Resonant frequency,

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{0.3\times 4\times 10^{-6}} - \frac{15^2}{(0.3)^2}} = 145.27\ Hz$$

Dynamic impedance,

$$Z_r = \frac{L}{CR} = \frac{0.3}{4\times 10^{-6}\times 15} = 5000\ \Omega$$

$$Q\text{-factor} = \frac{2\pi f_r L}{R} = \frac{2\pi \times 145.27 \times 0.3}{15}$$

$$= 18.255$$

POLYPHASE CIRCUIT (3 Phase)

Polyphase circuit means, the circuits to having more than one phases or windings. Each phase having a single alternating voltage of the same magnitude & frequency. Hence, a polyphase system is essentially a combination of two or more than two voltages having same magnitude & frequency but displaced from each other by equal electrical angle. This angular displacement between the adjacent voltages is called phase difference & depends upon the no of phases.

$$\text{Phase diff.} = \frac{360 \text{ electrical degree}}{\text{No of phases.}}$$

But, for 2 phase system the above is wrong where the voltages are displaced by 90° electrical.

Thus, an ac system having a group of equal voltages of same frequency arranged to have equal phase difference b/w adjacent emfs is called a **Polyphase System.**

The polyphase system may be two phase, three phase or six phase system. But for all practical purposes, 3-phase system is invariably employed.

Advantages of 3 phase system over 1-ϕ system.

1) In single phase, the power delivered is pulsating. Even when the voltage & current are in phase, the power is zero twice in each cycle. [illegible]

polyphase system, power delivered almost constant when the loads are balanced.

2) The rating (output) of a 3-phase machine is nearly 1.5 times the rating (output) of a single phase machine of the same size.

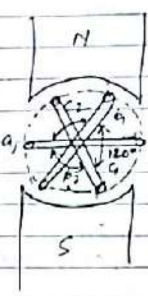
3) Power transmission in 3-phase system is economic.

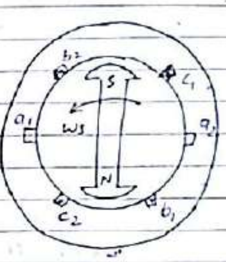
4) The 3-phase IM is ~~more~~ superior than 1-phase IM. because (i) 3-phase IM are self starting whereas 1 phase IM have no starting torque without using auxiliary means. (ii) 3-phase IM have higher power factor & efficiency than that of 1 phase IM.

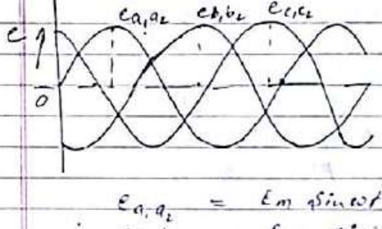
Generation of 3-phase EMF's

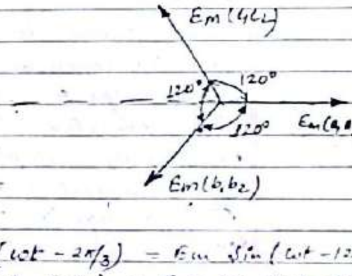
In a 3-ϕ system, there are 3 equal voltages of the same magnitude & frequency & ~~phase~~ having a phase difference of 120°. When the windings are rotated in a stationary magnetic field or when the windings are kept stationary & the magnetic field is rotated, an emf is induced in each ~~coil~~ winding or phase. These emfs are of same magnitude & frequency but displaced from one another by 120° electrical.

Consider three identical coils a_1a_2, b_1b_2 & c_1c_2. a_1, b_1 & c_1 are start terminals & a_2, b_2 & c_2 are finish terminals. Phase difference between them is 120°. The 3 coils are mounted on same axis & rotated in anticlockwise direction at ω radians/second. The emf's induced in the coils are as follows:









$$e_{a_1a_2} = E_m \sin \omega t$$
$$e_{b_1b_2} = E_m \sin(\omega t - 2\pi/3) = E_m \sin(\omega t - 120°)$$
$$e_{c_1c_2} = E_m \sin(\omega t - 4\pi/3) = E_m \sin(\omega t - 240°)$$

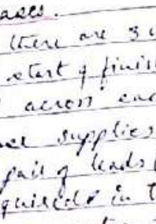
Phase Sequence The order in which the voltages or emfs in the three phases attain their maximum positive value is called the phase sequence. It is essentially ~~because~~ in the following application.

1) The direction of rotation of 3-phase induction motors depends upon the phase sequence of 3-ϕ supply. To reverse the direction of rotation, the ϕ sequence of the supply given to the motor has to be changed.

2) The parallel operation of 3-phase alternators & transformers is only possible if phase sequence is known.

Inter connection of Three Phases.

In a 3-ϕ ac generator, there are 3 wdgs. Each wdg has 2 terminals (start & finish). If a seperate load is connected across each phase wdg, then each phase supplies as independent load through a pair of leads (wires). Thus, six wires will be required in this case, to connect the load to generator. This will make the whole system complicated & expensive.

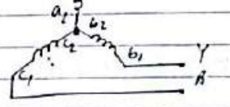


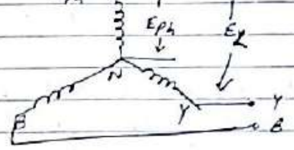
In order to reduce the no. line conductors, the three phase wdgs are suitably inter connected. The two universally adopted methods of inter connection of 3 phases are

1) Star or wye (Y) connection
2) Mesh or Delta (Δ) connection

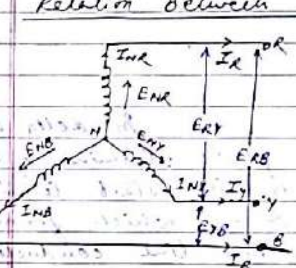
These methods are also beneficial for some important applications.

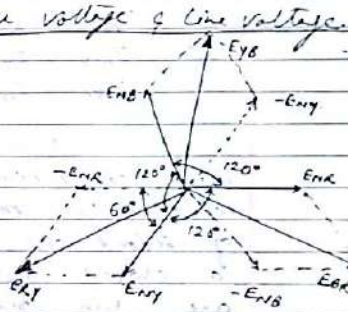
1) Star or Wye (Y) connection :— In star or Y connection, the similar ends of the 3 wdgs. are connected to a common point called star or neutral point. The 3 line conductors are run from the remaining 3 free terminals called line conductors. Generally 3 wires are carried to the external ckt giving 3 phase 3-wire star connected system. However some times a fourth wire is carried out from the star point to the external ckt. called neutral wire giving 3 phase, 4 wire star connected system.





Relation Between phase voltage & line voltage.





Here, $E_{NR} = E_{NY} = E_{NB} = E_{ph}$ (in magnitude)

Between two lines there are 2 phase voltg.

Taking the loop NRYN, we get

$\overline{E_{NR}} + \overline{E_{RY}} - \overline{E_{NY}} = 0$

$\overline{E_{RY}} = \overline{E_{NY}} - \overline{E_{NR}}$ (Vector Diff.)

To find the vector sum of E_{NY} & $-E_{NR}$, reverse the vector E_{NR} & add it vectorially with E_{NY}.

$\therefore \quad E_{RY} = \sqrt{E_{NY}^2 + E_{NR}^2 + 2 E_{NY} E_{NR} \cos 60^\circ}$

$E_L = \sqrt{E_{ph}^2 + E_{ph}^2 + 2 E_{ph} E_{ph} \cdot \frac{1}{2}}$

$= \sqrt{3 E_{ph}^2}$

$E_L = \sqrt{3}\, E_{ph}$ (in magnitude)

Similarly for $\bar{E}_{YR} = \bar{E}_{NB} - \bar{E}_{NY}$
& $\bar{E}_{BR} = \bar{E}_{NR} - \bar{E}_{BN}$ also we get.

$$\boxed{E_L = \sqrt{3}\, E_{ph}}$$

In phase current & line current

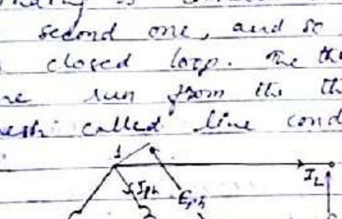
$I_L = I_{ph}$
line current = Phase current

$I_{NR} = I_R$

$I_{NY} = I_Y$

$I_{NB} = I_B$.

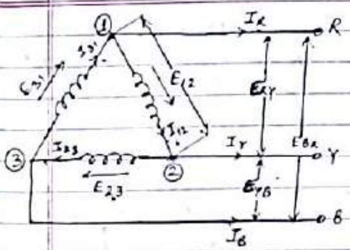
2) <u>Delta or Mesh connection</u> :- In delta or mesh connection, the finish terminal of one winding is connected to the start terminal of second one, and so on which forms a closed loop. The three line conductors are run from the three nodes of the mesh called line conductors. Delta connection does not contain any neutral point. So that it contains only 3 phase 3 wire system.

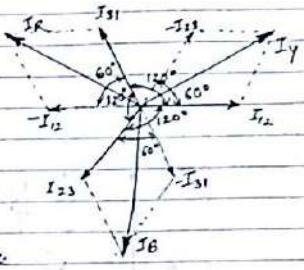


<u>Relationship Between Phase voltage & line voltage and Phase current & line current.</u>

∴ The phase voltages & line voltages in the delta connected system is same

∴ $E_{12} = E_{RY}$; $E_{23} = E_{YB}$ & $E_{31} = E_{BR}$





Line Voltage = phase Voltage

But in case of current,

$I_{12} = I_{23} = I_{31} = I_{ph}$ (in magnitude)

current is divided at every node 1, 2 & 3

Now, by applying Kirchoff's law at node 1.

$\bar{I}_{31} = \bar{I}_R + \bar{I}_{12}$

or $\bar{I}_R = \bar{I}_{31} - \bar{I}_{12}$ (vector Diff^n)

To find the vector sum of $\bar{I}_{31}$ & $-\bar{I}_{12}$ reverse the vector I_{12} & add it vectorially with $\bar{I}_{31}$.

$$\therefore\ I_R = \sqrt{I_{31}^2 + I_{12}^2 + 2 I_{31} I_{12} \cos 60°}$$

$$I_L = \sqrt{I_{ph}^2 + I_{ph}^2 + 2 I_{ph} I_{ph} \cdot \tfrac{1}{2}}$$

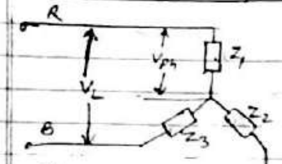
$$I_L = \sqrt{3 I_{ph}^2}$$

$$\boxed{I_L = \sqrt{3}\, I_{ph}}$$

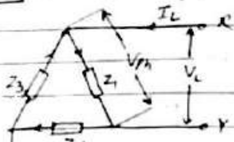
Similarly, for $I_Y = I_{12} - I_{23}$,
& $I_B = I_{23} - I_{31}$, we get

$$\boxed{I_L = \sqrt{3}\, I_{ph}}$$

Power in 3φ Circuits :-



Load Connection in Star



Load connection in Delta.

Power in 1-phase system is
$$P = VI\cos\phi$$
Now, Power in 3-phase system is sum of power in three phases

$\therefore \quad P = 3VI\cos\phi = 3V_{ph}I_{ph}\cos\phi$.

In star connection,
$$P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \cos\phi$$
$$P = \sqrt{3}\,V_L I_L \cos\phi \quad \left[\because V_{ph} = V_L/\sqrt{3} \text{ \& } I_L = I_{ph}\right]$$

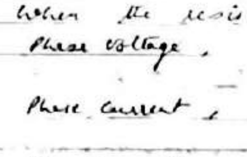
In delta connection,
$$P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \cos\phi.$$
$$= \sqrt{3}\,V_L I_L \cos\phi. \quad \left[\because V_{ph} = V_L \text{ \& } I_{ph} = I_L/\sqrt{3}\right]$$

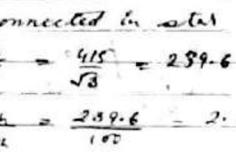
Thus, the total power in a 3 phase balanced load, irrespective of their connections (star or delta) is given by the relation $\sqrt{3}\,V_L I_L \cos\phi$.

Its unit is kW. or Watts.

Now, Active Power, $P = \sqrt{3}\,V_L I_L \cos\phi$ kW.
Reactive Power, $Q = \sqrt{3}\,V_L I_L \sin\phi$ KVAR.
Apparent Power, $S = \sqrt{3}\,V_L I_L$ KVA.

Ques Three 100 Ω resistors are connected first in star & then in delta across 415 V, 3-phase supply. Calculate the line & phase currents in each & also the power taken from the source.





Soln:- When the resistors are connected in star

Phase voltage, $V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6\,V$

Phase current, $I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{239.6}{100} = 2.396\,A$

Line current, $I_L = I_{ph} = 2.396\,A$

Power drawn, $P = 3 I_{ph}^2 R = 3 \times (2.396)^2 \times 100$
$= 1722\,W.$

When resistors are connected in delta.

$V_{ph} = V_L = 415\,V$

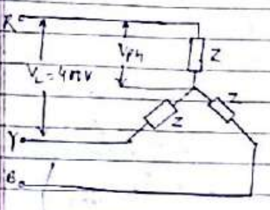
$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{415}{100} = 4.15\,A.$

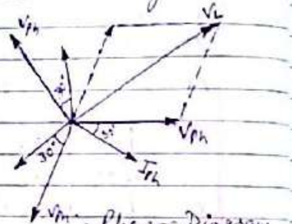
$I_L = \sqrt{3}\,I_{ph} = 4.15 \times \sqrt{3} = 7.188\,A$

$\therefore$ Power drawn, $P = 3 I_{ph}^2 R_{ph} = 3 \times (4.15)^2\, 100$
$= 5166\,W.$

Ques A balanced star connected load is supplied from a balanced 3 phase, 400V, 50Hz system. The current in each phase is 30A & lags 30° behind the phase voltage. Find →

i) The total power
ii) Phase voltage. Draw the phasor diagram





V_{ph}. Phasor Diagram

Phase Voltage, $V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231V$

Power factor, $\cos\phi = \cos 30° = 0.866$ lag

Line Current $I_L = I_{ph} = 30A$

Total Power, $P = \sqrt{3} V_L I_L \cos\phi$

$= \sqrt{3} \times 400 \times 30 \times 0.866$

$= 18000 W = 18 kW$

Power Measurement in 3-phase circuits :-

In ac circuits, power is measured with the help of a wattmeter. A wattmeter is an instrument which consists of two coils called current coils & potential coil. The current coil having low resistance is connected in series with loads so that it carries the load current. The potential coil having high resistance is connected across the load and carries the current proportional to the potential difference.

For measuring power in a polyphase system, more than one wattmeter is required or more than one readings are made by one wattmeter. The first method is more convenient & the no. of wattmeters required to measure power in a given polyphase system is determined by "Blondel's theorem".

When power is supplied by K-wire ac system, the no. of wattmeters required to measure power is one less than the no. of wires ie (K-1), regardless the load is balanced or unbalanced.

Hence three wattmeters are required to measure power in three phase, 4 wire system whereas only two wattmeters are required to measure power in 3 phase, 3 wire system.

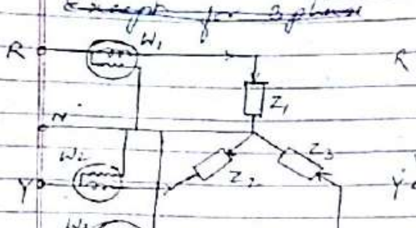
Three Wattmeter Method :-

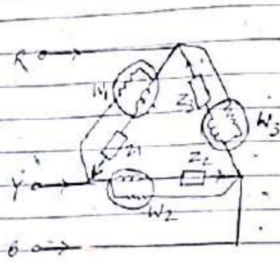
This method is employed to measure power in 3 phase, 4 wire system. However, this method can also be employed in 3 phase 3 wire delta connected load, where power consumed by each load is required to be determined seperately. The connections for both star & delta connected loads are shown in figure.

The total power is given by the algebraic sum of the readings of three wattmeters ie

Total Power, $\boxed{P = W_1 + W_2 + W_3}$

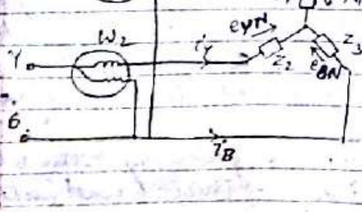
~~Except for 3 phase~~

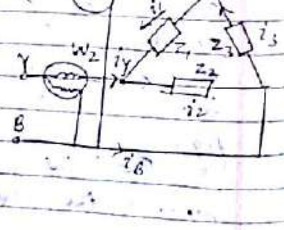




Two Wattmeter Method → This method can be employed to measure power in a 3 phase, 3 wire star or delta connected balanced or unbalanced load. In this method, the current coils of the wattmeters are connected in any two lines, say R & Y and the potential coil of each wattmeter is joined across the same line and the ~~third~~ third line (i.e. B).

It can be proved that the sum of the powers measured by the two wattmeters W_1 & W_2 is equal to the total instantaneous power absorbed by the three loads Z_1, Z_2, Z_3.





Considering star connections:-

Instantaneous current through current coil of $W_1 = i_R$

Instantaneous — " — $W_2 = i_Y$

Instantaneous P.d. across potential coil of $W_1 = (e_{RN} - e_{BN})$

Instantaneous Power measured by $W_1 = i_R(e_{RN} - e_{BN})$

Instantaneous P.d. across potential coil of $W_2 = (e_{YN} - e_{BN})$

Instantaneous Power measured by $W_2 = i_Y(e_{YN} - e_{BN})$

$$\therefore W_1 + W_2 = i_R(e_{RN} - e_{BN}) + i_Y(e_{YN} - e_{BN})$$

$$= i_R e_{RN} + i_Y e_{YN} - e_{BN}(i_R + i_Y)$$

$$= i_R e_{RN} + i_Y e_{YN} + i_B e_{BN}$$

$(\because i_R + i_Y + i_B = 0)$

= Total power absorbed in the three loads at any instant (i.e. P)

or $\boxed{P = W_1 + W_2}$

Considering delta connections :-

Instantaneous current through current coil of $W_1 = i_R = i_1 - i_3$

Instantaneous P.d. across potential coil of $W_1 = e_{RB}$

$\therefore$ Instantaneous power measured by $W_1 = e_{RB}(i_1 - i_3)$

Instantaneous current through CC of $W_2 = i_Y = i_2 - i_1$

Instantaneous P.d across PC of $W_2 = e_{YB}$

$\therefore$ Instantaneous Power measured by $W_2 = e_{YB}(i_2 - i_1)$

$$\therefore W_1 + W_2 = e_{RB}(i_1 - i_3) + e_{YB}(i_2 - i_1)$$

$$= i_1 e_{RB} - i_3 e_{RB} + i_2 e_{YB} - i_1 e_{YB}$$

$$\cancel{= i_1 e_{RB} + i_2 e_{YB} - e}$$

$$= i_2 e_{YB} + i_3 e_{BR} - i_1(e_{YB} + e_{BR})$$

$(\because -e_{RB} = e_{BR})$

$$= i_1 e_{RY} + i_2 e_{YB} - i_3 e_{BR}$$

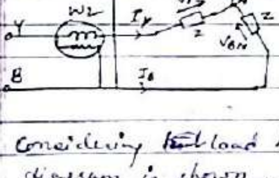
$(\because e_{RY} + e_{YB} + e_{BR} = 0)$

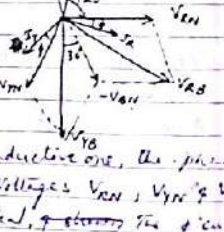
= Total power absorbed in the three loads at any instant (i.e. P)

$\therefore \quad P = W_1 + W_2$

It may be noted that power measured by two wattmeters at any instant is the instantaneous power absorbed by the three loads connected in three phases. In fact, this power is the average power drawn by the load ~~they~~ since the wattmeters read the average power because of the inertia of their moving system.

Two-Wattmeter Method (Balanced Load)





Considering ~~the~~ load to be an inductive one, the phasor diagram is shown. The 3ϕ voltages V_{RN}, V_{YN} & V_{BN} displaced by an angle 120° electrical, ~~& shown~~ the ϕ current lag behind their respective phase voltages by an angle ϕ.

Current through CC of $W_1 = I_R$

P.d. across PC of $W_1 = \bar{V}_{RB} = \bar{V}_{RN} - \bar{V}_{BN}$

To ~~obtain V_{RB}, reverse the phasor V_{BN} & add it vectorially to phasor V_{RN}~~. The phase difference b/w V_{RB} & I_R is $(30° - \phi)$.

$\therefore$ Power measured by wattmeter, $W_1 = V_{RB} I_R \cos(30° - \phi)$

current through CC of $W_2 = I_Y$

P.d. across PC of $W_2 = \bar{V}_{YB} = \bar{V}_{YN} - \bar{V}_{BN}$

$\therefore$ Power measured by wattmeter, $W_2 = V_{YB} I_Y \cos(30° + \phi)$

Since the load is balanced;

$$I_R = I_Y = I_B = I_L$$
$$V_{RY} = V_{YB} = V_{BR} = V_L$$

$\therefore$ Wattmeter reading $W_1 = V_L I_L \cos(30° - \phi)$

—— " —— $W_2 = V_L I_L \cos(30° + \phi)$

Sum of two wattmeter readings $= W_1 + W_2$

$$= V_L I_L \cos(30° - \phi) + V_L I_L \cos(30° + \phi)$$
$$= V_L I_L [\cos(30° - \phi) + \cos(30 + \phi)]$$
$$= V_L I_L [\cos 30° \cos\phi + \sin 30° \sin\phi + \cos 30° \cos\phi - \sin 30° \sin\phi]$$
$$= V_L I_L \, 2\cos 30° \cos\phi = V_L I_L \, 2 \cdot \frac{\sqrt{3}}{2} \cos\phi$$
$$= \sqrt{3} V_L I_L \cos\phi .$$

= Total Power absorbed by 3ϕ balanced load (P)

$\therefore \quad P = W_1 + W_2$

Thus, sum of the readings of two wattmeters is equal to the power absorbed in a 3phase balanced load.

Determination of Power factor by wattmeter readings.

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi . \quad \text{--- (1)}$$
$$W_1 - W_2 = V_L I_L [\cos(30° - \phi) - \cos(30° + \phi)]$$
$$= 2 V_L I_L \sin 30° \sin\phi = V_L I_L \sin\phi . \quad \text{--- (2)}$$

Divide eqn (2) by (1), we get

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi}$$

$$\tan\phi = \sqrt{3} \, \frac{W_1 - W_2}{W_1 + W_2}$$

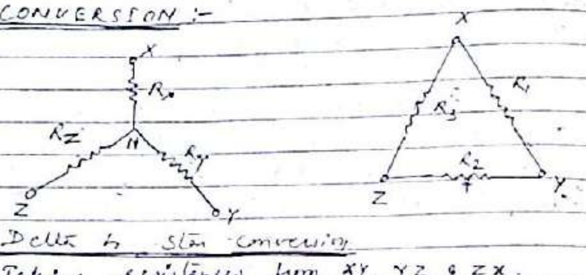
Power factor of load, $\cos\phi = \cos\left[\tan^{-1} \sqrt{3} \frac{(W_1 - W_2)}{(W_1 + W_2)}\right]$

Reactive Power $\Rightarrow$ Multiply eqn (2) by $\sqrt{3}$ we get

$$\sqrt{3}(W_1 - W_2) = \sqrt{3} V_L I_L \sin\phi = Q$$

$\therefore$ Reactive Power, $Q = \sqrt{3}(W_1 - W_2)$.

CONVERSION :-



Delta to Star Conversion

Taking resistances from XY, YZ & ZX.

In Star, $R_{XY} = R_X + R_Y$

$R_{YZ} = R_Y + R_Z$

$R_{ZX} = R_Z + R_X$

In Delta, $R_{XY} = R_1 // (R_2+R_3) = \frac{R_1R_2 + R_1R_3}{R_1+R_2+R_3}$

$R_{YZ} = R_2 // (R_1+R_3) = \frac{R_1R_2 + R_2R_3}{R_1+R_2+R_3}$

$R_{ZX} = R_3 // (R_1+R_2) = \frac{R_1R_3 + R_2R_3}{R_1+R_2+R_3}$

Equating cell equs we get.

$R_X + R_Y = \frac{R_1R_2 + R_1R_3}{R_1+R_2+R_3}$ --- (1)

$R_Y + R_Z = \frac{R_1R_2 + R_2R_3}{R_1+R_2+R_3}$ --- (2)

$R_Z + R_X = \frac{R_1R_3 + R_2R_3}{R_1+R_2+R_3}$ --- (3)

equ (1) − (2) we get

$R_X - R_Z = \frac{R_1R_3 - R_2R_3}{R_1+R_2+R_3}$ --- (4)

equ (4) + (3) we get.

$2R_X = \frac{R_1R_3 + R_2R_3 + R_1R_3 - R_2R_3}{R_1+R_2+R_3}$

$\boxed{R_X = \frac{R_1R_3}{R_1+R_2+R_3}}$ — (a)

Similarly, $\boxed{R_Y = \frac{R_1R_2}{R_1+R_2+R_3}}$ — (b)

$\boxed{R_Z = \frac{R_2R_3}{R_1+R_2+R_3}}$ — (c)

Star to Delta Conversion

Now, take the above equation & multiply each with other & add them, we get

$R_XR_Y + R_YR_Z + R_ZR_X = \frac{R_1R_2R_1R_3 + R_1R_2R_2R_3 + R_1R_3R_2R_3}{(R_1+R_2+R_3)^2}$

$R_XR_Y + R_YR_Z + R_ZR_X = \frac{R_1R_2R_3(R_1+R_2+R_3)}{(R_1+R_2+R_3)^2}$

$= \frac{R_1R_2R_3}{(R_1+R_2+R_3)}$

Now, divide whole equation by R_X we get

$R_Y + R_Z + \frac{R_YR_Z}{R_X} = \frac{R_1R_2R_3}{R_X(R_1+R_2+R_3)}$

$= \frac{R_1R_2\cancel{R_3}}{(R_1+R_2+R_3)} \times \frac{(R_1+R_2+R_3)}{R_1R_3}$

$= R_2$

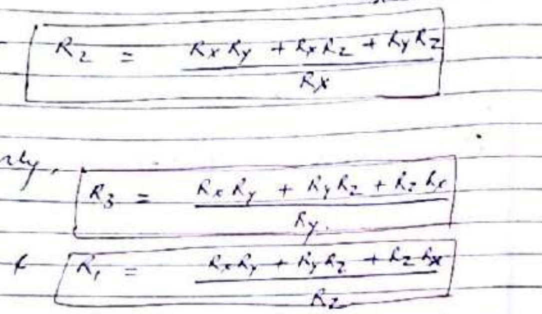
$R_2 = R_y + R_z + \frac{R_y R_z}{R_x}$

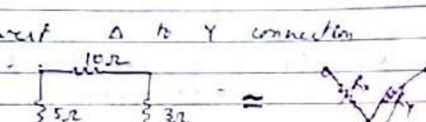
or $\boxed{R_2 = \frac{R_x R_y + R_x R_z + R_y R_z}{R_x}}$

Similarly, $\boxed{R_3 = \frac{R_x R_y + R_y R_z + R_z R_x}{R_y}}$

& $\boxed{R_1 = \frac{R_x R_y + R_y R_z + R_z R_x}{R_z}}$

Ques. Convert Δ to Y connection





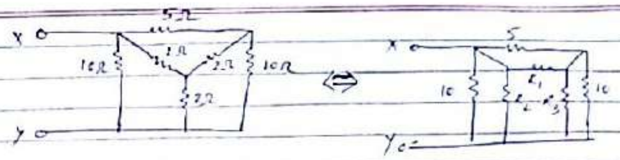
~~$R_x = 3\times5$~~

$R_x = \frac{10\times5}{5+10+3} = 2.78\,\Omega$

$R_y = \frac{10\times3}{5+10+3} = 1.67\,\Omega$

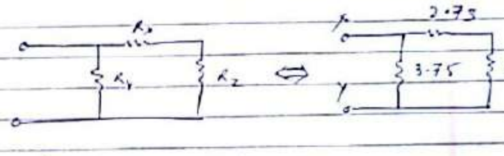
$R_z = \frac{5\times3}{5+10+3} = 0.83\,\Omega$

Ques. Find the equivalent resistance b/w terminals x-y in the resistive network of given figure.



$R_1 = \frac{2\times2 + 2\times2 + 2\times2}{2} = \frac{12}{2} = 6\,\Omega$

$R_2 = 6\,\Omega$ & $R_3 = 6\,\Omega$



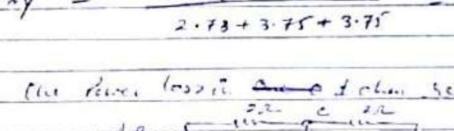
$R_x = \frac{5\times6}{5+6} = 2.73\,\Omega$

$R_y = \frac{10\times6}{10+6} = 3.75\,\Omega$

$R_z = \frac{10\times6}{10+6} = 3.75\,\Omega$

$R_{xy} = \frac{3.75\,(2.73+3.75)}{2.73+3.75+3.75} = 2.375\,\Omega$

Ques. Find the power loss in 1 ohm resistor



Sol.