UNIT – II

A.C. CIRCUITS

Date: 04/10/07

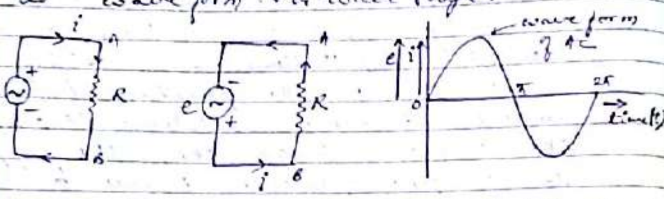
A.C. Circuits :- The path for the flow of alternating current is called an AC circuit. Alternating supply is invariably used for domestic ~~applications.~~ & industrial applications. In dc. circuits, the opposition to the flow of current is due to resistance (R) only of the ckt. whereas, in ac ckt, the opposition to the flow of current is due to resistance (R), inductive reactance ($X_L = 2\pi f L$) & ~~capacitance~~ Capacitive reactance ($X_C = 1/2\pi f C$) of the ckt. In ac ckts frequency plays an important role. In these circuits current & voltage are represented with magnitude & direction (phasors). The voltage & current may or may not be inphase with each other depending upon the parameters (R, L & C) of the circuit. In ac ckts the current & voltages are added & substracted vectorially.

~~AI~~ Alternating Voltage & Current:

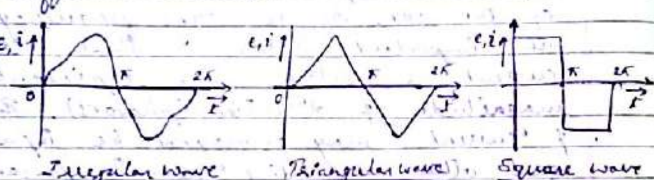
Voltage ~~and current~~ ~~those~~ that changes its polarity & magnitude at regular intervals of time is called an alternating Voltage. When an alternating voltage source is connected across a load resistance R, the current first flows through it in one direction & then in opposite direction, when the polarity is reversed.

Wave form :- The graph representing the

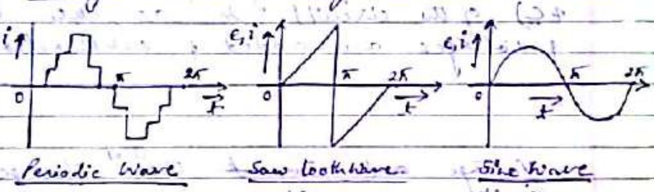
manner in which an alternating voltage or current changes w.r.t. time is known as wave form or wave shape.



The alternating voltage & current may vary in different manner.



Irregular wave — Triangular wave — Square wave



Periodic Wave — Saw tooth wave — Sine wave

Sinusoidal Alternating Quantity

Alternating quantity (voltages or current) which varies according to sine of angle θ ($\theta = \omega t$) is known as sinusoidal alternating quantity. For generation of electrical power, sinusoidal voltages & currents are selected all over the world.

Because

i) The sinusoidal voltages and currents produce low iron & copper losses in ac rotating machines & transformers. This improves the efficiency of ac machines.

ii) The sinusoidal voltages & currents offer less interference to nearby communication systems (telephone lines etc).

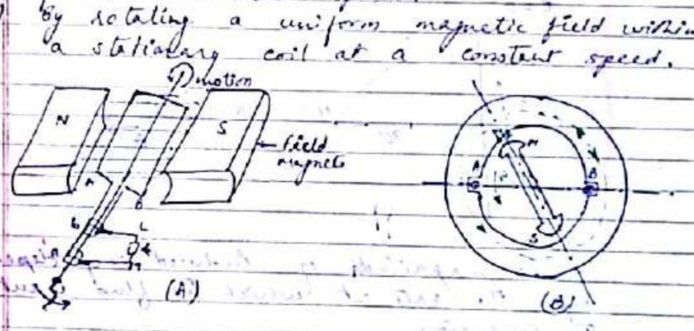
iii) They produce least ~~distrubea~~ disturbance in the electrical circuits.

Generation of Alternating Voltage & Current

Alternating voltage can be generated by either

i) By rotating a coil in a uniform magnetic field at constant speed.

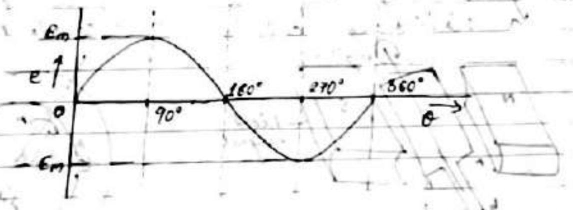
ii) By rotating a uniform magnetic field within a stationary coil at a constant speed.



(A) (B)

The first method is generally applied in small ac generators, whereas second method is applied in large ac generators due to economical considerations. In both the cases, magnetic field is cut by the conductors (or coil sides) & an emf is induced in them. The direction of magnitude of the induced emf in the conductors depend upon the position of the conductors.

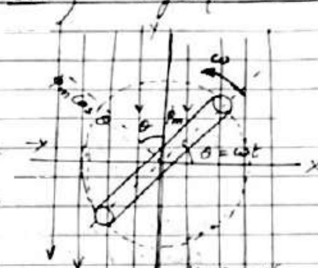
Let LM load connected through brushes & slip rings.
It rotated in anticlock wise direction at a constant angular velocity of ω radians per second. & an emf is induced in the coil



The magnitude of induced emf depends upon the rate at which the flux cut by the conductors.

Equations of the Alternating Voltages & Currents

Consider a rectangular coil,
No of turns = N, rotating in a mag. field,
angular velocity = ω rad/sec
ϕ_m → maximum flux
In time t sec, this coil rotates through an angle $\theta = \omega t$
In this deflected position, the component of the flux which is perpendicular to the plane of the coil is $\phi = \phi_m \cos\omega t$. Hence, flux linkages of the coil at any time are $N\phi = N\phi_m \cos\omega t$



According to Faraday's laws of electromagnetic induction, the emf induced in the coil is given by the rate of change of flux-linkage of the coil. Hence, the value of the induced emf at this instant (i.e. when $\theta = \omega t$) or the instantaneous value of the induced emf is.

$$e = \frac{-d}{dt}(N\phi) = -N\frac{d}{dt}(\phi_m \cos\omega t)$$

$$= -N\phi_m\,\omega(-\sin\omega t)$$

$$= N\phi_m\,\omega \sin\omega t \text{ Volts}$$

$$e = N\phi_m\,\omega \sin\theta \text{ Volts}$$

When the coil has turned through 90° i.e. when $\theta = 90°$ then $\sin\theta = 1$ hence e ...

maximum value say E_m

$\therefore \quad E_m = \omega N \phi_m = \underline{\omega N B_m A}$ Volts

B = maxi flux density in Wb/m²

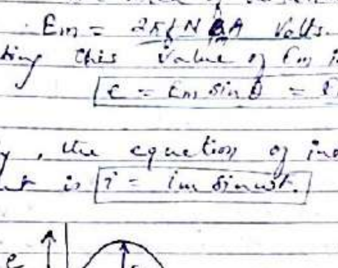
A = Area of the coil in m².

$E_m = \underline{2\pi f N B_m A}$ Volts.

Substituting this value of E_m in Equation

$$\boxed{e = E_m \sin\theta = E_m \sin\omega t}$$

Similarly, the equation of induced alternating current is $\boxed{i = i_m \sin\omega t}$.



~~Class~~ Definitions :-

1) Cycle → One complete set of +ve & -ve values of alternating quantity is known as cycle ~~Hence each diagram~~ A cycle may be sometimes specified in terms of angular measure. In that case, one complete cycle is said to spread over 360° or 2π radian.

2) Time Period :- The time taken by an alternating quantity to complete one cycle is called its time period, T. For Example 50 Hz alternating current has a time period of 1/50 sec.

3) Frequency :- The no. of cycles/sec is called the frequency of the alternating quantity. unit is hertz (Hz).

$$\boxed{f = \frac{PN}{120}} \quad \text{and} \quad f = \frac{1}{T}$$

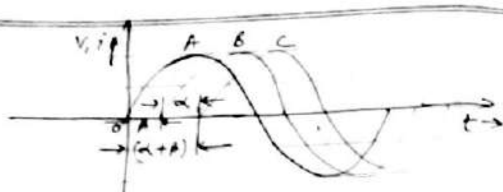
4) Amplitude :- The maximum value, +ve or -ve of an alternating quantity is known as amplitude.

5) Phase & ~~Phase angle~~ :- The phase of an wave may be defined as its position with respect to a reference axis or reference wave ~~& phase angle as the angle of lead or lag with respect to reference axis or another reference wave~~

6) Phase Difference :- It is defined as the difference in angle between the coils. In case of 3 coils, the values of induced emfs in three coils are the same, but there is only the difference in phase, i.e. emfs of these coils do not reach their maximum or zero values simultaneously.

The quantity is one which reaches maximum value earlier as compared to the other is called leading quantity.

The quantity is one which reaches maximum value ~~at least~~ later as compared to the other is called lagging quantity.



Root Mean Square (RMS) Value :- The rms value of an alternating current is given by that steady (d.c.) current which when flowing through a ckt for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time.

$i = I_m \sin\theta$

The mean of squares of the instantaneous values of current over half cycle is.

$$I^2 = \frac{1}{\pi}\int_0^{\pi} i^2 \, d\theta$$

$$I^2 = \frac{1}{\pi}\int_0^{\pi} (I_{max}\sin\theta)^2 \, d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} I_{max}^2 \sin^2\theta \, d\theta$$

$$= \frac{I_{max}^2}{\pi}\int_0^{\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \frac{I_{max}^2}{2\pi}\int_0^{\pi} (1-\cos 2\theta)\, d\theta$$

$$= \frac{I_{max}^2}{2\pi}\left|\theta - \frac{\sin 2\theta}{2}\right|_0^{\pi}$$

$$= \frac{I_{max}^2}{2\pi} \times \pi = \frac{I_{max}^2}{2}$$

$$I = \sqrt{\frac{I_{max}^2}{2}} = \frac{I_{max}}{\sqrt{2}}$$

$$\boxed{I = 0.707\, I_{max}}$$

Average Value or Mean Value - The average value of an alternating current is expressed by that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time.

The mean value is only of the use in connection with processes where the results depend on the current only, irrespective of the voltage such as electroplating or battery charging.

9) Form Factor :- The ratio of rms (or effective) value to average value is the form factor (K_f) of the wave form. It has use in voltage generation & instrument correction factor.

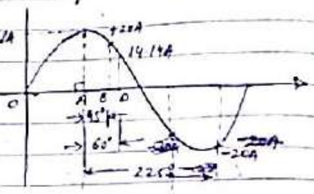
10) Peak Factor :- The ratio of maximum value to the rms value is the peak factor (K_p) of the wave form.

Q1 An alternating current varying sinusoidally with a frequency of 50 Hz has an RMS value of 20A. Write down the equation for the instantaneous value and find the value (a) 0.0025 second (b) 0.0125 sec. after passing through a +ve maxi value. At what time, measured from a +ve maxi value, will be instantaneous current be 14.14 A?

Sol^n:- $I_m = 20\sqrt{2} = 28.2A$, $\omega = 2\pi \times 50 = 100\pi$ rad/s

Eqn of the sinusoidal current wave

$i = 28.28 \sin 100\pi t$ amp



Since, time values are given from pt. A where voltage has the maximum value, the equation may itself be reffered to pt A. In the case, the eqn becomes:-

$i = 28.2 \cos 100\pi t$

i) When $t = 0.0025$ sec.

$i = 28.28 \cos(100\pi \times 0.0025)$ (angle in radians)

$= 28.28 \cos(100 \times 180° \times 0.0025)$ (angle in degree)

$= 28.28 \cos 45° = 28.2 \times \frac{1}{\sqrt{2}} = 20A$

ii) When $t = 0.0125$ sec.

$i = 28.2 \cos(100\pi \times 0.0125)$

$= 28.2 \cos 225°$

$= 28.2 \times \left(\frac{-1}{\sqrt{2}}\right) = -20A$

iii) Here, $i = 14.14A$

$\therefore 14.14 = 28.28 \cos(100\pi t)$

$\frac{14.14}{28.28} = \cos(180 \times 100t)$

$\frac{1}{2} = \cos(180 \times 100t)$

$\cos^{-1}\left(\frac{1}{2}\right) = 18000t$

$60 = 18000t$

$t = \frac{1}{300}$ sec. Ans

Ques An alternating current is represented by $i = 70.7 \sin 520t$. Determine (i) the frequency, (ii) the current 0.0015 sec after passing through zero, increasing positively, (iii) rms & average value.

Sol^n:- Comparing equation with the standard equation

$i = I_m \sin \omega t$ & $i = 70.7 \sin 520t$

here, $I_m = 70.7A$ $\omega = 520$

Now, $\omega = 2\pi f$

$520 = 2\pi f \Rightarrow f = \frac{520}{2\pi} = 83Hz$

Current at $t = 0.0015$ sec

$i = 70.7 \sin 520 \times 0.0015$

$= 70.7 \sin(0.78) = 70.7 \times 0.7033$

$\boxed{i = 49.72 A}$

$I_{rms} = I_m/\sqrt{2} = \frac{70.7}{\sqrt{2}} = 50 A$

Average value, $I_{av} = I_m \times \frac{2}{\pi} = 70.7 \times 0.$

$\boxed{I_{av} = 45.03A}$

1) **AC Through Pure Ohmic Resistance**

In fig. let the applied voltage be given by

$V = V_m \sin\theta = V_m \sin \omega t$ — (1)

R = ohmic resistance

i = instantaneous current

$V = iR$

Putting the value of v from above, we get

$$V_m \sin\omega t = iR$$
$$i = \frac{V_m}{R}\sin\omega t$$

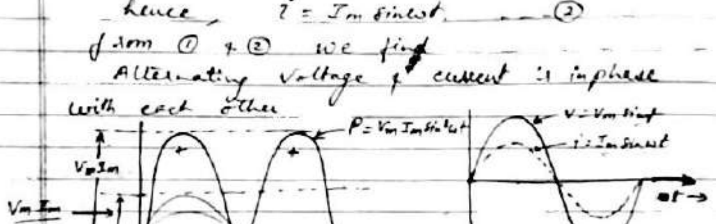
i is maxi when $\sin\omega t$ is unity

$$\therefore\ I_m = \frac{V_m}{R}$$

hence, $i = I_m \sin\omega t$ ——— ②

from ① & ② we find

Alternating voltage & current is inphase with each other



Power :- Instantaneous power, $P = vi = V_m I_m \sin^2\omega t$

$$= \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$
$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t$$

Power consists of a constant part $\frac{V_m I_m}{2}$ & fluctuating part $\frac{V_m I_m}{2}\cos 2\omega t$ of freq. double that of voltage & current waves. For a complete cycle, the average value of $\frac{V_m I_m}{2}\cos 2\omega t$ is zero.

Hence, Power for the whole cycle is

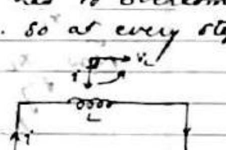
$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$
$$P = V_{rms} I_{rms} = VI \text{ Watt}$$

Note :- It is seen from fig that no part of the power cycle becomes negative at any time. ~~In other~~ This is so because the instantaneous values of voltage & current are always either both +ve & -ve & hence the product is always positive.

2) AC through pure Inductance Alone :-

whenever an alternating voltage is applied to a purely inductive coil, a back emf is produced due to self-inductance of the coil. The back emf at every step, opposes the rise & fall of current through the coil. As there is no ohmic voltage drop, the applied voltage has to overcome this self-induced emf only. So at every step

$$v = L\frac{di}{dt}$$



$$v = V_m \sin\omega t$$
$$\therefore\ V_m \sin\omega t = L\frac{di}{dt}$$
$$di = \frac{1}{L}V_m \sin\omega t\, dt$$

integrating both sides.

$$\int di = \int \frac{1}{L} V_m \sin\omega t\,dt$$
$$i = \frac{V_m}{L}\int \sin\omega t\, dt$$
$$= \frac{V_m}{\omega L}(-\cos\omega t)$$
$$= \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$
$$= \frac{V_m}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

maxi. value of i is $I_m = \frac{V_m}{\omega L}$ when $\sin(\omega t - \frac{\pi}{2})$ is unity

Hence equation of the current becomes
$$\boxed{i = I_m \sin(\omega t - \frac{\pi}{2})}$$
Now we get, For applied voltage
$$V = V_m \sin\omega t.$$
The current through ~~inductor~~ purely inductive ckt is $i = I_m \sin(\omega t - \frac{\pi}{2})$

Clearly, the current lags behind the voltage by $\pi/2$ (phase diff).

$I_m = V_m/\omega L = V_m/X_L$.

X_L = inductive reactance of the coil.

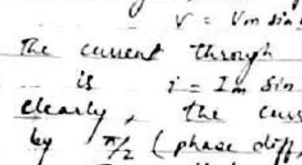
$X_L = \omega L = 2\pi f L$

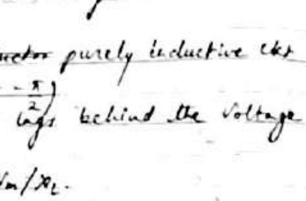
Now $\boxed{X_L \propto f}$

Power :- Instantaneous Power, $P_i = V_m I_m \sin\omega t \sin(\omega t - \frac{\pi}{2})$
$$= -V_m I_m \sin\omega t \cos\omega t.$$
$$= -\frac{V_m I_m}{2}\, 2\sin\omega t\cos\omega t.$$
$$= -\frac{V_m I_m}{2}\,(\sin 2\omega t)$$

Power for whole cycle is
$$P = \frac{-V_m I_m}{2}\int_0^{2\pi} \sin 2\omega t\, dt = 0$$

It is clear from fig. Average power from the

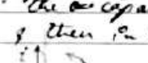




supply for a complete cycle is zero. In the figure power wave is the sine wave of freq. double that of the voltage & current waves. The maximum value of instantaneous power is $V_m I_m/\sqrt{2}$.

3) AC through Pure Capacitance Alone :-

~~In the ckt~~ when an alternating voltage is applied to the plates of a capacitor, the ~~ac~~ capacitor is charged first in one direction & then in the opposite direction.

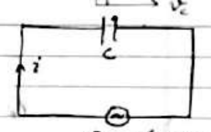


$\therefore v = V_m \sin\omega t$ — (1)

v = Pot. Diff. b/w plates at any instant

q = charge on plates

C = capacitance



Now, $v = q/C$ — (2)

from (1) & (2) we get.

$$\frac{q}{C} = V_m \sin\omega t$$

$i = \frac{d}{dt}(Cv)$ $$\boxed{q = C V_m \sin\omega t} \quad — (3)$$

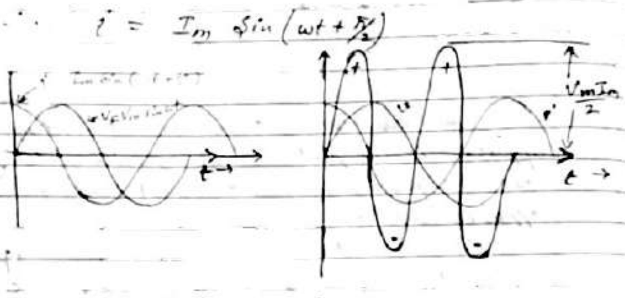
i is given by the rate of ~~change~~ flow of charge

$$i = \frac{dq}{dt} = \frac{d}{dt}(C V_m \sin\omega t)$$
$$i = \omega C V_m \cos\omega t$$
$$i = \frac{V_m}{1/\omega C}\cos\omega t$$
$$= \frac{V_m}{1/\omega C}\sin(\omega t + \frac{\pi}{2})$$

$\therefore$ current is maximum when $\sin(\omega t + \pi/2)$ is unity.

$\therefore\ I_m = \frac{V_m}{1/\omega C} = \frac{V_m}{X_C}$

$\therefore\ i = I_m \sin(\omega t + \pi/2)$



Power → Instantaneous Power.

$$P_i = vi = V_m I_m \sin\omega t \sin(\omega t + 90^\circ)$$

$$= V_m I_m \sin\omega t \cos\omega t$$

$$= \frac{V_m I_m}{2}\, 2\sin\omega t \cos\omega t$$

$$P = \frac{V_m I_m}{2} \sin 2\omega t$$

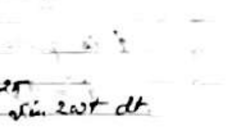
Now,
Power for whole cycle

$$P = \frac{1}{2} V_m I_m \int_0^{2\pi} \sin 2\omega t \, dt$$

$$= 0$$

Ques The voltage applied across 3-branched ckt is given by $v = 100 \sin(5000t + \pi/4)$. Calculate the branch currents & total current

$R = 25\ \Omega$ $L = 2\,mH$ $C = 30\,\mu F$



The total instantaneous current is the vector sum of the three branch currents.

$$i = i_R + i_L + i_C$$

$$i_R = \frac{v}{R} = \frac{100 \sin(5000t + \pi/4)}{25}$$

$$= 4 \sin(5000t + \pi/4)$$

$$i_L = \frac{1}{L}\int v\, dt$$

$$= \frac{1000}{2}\int 100 \sin(5000t + \pi/4)$$

$$= -10 \cos(5000t + \pi/4)$$

$$i_C = C\frac{dv}{dt}$$

$$= \frac{30}{10^6}\frac{d}{dt}\left[100 \sin(5000t + \pi/4)\right]$$

$$= \frac{3 \times 5000}{10^3}\left[\cos(5000t + \pi/4)\right]$$

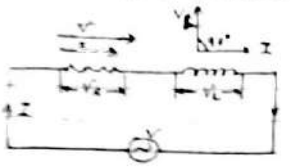
$$= 15 \cos(5000t + \pi/4)$$

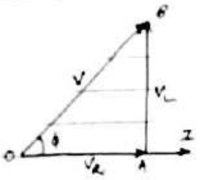
$$i = 4\sin(5000t + \pi/4) + 15\cos(5000t + \pi/4) - 10\cos(5000t + \pi/4)$$

$$\boxed{i = 4\sin(5000t + \pi/4) + 5\cos(5000t + \pi/4)}$$

AC Circuit Resistance & Inductance

A pure Resistance & a pure Inductive coil are connected in series





V = rms voltage, I = rms current.

Voltage drop across R (in phase with I), $V_R = IR$

Voltage drop across coil (ahead of I by 90°), $V_L = I X_L$

In Δ OAB, OA vector represents ohmic voltage

AB ——"—— Inductive drop

OB ——"—— applied voltage (vector sum of two)

Now, $V = \sqrt{(V_R^2 + V_L^2)} = \sqrt{(IR)^2 + (I X_L)^2}$

$= I\sqrt{R^2 + X_L^2}$

$I = \dfrac{V}{\sqrt{R^2 + X_L^2}}$

$I = \dfrac{V}{Z}$

where, $Z = \sqrt{R^2 + X_L^2}$ is impedance of the ckt.

$\therefore Z^2 = R^2 + X_L^2$

It is clear that the applied voltage V leads the current I by an angle ϕ

$\therefore \tan\phi = \dfrac{V_L}{V_R} = \dfrac{I X_L}{I R} = \dfrac{X_L}{R}$

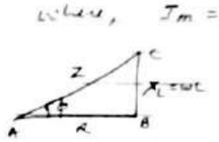
$$\boxed{\phi = \tan^{-1}\frac{X_L}{R}}$$

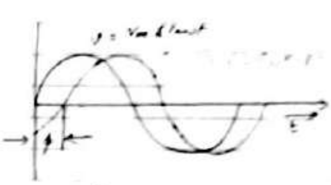
Now, I is lags behind V by angle ϕ.

If Applied Voltage, $v = V_m \sin\omega t$

then current $i = I_m \sin(\omega t - \phi)$

where, $I_m = \dfrac{V_m}{Z}$





Power – Instantaneous Power, $P = vi$

$P = V_m \sin\omega t \cdot I_m \sin(\omega t - \phi)$

$= \dfrac{V_m I_m}{2}\, 2\sin\omega t \cdot \sin(\omega t - \phi)$

$= \dfrac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t - \phi)\right]$

$= \dfrac{V_m I_m}{2}\cos\phi - \dfrac{V_m I_m}{2}\cos(2\omega t - \phi)$

Average Power consumed in the ckt over a complete cycle

$P = \dfrac{V_m I_m}{2}\cos\phi$

The constant part $\dfrac{V_m I_m}{2}\cos\phi$ which contributes to real power. ~~$\frac{V_m I_m}{2}\cos(2\omega t - \phi) = 0$~~

The pulsating component $\dfrac{V_m I_m}{2}\cos(2\omega t - \phi)$ which has a frequency twice that of the voltage & current. It does not contribute to actual power since its average value over a complete cycle is zero.

Average Power, $P = \dfrac{V_m I_m}{2}\cos\phi - 0$

$$P = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi$$

$$= V_{rms} I_{rms} \cos\phi = VI\cos\phi$$

where, $\cos\phi$ is called the Power factor of the circuit.

Power factor :- It is defined as the cosine of the angle b/w voltage & current in ac ckt. It may also be defined as the ratio of resistance to impedance of an ac ckt.

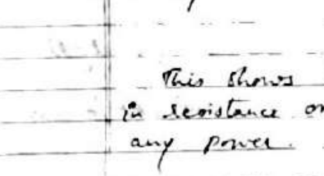
From phasor diagram

$$\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$

$\therefore$ Avg Power, $P = VI\cos\phi = (IZ) I \frac{R}{Z}$

$$= I^2 R$$

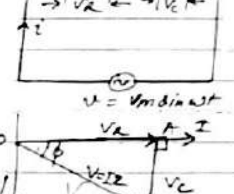
This shows that power is actually consumed in resistance only, inductance does not consume any power.



It is clear that power is -ve b/w angle 0 & ϕ & b/w 180° & $(180+\phi)$. During rest of the cycle the power is +ve. Since the area of under +ve loops is greater than that under the -ve loops, the net power over the complete cycle is positive. Hence, a definite quantity of power is utilized or consumed by this ckt.

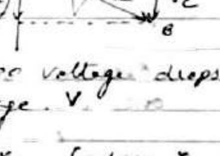
5) AC through Resistance & Capacitance :-

Resistance of R ohm & capacitance of C farad is connected in series in the ckt.



$v = V_m \sin\omega t$

Voltage drop in the Resistance
$V_R = IR$ (in phase with current)
Voltage drop in the Capacitance
$V_C = IX_C$ (90° behind the current)
~~voltage drop in the~~



The vector sum of these two voltage drops is equal to the applied voltage. V

$V_R = IR$ & $V_C = IX_C$ (where $X_C = \frac{1}{2\pi fC}$)

In Δ OAB.

$$V = \sqrt{(V_R)^2 + (V_C)^2}$$

$$= \sqrt{(IR)^2 + (IX_C)^2} = I\sqrt{R^2 + X_C^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

From the phasor diagram it is clear that current in the ckt. leads the applied voltage by an angle ϕ. And ϕ is called as phase angle

$$\boxed{i = I_m \sin(\omega t + \phi)}$$

From phasor diagram.

$$\tan\phi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R}$$

$$\boxed{\phi = \tan^{-1}\frac{X_C}{R}}$$

Power :- If the alternating

$$P = Vi$$

$$= V_m \sin\omega t \cdot I_m \{\sin(\omega t + \phi)\}$$

$$= \frac{V_m I_m}{2} \, 2 \sin\omega t \, \sin(\omega t + \phi)$$

$$= \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t + \phi)\right]$$

$$= \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t + \phi)$$

Average power consumed ~~over~~ in a ckt over a complete cycle,

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\phi \quad -(1)$$

$$P = V_{rms} I_{rms} \cos\phi$$

$$\boxed{P = VI\cos\phi}$$

where, $\cos\phi$ is power factor of the ckt

from phasor diagram $\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ}$

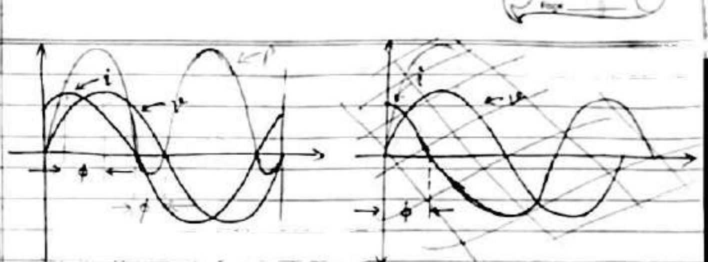
$$\boxed{\cos\phi = \frac{R}{Z}}$$

Now $P = VI\cos\phi$.

$$= (IZ)\, I \frac{R}{Z} = I^2 R.$$

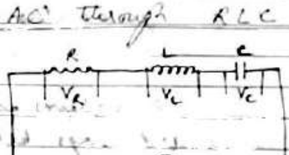
This shows that power is consumed in resistance only, capacitor does not consume any power.

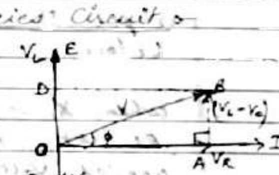
It is clear from the figure that power is -ve b/w angle ~~(180-φ)~~ and 180° & b/w $(360-\phi)$ & 360°. During rest of the cycle, the power is +ve.



Since, the area under the +ve loops is greater than that under the -ve loops, the net power over a complete cycle is +ve. Hence a definite quantity of power is utilized or consumed by this ckt.

(6) AC through RLC series Circuit :-





Now, $V_R = IR$ voltage across R (inphase with I)

$V_L = IX_L$ voltage across L (leads I by 90°)

$V_C = IX_C$ voltage across C (Lags I by 90°)

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2}$$

$$= \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2} = IZ$$

$$\boxed{I = \frac{V}{Z}}$$

$$\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$

$$\tan\phi = \frac{V_L - V_C}{V_R} = \left(\frac{X_L - X_C}{R}\right)\frac{I}{I}$$

$$\boxed{\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)}$$

Power,

Avg Power $P = VI\cos\phi = I^2R$

Power factor, $\cos\phi = \frac{V_R}{Z} = \frac{R}{Z}$

$v = V_m \sin\omega t$

The ckt current is represented by the eqn as per the constants & or parameters explained below:-

Case 1 when $X_L > X_C$, the ckt behaves as an RL ckt. The ckt current lags behind the applied voltage & Pf is lagging.

$$\boxed{i = I_m \sin(\omega t - \phi)}$$

Case 2 when $X_L < X_C$, the ckt behaves as an RC series ckt. $I \rightarrow$ leading Pf :- leading

$$\boxed{i = I_m \sin(\omega t + \phi)}$$

Case 3 when $X_L = X_C$, the ckt behaves as an R ~~series~~ ckt. $I \rightarrow$ inphase Pf $\rightarrow$ unity

$$\boxed{i = I_m \sin\omega t}$$

AC series Ckt

Type of Impedance	Value of Z	ϕ angle of current	Power factor
R	R	$0°$	1
L	$\omega L = X_L$	$90°$ lag	0
C	$1/\omega C = X_C$	$90°$ lead	0
R & L	$\sqrt{R^2 + (\omega L)^2}$	$0 < \phi < 90°$ lag	$1 > P_f > 0$ lag
R & C	$\sqrt{R^2 + (1/\omega C)^2}$	$0 < \phi < 90°$ lead	$1 > P_f > 0$ lead
RLC	$\sqrt{R^2 + (\omega L \sim 1/\omega C)^2}$	$0 \le \phi \le 90°$ lead, $90°$ lag	$0 \le P_f \le 1$ lead or lag

Power in AC circuit

1) Apparent Power (S) :- It is the product of rms value of the applied voltage & circuit current. It is also called wattless or idle power.

Apparent Power (S) $= VI = IZI = I^2Z$ VA

This power is represented when the system power factor (Pf) is not known. The alternators & transformers are rated in apparent power.

2) Active Power (P) :- It is the power which is dissipated in the ~~current~~ ckt resistance. It is has seen that the power is consumed only in resistance. A pure inductor & a pure capacitor do not consume any power, since in a half cycle power is received from source by these components & in next cycle the same is returned to the source.

Active Power (P) $= VI\cos\phi = I^2R$ Watts

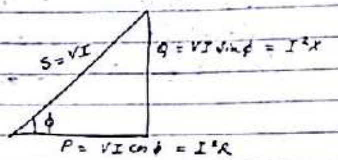
3) Reactive Power (Q) :- It is the power developed in the reactance of the circuit.

Reactive Power (Q) $= VI\sin\phi = I^2X$ Volt Ampere Reactive (VAR)

These powers are related as

$$S = P + jQ$$

$$S = \sqrt{P^2 + Q^2}$$



Power factor & its Importance :-

It is defined as the cosine of the angle b/w voltage & current phasor (ϕ). It is the ratio of resistance & impedance & also the ratio of active power to the apparent power.

$$\boxed{\text{Power factor (Pf)} \ \cos\phi = \frac{\text{Active Power (P)}}{\text{Apparent Power (S)}} = \frac{R}{Z}}$$

The value of power factor (Pf) is vary from 0 to 1. It can be never less & more than 1. We have seen different values of Pf in series ckts. i.e. in case of Resistive, Inductive, capacitive, RL, RC & RLC series ckts.

Usually the word lagging or leading is attached with the numerical value of Pf to signify whether the current lags or leads the voltage. In inductive ckt, current always lags the voltage, whereas in capacitive ckt current always leads the voltage. So, power factor is always mentioned lagging & leading respectively.

Importance of Power factor :-

$$P = VI \cos\phi \Rightarrow I = \frac{P}{V\cos\phi}$$

from above relation, it is clear that for a fixed power at constant voltage, the current drawn by the ckt increases with decrease in Pf.

Thus at low Pf ac ckt. draw more current from their mains & results in the following disadvantages.

1) Greater Conductor Size :- At low Pf conductors are to carry more current for same power, therefore, they require larger area of cross section.

2) Poor Efficiency :- At low power factors, the conductors have to carry larger current which increases copper losses (I^2R) & results in poor efficiency.

3) Larger Voltage Drop :- At lower Pf, cond^r have larger current which increases voltage drop (IR) in the system & results in poor regulation.

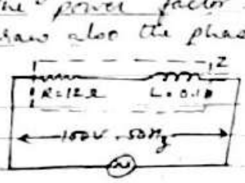
4) Larger KVA rating of Equipment :- The KVA rating of electrical m/c & equipment connected in power system such as alternators, transformers, switch gears etc. will be more at lower Pf. since it is inversely proportional to Pf (i.e. $KVA = KW/\cos\phi$)

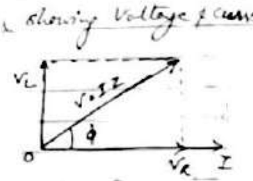
Note:- To improve the power factor of an ac ckt a capacitor is connected across the ckt.

Ques A coil having a resistance of 12 Ω & an inductance of 0.1 H is connected across a 100 V, 50 Hz supply. Calculate

i) The reactance & the impedance of the coil
ii) The current
iii) The phase diff. b/w the current & the applied voltage
iv) The power factor.

Draw also the phasor diagram showing voltage & current





Solution:- i) $R = 12\,\Omega$

$X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 0.1 = 31.4\ \Omega$

$Z = \sqrt{R^2 + X_L^2}$
$= \sqrt{12^2 + 31.4^2} = 33.63\ \Omega$

ii) $I = \frac{V}{Z} = \frac{100}{33.63} = 2.97\ A$

iii) Phase difference, $\phi = \cancel{\tan^{-1} \frac{33.63}{31.4}}$

$\phi = \tan^{-1} \frac{X_L}{R} \Rightarrow \tan^{-1} \frac{31.4}{12}$

$= \tan^{-1} 2.616 = 69.1°$

iv) p.f., $\cos\phi = \cos 69.1°$
$= 0.3568$ lag

Ques A voltage $e = 200 \sin 100\pi t$ is applied to a coil having $R = 200\ \Omega$ & $L = 638$ mH. Find the expression for the current & also determine the power taken by the coil.

Solⁿ:- $R = 200\ \Omega$

$L = 638\ mH = 638 \times 10^{-3}\ H$

Applied Voltage $= 200 \sin 100\pi t$ — ①

Standard Equation $= E_m \sin \omega t = E_m \sin 2\pi f t$ — ②

comparing eqn ① & ②, we get

$E_m = 200\ V$

$2\pi f t = 100\pi t$

$f = 50\ Hz$

Inductive Reactance, $X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 0.638$

$= 200.43\ \Omega$

Impedance, $Z = \sqrt{R^2 + X_L^2} = \sqrt{(200)^2 + (200.43)^2}$

$Z = 283.15\ \Omega$

$I_m = \frac{E_m}{Z} = \frac{200}{283.15} = 0.706\ A$

$\phi = \tan^{-1} \frac{X_L}{R} \Rightarrow \phi = \tan^{-1} \frac{200.43}{200}$

$\phi = 45.06°$

Expression for Current, $i = I_m \sin(100\pi t - 45.06°)$

RMS Value of Voltage & current

$E = \frac{E_m}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 141.42\ V$

$I = \frac{I_m}{\sqrt{2}} = \cancel{\frac{283.15}{}}\ \frac{0.706}{\sqrt{2}} = 0.5\ A$

Power factor of coil, $\cos\phi = \cos 45.06°$
$= 0.7064$ lag

Power, $P = VI\cos\phi$

$= 141.42 \times 0.5 \times 0.7064$

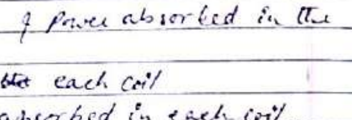
$= 50$ Watts

Ques Two coils are connected in series having resistance & inductive reactance 5Ω & 6Ω, 3 & 7Ω respectively. A sinusoidal voltage of 200V, 50Hz is applied across the combination. Calculate :-

i) Current, Power factor & Power absorbed in the whole ckt.

ii) Voltage drop across the each coil

iii) Power factor & Power absorbed in each coil



Solⁿ :- $R_1 = 5\Omega$, $R_2 = 3\Omega$

$X_{L_1} = 6\Omega$, $X_{L_2} = 7\Omega$

Applied Voltage, $V = 200V$

$Z_1 = \sqrt{R_1^2 + (X_{L_1})^2}$

$= \sqrt{5^2 + 6^2} = 7.81\Omega$

$Z_2 = \sqrt{R_2^2 + (X_{L_2})^2}$

$= \sqrt{3^2 + 7^2} = 7.606\Omega$

Total Resistance, $R = R_1 + R_2 = 8\Omega$

Total Reactance, $X_L = X_{L_1} + X_{L_2} = 13\Omega$

Total Impedance, $Z = \sqrt{R^2 + X_L^2}$

$= \sqrt{8^2 + 13^2} = 15.264\Omega$

i) Current in Ckt, $I = \frac{V}{Z} = \frac{200}{15.264} = 13.1A$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{8}{15.264} = 0.5241$

Power absorbed in the ckt, $P = VI\cos\phi$

$= 200 \times 13.1 \times 0.5241$

$= 1373$ Watt

ii) Voltage drop across coil-1, $V_1 = IZ_1$

$= 13.1 \times 7.81 = 102.3V$

—— " —— coil 2 $V_2 = IZ_2$

$= 13.1 \times 7.616 = 99.771$

iii) Pf of coil-1, $\cos\phi_1 = \frac{R_1}{Z_1} = \frac{5}{7.81} = 0.64$ lagging.

Power, $P_1 = V_1 I\cos\phi_1 = 102.3 \times 0.64 \times 13.1$

$= 858$ W

Pf of coil-2 $\cos\phi_2 = \frac{R_2}{Z_2} = \frac{3}{7.616} = 0.394$ lagging

Power, $P_2 = V_2 I\cos\phi_2 = 99.77 \times 13.1 \times 0.394$

$= 515$ W

Series Resonance :- In RLC series ckt, when ckt current is in phase with the applied voltage, the ckt is said to be in series resonance. This condition is obtained in an RLC ckt when $X_L = X_C$ (or $X_L - X_C = 0$)

At resonance, $X_L - X_C = 0$ or $X_L = X_C$

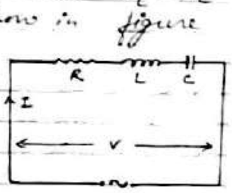
Impedance, $Z_r = \sqrt{R^2 + (X_L - X_C)^2} = R$

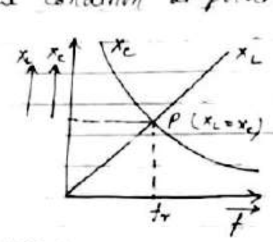
current, $I_r = \frac{V}{Z_r} = \frac{V}{R}$

Since, at resonance the opposition to the flow of current is only resistance (R) of the ckt, the circuit draws maxi current under this condition

Resonant Frequency

The value of $X_L = 2\pi f L$ & $X_C = \frac{1}{2\pi f C}$ can be changed by changing the supply frequency. When frequency increases the value of X_L increases, whereas, the value of X_C decrease & vice versa. Thus to obtain series resonance, the frequency is adjusted to f_r so that $X_L = X_C$ the condition at point P show in figure





$\therefore$ At series resonance

$$X_L = X_C$$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r^2 = \frac{1}{(2\pi)^2 LC}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

where, f_r is the resonant frequency in Hz & L & C are measured in Henry & Farad respectively.

Effects of Series Resonance

i) At resonance $X_L = X_C$ therefore, the impedance of the ckt is minimum & is reduced to the resistance of the ckt ie

$$\boxed{Z_r = R}$$

ii) Since impedance is minimum, the ckt current is maximum at resonance, ie.

$$\boxed{I_r = \frac{V}{Z_r} = \frac{V}{R} \text{ (maximum)}}$$

iii) Power taken by the ckt is maximum, as I_r is maxi

$$\boxed{P_r = I_r^2 R}$$

iv) As the current drawn by the ckt, at resonance is very large (maximum), the voltage drop across L ~~(X_L, V_L =~~ & C are also very large.

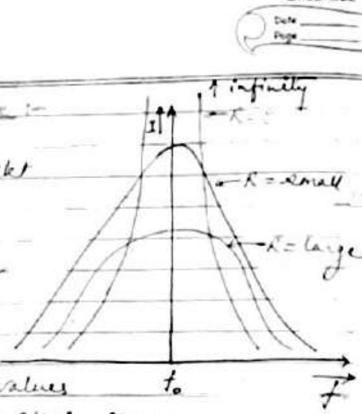
$$V_L = I X_L = I \times 2\pi f_r L$$

$$V_C = I X_C = \frac{I}{2\pi f_r C}$$

In power system, at resonance, the excess voltage built up across the inductive & capacit components (such as ckt breakers, reactors etc) may cause damage. Therefore, series resonance should be avoided in power system. However, in some of the electronic devices (such as antenna ckt of radio & TV receiver, tuning ckt etc). the principle of series resonance is used to increase the signal voltage; current at a desired frequency (f_r)

Since, a series resonance ckt has the capability to draw heavy current & power from the mains, its is often regarded as Acceptor circuit.

Resonance Curve :-

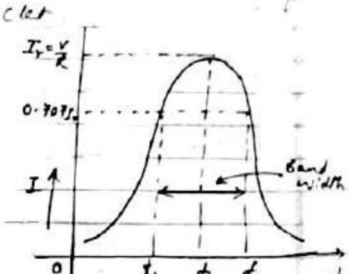


The curve, b/w ckt current & the frequency of the applied voltage, is known as resonance curve.

For smaller values of R, the resonance curve is sharply peaked & such a ckt is said to be sharply resonant or highly selective. However, ~~the~~ for larger values of R, resonance curve is flat & is said to have poor selectivity. The ability of a resonant ckt to discriminate between one particular frequency & all ~~the~~ others is called its selectivity.

Band Width :- The range of frequency over which ckt current is equal to or more than 70.7 % of maximum current (ie I_r current at resonance) is known as bandwidth of the series resonant ckt.

It is shown in figure b/w frequency range f_1 to f_2.



∴ Band Width

$$\boxed{BW = f_2 - f_1}$$

Here,

$f_1 \rightarrow$ lower cut-off frequency

$f_2 \rightarrow$ Upper cut off frequency

The bandwidth represents the frequency range at which the ckt offers low impedance to ckt current.

i) If the resonant frequency is not located at the centre of upper & lower cut off frequency then,

$$\boxed{f_r = \sqrt{f_1 f_2}}$$

ii) When the resonant frequency is located sufficiently near to the center of the two cut-off frequencies & Q of the ckt is ≥ 10, then

$$\boxed{f_1 = f_r - \frac{BW}{2}} \quad \& \quad \boxed{f_2 = f_r + \frac{BW}{2}}$$

Q - Factor of series Resonant Ckt :-

~~We have~~ At series resonance, the ckt draws largest current from the mains, this produces a heavy voltage across L or C. The factor by which the p.d. across L or C rises to that of the applied voltage is called the Q-factor of the series resonant ckt.

$$\therefore \text{Q-factor} = \frac{\text{Voltage across L or C}}{\text{Applied Voltage}}$$

$$= \frac{I_r X_L}{I_r R} = \frac{X_L}{R} = \frac{\omega_r L}{R}$$

where, $\omega_r = 2\pi f_r = 2\pi \frac{1}{2\pi\sqrt{LC}} = \frac{1}{\sqrt{LC}}$

$\therefore$ Q-factor $= \frac{L}{R} \times \frac{1}{\sqrt{LC}} \Rightarrow \frac{1}{R}\sqrt{\frac{L}{C}}$

Ques Determine the parameters of an RLC series ckt that will resonate at 1000Hz, has a band width of 100 Hz & draws 16W from a 200V generator operating at the resonant frequency of the circuit.

Sol^n:- At resonance: Voltage across resistor

V_R = supply voltage = 200

Now, $P = \frac{V_R^2}{R}$ $\therefore R = \frac{V_R^2}{P}$

$R = \frac{200^2}{16} = 2500\,\Omega$

Now, Q-factor $= \frac{f_r}{BW} = \frac{1000}{100} = 10$

But, $Q = \frac{V_L}{V_R} = \frac{I X_L}{IR} = \frac{2\pi f_r L}{R}$

$10 = \frac{2\pi \times 1000}{2500} \times L$

$L = 25/2\pi = 3.98\,H$

Now, $2\pi f_r L = \frac{1}{2\pi f_r C}$

$C = \frac{1}{4\pi^2 f_r^2 L} = \frac{1}{(2\pi \times 1000)^2 \times 3.98}$

$= 6.3 \times 10^{-9}$ Farad.

AC Parallel Circuits:- The ac circuits in which no. of branches are connected in such a way that the voltage across each branch is the same but current through them is different, are called ac parallel circuits. The parallel circuits are used more frequently in ac system because of foll. reasons:

i) Almost all the electrical appliances of different ratings are operated at the same supply voltages & are connected in parallel.

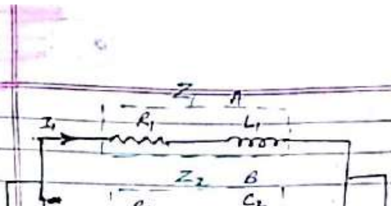
ii) Each device is required to be operated independently without disturbing the operation of other devices. Hence, connected in parallel.

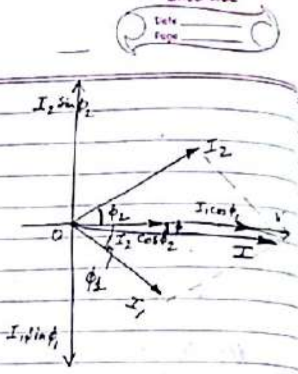
In parallel ckt, no. of branches are connected in parallel, & each branch is analysed seperately as a series ckt. & then the effects of seperate branches are combined together. The foll. methods are applied for solving ac parallel ckts.

i) Phasor or Vector method
ii) Admittance method.
iii) Method of phasor algebra or vector algebra.

(i) Phasor or Vector Method:-

Consider ~~an~~ a ckt having two reactors A & B have been joined in parallel across an rms supply of V volts. The voltage across A & B is same but the current ~~is different~~ through them is different.





Impedance of each branch of the ckt seperately,

At A, $Z_1 = \sqrt{R_1^2 + X_{L_1}^2}$

At B, $Z_2 = \sqrt{R_2^2 + X_{C_2}^2}$

where, $X_{L_1} = 2\pi f L_1$

$X_{C_2} = \frac{1}{2\pi f C_2}$

Now, current & phase angle ~~at~~ in branch A & B.

At branch A,

$I_1 = \frac{V}{Z_1}$ & $\phi_1 = \cos^{-1}\frac{R_1}{Z_1}$ (lagging)

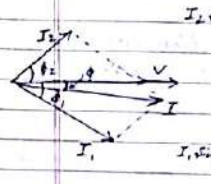
At branch B,

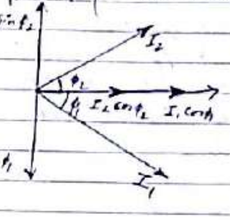
$I_2 = \frac{V}{Z_2}$ & $\phi_2 = \cos^{-1}\frac{R_2}{Z_2}$ (leading)

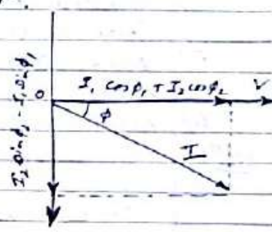
The resultant current I is the vector sum of the branch current I_1 & I_2 & can be found by

(i) Using // gram law of vectors. or

(ii) Resolving I_1 & I_2 into their x-y components (active and reactive components) respectively & then by combining these components. Generally method (ii) is preferable, as it is quick & convinient.







Sum of active components of I_1 & I_2

$I\cos\phi = I_1\cos\phi_1 + I_2\cos\phi_2$

Sum of reactive components of I_1 & I_2

$I\sin\phi = I_2\sin\phi_2 - I_1\sin\phi_1$ (-ve)

Resultant current, $I = \sqrt{(I\cos\phi)^2 + (I\sin\phi)^2}$

$$\boxed{I = \sqrt{(I_1\cos\phi_1 + I_2\cos\phi_2)^2 + (I_2\sin\phi_2 - I_1\sin\phi_1)^2}}$$

& $$\boxed{\tan\phi = \frac{I_2\sin\phi_2 - I_1\sin\phi_1}{I_1\cos\phi_1 + I_2\cos\phi_2}} = \frac{Y\text{ component}}{X\text{ component}}$$

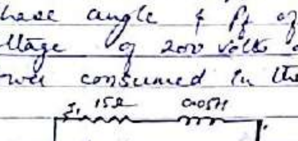
If $\tan\phi$ is +ve, then current I leads ~~the~~ & if $\tan\phi$ is -ve, then current I lags behind the applied voltage V.

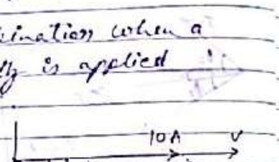
Power factor of the whole ckt is given by

$$\boxed{\cos\phi = \frac{I_1\cos\phi_1 + I_2\cos\phi_2}{I} = \frac{X\text{ component}}{I}}$$

Ques: A coil of resistance 15 ohm & inductance 0.05H is connected in parallel with a non-inductive resistance of 20 ohm. Find
i) current in each branch of the ckt.
ii) Total current supplied
iii) Phase angle & pf of combination when a voltage of 200 volts at 50Hz is applied
iv) Power consumed in the ckt.

Soln.-





At branch (1) $R_1 = 15\Omega$

$X_{L_1} = 2\pi f L_1 = 2\pi 50 \times 0.05 = 15.7\Omega$

$Z_1 = \sqrt{R_1^2 + X_{L_1}^2} = \sqrt{15^2+15.7^2} = 21.72\Omega$

$I_1 = \frac{V}{Z_1} = \frac{200}{21.72} = 9.2A$

$\phi_1 = \cos^{-1}\frac{R_1}{Z_1} = \cos^{-1}\frac{15}{21.72} = 46.3°$ lagging

At branch (2) $R_2 = 20\Omega$

$Z_2 = R_2$

$I_2 = \frac{V}{R_2} = \frac{200}{20} = 10A$

$\phi_2 = \cos^{-1}\frac{R_2}{Z_2} = 0$

I_2 is in phase with V.

Resolving the current horizontally & vertically in the phasor diagram

$I_x = I_2 + I_1\cos\phi_1$
$= 10 + 9.2\cos 46.3°$
$= 16.356\ A$

$I_y = 0 - I_1 \sin\phi_1$
$= 0 - 9.2\sin 46.3°$
$= -6.65\ A$

Total current supplied, $I = \sqrt{I_x^2 + I_y^2}$
$= \sqrt{(16.356)^2 + (-6.65)^2}$

$\boxed{I = 17.656\ A}$

Phase angle, $\phi = \tan^{-1}\frac{I_y}{I_x}$

$\phi = \tan^{-1}\left(\frac{-6.65}{16.356}\right) = -22.126°$

$\boxed{\phi = -22.126°}$

Power factor of the ckt.

$\boxed{\cos\phi} = \cos(-22.126) = 0.9264$ (lag)

Power, $P = VI\cos\phi$
$= 200 \times 17.656 \times 0.9264$

$\boxed{P = 3271.3\ Watt}$

5) Admittance Method.

Admittance of ~~ckt~~ a ckt is defined as the reciprocal of its impedance. It is represented by "Y".

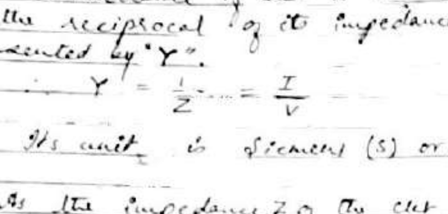
$$\therefore Y = \frac{1}{Z} = \frac{I}{V}$$

Its unit is Siemens (S) or mho

As the impedance Z of the ckt has two components X and R. Similarly, admittance Y also has two components g & b.

where $g \rightarrow$ conductance $= \frac{1}{R}$

& $b \rightarrow$ susceptance $= \frac{1}{X}$



Now, conductance, $g = Y\cos\phi$

$$g = \frac{1}{Z}\cdot\frac{R}{Z}$$

$$g = \frac{R}{Z^2} = \frac{R}{R^2+X^2}$$

Similarly, susceptance $b = Y\sin\phi = \frac{1}{Z}\cdot\frac{X}{Z}$

$$b = \frac{X}{Z^2} = \frac{X}{R^2+X^2}$$

Admittance $Y = \sqrt{g^2+b^2}$

Unit of g, b & Y is Siemens or mho.

We will regard the capacitive susceptance as +ve and inductive susceptance as −ve.

Solution of Parallel a.c. ckt by Admittance Method

Consider a Parallel ckt as shown in figure

~~Find~~ First, impedance in each branch

Now at branch A,

$$Z_1 = \sqrt{R_1^2 + X_{C_1}^2}$$

At B,

$$Z_2 = \sqrt{R_2^2 + X_{L_2}^2}$$

Now, find phase angle of each branch.

$$\phi_1 = \tan^{-1}\frac{X_{C_1}}{R_1}$$

$$\phi_2 = \tan^{-1}\frac{X_{L_2}}{R_2}$$

Find conductance, susceptance & admittance in each branch.

$$g_1 = \frac{R_1}{Z_1^2} \quad ; \quad b_1 = \frac{X_{C_1}}{Z_1^2}\ (+ve) \quad ; \quad Y_1 = \sqrt{g_1^2+b_1^2}$$

$$g_2 = \frac{R_2}{Z_2^2} \quad ; \quad b_2 = \frac{X_{L_2}}{Z_2^2}\ (-ve) \quad ; \quad Y_2 = \sqrt{g_2^2+b_2^2}$$

Now find the Algebraic sum of conductance & susceptance.

$$G = g_1 + g_2 \quad \& \quad B = b_1 + b_2$$

& Total Admittance of the ckt is

$$\boxed{Y = \sqrt{G^2+B^2}}$$

Now ~~find~~ branch currents.

$$\boxed{I_1 = VY_1}$$

$$\boxed{I_2 = VY_2}$$

$$\therefore \boxed{I = VY}$$

~~Total Phase angle~~

Resultant phase angle & power factor

$$\boxed{\phi = \tan^{-1}\frac{B}{G}}$$ (lagging if B is -ve) (leading if B is +ve)

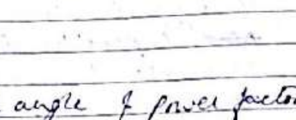
$$\boxed{P_f = \cos\phi = \frac{G}{Y}}$$

Ques. A parallel ckt has two branches. One branch contains a resistance of 8 ohms inductance of 19.1 mH in series & the other contains a resistance of 6 ohms & capacitor of 601.55 μF in series. This parallel ckt is connected across a supply voltage of 240V, 50Hz. Determine (i) current drawn by each branch.

ii) Total current drawn from the mains.

iii) Pf of the whole ckt.

Sol^n :- Branch 1,



$X_{L_1} = 2\pi f L_1$

$= 2\pi \cdot 50 \times 19.1 \times 10^{-3}$

$= 6\,\Omega$

$Z_1^2 = \sqrt{R_1^2 + X_{L_1}^2}$

$= \sqrt{64+36} = 10$

$g_1 = \frac{R_1}{Z_1^2} = \frac{8}{100} = 0.08\ ℧$

$b_1 = \frac{X_{L_1}}{Z_1^2} = \frac{6}{100} = 0.06\ ℧\ (-ve)$

$Y_1 = \sqrt{g_1^2 + b_1^2} = \sqrt{(0.08)^2 + (0.06)^2}$

$= 0.1$ mho.

$I_1 = VY_1 = 240 \times 0.1 = 24$ Amp.

Branch 2,

$X_{C_2} = \frac{1}{2\pi f C_2} = 5.291\ \Omega$

$Z_2 = \sqrt{R_2^2 + X_{C_2}^2} = 8\ \Omega$

$g_2 = \frac{R_2}{Z_2^2} = 0.09375\ ℧$

$b_2 = \frac{X_{C_2}}{Z_2^2} = \frac{5.291}{64} = 0.08268\ ℧$ (+ve)

$Y_2 = \sqrt{g_2^2 + b_2^2} = \sqrt{(0.09375)^2 + (0.08268)^2}$

$= \sqrt{0.0088 + 0.0068}$

$= ~~0.2778~~\ 0.125\ ℧$

$I_2 = VY_2 = 240 \times 0.125 = 30$ Amp.

Total Conductance, $G = g_1 + g_2 = 0.17375\ ℧$

— " — Susceptance, $B = -b_1 + b_2 = 0.02268\ ℧$

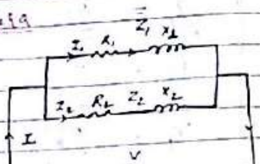
— " — Admittance $Y = \sqrt{G^2+B^2} = 0.1752\ ℧$

Total current, $I = VY = 240 \times 0.1752$

$= ~~42.048~~$ A

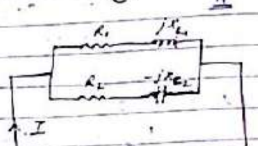
Pf, $\cos\phi = \frac{G}{Y} = \frac{0.17375}{0.1752} = 0.9917$ (leading)

3) Complex or Phasor Algebra



A

Z_1 & Z_2 are two impedances ~~have~~ of parallel branches having same Potential difference.



B

Now, $I_1 = \frac{V}{Z_1}$

& $I_2 = \frac{V}{Z_2}$

$$I = I_1 + I_2 = \frac{V}{Z_1} + \frac{V}{Z_2} = V\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)$$

$$= V(Y_1 + Y_2) = VY$$

Where, Total admittance $Y = Y_1 + Y_2$

~~Now the~~ The admittances of the impedances are complex quantities, all additions must be in complex form. Simple additions must not be attempted.

Consider the two parallel branches of figure B.

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_{L_1}} = \frac{R_1 - jX_{L_1}}{(R_1 + jX_{L_1})(R_1 - jX_{L_1})}$$

$$= \frac{R_1 - jX_{L_1}}{R_1^2 + X_{L_1}^2}$$

$$= \frac{R_1}{R_1^2 + X_{L_1}^2} - j\frac{X_{L_1}}{R_1^2 + X_{L_1}^2}$$

$$= \frac{R_1}{Z_1^2} - j\frac{X_{L_1}}{Z_1^2}$$

$$= g_1 - jb_1$$

where, g_1 is conductance of upper branch & b_1 is susceptance of upper branch.

similarly,

$$Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 - jX_{C_2}}$$

$$= \frac{R_2 + jX_{C_2}}{(R_2 - jX_{C_2})(R_2 + jX_{C_2})}$$

$$= \frac{R_2 + jX_{C_2}}{R_2^2 + X_{C_2}^2}$$

$$= \frac{R_2}{R_2^2 + X_{C_2}^2} + j\frac{X_{C_2}}{R_2^2 + X_{C_2}^2}$$

$$= g_2 + jb_2$$

Total Admittance $Y = Y_1 + Y_2$

$$Y = (g_1 - jb_1) + (g_2 + jb_2)$$

$$= (g_1 + g_2) - j(b_1 - b_2)$$

$$\boxed{Y = G - jB}$$

$$\boxed{Y = \sqrt{G^2 + B^2}}$$

$$\phi = \tan^{-1}\left(\frac{B}{G}\right) = \tan^{-1}\left(\frac{b_1 - b_2}{g_1 + g_2}\right)$$

The polar form for admittance is $Y = Y\angle\phi$

where ϕ is given above

$$Y = \sqrt{G^2+B^2} \angle \tan^{-1}(B/G)$$

Total current $I = VY$; $I_1 = VY_1$, $I_2 = VY_2$

If $V = V\angle 0°$ & $Y = Y\angle\phi$.

$$I = VY = V\angle 0° \times Y\angle\phi$$
$$= VY\angle\phi.$$

In general, if $V = V\angle\alpha$ & $Y = \angle\beta$

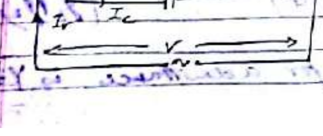
then $I = VY$

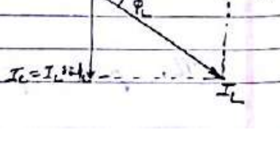
$$= V\angle\alpha \; Y\angle\beta$$
$$= VY\angle(\alpha+\beta)$$

Hence, it should be noted that when vector voltage is multiplied by admittance either in complex (rectangular) or polar form, the result is vector current in its proper phase relationship with respect to the voltage, regardless of the axis of which the voltage may be have been referred to.

Parallel Resonance :-

In a ckt an inductor & capacitor is said to be in parallel resonance when the ckt current is in phase with the applied voltage.





since at resonance, the reactive component of current is supressed, the ckt draws minimum current under this condition.

Resonant Frequency. The value of $X_L = 2\pi f L$ & $X_C = \frac{1}{2\pi f C}$ can be changed by changing the supply frequency.

As freq. increases, X_L increases so that Z_L also increases

$\therefore$ I_L decreases & lags the voltage V

On the other hand, X_C decrease & consequently value of I_C increases.

At frequency f_r,

$$I_C = I_L \sin\phi_L$$

$\therefore$ At parallel resonance $\boxed{I_C = I_L \sin\phi_L}$

where, $I_L = \frac{V}{Z_L}$; $\sin\phi_L = \frac{X_L}{Z_L}$ & $I_C = \frac{V}{X_C}$

$$\frac{V}{X_C} = \frac{V}{Z_L} \times \frac{X_L}{Z_L} \quad \text{or} \quad X_L X_C = Z_L^2$$

$$\therefore \frac{\omega L}{\omega C} = Z_L^2$$

$$\text{or} \quad \frac{L}{C} = R^2 + X_L^2$$

$$\frac{L}{C} = R^2 + (2\pi f_r L)^2$$

$$2\pi f_r L = \sqrt{\frac{L}{C} - R^2}$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

If R is very small as compared to L

$\therefore$ Resonant frequency $\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$

Effects of Parallel Resonance

At parallel resonance, line current $I_r = I_L \cos\phi$.

$$\therefore \frac{V}{Z_r} = \frac{V}{Z_L} \times \frac{R}{Z_L}$$

$$\frac{1}{Z_r} = \frac{R}{Z_L^2}$$

$$\frac{1}{Z_r} = \frac{R}{L/C} = \frac{CR}{L} \quad \left[\because Z_L^2 = \frac{L}{C}\right]$$

$\therefore$ ckt impedance, $Z_r = \frac{L}{RC}$

It shows that

i) Circuit impedance $Z_r \left(= \frac{L}{RC}\right)$ is a pure resistive because there is no frequency term present.

ii) The value of Z_r is very high because the ratio of $\frac{L}{C}$ is very large at parallel resonance.

iii) The ckt current I_r is very small because Z_r is very high.

iv) The current flowing through the capacitor & coil is much greater than the line ~~curr~~ current because the impedance of each branch is quite low then ckt impedance Z_r

Since a parallel resonant ckt can draw a very small current & power from the mains, it is often regarded as rejector circuit.

Q-factor of a Parallel Resonance circuit.

We have seen that at parallel resonance, the current circulating b/w the two branches is many times greater than the line current drawn from the mains. This current amplification produced by the resonance is called the Q factor of the parallel resonant ckt.

$$\therefore \text{Q-factor} = \frac{\text{Current circulating b/w L \& C}}{\text{Line current}} = \frac{I_C}{I_r}$$

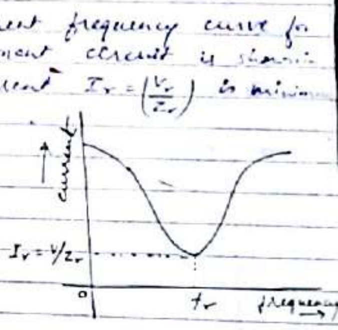
Now, $I_C = \frac{V}{X_C} = 2\pi f_r CV$ & $I_r = \frac{V}{\left(\frac{L}{CR}\right)}$

$$\text{Q-factor} = \frac{2\pi f_r CV}{V} \times \frac{L}{CR} = \frac{2\pi f_r L}{R}$$

or $$\text{Q-factor} = \frac{2\pi L}{R} \times \frac{1}{2\pi\sqrt{LC}}$$

or $$\text{Q factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Resonance curve: A current frequency curve for a typical parallel resonant circuit is shown in figure. The value of current $I_r = \left(\frac{V_r}{Z_r}\right)$ is minimum at resonance.



Ques. A parallel ckt consists of a coil having 15 Ω resistance & 300 mH inductance in parallel with a capacitor of capacitance 4 μF. Determine the

i) The resonant frequency

ii) Dynamic impedance of the ckt.

iii) Q factor of the ckt at resonance.

Sol i) Resonant frequency,

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{0.3 \times 4 \times 10^{-6}} - \frac{15^2}{(0.3)^2}} = 145.27\ Hz$$

ii) Dynamic impedance,

$$Z_r = \frac{L}{CR} = \frac{0.3}{4 \times 10^{-6} \times 15} = 5000\ \Omega.$$

iii) Q-factor $= \frac{2\pi f_r L}{R} = \frac{2\pi \times 145.27 \times 0.3}{15}$

$= 18.255$

POLYPHASE CIRCUIT (3 Phase)

Polyphase circuit means, the circuits to having more than one phases or windings. Each phase having a single alternating voltage of the same magnitude & frequency. Hence, a polyphase system is essentially a combination of two or more than two voltages having same magnitude & frequency but displaced from each other by equal electrical angle. This angular displacement between the adjacent voltages is called phase difference & depends upon the no of phases.

$$\text{Phase diff.} = \frac{360 \text{ electrical degree}}{\text{No of phases.}}$$

But, for 2 phase system the above is wrong, where the voltages are displaced by 90° electrical.

Thus, an ac system having a group of equal voltages of same frequency arranged to have equal phase difference b/w adjacent emf's is called a **Polyphase System**.

The polyphase system may be two phase, three phase or six phase system. But for all practical purposes, 3-phase system is invariably employed.

Advantages of 3 phase system over 1-ϕ system.

1) In single phase, the power delivered is pulsating. Even when the voltage & current are in phase, power is zero twice in each cycle. [illegible]