}}{2N}}$

where, $\boxed{\lambda_{avg} = \frac{\lambda_1 + \lambda_2}{2}}$

Q 10) Explain plain transmission diffraction grating. Derive the maxima and minima.

Ans Diffraction grating is an arrangement of parallel and equidistant slit.

It is made by ruling a large number of parallel and equidistant lines on an optically plane glass plate with a diamond point.

The ruled part scatters the light while the unruled part transmits the light.

Let us consider a N number of equidistant and parallel slits. Let 'e' be the width of each slit and 'd' be the space between each slit, then ~~the~~ (e+d) is called 'grating element'.

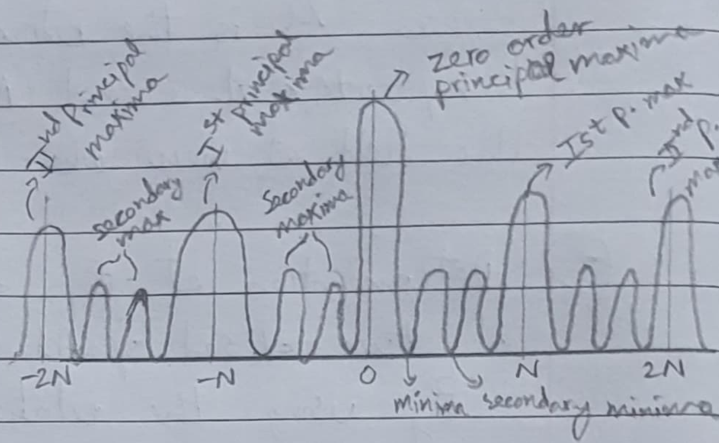
Intensity pattern for N-slits is given by,

$$I = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2 \left(\frac{\sin N\beta}{\sin\beta}\right)$$

where $\alpha = \frac{\pi}{\lambda} e \sin\theta$ and $\boxed{\beta = \frac{\pi}{\lambda}(e+d)\sin\theta}$ —①

$\therefore$ for single slit i.e $N=1$, $I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 \cancel{\frac{\sin\beta}{\sin\beta}}$

$$\boxed{I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2}$$



Condition for Maxima

For maximum intensity,

$$\sin\beta = 0$$

$$\therefore \beta = 0, \pi, 2\pi, ---$$

$$\boxed{\beta = \pm n\pi}$$

eq ①, $\beta = \frac{\pi}{\lambda}(e+d)\sin\theta$

$$\pm n\cancel{\pi} = \frac{\cancel{\pi}}{\lambda}(e+d)\sin\theta$$

$\boxed{(e+d)\sin\theta = \pm n\lambda}$ for Maxima where $n = 0, 1, 2, ---$

Condition for Minima

For minimum intensity,

$$\sin N\beta = 0$$

$$N\beta = 0, \pi, 2\pi, ---$$

$$N\beta = \pm m\pi$$

$$\frac{N\cancel{\pi}(e+d)\sin\theta}{\lambda} = \pm m\cancel{\pi}$$

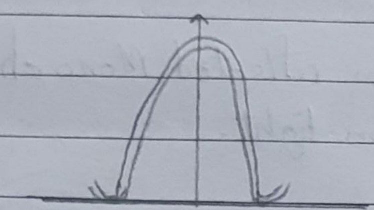
$\boxed{N(e+d)\sin\theta = \pm m\lambda}$ for Minima where $m = 1, 2, 3$, excluding $m = 0, N, 2N --- nN$ as at these values there will be zero order principal maxima, first order principal maxima, --- upto n^{th} order principal maxima.

Q 8) Explain Rayleigh criterion and resolving power of grating.

Ans Grating is an arrangement of parallel and equidistant ~~lines~~ slits on a glass plate and the ability of a grating to form separate diffraction maximas or spectral lines of close wavelengths is called the Resolving power of grating

~~So according to Rayleigh's criterion,~~

Let us consider two spectral lines of wavelength λ and $\lambda + d\lambda$



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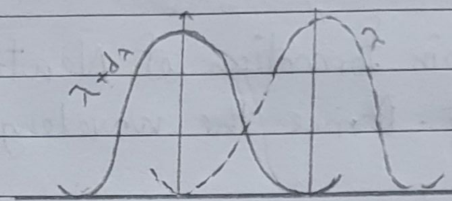
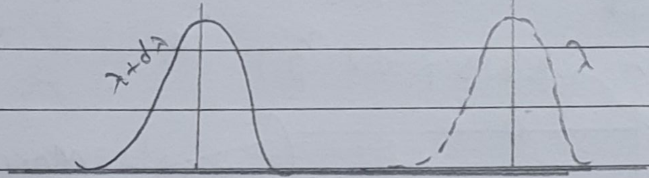
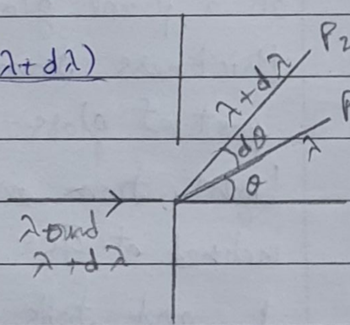


Images Just resolved



Well resolved images.

According to Rayleigh's criterion, the wavelength of two spectral lines, having wavelengths λ and $\lambda + d\lambda$, is said to be Just Resolved by the grating when the n^{th} maxima of one wavelength $(\lambda + d\lambda)$ falls over the minima of other wavelength (λ) in same direction



Condition for n^{th} Maxima in direction $(\theta + d\theta)$ of $(\lambda + d\lambda)$

$$\boxed{(e+d)\sin(\theta + d\theta) = n(\lambda + d\lambda)} \quad -①$$

Condition for adjacent minima of wavelength λ in direction of $(\theta + d\theta)$

$$N(e+d)\sin(\theta + d\theta) = m\lambda$$

when adjacent minima $m = nN + 1$,

$$N(e+d)\sin(\theta + d\theta) = (nN+1)\lambda \qquad [\text{from eq ①}]$$

$$N\,n(\lambda + d\lambda) = (nN+1)\lambda$$

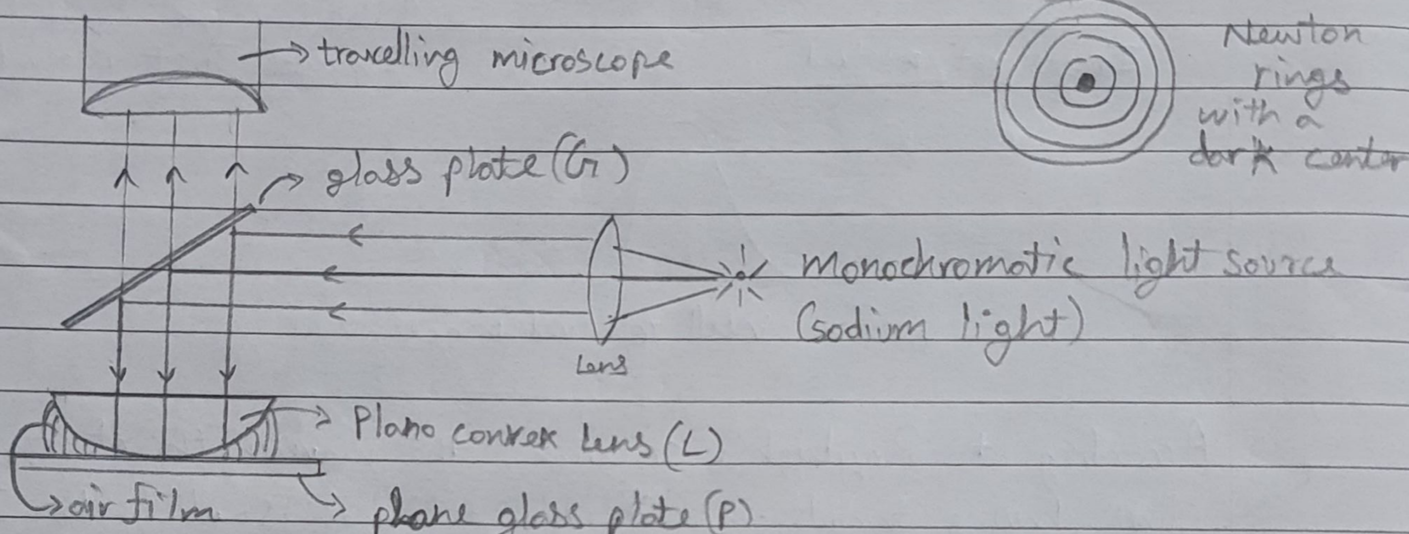
$$\cancel{nN\lambda} + nNd\lambda = \cancel{nN\lambda} + \lambda$$

$$nNd\lambda = \lambda$$

$$\boxed{nN = \frac{\lambda}{d\lambda}}$$ i.e the Resolving power of grating

The resolving power of grating is the product of total number of rulings (N) and the order of spectral lines (n)
and λ is the average of two nearby wavelengths
$d\lambda$ is the difference of two nearby wavelengths

2> Explain formation of Newton's ring in reflected Monochromatic light. Derive the wavelength of sodium light.



When a plano convex lens (L) of large radius of curvature is placed on a plane glass plate (P), an air film, of gradually increasing thickness from the point of contact, is formed between the upper part of glass plate and lower part of plano convex lens.
Light from monochromatic light source falls on glass plate G, inclined at 45° and get reflected from the front of plate towards L and falls normally on the airfilm enclosed between L and P.

Then this light gets reflected back upward from air film due to which interference takes place which results in the formation of bright and dark fringes observed by the help of a travelling microscope

These fringes are circular because the air film is symmetrical about the point of contact of the lens with ~~and~~ the glass plate.

Path difference of reflected ray, $\Delta = 2\mu t \cos r + \frac{\lambda}{2}$

for normal incidence of rays ($r=0$) and assembly in air ($\mu=1$), $\boxed{\Delta = 2t + \frac{\lambda}{2}}$ — (1)

for $t=0$, at centre, $\boxed{\Delta = \frac{\lambda}{2}}$ which is the condition for minimum intensity at the centre thus we get a dark ring at centre.

Condition of ~~Maxima~~ Bright Fringe

$$\Delta = n\lambda$$

$$2t + \frac{\lambda}{2} = n\lambda$$

$$2t = n\lambda - \frac{\lambda}{2} \qquad \boxed{2t = (2n-1)\frac{\lambda}{2}} \text{ — (A)}$$

Condition for Dark Fringe

$$\Delta = (2n+1)\frac{\lambda}{2}$$

$$2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

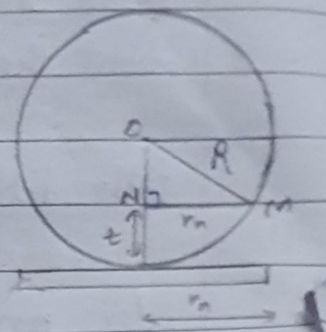
$$2t = (2n+1-1)\frac{\lambda}{2} = 2n\frac{\lambda}{2}$$

$$\boxed{2t = n\lambda} \text{ — (B)}$$

Diameter of fringes

Let us consider a convex lens of radius of curvature 'R' from which a plano convex lens of thickness 't' is cut.

r_n is the radius of newton's ring.



<u>In Δ ONM</u> by pythagoras theorem

$$(OM)^2 = (ON)^2 + (MN)^2$$
$$R^2 = (R-t)^2 + (r_n)^2$$
$$\cancel{R^2} = \cancel{R^2} + t^2 - 2Rt + r_n^2 \quad \left[\text{neglecting } t^2 \because r_n^2 >>> t^2\right.$$
$$2Rt = r_n^2$$
$$\boxed{2t = \frac{r_n^2}{R}} - (C)$$

→ For bright fringe, eq (A), $2t = (2n-1)\frac{\lambda}{2}$

$$\frac{r_n^2}{R} = (2n-1)\frac{\lambda}{2}$$

$$\left(\frac{D_n}{2}\right)^2 = (2n-1)\frac{\lambda R}{2}$$

$$\frac{D_n^2}{\cancel{4}_2} = (2n-1)\frac{\lambda R}{\cancel{2}}$$

$$D_n = 2\lambda R(2n-1)$$
$$\boxed{D_n = \sqrt{2n R(2n-1)}}$$

<u>or</u> $\boxed{D_n \propto \sqrt{2n-1}}$ where, $n = 1, 2, 3$

i.e $D_n \propto \sqrt{1}, \sqrt{3}, \sqrt{5}, \ldots$

$\therefore$ Diameter of n^{th} bright fringe is directly proportional to square root of odd numbers.

→ For dark fringe, eq (B), $2t = n\lambda$

$$\frac{r_n^2}{R} = n\lambda$$

$$\frac{D_n^2}{4} = n\lambda R$$

$D_n^2 = 4n\lambda R$ or $\boxed{D_n \propto \sqrt{n}}$ where, $n = 1, 2, 3$

$\boxed{D_n = \sqrt{4n\lambda R}}$ i.e $D_n \propto 1, 2, 3$

∴ The diameter of n^{th} dark fringe is directly proportional to square root of natural numbers n

⇒ Determination of wavelength of monochromatic light

Let D_n and D_{n+p} be the diameter of n^{th} and $(n+p)^{th}$ dark fringe

$$D_n^2 = 4n\lambda R$$

$$D_{n+p}^2 = 4(n+p)\lambda R$$

$$\therefore D_{n+p}^2 - D_n^2 = 4\lambda R(n+p) - 4\lambda R n$$

$$D_{n+p}^2 - D_n^2 = 4\lambda R(n+p-n)$$

$$D_{n+p}^2 - D_n^2 = 4p\lambda R$$

$$\boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}}$$

By using this equation we can find the wavelength of sodium light which comes out to be 5896 Å.

Q3) How can Newton's ring be used to determine the refractive index of liquid? Derive necessary formula.

Ans Path difference in thin film, $\Delta = 2\mu t \cos r + \frac{\lambda}{2}$

at $r = 0$, $\cos r = 1$

$$\Delta = 2\mu t + \frac{\lambda}{2}$$

where μ is the refractive index of liquid.

For bright fringe, $\Delta = n\lambda$

$$2\mu t + \frac{\lambda}{2} = n\lambda$$

$$2\mu t = (2n-1)\frac{\lambda}{2}$$

$$\boxed{2t = \frac{(2n-1)\lambda}{2\mu}}$$

By geometrical property of a circle, $2t = \frac{r_n^2}{R}$

$$\frac{(2n-1)\lambda}{2\mu} = \frac{r_n^2}{R}$$

$$\frac{D_n^2}{4_2} = (2n-1)\frac{\lambda R}{2\mu}$$

$$D_n^2 = \frac{2\lambda R(2n-1)}{\mu}$$

for $(n+p)^{th}$ fringe, $D_{n+p}^2 = \frac{2\lambda R(2(n+p)-1)}{\mu}$

$$\therefore D_{n+p}^2 - D_n^2 = \frac{2\lambda R}{\mu}\left[2n+2p-1-2n+1\right] = \frac{2\lambda R}{\mu}(2p)$$

$$(D_{n+p}^2 - D_n^2)_{liquid} = \frac{4p\lambda R}{\mu} \quad -(1)$$

$$(D_{n+p}^2 - D_n^2)_{air} = 4p\lambda R \quad -(2)$$

Dividing equation (1) & (2)

$$\frac{(D_{n+p}^2 - D_n^2)_{liquid}}{(D_{n+p}^2 - D_n^2)_{air}} = \frac{4p\lambda R \times \frac{1}{\mu}}{4p\lambda R}$$

$\therefore$ Refractive index of liquid i.e $\boxed{\mu = \frac{(D_{n+p}^2 - D_n^2)_{air}}{(D_{n+p}^2 - D_n^2)_{liquid}}}$

Q4> Explain briefly why Newton's rings are circular.

Explain difference between interference and diffraction.

Explain difference between fresnel's diffraction and fraunhofer diffraction

Q7> Explain difference between fresnel's diffraction and fraunhofer diffraction.

Ans Newton's rings are circular because the air film is symmetrical about the point of contact of plano convex lens with the glass plate.

Interference	Diffraction
i> The superpositioning of two waves is called interference	i> The bending of light is called Diffraction
ii> The interference fringes may or may not have same width.	ii> Diffraction fringes do not have same width.
iii> The region of minimum intensity is usually almost perfectly dark.	iii> The region of minimum intensity is not perfectly dark.
iv> All maxima are of same intensity.	iv> All maxima are of varying intensities.
v> Types of interference :- a> Constructive interference b> Destructive interference	v> Types of diffraction :- a> Fresnel's diffraction b> Fraunhofer diffraction

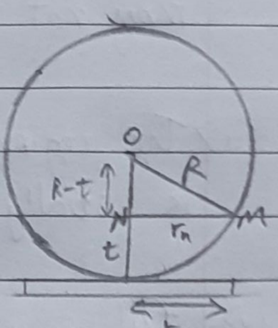
Fresnel Diffraction	Fraunhofer Diffraction
i> The source and screen are at finite difference	i> The source and screen are at infinite distance.
ii> The wavefronts are divergent either spherical or cylinderical	ii> The wavefronts are plane which is realised by using convex lens.
iii> No mirror or lenses are used for observation	iii> Diffracted rays are collected by a lens on a telescope for observation.
iv> for obtaining Fresnel's diffraction, zone plates are used	iv> For obtaining fraunhofer diffraction, single or double slits or diffraction grating is used

v) The center of diffraction pattern maybe bright or dark depending on the number of fresnel zones.	v) The center of the diffraction pattern is always bright for all paths parallel to axis of lens

Q12) In Newton's ring show that the diameter of bright rings is proportional to square root of odd natural number. The diameter of 10th dark ring due to wavelength 6000 A° in air is 0.5 cm. Find the radius of curvature of lens.

Ans Let us consider a convex lens of radius of curvature R from which a plano convex lens of thickness 't' is cut.

r_n is the radius of n^{th} newton's ring



In ΔONM by pythagoras theorem

$$(OM)^2 = (ON)^2 + (NM)^2$$

$$R^2 = (R-t)^2 + (r_n)^2$$

$$\cancel{R^2} = \cancel{R^2} + t^2 - 2Rt + r_n^2 \quad [\text{neglecting } t^2 \because r_n^2 >>> t^2]$$

$$2Rt = r_n^2$$

$$\boxed{2t = \frac{r_n^2}{R}}$$

For bright fringe, $\Delta = n\lambda$

$$2t + \frac{\lambda}{2} = n\lambda$$

$$2t = (2n-1)\frac{\lambda}{2}$$

$$\frac{r_n^2}{R} = (2n-1)\frac{\lambda}{2}$$

Path difference in thin film,

$$\Delta = 2\mu t \cos r + \frac{\lambda}{2}$$

$(r = 0, \mu = 1)$

$$\boxed{\mu = 2t + \frac{\lambda}{2}}$$

$$\frac{D_n^2}{\cancel{4}_2} = (2n-1)\frac{\lambda R}{\cancel{2}}$$

$$D_n^2 = 2\lambda R(2n-1)$$

$\boxed{D_n = \sqrt{2\lambda R(2n-1)}}$ or $\boxed{D_n \propto \sqrt{2n-1}}$ where $n = 1, 2, 3 \ldots$

i.e. $D_n \propto \sqrt{1}, \sqrt{3}, \sqrt{5}$

Thus, Diameter of bright rings is proportional to square root of odd nature numbers

(Sol 12) Given, $n = 10$

$D_{10} = 0.5\ cm = 5 \times 10^{-1}\ cm$

$\lambda = 6000\ A^{\circ} = 6 \times 10^{3} \times 10^{-10} \times 10^{2} = 6 \times 10^{-5}\ cm$

$R = ?$

For dark fringe, $D_n = \sqrt{4 n \lambda R}$

$D_{10} = \sqrt{4 \times 10 \times 6 \times 10^{-5} \times R}$

$(5 \times 10^{-1})^2 = 24 \times 10^{-4} \times R$

$\frac{25 \times 10^{-2}}{24 \times 10^{-4}} = R$

Radius of curvature i.e $\boxed{R = 1.041 \times 10^{-2}\ cm}$

Q 11) In Newton's ring shows the diameter of 4th and 12th dark rings are .400 cm and .700 cm. Find the diameter of 20 dark rings

Sol Given, $D_4 = 0.4\ cm$

$D_{12} = 0.7\ cm$

$D_{20} = ?$

$D_{n+p}^2 - D_n^2 = 4 p \lambda R$

$\lambda R = \frac{D_{4+8}^2 - D_4^2}{4(8)} = \frac{0.49 - 0.16}{32} = \frac{33 \times 10^{-2}}{32}$

$\boxed{\lambda R = 1.031 \times 10^{-2}}$

for Dark fringe, $D_n^2 = 4 n \lambda R$

$D_{20} = \sqrt{4 \times 20 \times 1.031 \times 10^{-2}}$

$D_{20} = \sqrt{8248 \times 10^{-4}}$

$D_{20} = 90.81 \times 10^{-2}$

Diameter of 20 dark rings $\boxed{D_{20} = 0.9081\ cm}$

Q 13) A parallel beam of sodium light is allowed to be incident normally on a plane grating having 4250 lines per/cm and a second order spectral lines is observed to be deviated through 30°. Calculate the wavelength of spectral line.

Sol

$e+d = 4250$ lines per cm

$n = 2$

$\theta = 30°$

$\lambda = ?$

For ~~bright f~~ maxima, $(e+d)\sin\theta = n\lambda$

$$\lambda = \frac{4250 \sin 30°}{2}$$

$$\lambda = 2125 \times \frac{1}{2}$$

$$\boxed{\lambda = 1062.5 \text{ cm}}$$

Q14) In Michelson's Interferometer 200 fringes cross the field of view when movable mirror is displaced through 0.0589 mm. Calculate the ~~wate~~ wave length of monochromatic light used.

Ans

$N = 200$

$l = 0.0589$ mm

$\lambda = ?$

$$\lambda = \frac{2l}{N} = \frac{2 \times 0.0589}{200} = \frac{589 \times 10^{-4}}{100} = 589 \times 10^{-6}$$

$$\boxed{\lambda = 5.89 \times 10^{-4} \text{ mm}}$$