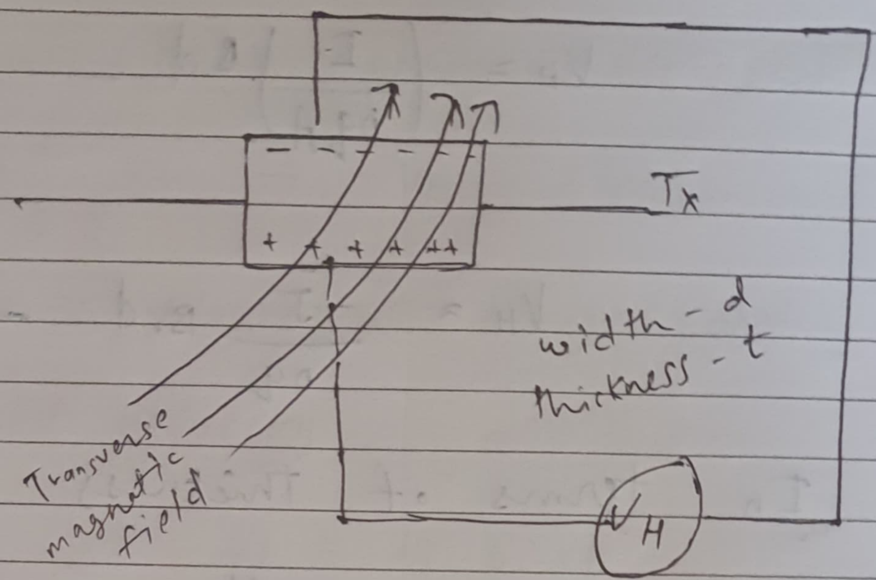
Unit - 3

Introduction to solids

~~Semi - Conductor~~

Hall effect



Use -

→ 1) it can be used to determine the type of semi conductor.

→ 2) what is the sign of majority charge failure

3) Sign of majority charge carier

4) Mobility of charge carrier concentration.

5) mean drift velocity of charge carrier.

Hall effect -

$$F_E = F_B$$

$$q E_H = q v_d B$$

$$E_H = v_d B$$

(v_d → drift velocity)

$$\frac{V_H}{d} = v_d B$$

$$\boxed{V_H = v_d B d} \quad -(1)$$

$\because \quad V_d = \left(\frac{I}{nqA}\right)$

$V_H = \left(\frac{I}{nqA}\right) B d$ —— (2)

$\left(J = \frac{I}{A}\right)$

$V_H = \frac{J}{nq} B.d$ —— (3)

In terms of thickness

Area $= dt$

$V_H = \frac{I}{nq(dt)} . B.\cancel{d}$

$V_H = \frac{I.B}{nqt}$

hall cofficient R_H

$R_H = \frac{E_H}{JB} = \frac{V_H/d}{JB}$

from (3)

$= \frac{\frac{\cancel{J}\,\cancel{B}\,\cancel{d}}{nq}/\cancel{d}}{\cancel{J}\,\cancel{B}}$

$\boxed{R_H = \frac{1}{nq}}$ —— (4)

If $R_H \rightarrow +ve$, holes are dominat charge carrier (p-type)

if $R_H \rightarrow -ve$ electron will be predominant charge carrier

drift velocity

$V_H = V_d . B. d$

$\frac{V_H}{B.d} = V_d$

Hall mobility (μ_H) - drift velocity required in unit electric field.

$\mu_H = \sigma R_H$

hall angle -

$\tan\theta_H = \frac{E_H}{E_x}$

$\theta_H = \tan^{-1}\frac{E_H}{E_x}$

$E_H = \frac{J}{\sigma}$, $E_H = \frac{V_H}{d} = \frac{JB}{nq}$

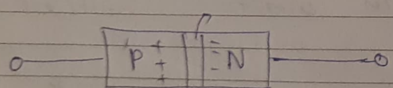
$$\theta_H = \tan^{-1} \frac{\cancel{J} B / nq}{\cancel{J} / \sigma}$$

$$\theta_H = \tan^{-1} \frac{B\sigma}{nq}$$

$$\boxed{\theta_H = \tan^{-1} R_H B\sigma}$$

I-V characteristic of P-N junction diode-

P-N junction



Semiconductor

- Intrinsic / pure (Ge, Se)
- Extrinsic
  - P-type
  - N-type

Conductor	Semi-Conductor	Insulator
1) there is a large no of free electron hence electric Conduction is possible Ex-> Silver, Copper	The ~~no~~ number of free electron neither very large nor very low. Ex - Carbon, Silicon	The ~~no~~ number of free electron very low almost negligible Ex - wood, glass.
2) The resistivity is very low 10^{-8} to 10^{-6} $\Omega \times m$	the resistivity Is 10^{-1} $\Omega \times m$	the resistivity is very high 10^{9} to 10^{11}
3) On Increasing temperature the resistivity increase and conductivity decrease	In temp. Inc their resistance decrease. Conductivity Inc.	
4) forbidden energy gain $\Delta E = 0$ conduction band overlap with the valance band	the forbidden gap between conduction and valance band $\Delta E = 1\,eV$	the valance band completely fill and conduction band empty

Semi-conductor - a solid substance that allows heat or electricity to pass through it or along it in particular conditions.

or

any class of crystalline solids intermediate in electrical conductivity between a conductor and an insulator.

Types of semi-conductor -

(1) **Intrinsic Semiconductor** - $Si\ (z=14), Ge(z=32)$

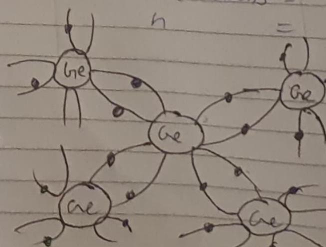
These atoms have four electrons in their outer most orbit. These are called the valance electrons.

An intrinsic (pure) semiconductor or i type semiconductor is pure semiconductor without any significant dopant species present.

no of excited electrons = number of holes

$n = p$

Intrinsic →
(i type)



Mobility of charge carries - when charged particle move in a crystal under the action of an external electric field, their velocity is directly proportinal to the intensity of external electric field.

$v \propto E$, $v = \mu E$

v is the drift velocity.
E is the intensity of electric field
μ is mobility of charge carries which is measured in $m^2 V^{-1} s^{-1}$.

if mobility of electron in the conduction band ~~to~~ is μ_n and hole in valance band is μ_p.

the Intrinsic semiconductor $\mu_n > \mu_p$

Conduction in intrinsic Semi Conductor - Let the number of electron per unit volume in the Conduction band of semiconductor is n. the charge per unit volume is ne charge will move with drift velocity v_n

$$V_n = -\mu_n E$$

Current density of electron

$$J_n = -n e V_n$$

(-) sign show that the direction of In is opposite to the direction of motion of electrons)

$J_n = ne\mu_n E$

if the no of holes per unit volume In the valance band p and mobility of holes μ_p.

current density
$$J_p = pe\mu_p E$$
Hence total current density

$$J = J_p + J_n$$
$$J = ne\mu_n E + pe\mu_p E$$
$$J = (ne\mu_n + pe\mu_p)E$$
$$J = \sigma E$$

σ is conductivity of crystal

$$\boxed{\sigma = ne\mu_n + pe\mu_p}$$

Fermi-level - The energy level which divides the field and empty level is called fermi level and the corresponding energy of that level is known as Fermi level.

$$\frac{1}{X^2}\frac{\partial^2 X}{\partial x^2} + \frac{2m}{\hbar^2}E = 0$$

$$\frac{1}{X_2}\frac{\partial^2 X}{\partial x^2} = k_1^2$$

$$k_1^2 = -\frac{2m_1}{\hbar^2}E$$

$$E = \frac{\hbar^2 k_1^2}{2m_1}$$

according to fermi diract distribution

$$f_E = \frac{1}{e^{(E-E_F)(kT+1)}}$$

AT absolute $T = 0$

$$\frac{E-E_F}{kT} = -\infty \quad \text{if } E < E_F$$

$$\frac{E-E_F}{kT} = \infty \quad \text{if } E > E_F$$

Fermi distribution function

$$f_E = \frac{1}{e^{-\infty}+1} = 1, \quad E < E_F$$

all state energy level filled.

$$f_E = \frac{1}{e^{\infty}+1} = 0, \quad E > E_F$$

all state energy level empty.

if $T > 0$ and $E = E_F$

$$f_{(E)} = \frac{1}{e^{(E-E_F/kT+1)}} = \frac{1}{e^0+1} = \frac{1}{2}$$

2) ~~Ext~~ Intrinsic Semiconductor - An ~~ex~~ Intrinsic semi conductor is one that has been doped, during manufacture of the semiconductor crystal a trace element or doping agent has been incorporated chemically into the crystal.

For the purpose of giving it different electrical properties than the i semiconductor is called ~~extrin~~ intrinsic semiconductor.

(II) extrinsic semi conductor - In extrinsic Semiconductor it is these foreign dopant atoms in the crystal lattic that mainly provides the charge carrier which carry electric current through the crystal.

The doping agent used are two types.

1) p type 2) n type

① p type semi conductor - An extrinsic semiconductor which has been doped with electron acceptor atoms is called p-type semi conductor beacuse the majority of charge carriers in the crystal are positive holes.

N-type semi conductor - An extrinsic semiconductor which has been doped with electron donor atoms is called an n-type semiconductor beacuse the majority of charge carriers in the crystal are negative electrons.

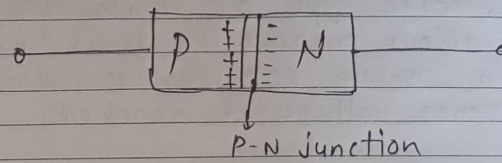
N-Type	P-Type
1) Here pentavalant impurity As, Sb are added in pure semiconductor.	Here trivalant impurities like Al, Ga are added to pure semiconductor
2) These are also called donor type impure semi conductor	These are also called exceptor type impure semiconductor
3) Here electrons are the majority carriers and holes is the minerity carriers.	Here hole are the majority carriers and electron is the minerity carries.
4) In n-type semi conductor fermi level lies near the bottom of conduction band	in p type semi conductor fermi level lies near top of valance band.

Intrinsic semiconductor	Extrinsic semiconductor
1) Intrinsic are pure semi conductor.	Extrinsic are doped semi conductor.
2) They behaving as an insulator at T = 0K	They show conductivity in between insulator and conductor
3) Conductivity of intrinsic semicondctor is poor.	Conductivity of extrinsic semiconductor is large
4) Fermi level lies at centere of forbidden energy gap.	Fermi level lies near the bottom of conduction band n type or near the top of valance band.

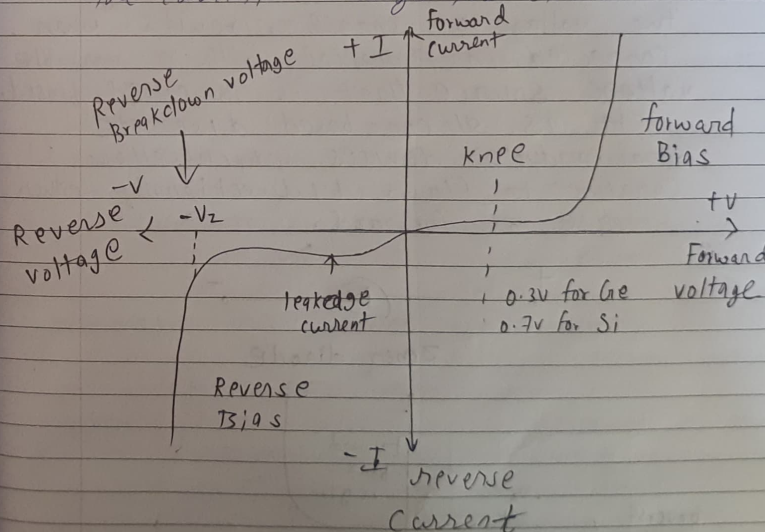
P-N Junction diaode - P-N junction diode is a basic semiconductor device that control flow of electric current in circuit. it has positive (p) side and negative (n) side.

in pure silicon semiconductor, there's an equal number of holes to electron.

I-V characterstic of P-N junction diode



Volt ampere characterstic of a pn junction or semiconductor diode is the curve between voltage across the junction and the current through the circuit.



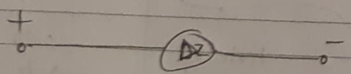
VI characteristic of PN Junction

Zener diode - A zener diaode is a special type of diade designed to reliably allow current to flow "backwords" when a certain set reverse voltage known as zener voltage is reached.

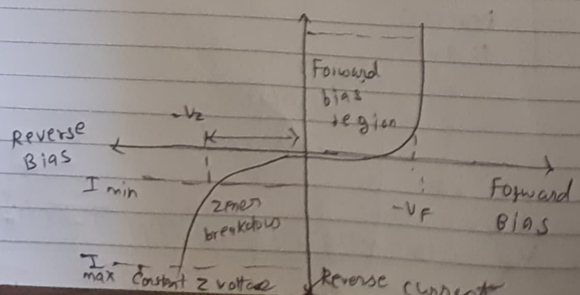
or

A form of semiconductor diode in which at a critical reverse voltage a large reverse current can flow.

Zener diade are used to regulate the voltage on small circuits, when connected in parallel with a variable voltage source that is reversed biased. It is silicon-based discrete semi conductor devices which allows current to flow bidirectionally - either reverse or forward.



Zener diaode



Advantages of zener diaode -

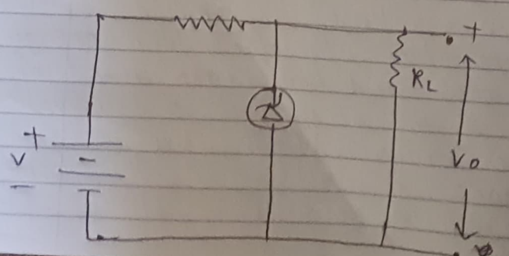
1) Less expensive than other diode
2) Ability to shift voltage
3) Easily compatible and obtainable across systems.
4) High performance standered
5) protection from over voltage
6) Ability to regulate and stabilize Circuit voltage
7) Greater control overflowing current
8) usable in smaller circuits.

Working - The junction of zener diode is thin and so strong electric field is produced even when a low reverse bias voltage is applied on, increasing the temperature, the forbidden energy gap reducess and the zener breakdown now begins to occure at a low voltage at the zener voltage, the resistance of diaode a falls and therefore the reverse current suddenly Inc when the reverse voltage is with drawn the junction of diaode regains back to orignal state.

Use of zener diode as a voltage regulator -

The value of supply voltage V and the resistance R are chosen such that diode current remains within a definite limit and the diode operates in the breakdown region. The potential difference Vo across the load resistance R remain constant even if the supply voltage or the load resistance R is changed.

The supply voltage V is obtained from the d.c power which contains some ripples. Thus the voltage V is applied on the zener diaode such that it is in reverse bias.



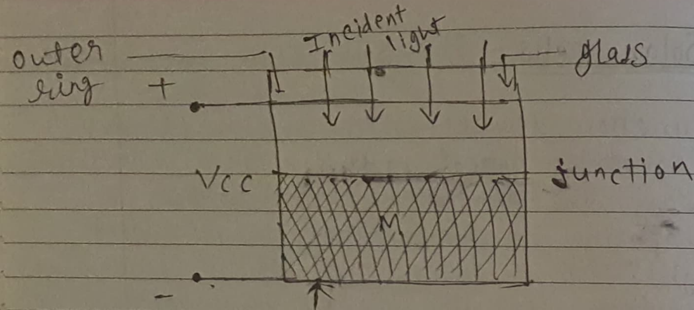
Solar cells - The solar cells are semiconductor junction devices which are used for converting optical radiation into electrical energy. The generated electric voltage is proportinal to the intensity of incident light. Due to their capability of generating voltage. They are called photovoltanic or solar cells.

A solar cell is P-N junction device with no voltage applied directly across the junction this convert power into electric power.

Construction - The cell is a p-N junction diode with approtridtely doped semiconductor. The solar cells are made from semi conductor material like silicon.

The top p-type layer is made very thin so that the light radiation may pendrate to fall on junction.

The doping level of P doped semiconductor is very high. P-type material is surrounded by a nickel plated ring which serves as positive terminal of the cell. and N type acts as negative cell.



Metallic contact solar cell

Solar cell

Working – when a photon of light energy colliads with the valance electron either in p type or n type material it impart half of the energy to the electron to leave it atom. As a result, free electron and holes are generated on each side of the junction in p type material the newly generated electron are minority carriers here electrons move freely across the junction with no applied bias. similarly in n type.

Advantages –

1) The solar cell is self generating device. it does not require any external power device.

2) it is pollution free energy conversion system.

limitation –

It doesn't convert all solar radiation into electric energy the efficiency is temperacture defends.

Bloch theorem – Bloch's theorem states that solutions to the schrodinger equation in a periodic potential take the form of a plane wave modulated by a periodic function.

Bloch function

$$\boxed{\cancel{\psi(r) = e^{ik\cdot r} u(r)}}$$

$$\cancel{\psi(r) = e^{ik\cdot r} u(r)}$$

$r \rightarrow$ position
$\psi \rightarrow$ wave function
$u \rightarrow$ periodic function
$k \rightarrow$ crystal momentum vector
$e \rightarrow$ Euler's number
$i \rightarrow$ imaginary unit

Bloch function $\rightarrow$ $\boxed{\psi(r) = e^{ik\cdot r} u(r)}$

Free electron theory of metal -

All metals contain large number of free electrons which move freely through the positive ionic core of the metals.
Since these free electron causes conduction in metal under the application of electrical field, they are called a conduction electrons.

Density of states -

The density of state is essentially the number of different states at a particular energy level that electrons are allowed, occupy.

The density of states in semiconductor equals to the number of states per unit energy and per unit volume.

$$g(E)\,dE = \frac{\#\ \text{state}}{\text{volume}}\,dE$$

$$g(E) = \frac{4\pi\,(2m)^{3/2}\sqrt{E}}{h^3}$$

Kronig penny model - it is simplified

model for an electron in one-dimensional periodic potential.

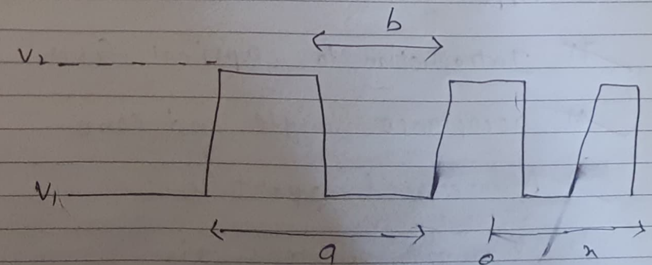
The possible states that the electron can occupy are determined by Schrodinger equation and the electron density of states.

or

it is strongly simplified one-D quantum mechanical model of a crystal.

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\,\psi = E\,\psi$$

In the case of Kroning penny model the potential $V(x)$ is periodic square wave.



K-P model

Unit - IV

laser

1) Einstein's theory of matter radiation interaction A and B cofficients.

2) Amplificotion of light by population inversion

3) Different types of lasers

4) property of laser beams

5) mono-chromaticily, coherence, directionaly and brightness, laser speckles,

6) Application of laser in science

7) Introduction to optical fiber

8) acceptance angle and cone

9) Numerical aperture

10) V number, attenuation

Laser - a device that produces a controlled ray of very powerful light that can be used as a tool.

or

laser is light amplification by the stimulated emission of radiation.

A laser is created when electrons in the atoms in optical material absorb the energy form an electrical current or a light.

That extra energy "excites" the electrons enough to move from a lower-energy orbit to a higher-energy orbit around the atom's nucleus.

Types of laser -

1) Solid-state laser
2) Gas laser
3) liquid laser
4) Semiconductor laser

Einstein's theory of matter radiation

* ~~Absorption of radiation~~

Einstein theory of radiation analyzes the processes by which the energy and momentum states of a gas of atoms achieve equillibrium with a thermal radiation field.

Einstein theory predicted that as light passes through a substance it could stimulate the emission of more light.

* Interaction A & B coffilcent -

E_2 ———————— N_2

E_1 ———————— N_1

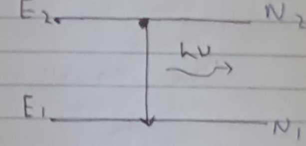
N_1 & N_2 → number of atoms present in energy state E_1 & E_2

$$\boxed{\frac{N_2}{N_1} = e^{-h\nu/kT}}$$

1) Rate of spontaneous equation -

no of atoms falling per second from E_2 to E_1

ie $\frac{dN_2}{dt} \propto N_2$



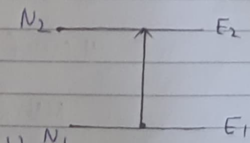
$$\frac{dN_2}{dt} = N_2 \cdot A_{21} \quad \text{——— (i)}$$

A_{21} = Constant proportinality

2) Rate of stimulated absorption:

no of atoms excited per second from E_1 to E_2

ie $\frac{dN_2}{dt} \propto N_1$



$$\frac{dN_2}{dt} = B_{12} N_1 u(\nu) \quad \text{——— (ii)}$$

$u(\nu)$ = energy density
B_{12} = constant

3) Rate of stimulated emission -

no of atoms falling per second E_2 to E_1

$$\frac{dN_2}{dt} \propto N_2$$

$$\frac{dN_2}{dt} = N_2 B_{21}\, u(\nu) \quad \text{--- (3)}$$

In equillibrium condition

eq (2) = eq (1) + eq (3)

$$B_{12} N_1\, u(\nu) = A_{21} \cdot N_2 + B_{21} N_2\, u(\nu)$$

$$B_{12} N_1\, u(\nu) = N_2 (A_{21} + B_{21}\, u(\nu))$$

$$B_{12} N_1\, u(\nu) = N_2 A_{21} + N_2 B_{21}\, u(\nu)$$

$$u(\nu)(B_{12} N_1 - B_{21} N_2) = N_2 A_{21}$$

$$u(\nu) = \frac{N_2 A_{21}}{B_{12} N_1 - B_{21} N_2}$$

$$= \frac{\frac{N_2}{N_1} \cdot A_{21}}{\frac{B_{12} N_1 - B_{21} N_2}{N_1}}$$

$$= \frac{\frac{N_2}{N_1} \cdot A_1}{B_{12} - B \ldots}$$

$$\therefore \frac{N_2}{N_1} = e^{-h\nu/KT}$$

$$u(\nu) = \frac{e^{-h\nu/KT} \cdot A_{21}}{B_{12} - e^{-h\nu/KT} \cdot B_{21}}$$

$$u(\nu) = \frac{A_{21}}{\frac{B_{12}}{e^{-h\nu/KT}} - B_{21}}$$

$$u(\nu) = \frac{A_{21}}{B_{12} \cdot e^{h\nu/KT} - B_{21}} \quad \text{--- (4)}$$

$$= \frac{A_{21}}{B_{12}\left(e^{h\nu/KT} - \frac{B_{21}}{B_{12}}\right)} \quad \text{--- (5)}$$

From plank's formula for black body radiation- $u(\nu)$

$$u(\nu) = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{e^{h\nu/KT} - 1} \quad \text{--- (6)}$$

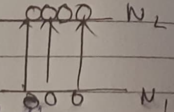
$$B_{21} = B_{12} = B$$

$$A_{21} = A$$

$$\boxed{\frac{A}{B} = \frac{8\pi h\nu^3}{c^3}}$$

Q) Amplification of light by population Inversion.

The non equillibrium state in which the no of atom in excited state N_2 is greater then no of atoms in ground state N_1.

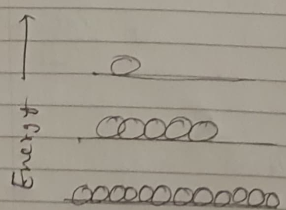


$N_2 > N_1$

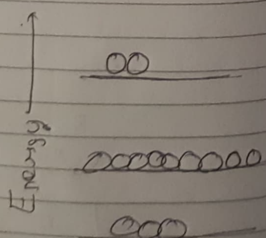
This state is called population inversion.

when beam of light passes through the amplifier, more photones will be stimulated than absorbed, thereby resulting in a net increase in the number of photones or an amplification of the beam.

it is requied in laser so that stimulated emission is more probable than induced absorption.



Normal distribution



Population Inversion

pumping - The process by which atoms are raised from the lower level to the upper level is called pumping.

A) Optical Pumping - optical pumping is the use of photones to excite the atoms.

optical pump sources are flash discharge tubes, Contineously operating lamps.

optical pump is used for solid state laser.

2) Electrical pumping - electrical pumping can be used only in case of laser matereals that can conduct electricity without destroying lasing activity.

In case of a gas laser, a high voltage pulse initially ionizes the gas so that it conducts electricity.

Different types of lasers-

1) solid - state laser

2) Gas laser

3) liquid laser

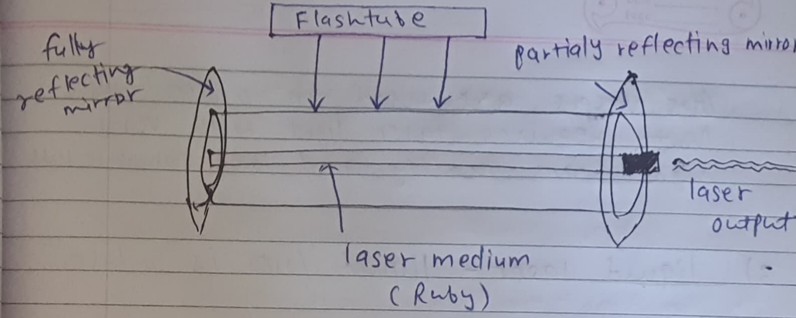
4) semiconductor laser.

① Solid- state laser - A solid state laser that uses a gain medium that is solid, rather than liquid or gas.

The inherent advantages of solid-state lasers are very high beam quality, nearly diffraction limited beam divergence, high beam pointing and amplitude stability, high peak power.

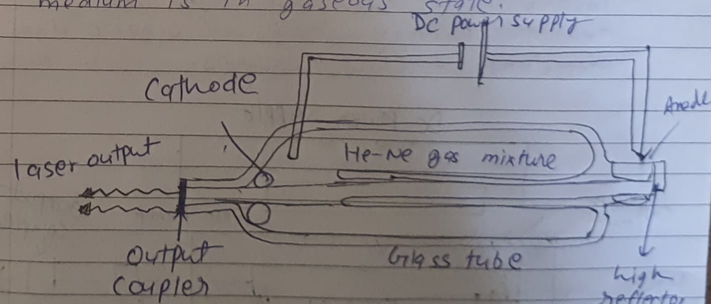
due to efficiency which the light can be coupled into the neodymium atoms.

Solid state laser are widly used in metal processing, such as red-green blue light sources in laser printers and projectors.



Solid - state laser

② Gas laser - A Gas laser is a laser in which an electric current is discharged through a gas inside the laser medium to produce laser light in gas laser the medium is in gaseous state.



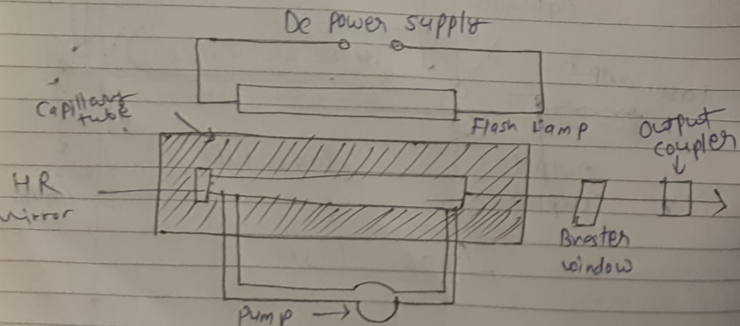
Gas laser

Gas lasers are used in applications that requires laser light with very high beam quality and long coherence length.

3) liquid laser - liquid laser is a laser that uses the liquid as laser medium. In liquid laser, light supplies energy to the laser medium.

A dye laser is example of the liquid laser.

A dye laser is made up of an organic dye mixed with a solvent. These lasers generate laser light from the excited energy states of organic dyes dissolved in liquid solvents.



liquid laser

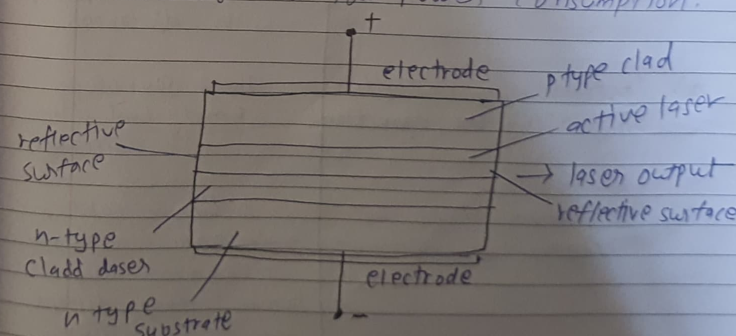
4) Semi - Conductor laser - A semi conductor laser is a device that causes laser oscillation by flowing an electric current to semi conductor.

diode laser is example of semi conductor lasers.

Semi conductor lasers used for optical data storage, metrology, spectroscopy, material processising, pumping of other lasers.

This laser is not suited for many applications due to its low power production.

This is small, volume, lightweight, good reliability and low power consumption.



Semi conductor laser

Difference between lases light and ordinary light.

Laser light	ordinary light
1) The light emitted from a lamp, flame incohrenet light.	laser light is cohrent light
2) lases beam is highly monochromatic.	ordinary light source spread over a wavelength rang 100 to 100A°
3) laser ~~light is~~ beam is extremly intense	Intensity of ordinary light source dicrease rapidelly with distance.
4) laser source radiation only along one direction	light source emit radiation in all direction.
6) lases beam does not divergen.	ordinary light source are highly divergent light travel long distane with spreading.

Spontaneous emission	Stimulated emission
1) Spontaneous emission is a random process.	it is not random process
2) it can't be controlled from outside	it can be controlled from outside
3) The photon emitted by the spontaneous emission travel in any direction.	The photon emitted by the stimulated emission travels in the direction of incident photons.
4) The light produce by this process is incohrent	The light produes coherent
5) There is no application of light in the process.	light application occur due to multiplication of photones.
6) light from the souce is unpolyrized	light from the source is polyrized.

property of laser beams :-

1) **Directionality** - An ordinary light source emit light waves in all direction. In a laser the photon of a partical direction only are allowed for multiple reflection. Thus the laser is highly directional.

2) **Intensity** - The intensity of ordinary light decrease as it travels in the space this is beacuse of its spreading.

The laser light does not spread with distance. it propagates in space in the form of normal beam and its intensity remains almost constant over long distance.

3) **Coherence** - The laser light is highly coherent. A laser emits the light waves of same wavelength and in same phase. The ordinary light emits wave of different wavelengths without any phase relationship.

4) **Divergence** - Divergence of light is a measure of its spread with distance. The angular spread in ordinary light is very high beacuse of its propagation in the divergence in laser beam is naglible. A very small divergence is due to the diffraction of laser light when it enverges out from the partially silvered mirror.

Optical fiber

Optical fibers are used most often as a means to transmit light between the two ends of the fiber and wide usage in fiber optic communications.

Optical fiber are extremly thin strands of very high purity silica which transmit light from one end to other with minimal loss.

There are two types of fibre

1) multimode fiber
2) singlemode fiber.

In optical fiber light travels down a fiber optic cable by bouncing off the walls of the cable repeatly.

Acceptance angle - where optical fiber accept signal.

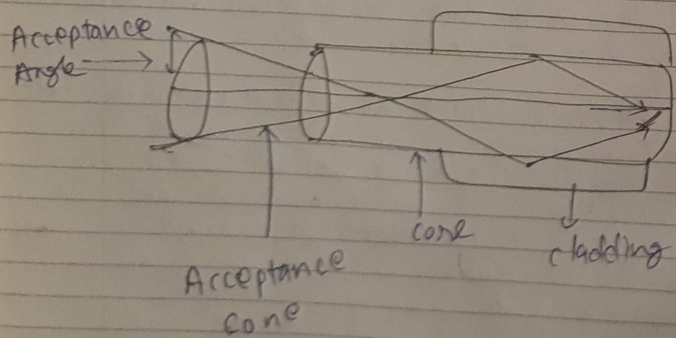
Acceptance angle & cone -

Acceptance angle is the maximum angle at which incoming sunlight canbe captured by a solar concentrator,

or

The acceptance angle of an optical fiber is defined based on a purely geometrical consideration.

it is the maximum angle of a ray hitting the fiber core which allows the incident light to be guided by the core.



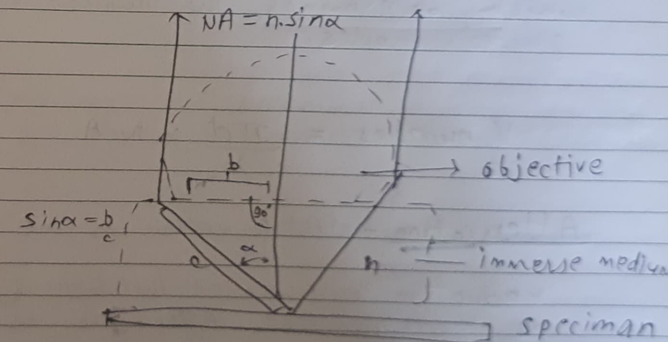
Acceptance angle & cone

Numerical aperture -

The numerical aperture of an optical system is a dimensionless number that characterise the range of angles over which the system can accept or emit light

The numerical aperture

It is the waveguide or fiber is the sine of the maximum angle of an incident beam, as required for efficent launching.



$$\boxed{NA = n \cdot \sin\alpha}$$

Numerical aperture

V number - V number determines the fraction of the optical power in a certain mode which is confined to the fiber core. for single-mode fibers, that fraction is low for low V values and reaches ≈ 90% near the single mode cut off.

or

The V number of step index fiber is a normalized frequency parameter which determines the number of guided modes.

$$V \text{ number} = \frac{\pi d}{\lambda} \times N.A$$

Attenuation - Attenuation is the reduction in power of the light signals as it transmitted. Attenuation is casued by passive media components such as cables, and connectors.

or ratio of optical output power to the input power in the fiber of length L.

$$\alpha = 10 \log_{10} P_i / P_o \text{ [in db/km]}$$

α = Attenuation constant.

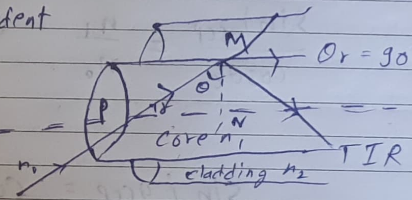
Acceptance angle of optical fiber -

Consider an optical fiber.

Core R.I is n_1

Cladding refrective index is n_2

i = angle of incident outside



$r \rightarrow$ angle of refraction at core

$\theta \rightarrow$ angle of incident at four cladding boundary

$$\frac{\sin i}{\sin r} = \frac{n_1}{n_0} \quad \text{--- (1)}$$

In PNM

$$r + 90° + \theta = 180°$$

$$r = 90° - \theta;$$

substitute r in eq (1)

$$\frac{\sin i}{\sin(90 - \theta_i)} = \frac{n_1}{n_0}$$

$$\frac{\sin i}{\cos \theta_i} = \frac{n_1}{n_0} \quad \text{--- (2)}$$

$i = i_{accp}$

$\theta = \theta_c$ when $\theta_r = 90°$

$\theta_c > TIR$

$$\frac{\sin i_{accp}}{\cos\theta_c} = \frac{n_1}{n_0} \quad \text{--- (3)}$$

$$\sin i_{accp} = \frac{\cos\theta_c \times n_1}{n_0}$$

Acceptance angle $\quad i_{accp} = \sin^{-1}\left[\frac{\cos\theta \times n_1}{n_0}\right]$

again applying

$\theta = \theta_c$
$\theta_r = 90°$, $\frac{\sin\theta_i}{\sin\theta_r} = \frac{n_2}{n_1}$

$$\sin\theta_c = \frac{n_2}{n_1}$$

$$\cos\theta_c = \sqrt{1-\sin^2\theta_c}$$

$$\cos\theta_c = \sqrt{1-\left(\frac{n_2}{n_1}\right)^2}$$

$$\cos\theta_c = \sqrt{\frac{n_1^2-n_2^2}{n_2^2}}$$

$$\cos\theta_c \quad \frac{1}{n}\sqrt{n_1^2-n_2^2}$$