Mechanics Mid Sem - I

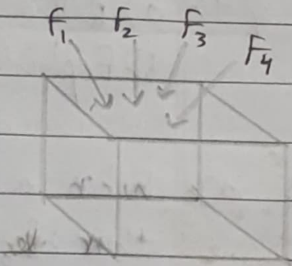
Q3) Explain System of Forces.

Ans When two or more forces act on a body then they are said to have formed a system of forces.

Types of System of forces:

1) Coplanar system of forces

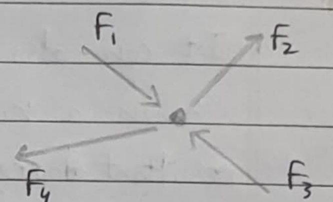
When a number of forces act simultaneously on the same plane then they are called coplanar forces.



2) Concurrent force system

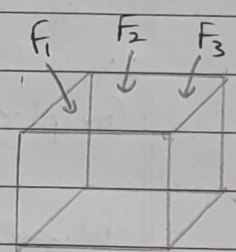
~~When a number of forces a~~

When the line of action of a number of forces pass through the same point then they are called coplanar force system.



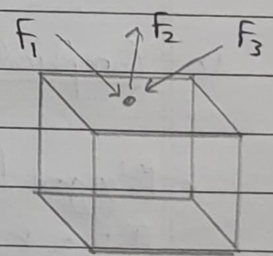
3) Coplanar Non-concurrent force system

When a number of forces act on the same plane but their line of actions do not pass through a common point then they are coplanar non-concurrent force system.



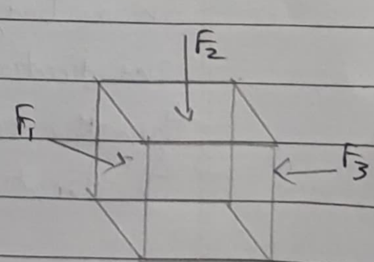
4) Coplanar Concurrent force system

When a number of forces act on the same plane and their line of actions pass through a common point then they are coplanar concurrent forces.



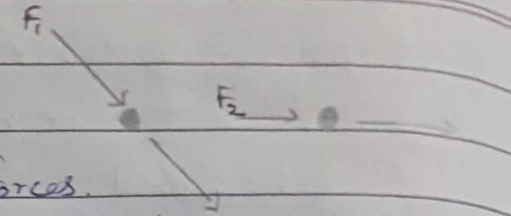
5) Non-coplanar Non-concurrent force system

When a number of forces do not act on the same plane and their line of action do not pass from a common point then they are called Non-coplanar non-concurrent forces.



6> <u>Colinear force system</u>

When line of action of number of forces are same, they form a colinear system of forces.



7> <u>Non coplanar concurrent forces</u> → LOA doesn't lie on same plane but they pass thru same point

Q4> a> Define resultant force and write about the methods to find it.

b> Explain triangle law of force and Polygon law of forces

Ans a> When a number of forces act simultaneously on a body then their algebraic or vector sum is called their resultant force.

It can also be defined as :

When a number of forces act simultaneously on a body then their resultant force is the force which when applied on the body will give the same effects as all the forces together.

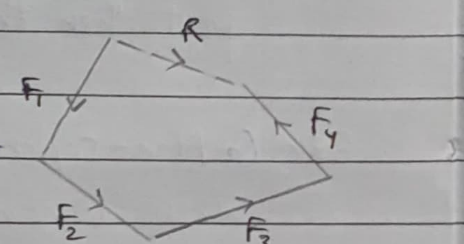
Methods to find resultant force

- Graphical method
  - Polygon law of forces
- Analytical method
  - Parallelogram law of forces
  - Resolution of forces

1> <u>Graphical Method</u>

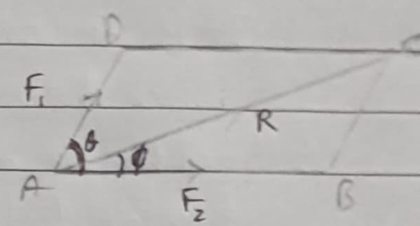
<u>Polygon law of forces</u>:

If a number of forces acting simultaneously on a body maybe be represented in magnitude and direction by the sides of a polygon in an order then the resultant may be represented in magnitude & direction by ~~completing~~ closing side of polygon but in opposite direction.



2) <u>Analytical Method</u>

i) <u>Parallelogram law of forces</u>

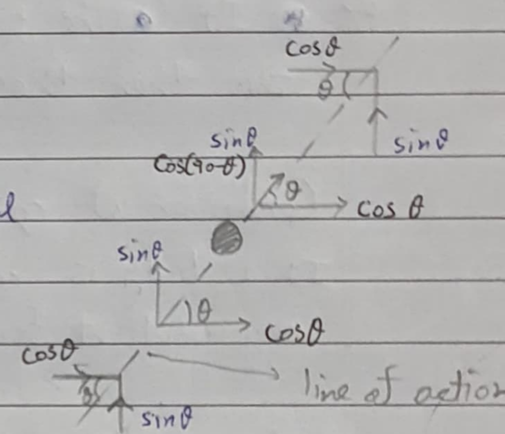


When two forces act simultaneously on a body maybe represented by adjacent sides of a parallelogram ~~then their~~ in magnitude & direction, then the diagonal of parallelogram will represent their resultant force in magnitude & direction

i.e Resultant force $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

and the direction is given by, $\tan\phi = \dfrac{F_1\sin\theta}{F_1\cos\theta + F_2}$

ii) <u>Resolution of forces</u>

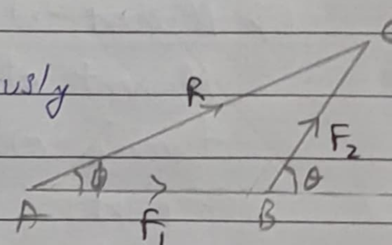


In resolution of forces, each forces is resolved / split into two components: horizontal and vertical component and then ΣH & ΣV are calculated so as to determine the resultant force by using the formula,

Resultant force i.e $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

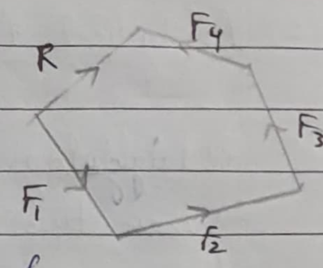
and direction is given by $\tan\theta = \dfrac{\Sigma V}{\Sigma H}$

b) <u>Triangle law of force</u>:



It states that "If two forces act simultaneously on a body be represented by two sides of of a triangle taken in order in magnitude and direction then the resultant of all these forces may be represented in magnitude and direction by the third side of the triangle taken in opposite order.

Polygon law of forces :



It is an extension of triangle ~~of~~ law of forces, which states that 'If a number of forces acting on a body be represented by, in magnitude & direction by the sides of a polygon taken in order then the resultant force maybe represented by the closing side of polygon taken in opposite order. (in magnitude and direction)

Q8> a> Explain conditions of equilibrium

b> Explain moment of a force & varignon's principle of moments.

Ans

a> If the resultant of number of forces acting on a body is zero then the body is said to be in equilibrium.

Condition of equilibrium :-

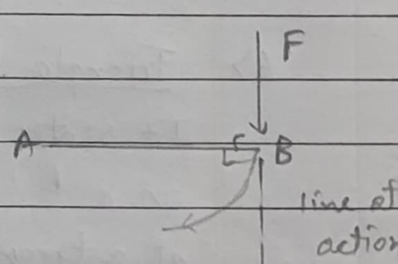
If a body is completely at rest then it means that there is neither any resultant force nor a couple acting on it. Thus,

i> $\Sigma H = 0$, The algebraic sum of all horizontal forces must be zero.

ii> $\Sigma V = 0$, The algebraic sum of all vertical forces must be zero.

iii> $\Sigma M = 0$, The algebraic sum of moment of all forces about any point must be zero.

b> Moment of a force



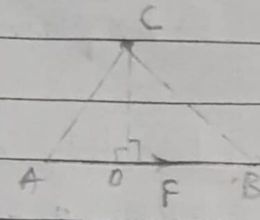
Moment of a force can be defined as the turning effect produced by the force about a point.

It is equal to the product of the force and the perpendicular distance between the point, about which moment is required, and the line of action of force.

i.e Moment of force about point A = force × ⊥ar distance = F × AB

Its unit is Newton metres. (Nm)

Graphical representation of a Moment:

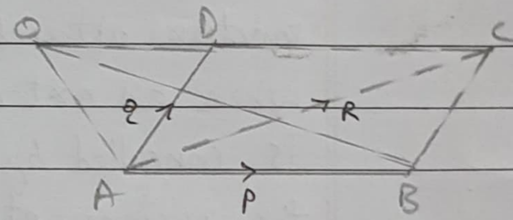


Moment of force F about point C
= 2 × area of Δ ABC

Varignon's principle of Moment

It states that 'If a number of coplanar forces are acting simultaneously on a body then the algebraic sum of all the forces about one point point is equal to the moment of resultant force about the same point'

i.e



Moment of force R about point O
= moment of P about O + moment of Q about O

Proof → Moment of R about O = 2× Area of Δ AOC
Moment of P about O = 2× Area of Δ AOB
Moment of Q about O = 2× Area of Δ AOD

Area of Δ AOB = Area of Δ ADC = Area of Δ AOC — (1)
(because they have same base and are b/w same parallel lines)

Area of Δ AOC = Area of Δ AOD + Area of Δ ADC
[From eq (1), Ar Δ ADC = Ar Δ AOB]
Multiplying both sides by 2

2× Area of Δ AOC = 2× Area of Δ AOD + 2× Area of Δ AOB

∴

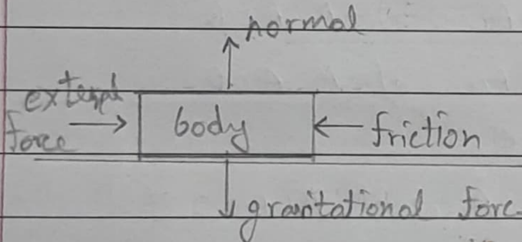
$\boxed{\text{Moment of R about O} = \text{Moment of Q about O} + \text{Moment of P about O}}$

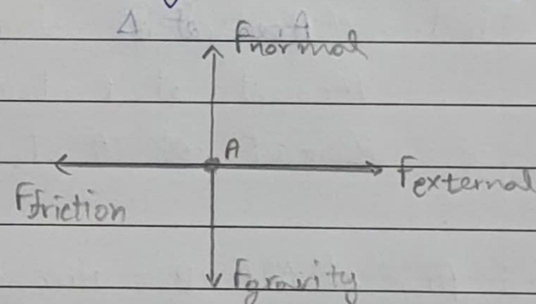
Q5> a> Explain free body diagram with neat examples.
b> Explain Lami's theorem.

Ans a> Free body diagram

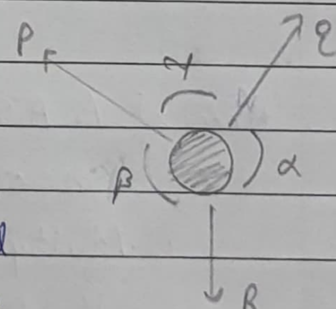
When a system ~~of~~, consisting of number of bodies, is in equilibrium then the diagram of each body, separately, which shows the forces and reactions is called free body diagram.
It follows bow's notation.

eg → Let us consider a body lying completely on rest on the surface of the table. Gravitational force is acting vertically downwards on this body which is cancelled out by the normal reaction offered vertically upwards by the table surface.
when an external force is applied horizontally on the body, it is cancelled by the equal and opposite friction offered by the surface of the table.
Then the free body diagram of this body, which is in equilibrium is as follows





Lami's Theorem

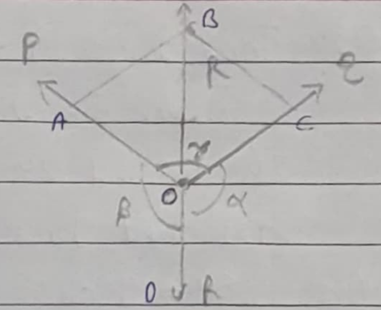


Lami's theorem states that:
If a body in equilibrium is acted upon by three forces then each force is ~~dir~~ proportional to the sine of angle between the other to forces
i.e $P \propto \sin\alpha$, $Q \propto \sin\beta$, $R \propto \sin\gamma$

$$\therefore \boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

Proof → In ΔAOB, $\boxed{\angle AOB = 180° - \beta}$

$\boxed{\angle ABO = \angle BOC = 180° - \alpha}$



$\angle AOB + \angle ABO + \angle OAB = 180°$

$180° - \beta + 180° - \alpha + \angle OAB = 180°$

$\angle OAB = \alpha + \beta - 180°$

$\angle OAB = 360° - \gamma - 180°$

$\boxed{\angle OAB = 180° - \gamma}$

$\alpha + \beta + \gamma = 360°$

$\boxed{\alpha + \beta = 360° - \gamma}$

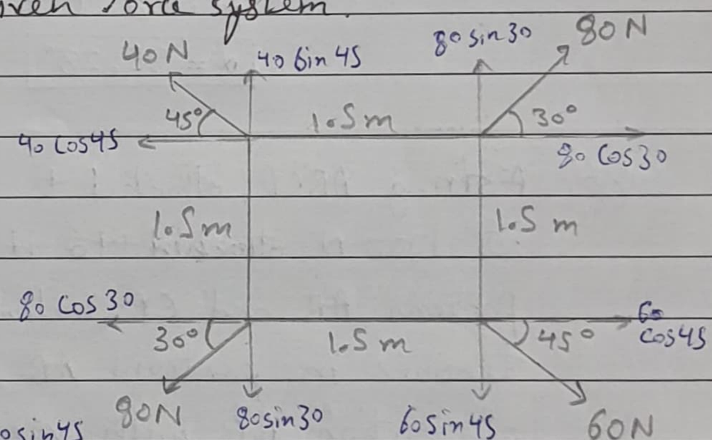
Using sine law in ΔAOC,

$$\frac{OA}{\sin(180-\alpha)} = \frac{AB}{\sin(180-\beta)} = \frac{OB}{\sin(180-\gamma)}$$

$[\because \sin(180-\theta) = \sin\theta]$

$$\therefore \boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

Q1) A body is under action of four coplanar forces as shown in Fig 1. Find magnitude of resultant of given force system.



Sol

$\Sigma H = \cancel{80\cos 30} + 60\cos 45 - \cancel{80\cos 30} - 40\cos 45$

$= \cos 45\,(60-40) = 0.707 \times 20$

$\boxed{\Sigma H = 14.14}$

$\Sigma V = \cancel{80\sin 30} + 40\sin 45 - \cancel{80\sin 30} - 60\sin 45$

$\Sigma V = (40-60)\sin 45 = -20 \times 0.707$

$\boxed{\Sigma V = -14.14}$

Resultant force i.e $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(14.14)^2 + (-14.14)^2}$

$= \sqrt{199.93 + 199.93} = \sqrt{399.87}$

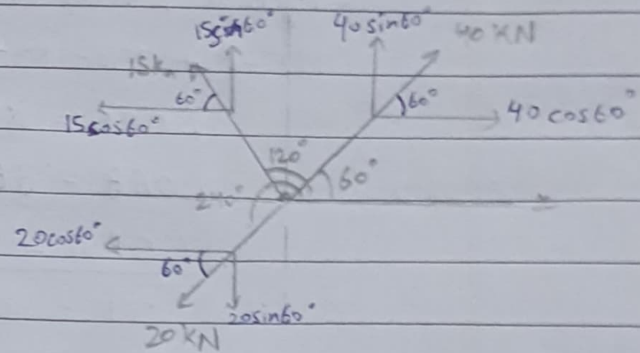
$\boxed{R = 19.99\ N}$

Direction is given by $\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{14.14}{14.14}\right) = \tan^{-1}(1) = 45°$

$\because \Sigma H$ is +ve & ΣV is -ve $\therefore$ actual α lies between ~~360~~ 270° and 360°

Actual angle $= 360 - \alpha = 360 - 45 = \boxed{315°}$

Q2> Three forces of magnitude 40kN, 15 kN, 20kN are acting at point O as shown in fig, the angles made by these forces with x-axis are 60, 120 and 240 Degrees resp. Determine the magnitude and direction of resultant force.



Ans $\Sigma H = 40\cos60 - 15\cos60° - 20\cos60°$

$\Sigma H = (40-15-20)\cos60° = 5 \times \frac{1}{2}$

$\boxed{\Sigma H = 2.5}$

$\Sigma V = 40\sin60 + 15\sin60 - 20\sin60$

$\Sigma V = (40+15-20)\sin60 = 35 \times 0.866$

$\boxed{\Sigma V = 30.31}$

Resultant i.e $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(2.5)^2 + (30.31)^2} = \sqrt{6.25 + 918.69}$

$R = \sqrt{924.94}$

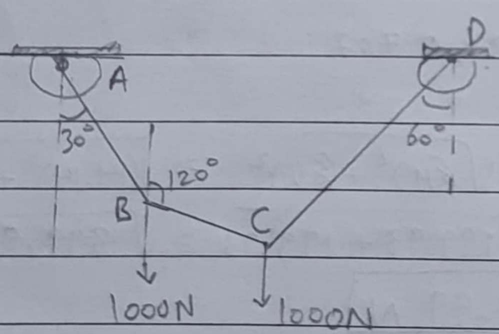
$\boxed{R = 30.41\,kN}$

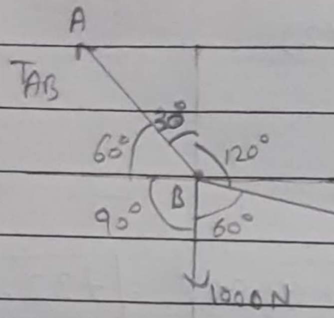
$\alpha = \tan^{-1}\frac{\Sigma V}{\Sigma H} = \tan^{-1}\left(\frac{30.31}{2.5}\right) = \tan^{-1}(12.124) = 85.28°$

$\because \Sigma H$ is positive & ΣV is positive $\therefore$ actual angle is between 0° and 90°

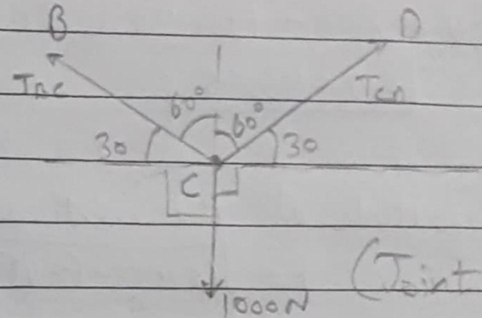
$\boxed{\alpha = 85.28°}$

Q6> A string ABCD, attached to fixed points A and D has two equal weights of 1000 N attached to it at B & C. The weights rest with the portions AB and CD inclined at angles as shown in fig. Find the tensions in portions AB, BC and CD of string, if inclination of portion BC with vertical is 120°.





(Joint B)



(Joint C)

T_{AB} = Tension in the portion AB of the string

T_{BC} = Tension in portion BC of string

T_{CD} = Tension in portion CD of string.

By Lami's theorem

$$\frac{T_{AB}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 150^\circ} = \frac{1000}{\sin 150^\circ}$$

$$T_{AB} = \frac{1000 \times \sin 60^\circ}{\sin 150^\circ}$$

$$T_{BC} = \frac{1000\ \cancel{\sin 150^\circ}}{\cancel{\sin 150^\circ}}$$

$$\boxed{T_{BC} = 1000\ N}$$

$$T_{AB} = \frac{1000 \times 0.866}{1/2}$$

$$\boxed{T_{AB} = 1732\ N}$$

By Lami's theorem

$$\frac{T_{BC}}{\sin 120} = \frac{1000}{\sin 120} = \frac{T_{CD}}{\sin 120}$$

$$T_{BC} = \boxed{T_{CD} = 1000 N}$$

Q7) The lever ABC of a component of a machine is hinged at B, and is subjected to a system of coplanar forces as shown in Fig.

Ans By Varginon's theorem,

Moment of P about B = Moment of 200 about B + Moment of 300 about B

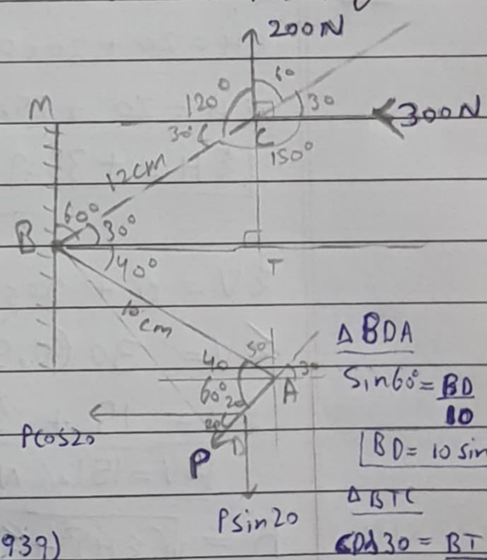
$$P \times 10 \sin 60^\circ = 200 \times 12 \cos 30 + 300 \times 12 \cos 60$$

$$P \times 10 \times 0.866 = 200 \times 12 \times 0.866 + 300 \times 12 \times 0.5$$

$$8.66\ P = 2078 + 1800$$

$$P = \frac{3878}{8.66}$$

$$\boxed{P = 447.8\ N}$$



ΔBDA

$\sin 60^\circ = \frac{BD}{10}$

$\boxed{BD = 10 \sin 60}$

ΔBTC

$\cos 30 = \frac{BT}{12}$

$\boxed{BT = 12 \cos 30^\circ}$

ΔBMC

$\cos 60 = \frac{MC}{12}$

$\boxed{MC = 12 \cos 60}$

At B, $\Sigma H = 300 + P\cos 20 = 300 + 447.3\,(0.939)$

$$\boxed{\Sigma H = 720.8}$$

$\Sigma V = 200 - P\sin 20 = 200 - 447.8(0.342)$.

$\boxed{\Sigma V = 46.85}$

Magnitude of the reaction at B

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(720.8)^2 + (46.85)^2} = \boxed{722.3\,N}$

Direction of reaction at B

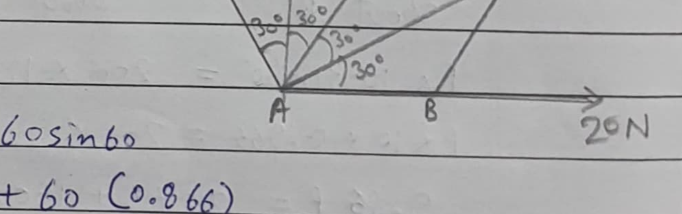
$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{46.85}{720.8}\right) = \tan^{-1}(0.065)$

$\boxed{\theta = 3.7^\circ}$

$\because$ ΣH and ΣV are both +ve $\therefore$ θ lies between 0° to 90°.

Q7> Neglecting friction, find the magnitude of the force (P) to keep the (Continue) Lever in equilibrium. Also determine the magnitude and direction of the reaction at B.

Q 9> The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting at one of the angular points of a regular hexagon, towards the other five angular points, taken in order. find the magnitude and direction of resultant force.



Sol $\Sigma H = +20 + 30\cos 30 + 40\cos 60 + 0 - 60\cos 60$

$\Sigma H = 20 + 30 \times 0.866 + (40 - 60) \times 0.5$

$\Sigma H = 20 + 25.98 - 10$

$\boxed{\Sigma H = +35.9\,N}$

$\Sigma V = 0 + 30\sin 30 + 40\sin 60^\circ + 50 + 60\sin 60$

$\Sigma V = 30(0.5) + 40(0.866) + 50 + 60(0.866)$

$\Sigma V = 15 + 34.64 + 50 + 51.96$

$\boxed{\Sigma V = 151.6\,N}$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(35.9)^2 + (151.6)^2} = \sqrt{1288.81 + 22982.56}$

$R = \sqrt{24271.37}$

$\boxed{R = 155.7\,N}$ i.e resultant force

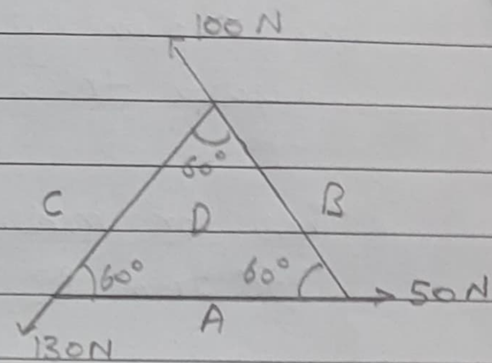
Direction is given by, $\tan\theta = \frac{\Sigma V}{\Sigma H}$

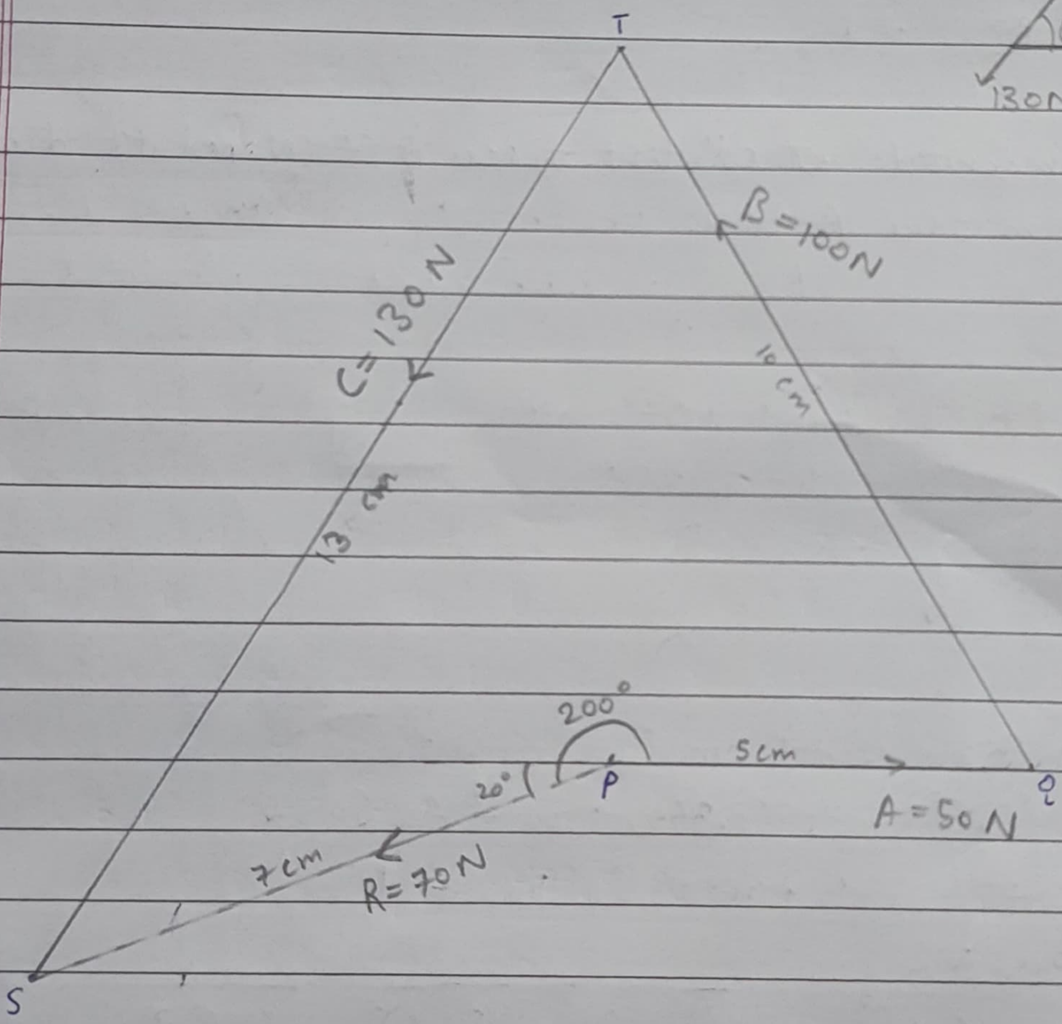
$$\theta = \tan^{-1}\left(\frac{151.6}{35.9}\right) = \tan^{-1}(4.22)$$

$$\boxed{\theta = 76.67}$$

$\because$ ΣH & ΣV are both +ve $\therefore$ θ lies b/w 0° to 90°

Q10) A particle is acted upon by three forces equal to 50N, 100 N and 130 N, along the three sides of an equilateral triangle, taken in order. Find graphically the magnitude and direction of the resultant force.





* First let $10\,N = 1\,cm$ be the scale

  $\therefore$ $A = 50\,N = 5\,cm$, $B = 100\,N = 10\,cm$, $C = 130\,N = 13\,cm$

* Draw force A as PQ of 5 cm
* Draw force B as QT of 10 cm at 60° from PQ
* Draw force C as TS of 13 cm at 60° from QT.
* Complete the polygon by joining SP.

According to polygon's law of forces,
If a number of forces acting simultaneously on a body be given by sides of a polygon in magnitude and direction, taken in order, then their resultant may be given by the closing side of polygon, in magnitude and direction, taken in opposite order.

* Thus on measuring SP, $SP = 7\,cm = 70\,N$ at an angle of 200° from PQ

$\therefore$ The magnitude of resultant force is 70 N and it's direction is 200° from force $A = 50\,N$.