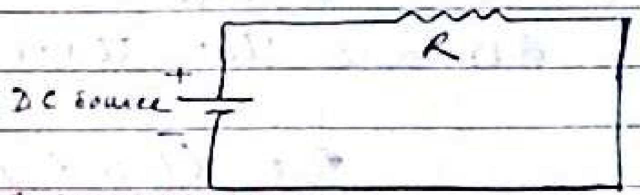
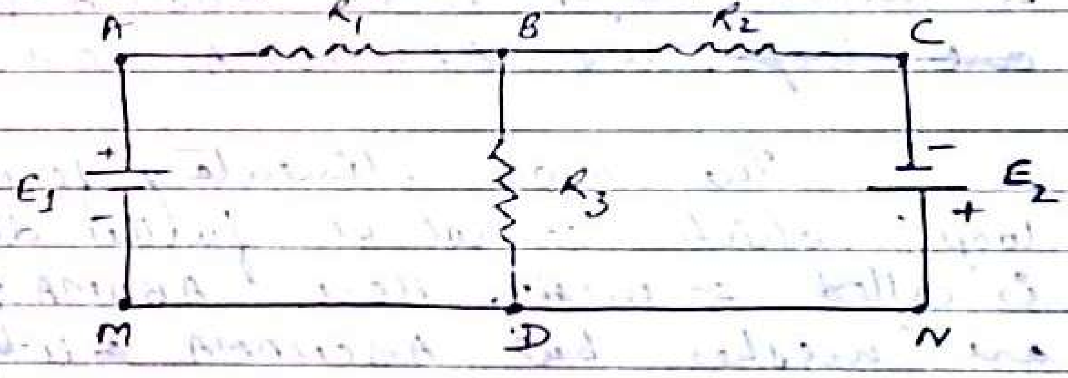
Date 17|09|07

UNIT - 1
D.C. NETWORK

DC Network :- The closed path followed by direct current (dc) is called a dc circuit or DC Network. The dc circuit essentially consists of a source of dc power (eg- battery, dc generator, etc).



NETWORK TERMINOLOGY



1) Active Element :- The element which supplies energy to the circuit is called active element. E_1 & E_2 are active element in above figure or circuit.

2) Passive Element :- The element which receives energy is called passive ~~elect~~ element (such as resistor, inductor & capacitor)

3) Node :- It is a point in the network where two or more circuit elements are joined. Eg A, B, C & D are nodes.

4) Junction :- A point in the network where three or more circuit elements are joined is called as junction.

5) Branch :- The part of a network which lies between two junction points is called branch. Here DAB, BCD & BD are the three branches.

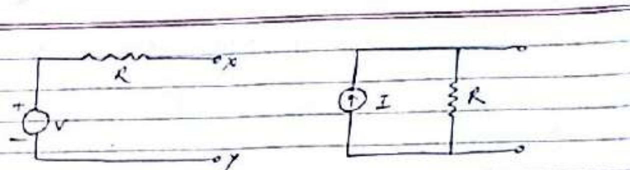
6) Loop :- The closed path of the network is called loop. Here, ABDMA & BCNDB ~~are loops~~ and ABCNDMA are loops.

7) Mesh :- The most elementary form of a loop which cannot be further divided is called a mesh. Here, ABDMA & BCNDB are meshes but ABCNDMA is loop.

CLASSIFICATION OF NETWORK ELEMENTS

1) Active Elements
2) Passive Elements

1) Active Elements :- The elements which supply energy to the network are called active element. These active elements may be constant voltages sources or constant current sources. Batteries and d.c. generators are normally used as voltage sources while most of the semiconductor devices like transistor etc. are treated as current sources.



Voltage source   current source.

2) Passive Elements :- The elements which receive energy from the network are called passive elements. Such as resistor, inductor & capacitor.

In DC network only resistances are relevant. The inductance & capacitance are relevant only in AC network.

Resistance :- Electrical resistance is the property of a material by virtue of which it opposes the flow of electrons through the material. Unit - Ohm (Ω)

Inductance :- Inductance is the property of a material by virtue of which it oppose any change of magnitude or direction of electric current passing through it. Unit - Henry (H)

Capacitance :- It is the capability of an elements to store electric charge within it. Unit → Farad (F)

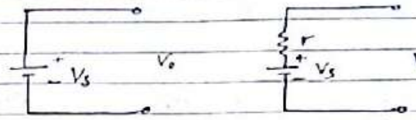
Voltage Current Rel^n ship of Network Elements

Network Elements	Voltage (volts)	Currents (amperes)	Power (watts)
R (Ω)	$V = Ri$	$i = V/R$	$P = i^2R$ $E = i^2Rt$
L (H)	$V = L\frac{di}{dt}$	$i = \frac{1}{L}\int V\,dt$	$P = Li\frac{di}{dt}$ $E = \frac{1}{2}Li^2$
C (F)	$V = \frac{1}{C}\int i\,dt$	$i = C\frac{dv}{dt}$	$P = Cv\frac{dv}{dt}$ $E = \frac{1}{2}Cv^2$

VOLTAGE SOURCES

A voltage source is a two terminal device whose voltage at any instant of time is independent of the current flowing through its terminals. It maintains the magnitude of ~~this~~ waveform of its ~~term~~ terminal voltage irrespective of the types of network connected to ~~the~~ its terminals. Such type of voltage sources are called ideal voltage sources & such ~~as~~ a voltage source must possess zero internal resistance, so that internal voltage drop in the source is zero.

In practice none such ideal constant voltage source can be obtained. However, smaller the internal resistance r of a voltage source, closer it comes to an ideal source.



Ideal Voltage sources   Practical Voltage source

Let, battery $= 12\,V = V_s$

Internal resistance $= 0.01\,\Omega$

When current is not supplied

$V_o = 12\,V$ in both the cases.

When current is supplied (say 100A)

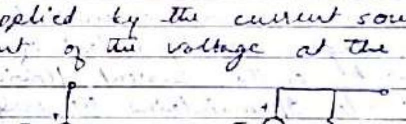
The drop across int. resistance is 1V

Hence; $V_o = 12V - 1V$ in Practical Voltage source

$V_o = 12V$ in Ideal voltage source

CURRENT SOURCE

An independent ideal current source is a 2 terminal ckt. element that will supply the same current to any load ~~terminal~~ resistance connected across its terminals. The current supplied by the current source is independent of the voltage at the source terminal.



Examples :- photo electric cells used in light meters, meta type gen.

Ideal current source   Practical Current source

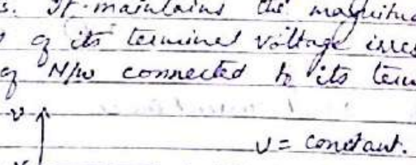
The current ~~curre~~ source in parallel add algebraically. When current is in the same direction their eqvt. source is sum of two individual sources. If dir^n is opposite eqvt source is diff. of the source current.

INDEPENDENT AND DEPENDENT SOURCE

1) Independent voltage sources :- The independent source is not dependent on any other quantity in the ckt it has a const. value i.e. the strength of a voltage or current is not changed by any variation in the connected n/w. The independent sources are of two types: Independent voltage sources & current sources.

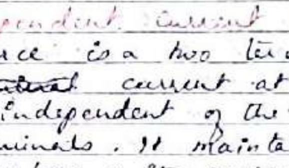
The term independent sources indicates that the magnitude of the source is independent of the network to which it is applied & that it exhibits its terminal characteristics even if completely isolated.

Independent voltage source :- An independent voltage source is a two terminal device whose voltage at any instant of time is independent of the current flowing through its terminals. It maintains the magnitude & waveform of its terminal voltage irrespective of the type of N/w connected to its terminals.



The independent voltage source is an ideal source. It does not represent exactly any physical device, because the ideal source could theoretically deliver an infinite amount of energy from its terminals, an automobile storage battery has a terminal voltage of 12V, that remains essentially constant as long as the current drawn from it does not exceed a few amperes.

Independent current source :- A current source is a two terminal device whose current at any instant of time is independent of the voltage across its terminals. It maintains the magnitude & waveform of its current irrespective of the type of the network connected to its terminals.



Such a source is non existence in the real world. However there are some sources which may be very closely approximated to an ideal current source. For Eg:- The electron beam of synchrotron operating at a constant beam current, a pentode vacuum tube amplifier & certain transistor circuit can deliver a constant current to a wide range of loads.

Dependent or Controlled Sources:- It is that source whose magnitude is governed by a current or voltage of the system in which it is situated. In a dependent source the output voltage (or current) depends on another voltage (or current).

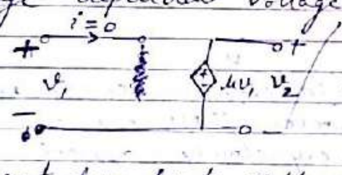
collector current of a bipolar (transistor)

In electronic ckt, the current through an element (transistor) is dependent on a current through some other element or in a MOSFET it is dependent on the voltage across some other elements.

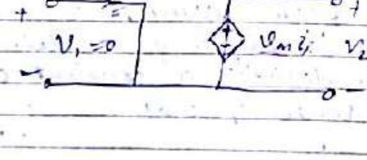
The dependent source ~~a~~ is basically a three terminal device. The three terminals are paired, with one common terminal & one pair is referred as input while the other pair as the o/p.

Dependent sources are classified into four categories.

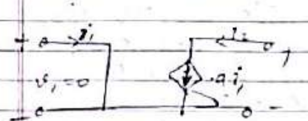
1) Voltage dependent Voltage ~~volt~~ source.



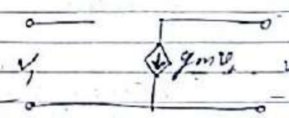
2) ~~V~~ Current dependent voltage sources:



3) Current Dependent current source



4) Voltage Dependent current source.



The quantities of input (v_1 & i_1) are controlling quantities whereas the output quantities (v_2 & i_2) are controlled (dependent) quantities. Thus a controlled source is unilateral in the sense that an input variable controls one of the quantities of the output, whereas the output quantities don't ~~flo~~ influence the i/p quantities.

Some physical devices operate almost like ideal dependent source, for eg. an operational Ampl^r is a voltage dependent voltage source, a common base transistor a current dependent current source & a field ~~of~~ effect transistor (FET) a voltage dependent current source.

A dc shunt generator operating at a constant speed and in the linear characteristic of field magnetisation is an example of current dependent voltage source.

Ques(1) A 200 Ω resistance is directly ~~p~~ switched on across a 20V battery. What is the current through the resistor? How much is the power loss? Also find ~~the~~ energy consumed in 5 sec.

Sol^n

$V = \cancel{200}\ 20V$

$R = 200\ \Omega$

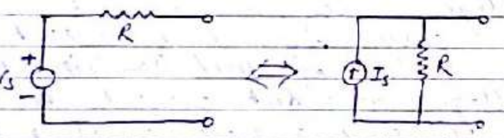
$$I = \frac{V}{R} = \frac{20}{200} = 0.1 \text{ Amp}$$

Power loss, $P = I^2R = (0.1)^2 \times 200$
$= 0.01 \times 200$
$= 2$ watts.

Energy consumed in 5 sec $= I^2Rt$
$= 2 \times 5 = 10$ Joules.

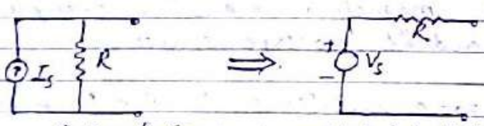
SOURCE CONVERSION:

A practical voltage source can be converted or transformed to an equivalent practical current source. Similarly, a practical current source can be converted to an equivalent practical voltage source. Replacing of one source by an equivalent source is called a source transformation or source conversion.



$$I_s = \frac{V_s}{R}.$$

Voltage source converted to current source.

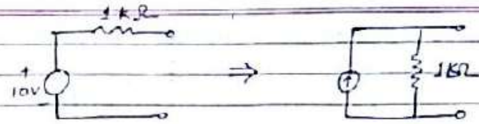


$$V_s = I_s \times R.$$

Current source converted into Voltage source

Ques(1) Convert the voltage source to a current source.

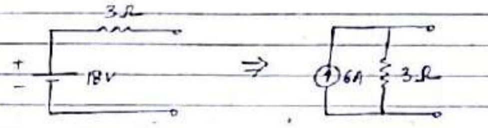
i)



$$I = \frac{V}{R} = \frac{10}{1 \times 10^3} = \frac{1}{100} = 0.01A$$

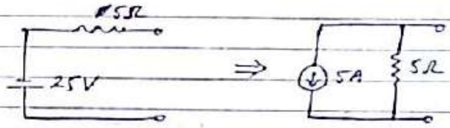
$\therefore I = 10 \text{ mA}.$

ii)



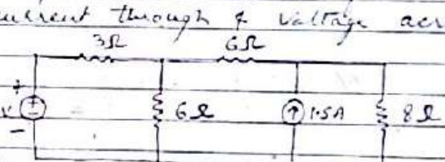
$$I_s = \frac{V_s}{R} = \frac{18}{3} = 6A$$

iii)

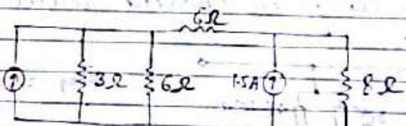


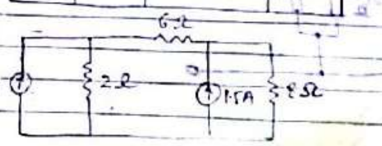
$$I_s = \frac{25}{5} = 5A$$

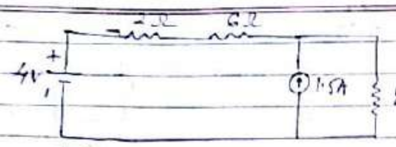
Ques. In the N/w, use source transformation to determine the current through & voltage across 8Ω resistor.

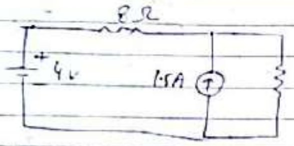


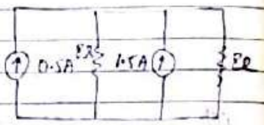
Sol^n:

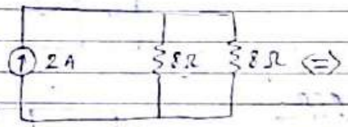


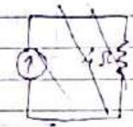


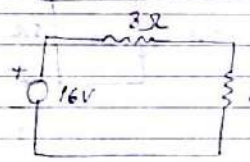












$I_{(8\Omega)} = 1A$

$V_{(8\Omega)} = I \times R = 1 \times 8 = 8V$

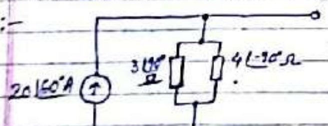
Current through ckt, $i = \frac{16}{16} = 1$ Amp.

Voltage across 8Ω resistor

$V = 8 \times i = 8V.$

Ques: Convert the given current source to voltage source. (for AC)

Sol^n:-



$$Z_{eq.} = \frac{3\angle 90^\circ \times 4\angle -90^\circ}{3\angle 90^\circ + 4\angle -90^\circ}$$

$$= \frac{j3 \times (-j4)}{j3 - j4}$$

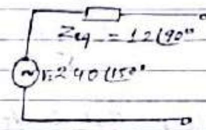
$$= \frac{-j^2 12}{-j}$$

$$= \frac{12}{-j} = 12\angle +90^\circ$$

$Z_{eq} = 12\angle 90^\circ$

$V = I \times Z_{eq} = 20\angle 60^\circ \times 12\angle 90^\circ$

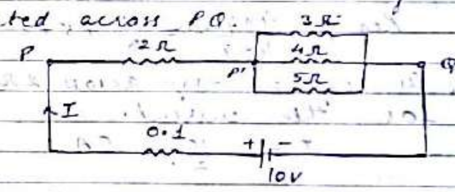
$= 240\angle 150^\circ$ Volts.



Ques: Three resistances of 3, 4 & 5 Ω are connected in parallel and this combination is put in series with 2Ω resistor.

(a) Find the equi. resistance of the comb^n (at terminal PQ)

(b) Obtain current in each ckt when a battery of 10V with internal resistance of 0.1Ω is connected across PQ.



Sol^n:- (a) $R_{eq(PQ)} = \frac{1}{\frac{1}{3}+\frac{1}{4}+\frac{1}{5}} + 2 = 3.28\ \Omega$

(b) $I = \frac{10}{3.28 + 0.1} = 2.96A$

Voltage across $PQ = 10 - 2.96 \times 0.1$

$= 9.704V$

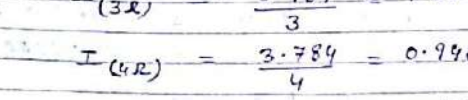
$$\text{Drop in } P'Q = 9.704 - 2.96 \times 2$$
$$= 3.784\ V$$

$$I_{(3\Omega)} = \frac{3.784}{3} = 1.26\ A$$

$$I_{(4\Omega)} = \frac{3.784}{4} = 0.946\ A$$

$$I_{(5\Omega)} = \frac{3.784}{5} = 0.7568\ A$$

Ques Calculate the voltage that is to be connected across terminal x-y in fig. such that the voltage across the 2 Ω resistor is 10V. Also find I_a & I_b. What is the power loss in the ckt.



Solⁿ:- Eqvt. resistance across the terminals x-y.

$$R_{eq} = \frac{4 \times 4}{4+4} + 5 + 2 = \frac{16}{8} + 5 + 2 = 9\ \Omega$$

If the voltage drop across 2 Ω resistor is to 10 V, the current.

$$I = \frac{10}{2} = 5A$$

Now voltage across x-y terminals

$$= 5 \times 9 = 45\ V.$$

Voltage across m-n terminals

$$V_{mn} = 45 - 5 \times (2+5) = 10\ V$$

$$\therefore I_a = \frac{10}{4} = 2.5A,\ I_b = \frac{10}{4} = 2.5A$$

The total power loss in the ckt. is I^2R.

i.e. $5^2 \times 9 = 225 W$.

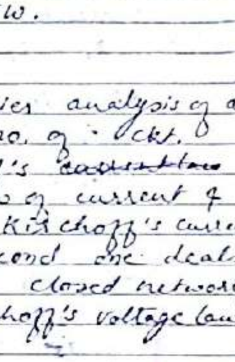
Kirchoff's law:-

The law enables easier analysis of an interconnection of any no. of ckt. elements. The Kirchoff's first law deals with flow of current & is popularly known as Kirchoff's current law (KCL) while the second one deals with voltage drop in a closed network and is known as Kirchoff's voltage law (KVL).

1) Kirchoff's current law:- It is a Kirchoff's first law. It states that, the algebraic sum of all currents at any node of a circuit is zero.

According to KCL,

$$i_1 + i_2 - i_3 - i_4 - i_5 = 0$$



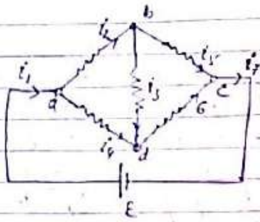
The direction of incoming currents to a node being +ve, the outgoing currents should be taken -ve. incoming current = outgoing current

Thus, $i_1 + i_2 = i_3 + i_4 + i_5$

i.e. the algebraic sum of currents entering a node must be equal to the algebraic sum of currents leaving a node.

Ques Find the magnitude & direction of the unknown currents in Fig. Given $i_1 = 10A$, $i_2 = 6A$, $i_5 = 4A$

Sol^n :- $i_1 = i_7 = 10A$



At node a,

$$i_1 = i_2 + i_4$$

$$i_4 = i_1 - i_2$$

$$= 10 - 6 = 4A$$

At node b,

$$i_2 = i_3 + i_5$$

$$i_3 = i_2 - i_5$$

$$= 6 - 4 = 2A$$

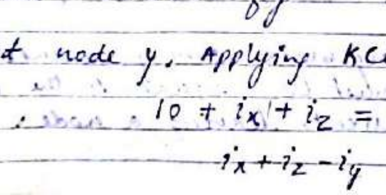
At node c,

$$i_7 = i_5 + i_6$$

$$i_6 = i_7 - i_5$$

$$= 10 - 4 = 6A$$

Ques. Find 'V' and the magnitude & direction of the unknown currents in the branches xn, yn, zn.



Sol^n :-

Let the unknown currents in branches xn, yn, & zn be i_x, i_y & i_z resp. & their dir^n shown in fig.

At node y, Applying KCL,

$$10 + i_x + i_z = i_y + 2$$

$$i_x + i_z - i_y = (-8) \quad \text{--- (1)}$$

Now by ohms law,

$$i_x = \frac{V}{5} A, \quad i_y = \frac{-V}{2} A \;\&\; i_z = \frac{V}{4} A$$

Now putting these values in equ (1) we get

$$\frac{V}{5} + \frac{V}{2} + \frac{V}{4} = -8$$

$$\frac{4V + 10V + 5V}{20} = -8$$

$$\cancel{-V = -160}$$

$$\cancel{V = 160 \text{ Volts}}$$

$$19V = -160$$

$$\boxed{V = -8.42 \text{ Volts}}$$

−ve sign indicates that node n to be +ve & at higher potential.

$$V = -8.42 \text{ Volts}$$

$$i_x = \frac{-8.42}{5} = -1.684 A$$

$$i_y = -\left(\frac{-8.42}{2}\right) = 4.21 A$$

$$i_z = \frac{-8.42}{4} = -2.1 A$$

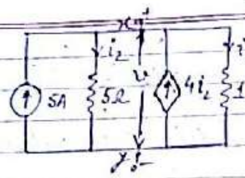
Dir^n of currents

i_x = n to x

i_y = n to y

i_z = n to z

Ques Find i_1 & i_2



Sol^n:- Applying KCL at x.

$$i_2 + i_1 = 5 + 4i_2$$

$$i_1 - 3i_2 = 5 \quad \text{--- (1)}$$

Also, $i_2 = \frac{v}{5} A$ & $i_1 = \frac{v}{1} A$

Putting in eqn (1).

$$-3\frac{v}{5} + v = 5$$

$$-3v + 5v = 25$$

$$2v = 25$$

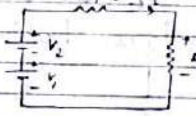
$$v = 12.5 V$$

$$i_1 = 12.5 A$$

$$i_2 = \frac{12.5}{5} = 2.5 A.$$

Kirchhoff's Voltage law (KVL)

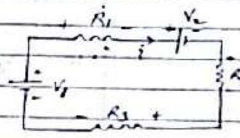
The algebraic sum of voltages (or voltage drops) in any closed path of network that is transversed in a single direction is zero.



$$iR_1 + iR_2 = V_1 + V_2$$

$$i(R_1 + R_2) = V_1 + V_2$$

$$i = \frac{V_1 + V_2}{(R_1 + R_2)}$$

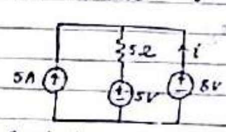


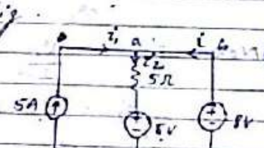
$$-V_1 + iR_1 + V_2 + iR_2 + iR_3 = 0$$

$$i(R_1 + R_2 + R_3) + V_2 = V_1$$

$$i = \frac{V_1 - V_2}{(R_1 + R_2 + R_3)}$$

Ques Find current 'i' in fig.





Sol^n:- Applying KCL,

$$i_2 = i_1 + i$$

$$i_2 = 5 + i \quad \text{--- (1)}$$

KVL in loop abcd.

$$5i_2 + 5 = 8$$

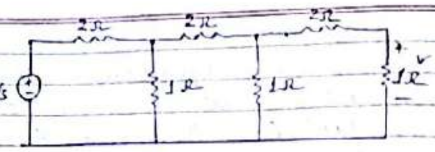
$$i_2 = 3/5 \, A \quad \text{--- (2)}$$

Using eqn (1) & (2) we get

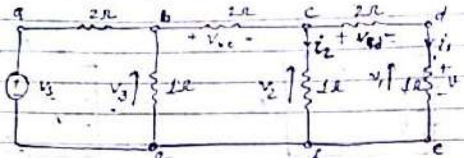
$$\cancel{i_2 = 5 + 3/5 = \frac{28}{5}}$$

$$\frac{3}{5} - 5 = i \Rightarrow i = \frac{-22}{5} = -4.5 A$$

Que. Find V_s.



Sol^n:-



$i_1 = \frac{v}{1} = v\ A$

In mesh cdef,

$v_2 = v_{cd} + v_1$

$= 2i_1 + v$

$v_2 = 2v + v = 3v$

Now $i_2 = \frac{v_2}{1} = v_2$

$i_2 = 3v\ A$

$v_3 = v_{bc} + v_2$

$= 2(i_1+i_2) + 3v$

$= 2(4v) + 3v = 11v$.

Now, $i_{ab} = i_{bg} + i_1 + i_2$

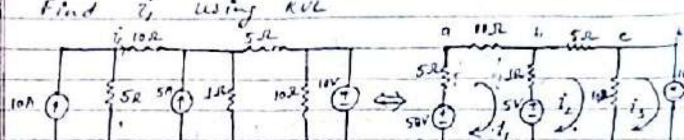
$= \frac{v_3}{1} + 4v$

$= 11v + 4v = 15v$.

$V_s = V_{ab} + V_3 \Rightarrow 15v \times 2 + 11v$

$V_s \Rightarrow 30v + 11v = 41v$.

Que. Find i_1 using KVL



Sol^n:-

In loop abcd,

$5i_1 + 10i_1 + 1(i_1 - i_2) + 5 - 50 = 0$

$16i_1 - i_2 - 45 = 0$

$16i_1 - i_2 = 45$ — (1)

In loop bchc,

$5i_2 + 10(i_2 - i_3) - 5 + i_2 - i_1 = 0$

$16i_2 - i_1 - 10i_3 = 5$ — (2)

In loop efgh,

$10 + 10(i_3 - i_2) = 0$

$10i_3 - 10i_2 = -10$ — (3)

(2) + (3)

$-i_1 + 16i_2 - 10i_3 = 5$

$-10i_2 + 10i_3 = -10$

$-i_1 + 6i_2 = -5$ — (4)

6 × equation (1)

$96i_1 - 6i_2 = 270$ — (5)

$-i_1 + 6i_2 = -5$ — (6)

$95i_1 = 265$

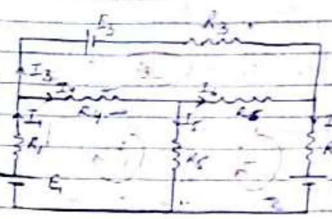
$i_1 = 2.79\ A$

$6i_2 = -5 + i_1$

$i_2 = \frac{-5 + 2.79}{6} = -0.37\ A$

$i_3 = -1.37\ A$

Mesh Current Method



I_1, I_2, I_3, I_4, I_5 & I_6 are branch currents
i_1, i_2 & i_3 are mesh currents.

$I_1 = i_1 \qquad I_4 = i_1 - i_3$
$I_2 = i_2 \qquad I_5 = i_1 - i_2$
$I_3 = i_3 \qquad I_6 = i_2 - i_3$

By Applying KVL in all the 3 meshes.

$$i_1(R_1 + R_4 + R_5) - i_2 R_5 - i_3 R_4 = E_1$$
$$-R_5 i_1 + (R_2 + R_5 + R_6) i_2 - R_6 i_3 = -E_2$$
$$-R_4 i_1 - R_6 i_2 + (R_3 + R_4 + R_6) i_3 = -E_3$$

Taking it in matrix form, we get.

$$\begin{bmatrix} R_1 + R_4 + R_5 & -R_5 & -R_4 \\ -R_5 & R_2 + R_5 + R_6 & -R_6 \\ -R_4 & -R_6 & R_3 + R_4 + R_6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ -E_2 \\ -E_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} R_{11} & -R_{12} & -R_{13} \\ -R_{21} & R_{22} & -R_{23} \\ -R_{31} & -R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} \Sigma V_1 \\ \Sigma V_2 \\ \Sigma V_3 \end{bmatrix}$$

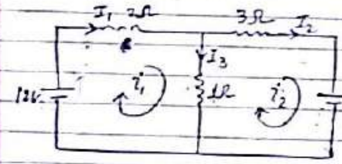
$R_{11} \rightarrow$ Sum of all resistances in mesh 1
$R_{22} \rightarrow$ —— " —— 2
$R_{33} \rightarrow$ —— " —— 3

$R_{12} = R_{21} \rightarrow$ Resistance common to mesh 1 & 2.
$R_{13} = R_{31} \rightarrow$ —— " —— 1 & 3.
$R_{23} = R_{32} \rightarrow$ —— " —— 2 & 3.

$\Sigma V_1 \Rightarrow$ Algebraic sum of sources Emf in mesh 1
$\Sigma V_2 \rightarrow$ —— " —— 2
$\Sigma V_3 \rightarrow$ —— " —— 3.

$$i_1 = \frac{\Delta_1}{\Delta}, \quad i_2 = \frac{\Delta_2}{\Delta}, \quad i_3 = \frac{\Delta_3}{\Delta}$$

Ques



Find I_1, I_2 & I_3

Solⁿ:- In general matrix Eqn.

$$\begin{bmatrix} R_{11} & -R_{12} \\ -R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \Sigma V_1 \\ \Sigma V_2 \end{bmatrix}$$

$R_{11} = 2 + 1 = 3\,\Omega \qquad \Sigma V_1 = 12$
$R_{12} = R_{21} = 1\,\Omega.$
$R_{22} = 3 + 1 = 4\,\Omega. \qquad \Sigma V_2 = -6$

$$\begin{bmatrix} 3 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}$$

$$i_1 = \frac{\Delta_1}{\Delta} \qquad i_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta_1 = \begin{vmatrix} 12 & -1 \\ -6 & 4 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} 3 & 12 \\ -1 & -6 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -1 \\ -1 & 4 \end{vmatrix}$$

$\Delta_1 = 42, \quad \Delta_2 = -6, \quad \Delta = 11$

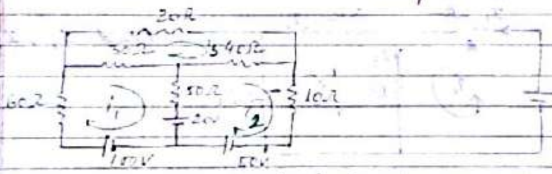
$i_1 = \frac{42}{11}, \quad i = 3.81$

$i_2 = \frac{-6}{11} = -0.54$

$I_1 = 3.81\,A$

$I_2 = -0.54\,A$

$I_3 = I_1 - I_2 = 3.81 + 0.54 = 4.35\,A$



Sol^n -

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} \Sigma V_1 \\ \Sigma V_2 \\ \Sigma V_3 \end{bmatrix}$$

$$\begin{bmatrix} 140 & -50 & -30 \\ -50 & 100 & -40 \\ -30 & -40 & 90 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 80 \\ 70 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 140 & -50 & -30 \\ -50 & 100 & -40 \\ -30 & -40 & 90 \end{vmatrix} = 140(9000-1600) + 50(-4500-1200) + 30(2000+3000)$$

$= 140 \times 7400 + 50 \times 6700 + 30 \times 5000$

$= 1036000 + 335000 + 150000$

$\Delta = 1521000$

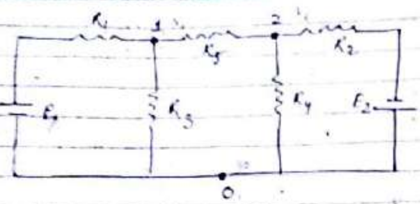
$$\Delta_1 = \begin{vmatrix} 80 & -50 & -30 \\ 70 & 100 & -40 \\ 0 & -40 & 90 \end{vmatrix} = 80(9000-1600) + 50(6300) - 30(2800)$$

$= 80 \times 7400 + 50 \times 6300 - 30 \times 2800$

$= 592000 + 315000 - 84000$

$= 723000$

NODE VOLTAGE METHOD



KCL At Node 1,

$$\frac{V_1 - E_1}{R_1} + \frac{V_1 - V_2}{R_5} + \frac{V_1}{R_3} = 0$$

KCL At Node 2,

$$\frac{V_2 + E_2}{R_2} + \frac{V_2 - V_1}{R_5} + \frac{V_2}{R_4} = 0$$

Rearranging,

$$\frac{V_1}{R_1} + \frac{V_1}{R_5} + \frac{V_1}{R_3} - \frac{V_2}{R_5} - \frac{E_1}{R_1} = 0$$

$$\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_5}\right) V_1 - \frac{1}{R_5} V_2 = \frac{E_1}{R_1}$$

$$(G_1 + G_3 + G_5) V_1 - G_5 V_2 = G_1 E_1 \quad \text{— (1)}$$

$$\left(\frac{1}{R_2} + \frac{1}{R_5} + \frac{1}{R_4}\right) V_2 - \frac{V_1}{R_5} = \frac{-E_2}{R_2}$$

$$(G_2 + G_4 + G_5) V_2 - G_5 V_1 = -G_2 E_2 \quad \text{— (2)}$$

$$\boxed{\begin{aligned} I_1 &= \frac{E_1}{R_1} = G_1 E_1 \\ \& \; I_2 &= \frac{E_2}{R_2} = G_2 E_2 \end{aligned}}$$

Equations (1) & (2) can be written in matrix form as

$$\begin{bmatrix} G_1 + G_3 + G_5 & -G_5 \\ -G_5 & G_2 + G_4 + G_5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} G_1 E_1 \\ -G_2 E_2 \end{bmatrix}$$

$$\begin{bmatrix} G_{11} & -G_{12} \\ -G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \Sigma I_1 \\ \Sigma I_2 \end{bmatrix}$$

Where,

$G_{11} \rightarrow$ sum of all conductances connected at node 1

$G_{22} \rightarrow$ —— " —— 2

$G_{12} = G_{21} \rightarrow$ sum of all conductance connected b/w node 1 & 2

$\Sigma I_1 \rightarrow$ Algebraic sum of all current sources connected at the node 1

$\Sigma I_2 \rightarrow$ —— " —— node 2

Similarly, for 3 nodes we have

$$\begin{bmatrix} G_{11} & -G_{12} & -G_{21} \\ -G_{21} & G_{22} & -G_{23} \\ -G_{31} & -G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \Sigma I_1 \\ \Sigma I_2 \\ \Sigma I_3 \end{bmatrix}$$

Ques

Sol^n

$$\begin{bmatrix} G_{11} & -G_{12} \\ -G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \Sigma I_1 \\ \Sigma I_2 \end{bmatrix}$$

$G_{11} = \frac{1}{20} + \frac{1}{10} + \frac{1}{15} = \frac{65}{300} = \frac{13}{60}$

$G_{22} = \frac{1}{15} + \frac{1}{10} + \frac{1}{10} = \frac{40}{150} = \frac{4}{15}$

$G_{12} = G_{21} = \frac{1}{15}$

$\Sigma I = \frac{100}{20} = 5\ A$

$\Sigma I_2 = -\frac{80}{10} = \cancel{8A}\ -8A$

$$\begin{bmatrix} \frac{13}{60} & -\frac{1}{15} \\ -\frac{1}{15} & \frac{4}{15} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$$

On solving matrix

$V_1 = 15\ V \qquad V_2 = -26.25\ V$

$I_1 = \frac{100 - V_1}{20} = \frac{100 - 15}{20} = 4.25\ A$

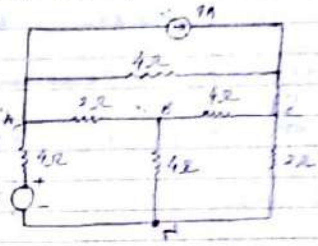
$I_5 = \frac{V_1}{10} = \frac{15}{10} = 1.5\ A$

$I_3 = \frac{V_1 - V_2}{15} = \frac{15 - (-26.25)}{15} = 2.75\ A$

$I_4 = \frac{V_2}{10} = \frac{-26.25}{10} = -2.625\ A$

$I_2 = \frac{V_2 + 80}{10} = \frac{-26.25 + 80}{10} = 5.375\ A$

Ques Use nodal analysis to determine the voltage across BC & the current in the 12V source.



At node A,

$\frac{V_1 - 12}{4} + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} + 9 = 0$

$-\frac{V_3}{4} + V_1\left(\frac{1}{4} + \frac{1}{2} + \frac{1}{4}\right) - \frac{V_2}{2} + 9 - \frac{12}{4} = 0$

$-\frac{V_3}{4} + V_1 - \frac{V_2}{2} + \frac{24}{4} = 0$

$-V_3 + 4V_1 - 2V_2 + 24 = 0$

$4V_1 - 2V_2 - V_3 = -24$ — (1)

At node B,

$\frac{V_2 - V_1}{2} + \frac{V_2}{4} + \frac{V_2 - V_3}{4} = 0$

$V_2\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{4}\right) - \frac{V_1}{2} - \frac{V_3}{4} = 0$

$V_2 - \frac{V_1}{2} - \frac{V_3}{4} = 0$

$4V_2 - 2V_1 - V_3 = 0$

$+2V_1 - 4V_2 + V_3 = 0$ — (2)

At node C,

$\frac{V_3 - V_2}{4} + \frac{V_3 - V_1}{4} + \frac{V_3}{2} - 9 = 0$

$$V_3\left(\frac{1}{4}+\frac{1}{2}+\frac{1}{4}\right) - \frac{V_2}{4} - \frac{V_1}{4} + 6 - 9 = 0$$

$$V_3 - \frac{V_2}{4} - \frac{V_1}{4} = 3\;9$$

$$4V_3 - V_2 - V_1 = 12\;36$$

$$V_1 + V_2 - 4V_3 = -36 \quad (3)$$

$$4V_1 - 2V_2 - V_3 = -24$$
$$2V_1 - 4V_2 + V_3 = 0$$
$$6V_1 - 6V_2 = -24$$
$$V_1 - V_2 = -4 \quad (4)$$

multiplying eqn (2) by 4, we get.

$$8V_1 - 16V_2 + 4V_3 = 0$$

Now add with $V_1 + V_2 - 4V_3 = -12$

$$9V_1 - 15V_2 = -12$$
$$3V_1 - 5V_2 = -4 \quad (5)$$

eqn (4) $\times$ 3:

$$3V_1 - 3V_2 = -12$$
$$3V_1 - 5V_2 = -4$$
$$2V_2 = -8$$
$$\boxed{V_2 = -4\text{ V}}$$

$$V_1 + 4 = -4$$
$$\boxed{V_1 = -8}$$

$$-8 - 4 - 4V_3 = -12$$
$$-4V_3 = 0$$
$$\boxed{V_3 = 0}$$

Voltage across BC $= V_2 - V_3 = -4\text{ V}$.

Current in 12V source $= \frac{V_1 - 12}{4} = \frac{-8-12}{4} = -5\text{ A}$

Thevenin's Theorem

Statement → Any two terminal bilateral linear dc circuit can be replaced by an equivalent circuit consisting of a voltage source & a series resistor.

Let, a simple dc ckt, we are to find I_L by Thevenin's theorem.

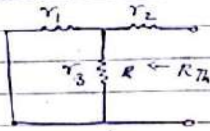
Steps

1) Remove the load resistance R_L & find the open ckt voltage (V_{oc}) across the open circuited load terminals by formula

$$\boxed{V_{oc} = I r_3 = \frac{V_S}{r_1 + r_3} \times r_3}$$

2) Deactivate the constant sources & find the internal resistance of or Thevenin's resistance of the source side looking through the open ckt load terminals.

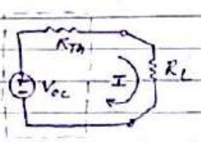
$$R_{Th} = r_2 + \frac{r_1 r_3}{r_1 + r_3}$$



3) Obtain Thevenin's Equivalent ckt by placing R_{Th} in series with V_{oc}

4) Reconnect R_L across the load terminals.

Eqvt source



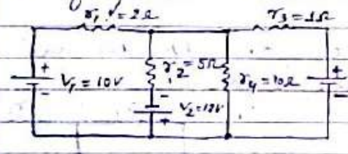
Now,

Load current

$$\boxed{I_L = \frac{V_{oc}}{R_{Th} + R_L} \text{ Amp.}}$$

Ques Find the current through the 10Ω resistor utilizing Thevenin's Theorem.

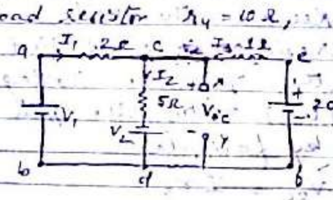
Sol^n:-



Remove the load resistor $r_4 = 10Ω$,

KCL At Node C,

$$I_1 + I_3 - I_2 = 0$$



$$\frac{V_1 - V_{oc}}{2} + \frac{V_3 - V_{oc}}{1} - \frac{V_{oc} + V_2}{5} = 0$$

$$\frac{10 - V_{oc}}{2} + \frac{20 - V_{oc}}{1} - \frac{V_{oc} + 12}{5} = 0$$

$$-V_{oc}\left(\frac{1}{2} + 1 + \frac{1}{5}\right) + \frac{10}{2} + 20 - \frac{12}{5} = 0$$

$$-V_{oc}\left(\frac{5 + 10 + 2}{10}\right) + \frac{50 + 200 - 24}{10} = 0$$

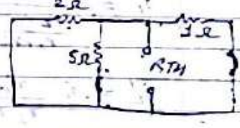
$$-V_{oc} \times \frac{17}{10} + \frac{113}{5} = 0$$

$$-V_{oc} = \frac{-113}{5} \times \frac{10}{17} = \frac{-226}{17}$$

$$V_{oc} = 13.29 \text{ V}$$

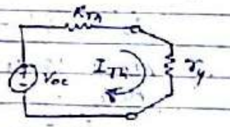
Now, Remove voltage sources by short ckts.

$$R_{TH} = \frac{1}{\frac{1}{2} + \frac{1}{5} + \frac{1}{1}} = \frac{10}{17}\ Ω$$



Thevenin's Eqvt Ckt.

$$I_4 = I_{Th} = \frac{V_{oc}}{R_{Th} + r_4}$$

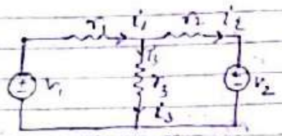


$$I_4 = \frac{13.29}{\frac{10}{17} + 10} = 1.26\,A$$

Superposition theorem

<u>Statement</u> :- If a no. of voltage or current sources are acting simultaneously in a linear network, the resultant current in any branch is the algebraic sum of the currents that would be produced in it, when each source acts alone replacing all other independent sources by their internal ~~...~~ resistances.

Let the ckt.



V_2 is S/C

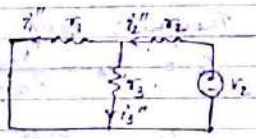
Here,

$$i_1' = \frac{V_1}{\frac{r_2 r_3}{r_2 + r_3} + r_1}$$

$$i_2' = i_1' \frac{r_3}{r_2 + r_3}$$

$$i_3' = i_1' - i_2'$$

Now, V_1 is shorted.



Here,

$$i_2'' = \frac{V_2}{\frac{r_1 r_3}{r_1 + r_3} + r_2}$$

$$i_1'' = i_2'' \frac{r_3}{r_1 + r_3} \quad \& \quad i_3'' = i_2'' - i_1''$$

According to superposition theorem,

$$i_3 = i_3' + i_3''$$

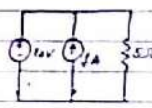
$$i_2 = i_2' - i_2''$$

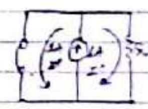
$$i_1 = i_1' - i_1''$$

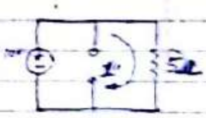
<u>Steps</u> :-

1) Take only one independent source of voltage/current and deactivate the other independent voltage/current sources (for voltage sources, remove the source and short the the respective ckt terminals & for current sources, just delete the source keeping the respective ckt terminals open). Obtain branch currents.
2) Repeat the above step for each of the independent sources.
3) To determine the net branch current utilizing superposition theorem, just add the currents obtained in step 1 & step 2 for each branch.

<u>Ques</u> Find current through 5Ω resistor.







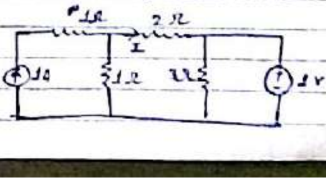
Soln:- Case 1, Voltage source is S/C (deactivating Vs)
Current in 5Ω resistor is zero
$I' = 0$

Case 2, Deactivating current source (O/C current source)
current in 5Ω resistor, $I'' = \frac{10}{5} = 2A$

By Principle of superposition
$I = I' + I''$

Current in 5Ω resistor, $I = 2A$.

<u>Ques</u>



Find I. !