# Important Questions

## UNIT-1

Q.1 Solve $\left(1+e^{x/y}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$

Q.2 Solve the differential equation $(D^2-2D+1)y=xe^x sinx$

Q.3 Solve the differential equation $(D^2+3D+2)y=4\cos^2 x$

Q.4 Solve the differential equation $(D^2-2D+1)y=xe^x sinx$

Q.5 Solve the differential equation $(D^2+4)y=e^{2x}+sin2x+x^2$

Q.6Solve $(1+e^y)cosx\,dx+e^y sinx\,dy=0$

Q.7 Solve $(1+y^2)dx-(\tan^{-1}y-x)dy=0$

Q.8 Solve the differential equation $x^2\frac{d^3y}{dx^3}+3x\frac{d^2y}{dx^2}+\frac{dy}{dx}=x^2logx$

Q.9Solve the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=(logx)sin(logx)$

Q.10 Solve the differential $\frac{dx}{dt}+y=sint$ , $\frac{dy}{dt}+x=cost$ given that $x=2\ and\ y=0\ when\ t=0$ .

## Unit -2

Q1 Solve the differential equation $x\frac{d^2y}{dx^2}-(2x-1)\frac{dy}{dx}(x-1)y=e^x$
Given that $y=e^x$ is one integral.

Q2 Using method of variation of parameters solve $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=\frac{e^{3x}}{x^2}$ .

Q3 Solve the differential equation $x\frac{d^2y}{dx^2}-\frac{dy}{dx}-4x^3y=x^5$.

Q4 Solve the differential equation $\frac{d^2y}{dx^2}-cotx-(1-cotx)y=e^x sinx$.

Q5. Solve by changing the independent variables

$$(1+x^2)^2\frac{d^2y}{dx^2}+2x(1+x^2)\frac{dy}{dx}+4y=0.$$

Q.6 Solve $cosx\frac{d^2y}{dx^2}+sinx\frac{dy}{dx}-2ycos^3x=2cos^5x$.

Q.7 Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$, given that $x+\frac{1}{x}$ is one integral.

Q.8 Solve in series $(1-x^2)\frac{d^2y}{dx^2}$ -2 x $\frac{dy}{dx}$ +n(n+1) y = 0 (Legendre's differential equation)

Q.9 Solve in series Frobenius method 9x(1-x)$\frac{d^2y}{dx^2}$ - 12 $\frac{dy}{dx}$ + 4 y = 0

Q.10 Prove that by series solution of Bessel's differential equation $x^2\frac{d^2y}{dx^2}$ + x $\frac{dy}{dx}$ + ($x^2$- $n^2$) y = 0.

## Unit -3 (Partial diff. Equation)

Q.1 Form the Partial differential equation by eliminating the arbitrary function from

$$Z = y^2 + 2\, f\left(\frac{1}{x} + \log y\right)$$

Q.2 Solve $y^2p - xyq = x(z-2y)$

Q.3 Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$

Q.4 Solve $(y^2 + z^2 - x^2)p - 2yxq + 2zx = 0$

Q.5 Solve (i) $x2\, p^2 + y^2 q^2 = z^2$ (ii) $z^2(p^2 + q^2) = x^2 + y^2$

Q.6 Solve $(D^2 - DD' - 2D'^2)z = (y-1)e^x$

Q.7 Solve$(p^2 + q^2)y = qz$. By using charpit's method.

Q.8 Solve $\frac{\partial^3 z}{\partial x^3} - 7\frac{\partial^3 z}{\partial x \partial y^2} - 6\frac{\partial^3 z}{\partial y^3} = \sin(x + 2y)$

Q.9 Solve $(D^2 + DD' + D' - 1)z = \sin(x + 2y)$.

Q.10 Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ by the method of separation of variable, where

$$u(x,0) = 6e^{-3x}.$$

## Unit -4 (Complex Analysis)

Q1 Show that the function $f(z) = e^x(cosy + isiny)$ is analytic find its derivative.

Q2 Using Cauchy's integral formula, evaluate $\int_C \frac{e^{2z}}{(z+1)^4}$ dz, where C is the circle $|z| = 3$.

Q3 Integrate $z^2$ along the straight line OA and also along the path OBA Consisting of two points z = 3+i

Q4 Apply the calculus of residue to show that $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$ , a>b>0

Q5 If f (z) is an analytic function and f' (z) is continuous at each point within and on a simple closed curve C, then $\int_c f(z)dz = 0$

Q6 Determine the pole and residues at each point .If $f(z) = \frac{1-e^{2z}}{z^4}$ .

Q7 Evaluate the integral$\int_0^{2+i} \bar{z}^2$ dz, along the real axis from z=0 to z=2 and then along a Parallel to y-axis from z=2 to z=2+i.

Q8 Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic .Find a function v such that $f(z) = u + iv$ is analytic. Also express f (z) in term of z.

Q9 Apply the Calculus of residue to prove that $\int_0^{2\pi} \frac{\cos 2\theta}{(5+4\cos\theta)}\, d\theta = \frac{\pi}{6}$

Q10 Evaluate the residues of $\frac{z^2}{(z-1)(z-2)(z-3)}$ at z = 1, 2, 3 and $\infty$ and show that their sum is 0.

## Unit -5 (Vector Calculus)

Q.1 Find the directional derivative of $\nabla.(\nabla\emptyset)$ at point (1,-2,1) in the direction of the normal surface $xy^2z = 3x+z^2$, where $\emptyset = x^3y^2z^4$

Q.2 Evaluate $\iint_s \vec{F}.\hat{n}dS$, where $\vec{F}$ =18zi-12j+3yk, and S is the surface of the plane 2x+3y+6z= 12 in the first octant.

Q.3 Evaluate $\iint_s \vec{F}.\hat{n}dS$ where $\vec{F}$=xi-yj+$(z^2-1)$k and S is a closed surface bounded by the planes z = 0, z = 1 and the cylinder $x^2 + y^2 = 4$. Also verify Gauss's divergence theorem.

Q.4 Verify Stokes theorem for $F = (x^2+y^2)\, i - 2xy\, j$ taken around the rectangle bounded by the lines x = ± a, y = 0 and y = b.

Q.5 Show that the vector field $V$ = (siny + z)i + (xcosy - z)j +(x- y)k is irrotational.

Q.6 Prove that (i) $\nabla r^n = nr^{n-2}\vec{r}$ Where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

(ii) $curl\, \overline{F} = 0$ Where $\overline{F} = grad(x^3 + y^3 + z^3 - 3xyz)$

Q.7 (i) Find a unit vector normal to the surface xyz=4 at the point (-1,-1,2).

(ii) Show that the vector $\overline{F} = (-x^2 + yz)i + (4y + 2x)j + (2xz - 4z)k$

Q.8 Use stokes's theorem to evaluate: $\int_c [(x + y)dx + (2x - z)dy + (y + z)dz)]$ where C is the boundary of the triangle with vertices (2,0,0),(0,3,0),(0,0,6).

Q.9 Evaluate $\iint_S F.n\, ds$ if $\overrightarrow{F} = 4x\, i - 2y^2\, j + z^2\, k$. and S is the surface boundary the region $x^2 + y^2 = 4$ z=0 and z=3 .

Q.10 Find the total work done by the field in moving a particle in a force field given by $F$ = 3xy i – 5zj +10xk along the curve $x= t^2+1$, $y=2t^2$, $z= t^3$ from t=1 to t=2.