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ASSIGNMENT 1

Q.1. Solve – $(e^{x/y}+1)\,dx + e^{x/y}(1-\frac{x}{y})\,dy$

$$\frac{dx}{dy} = \frac{-e^{x/y}(1-\frac{x}{y})}{1+e^{x/y}}$$

Put $x = vy$

$$\frac{dx}{dy} = y\frac{dv}{dy} + v$$

$$\frac{dx}{dy} = \frac{-e^{x/y}(1-x/y)}{1+e^{x/y}}$$

$$v + y\frac{dv}{dy} = \frac{-e^{v}(1-v)}{1+e^{v}}$$

$$y\frac{dv}{dy} = \frac{-e^{v}(1-v)+v(1+e^{v})}{1+e^{v}}$$

$$y\frac{dv}{dy} = \frac{-(e^{v}+v)}{1+e^{v}}$$

$$\int \frac{1+e^{v}}{(v+e^{v})}\,dv = \int \frac{-dy}{y} + \log c$$

$$\log(v+e^{v}) = -\log y + \log c$$

$$\frac{v+e^{v}}{y} = c$$

$$\left(\frac{x}{y}+e^{x/y}\right)y = c$$

$$\Rightarrow (x+e^{x/y}.y = c$$ Ans

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Q. 2 Solve :- $(D^2-2D+1)y = xe^x \sin x$

$AE \rightarrow m^2-2m+1=0$

$(m-1)^2=0$

$CF \rightarrow (c_1x+c_2)e^x$

$PI = \frac{1}{(D^2-2D+1)} xe^x \sin x$

$= e^x \frac{1}{(D-1)^2}(x \sin x)$

$\frac{e^x}{(D+1-1)^2} x \sin x = e^x \cdot \frac{1}{D^2}(x \sin x)$

$= e^x \iint x \sin x \, dx \, dx$

$e^x \left[\int [-x\cos x + \sin x]\right]$

$= e^x \left[(-x\sin x - \cos x) - \cos x\right]$

$= -e^x(x\sin x + 2\cos x)$

$y = (c_1x+c_2)e^x - e^x(x\sin x + 2\cos x)$

$[\, y = CF + PI \,]$ Ans

Q. 3 Solve $(D^2+3D+2)y = 4\cos^2 x$

$AE \rightarrow m^2+3m+2=0$

$(m+1)(m+2)=0$

$m = -1$ or -2

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$CF \rightarrow c_1 e^{-x} + c_2 e^{-2x}$

$PI \rightarrow \frac{1}{(D^2+3D+2)} (2)(1+\cos 2x)$

$= \frac{2}{(D^2+3D+2)} e^{0x} + \frac{2\cos 2x}{(D^2+3D+2)}$

$\frac{2}{2} + \frac{2\cos 2x}{D^2+3D+2} \Rightarrow 1+2 \frac{\cos 2x}{-4+3D+2}$

$1+2 \frac{\cos 2x}{3D-2} \Rightarrow 1+2 \frac{(\cos 2x)(3D+2)}{9D^2-4}$

$1 - \frac{2}{40}\left[-(2\sin 2x).3 + 2\cos 2x\right]$

$1 - \frac{1}{20}\left[2\cos 2x - 6\sin 2x\right]$

$y = CF + PI$

$y = c_1 e^{-x} + c_2 e^{-2x} + 1 - \frac{1}{2}$

$\left[2\cos 2x - 6\sin 2x\right]$

Ans

Q.4 Solve :- $(D^2 - 2D + 1)y = xe^x \sin x$

Same as question 2 [Refer to Q 2].

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Q 5. $(D^2+4)y = e^{2x} + \sin 2x + x^2$

$AE \to m^2+4=0 \Rightarrow m = \pm 2i$

$CF = c_1 \cos 2x + c_2 \sin 2x$

$$PI = \frac{1}{(D^2+4)}\left[e^{2x} + \sin 2x + x^2\right]$$

$$= \frac{e^{2x}}{(D^2+4)} + \frac{\sin 2x}{(D^2+4)} + \frac{x^2}{(D^2+4)}$$

$$\frac{e^{2x}}{5} + \frac{x \sin 2x}{2D} + \frac{x^2}{4\left(\frac{D^2}{4}+1\right)}$$

$$\frac{e^{2x}}{5} - \frac{x \cos 2x}{4} + \frac{x^2}{4}\left(\frac{D^2}{4}+1\right)$$

$$\frac{e^{2x}}{5} - x\frac{\cos 2x}{4} + \frac{x^2}{4}\left[1 - \frac{D^2}{4}\right]$$

$$\frac{e^{2x}}{5} - \frac{x \cos 2x}{4} + \frac{x^2}{4} - \frac{1}{2}$$

$y = CF + PI$

$$y = c_1 \cos 2x + c_2 \sin 2x + \frac{e^{2x}}{5} - \frac{x \cos 2x}{4} + \frac{x^2}{4} - \frac{1}{2}$$ Ans

Q 6. Solve :- $(1+e^y)\cos x\,dx + e^y \sin x\,dy = 0$

$(1+e^y)\cos x\,dx = -e^y \sin x\,dy$

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$$\int \frac{\cos x}{\sin x}\,dx = \int \frac{-e^y}{(1+e^y)}\,dy + \log c$$

$$\log \sin x = -\log(1+e^y) + \log c$$

$$(\sin x)(1+e^y) = c \quad \text{-Ans}$$

Q 7 Solve :- $(1+y^2)\,dx - (\tan^{-1}y - x)\,dy = 0$

$$(\tan^{-1}y - x)\,dy = (1+y^2)\,dx$$

$$\frac{dx}{dy} = \frac{\tan^{-1}y}{1+y^2} - \frac{x}{(1+y^2)}$$

$$\frac{dx}{dy} + \frac{x}{(1+y^2)} = \frac{\tan^{-1}y}{1+y^2}$$ which is LDE in x.

$$IF = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$$

Complete solution

$$x\,IF = \int Q\,IF + c$$

$$x(e^{\tan^{-1}y}) = \int \frac{\tan^{-1}y}{(1+y^2)}\,e^{\tan^{-1}y} + c$$

$$x(e^{\tan^{-1}y}) = e^{\tan^{-1}y}(\tan^{-1}y - 1) + c$$

$$x = \tan^{-1}y - 1 + c\,e^{-\tan^{-1}y} \quad \text{-Ans}$$

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Q.8 Solve :- $\frac{x^3 d^3 y}{dx^3} + \frac{3x^2 d^2 y}{dx^2} + \frac{x\, dy}{dx} = x^2 \log x$

Let $x = e^z$, $\frac{x^3 d^3 y}{dx^3} = D(D-1)(D-2)y$, $\frac{x^2 d^2 y}{dx^2}$
$= D(D-1)y$, $\frac{x\, dy}{dx} = Dy$

$[D(D-1)(D-2) + 3[D][D-1] + D]y = x^2 \log x$

AE $\rightarrow$ $m(m-1)(m-2) + 3(m^2-m) + m = 0$

$m = 0, 0, 0$

$CF = (c_1 z + c_2 z^2 + c_3)$

$PI = \frac{1}{D^3}(e^{2z} \cdot z)$

$\iiint (e^{2z} \cdot z)\, dz\, dz\, dz = \frac{1}{8} z e^{2z} - \frac{1}{4} e^{2z}$

$y = CF + PI$

Ans $y = c_1 \log x + c_2 \log x^2 + c_3 + \frac{1}{8} \log e^{2\log x} - \frac{1}{4}$

Q9. Solve :- $\frac{x^2 d^2 y}{dx^2} + x\frac{dy}{dx} + y = (\log x) \sin(\log x)$

Put, $x = e^z$, $\log x = z$

$D(D-1)y + Dy + y = z \sin z$

$(D^2+1)y = z \sin z$

AE $\rightarrow$ $m^2 + 1 = 0 \Rightarrow m = \pm i$

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$CF = c_1 \cos z + c_2 \sin z$

$c_1 \cos \log x + c_2 \sin \log x$

$PI = \frac{1}{(D^2+1)} z \sin z$

Now we will take imaginary part of above PI.

$PI = I.F \text{ of } \frac{1}{(D^2+1)} z e^{iz} \Rightarrow IP \text{ of } \frac{e^{iz}}{(D+i)^2+1} \cdot z$

$\frac{e^{iz}}{2iD} \left\{1 + \frac{D}{2i}\right\}^{-1} z$

$IP \text{ of } \frac{e^{iz}}{2iD} \left[1 - \frac{D}{2i}\right] z = IP \text{ of } \frac{e^{iz}}{2i} \frac{1}{D} \left[z - \frac{1}{2i}\right]$

$IP \text{ of } \frac{(\cos z + \sin z)}{2i} \left[\frac{z^2}{2} - \frac{z}{2i}\right]$

$IP \text{ of } \frac{-z^2 \cos z}{4} + \frac{z \sin z}{4}$

$PI = \frac{1}{4} z \left[\sin z - z \cos z\right]$

$y = CF + PI$

Ans

$y = c_1 \cos \log x + c_2 \sin \log x + \frac{1}{4} \log x \left[\sin \log x - \log x \cos \log x\right]$

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Solve $\frac{dx}{dt}+y=\sin t$ $\frac{dy}{dx}+x=\cos t$.

Q.10 Given that $x=2$ & $y=0$ when $t=0$

$$x=\cos t-\frac{dy}{dt}$$

$$\frac{d}{dt}\left[\cos t-\frac{dy}{dt}\right]+y=\sin t$$

$$\frac{-d^2y}{dt^2}+y=2\sin t$$

$$(D^2-1)y=-2\sin t$$

$$AE = m^2-1=0$$

$$m=1,-1$$

$$CF = c_1e^{t}+c_2e^{-t}$$

$$PI=\frac{1}{D^2-1}\cdot[-2\sin t]$$

$$\frac{1}{-2}\cdot -2\sin t=\sin t$$

$$y=CF+PI=c_1e^{t}+c_2e^{-t}+\sin t$$

Put y in x we get

$$x=\cos t-\frac{d}{dt}\left[c_1e^{t}+c_2e^{-t}+\sin t\right]$$

$$x=-c_1e^{t}+c_2e^{-t}$$

Given $x=2$, $y=0$ at $t=0$

$$c_1+c_2=0$$

$$-c_1+c_2=2$$

$$(+)\quad 2c_2=2$$

$$c_2=1 \ \& \ c_1=-1$$

$c_2=1$, $c_1=-1$ Ans

Assignment 2

Q.1 Solve $x\frac{d^2y}{dx^2} - (2x-1)\frac{dy}{dx} + (x-1)y = 0$

Given $y = e^x$

$$\frac{d^2y}{dx^2} - \left[2-\frac{1}{x}\right]\frac{dy}{dx} = \frac{R}{y}$$

v is given by

$$\frac{d^2v}{dx^2} + \left[P + \frac{2}{y}\frac{dy}{dx}\right]\frac{dv}{dx} = 0$$

put $\frac{dv}{dx} = t$ $\quad \frac{d^2v}{dx^2} = \frac{dt}{dx}$

$\frac{dt}{dx} + \frac{1}{x}t = 0$ which is LDE in t.

$$IF = e^{\int \frac{1}{x}dx} = e^{\log x} = x$$

$$t.x = \int 0.x + c$$

$$xt = c$$

$$x.\frac{dv}{dx} = c \Rightarrow \int dv = c\int \frac{dx}{x} \Rightarrow$$

$$v = c\log x + c_1$$

$y = uv \Rightarrow y = e^x[c\log x + c_1]$ Ans

Q.2 $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$ by VP.

AE $\rightarrow m^2 - 6m + 9 = 0$ $(m-3)^2 = 0, m = 3, 3$

$CF = (c_1 + c_2x)e^{3x}$

let, $y = (A + Bx)e^{3x}$

Now

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$$\cancel{A'}u + B'v = 0$$
$$A'u' + B'v = R$$

$$\cancel{A'}e^{3x} + B'xe^{3x} = 0 \quad \times 3$$
$$3A'e^{3x} + B'[3xe^{3x} + e^{3x}] = e^{3x}/x^2$$

Ans

$$3A' + 3B'x = 0$$
$$(-)\ \underline{3A' + B'[3x+1] = 1/x^2}$$
$$\int B' = \int \frac{1}{x^2}$$
$$B = \frac{-1}{x} + c_2$$

Put B in A

$$A' + \frac{1}{x^2} \cdot x = 0$$

$$A = -\log x + c_1$$

Ans $y = [(-\log x + c_1) - 1 + x c_2] e^{3x}$

Q.4

Q.3 $x\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = x^5$

$$\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} - 4x^2y = x^4$$

choosing z such that $-4x^2 = \left(\frac{dz}{dx}\right)^2 a_1$

$$z = x^2$$

Now the eq^n become $\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1 y = R_1$

$$P_1 = \frac{2 - \frac{1}{x} \times 2x}{4x^2} = 0 \quad Q_1 = \frac{-4x^2}{4x^2} = -1$$

Ans

$$R_1 = \frac{x^4}{4x^2} = \frac{x^2}{4} = \frac{z}{4}$$

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$$\frac{d^2y}{dz^2} - 1 = \frac{z}{4}$$

$AE = m^2 - 1 = 0, \quad m = 1, -1$

$CF = c_1 e^{z} + c_2 e^{-z}$

$$PI = \frac{1}{(D^2-1)} \frac{z}{4} = \frac{-z}{4}$$

Ans $y = CF + PI = c_1 e^{z} + c_2 e^{-z} - \frac{z}{4}$

$$= c_1 e^{x^2} + c_2 e^{-x^2} - \frac{x^2}{4}$$

Q.4 Solve :- $\frac{d^2y}{dx^2} - \cot x \frac{dy}{dx} - (1-\cot x) y = e^{x} \sin x$

$1 + P + Q = -\cot x - 1 + \cot x + 1 = 0$

Hence $y = e^{x}$

v is given by

$$\frac{d^2v}{dx^2} + \left[-\cot x + \frac{2}{e^{x}} e^{x}\right] \frac{dv}{dx} = \sin x \cdot \frac{e^{x}}{e^{x}}$$

let $\frac{dv}{dx} = t, \quad \frac{d^2v}{dx^2} = \frac{dt}{dx}$

$$\frac{dt}{dx} + [-\cot x + 2] t = \sin x$$

$$IF = e^{\int 2 - \cot x} = \frac{e^{2x}}{\sin x}$$

$$t \cdot \frac{e^{2x}}{\sin x} = \int \frac{e^{2x}}{\sin x} \cdot \sin x + c_1$$

$t = \frac{1}{2} \sin x + e^{-2x} \sin x \; c_1$

$\int \frac{dv}{v} = \frac{1}{2} \int \sin x \, dx + \int e^{-2x} \sin x \, c_1 + c_2$

$$= \frac{-1}{2} \cos x - \frac{c_1}{5} e^{-2x} [\cos x + 2 \sin 2x] + c_2$$

Ans $y = uv = e^{x} \left[\frac{-1}{2} \cos x - \frac{c_1}{5} e^{-2x} [\cos x + 2 \sin 2x] + c_2\right]$

Q 5. $(1+x^2)^2 \frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} + 4y = 0$ by IV

$$\frac{d^2y}{dx^2} + \frac{2x}{(1+x^2)}\frac{dy}{dx} + \frac{4}{(1+x^2)^2} = 0$$

choose z such that

$$\frac{\frac{4}{(1+x^2)^2}}{(dz/dx)^2} = 4 \Rightarrow \int \frac{1}{1+x^2}dx = \int dz$$

Now eq^n become

$$\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1 y = R_1$$

$$P_1 = \frac{-2x}{(1+x^2)^2} + \frac{2x}{(1+x^2)} \times \frac{1}{(1+x^2)} = 0$$

$$Q_1 = +4 \qquad R_1 = 0$$

$$\frac{d^2y}{dz^2} + 4y = 0$$

AE $\rightarrow$ $m^2 + 4 = 0$

$$m = \pm 2i$$

$$CF = y = c_1 \cos 2z + c_2 \sin 2z$$

Ans $y = c_1 \cos 2\tan^{-1}x + c_2 \sin 2\tan^{-1}x.$

Q. 6 $\cos x \frac{d^2y}{dx^2} + \sin x \frac{dy}{dx} - 2y\cos^3 x = 2\cos^5 x$

$$\frac{d^2y}{dx^2} + \tan x\frac{dy}{dx} - 2y\cos^2 x = 2\cos^4 x$$

choose z such that $\left(\frac{dz}{dx}\right)^2 = \cos^2 x$

OR $z = \sin x$

$$P_1 = \frac{-\sin x + \sin x}{\cos^2 x} = 0, \quad Q_1 = 2, \quad R_1 = 2\cos^2 x$$

eq^n becomes

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$$\frac{d^2y}{dz^2} + 2y = 2\cos^2 x$$

$AE \rightarrow m^2+2=0 \Rightarrow m = \pm 2i$

$CF = c_1 \cos 2z + c_2 \sin 2z$

$$PI = \frac{1}{(D^2+2)} 2(\cos^2 x) = \frac{2(1-\sin^2 x)}{(D^2+2)}$$

$$\frac{2e^{0x}}{D^2+2} - \frac{2\sin^2 x}{D^2+2} = 1 - \frac{2z^2}{2\left[1+\frac{D^2}{2}\right]}$$

$$= 1 - z^2\left[1+\frac{D^2}{2}\right]^{-1}$$

Ans $= 1 - z^2 + 1 = z^2$

$y = CF + PI \Rightarrow y = c_1 \cos 2 \sin x + c_2 \sin 2 \sin x + \sin^2 x.$

Q 7 $\quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ Given $x + \frac{1}{x}$

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = 0$$

v is given by

$$\frac{d^2v}{dx^2} + \left[\frac{1}{x} + \frac{2}{(x+\frac{1}{x})} \cdot \left(1-\frac{1}{x^2}\right)\right]\frac{dv}{dx} = 0$$

$$\frac{dt}{dx} + \left[\frac{-1}{x} + \frac{4x}{x^2+1}\right]t = 0$$

$$t = \frac{c_1 x}{(x^2+1)^2}$$

$$v = \frac{-1\, c_1}{2(x^2+1)} + c_2$$

$$y = uv \Rightarrow y = \left(x+\frac{1}{x}\right)\left[\frac{-c_1}{2(x^2+1)} + c_2\right]$$ Ans

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Q. 8 Solve $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$

$$y = x^m (a_0 + a x^{-1} + \dots)$$

$$y = \sum_{k=0}^{\infty} a_k x^{m-k}$$

$$\frac{dy}{dx} = \sum_{k=0}^{\infty} a_k (m-k) x^{m-k-1} \quad \frac{d^2y}{dx^2} = \sum_{k=0}^{\infty} a_k (m-k)$$

$$\sum_{k=0}^{\infty} (m-k)(m-k-1) a_k x^{m-k-2} - \sum_{k=0}^{\infty} \left[(m-k-n)(m-k+n+1)\right] a_k x^{m-k} = 0$$

eqn the zero to Cofficient we get

$$(m-n)(m+n+1)a_0 = 0$$

so, $m = n$ or $m = -(n+1)$, $a_0 \neq 0$

Then, $a_k = \dfrac{-(m-k+2)(m-k+1)}{k(2n-k+1)} a_{k-2}$

since $a_1 = 0 \Rightarrow a_3 = a_5 = a_7 \dots a_{2n-1} = 0$

when $m = n$

$$a_k = \frac{-(n-k+2)(n-k+1)}{k(2n-k+1} a_{k-2}$$

Put $k = 2, 4, 6 \dots$ we get $a_2 = \dfrac{-n(n-1)}{2(2n-1)} a_0$

$$a_4 = \frac{-(n-2)(n-3)}{4(2n-3)}, \quad a_6 = \frac{n(n-1)(n-2)(n-3)}{2.4(2n-1)(2n-3)} a_0$$

Hence one solution is

$$y_1 = a_0\left[x^n - \frac{(n)(n-1)}{2(2n-1)} x^{n-2} + \frac{n(n-1)(n-2)\dots}{2.4(2n-1)(2n-3)} x^{n-4}\right.$$

When $m = -(n+1)$ then

$$y_2 = a_0\left[x^{-n-1} + \frac{(n+1)(n+2)}{2(2n+3)} x^{-n-3} + \frac{n(n+1)(n+2)(n+3)(n+4)}{2.4(2n+3)(2n+5)}\right.$$

Ans

Q. 9

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Q.9 Solve $9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0$

$$y = \sum_{r=0}^{\infty} A_r x^{m+r}$$

$$\frac{dy}{dx} = \sum_{r=0} A_r (m+r) x^{m+r-1}$$

$$\frac{d^2y}{dx^2} = \sum_{r=0}^{\infty} A_r (m+r)(m+r-1) x^{m+r-2}$$

$$\sum_{r=0}^{\infty} A_r \left[(3m+3r-4)(3m+3r+1) x^{m+r} - 3 (m+r)(3m+3r-7)\right] = 0$$

Now equating to 0 we get

$$-3A_0 (m)(3m-7) = 0 \Rightarrow m = 0, 7/3$$

$$A_{p+1} = \frac{3m+3p+1}{3(m+p+1)} A_p$$

Put $p = 0, 1, 2$

$$A_1 = \frac{3m+1}{3(m+1)} A_0, \quad A_2 = \frac{(3m+4)(3m+1)}{3^2(m+2)(m+1)} A_0 \ldots$$

$$y = \sum_{r=0}^{\infty} A_r x^{m+r} =$$

$$A_0 x^m \left[1 + \frac{3m+1}{3(m+1)} x + \frac{(3m+1)(3m+4)}{3^2(m+2)(m+1)} x^2 + \frac{(3m+1)(3m+4)}{3^3(m+1)(m+2)(m+3)}\right.$$

Now when $m = 0$

$$y_1 = a\left[1 + \frac{1}{3}x + \frac{1}{3}\cdot\frac{4}{6}x^2 + \frac{1\cdot4\cdot7}{3\cdot6\cdot9}x^3 + \ldots\right]$$

Now when $m = \frac{7}{3}$

$$y_2 = bx^{7/3}\left[1 + \frac{8}{10}x + \frac{8\cdot11}{10\cdot13}x^2 + \frac{8\cdot11\cdot14}{10\cdot13\cdot16}x^3 + \ldots\right]$$

& also Complete sol^n is

$$y = y_1 + y_2$$

$$y = a\left[1 + \frac{1}{3}x + \frac{1\cdot4}{3\cdot6}x^2 + \frac{1\cdot4\cdot7}{3\cdot6\cdot9}x^3 + \ldots\right] +$$

$$bx^{7/3}\left[1 + \frac{8x}{10} + \frac{8\cdot11}{10\cdot13}x^2 + \frac{8\cdot11\cdot14}{10\cdot13\cdot16}x^3 + \ldots\right]$$

Ans

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Q.10 Prove $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-n^2)y = 0$

$y = \sum_{k=0}^{\infty} a_k x^{m+k}$ $\quad \frac{dy}{dx} = \sum_{k=0}^{\infty} (a_k)(m+k) x^{m+k-1}$

$\frac{d^2y}{dx^2} = \sum_{k=0}^{\infty} a_k (m+k)(m+k-1) x^{m+k-2}$

$\sum_{k=0}^{\infty} [(m+k)^2 - n^2] a_k x^{m+k} + \sum_{k=0}^{\infty} a_k x^{m+k+2} = 0$

Equating to zero we get

$[(m+1)^2 - n^2] a_1 = 0 \Rightarrow a_1 = 0$ for $m = \pm n$

$a_{k+2} = \frac{-a_k}{(m-n+k+2)(m+n+k+2)}$

Put $k = 1, 3, 5 \ldots (2n-1)$ we get $a_3 = a_5 = 0$

Put $k = 0, 2, 4 \ldots (2n)$ we get

$a_2 = \frac{-a_0}{(m-n+2)(m+n+2)}$

$a_4 = \frac{-a_2}{(m-n+4)(m+n+4)} = \frac{a_0}{[(m+2)^2-n^2][(m+4)^2-n^2]}$

$y = a_0 x^m \left[1 - \frac{x^2}{[(m+2)^2-n^2]} + \frac{x^4}{[(m+2)^2-n^2][(m+4)^2-n^2]}\right]$

For $m = n$ we get

$y_1 = a_0 x^n \left[1 - \frac{x^2}{2(2n+2)} + \frac{x^4}{2\cdot 4(2n+2)\cdot(2n+4)} \ldots\right]$

$y_1 = a_0 x^n \sum_{k=0}^{\infty} \frac{(-1)^k (n+1)!}{2^{2k} k! (n+k+1)!}$

for $m = -n$

$y_2 = a_0 x^{-n} \sum_{k=0}^{\infty} \frac{(-1)^k (-n+1)!}{2^{2k} k! (-n+k+1)!}$ Ans

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ASSIGNMENT 3

Q.1 Solve $y^2p - xyq = x(z-2y)$

$$\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x(z-2y)}$$

$$\frac{dx}{y^2} = \frac{dy}{-xy}$$

$$\int -x\,dx = \int y\,dy$$

$$\frac{-x^2}{2} = \frac{y^2}{2} + c$$

$$c_1 = \frac{y^2}{2} + \frac{x^2}{2}$$

$$\frac{dy}{-xy} = \frac{dz}{x(z-2y)}$$

$$\frac{dz}{dy} = \frac{2y-z}{y}$$

$$\frac{dz}{dy} + \frac{z}{y} = 2$$

$$P = \frac{1}{y} \qquad Q = 2$$

Integrating factor $I_f = e^{\int P\,dx} = e^{\int \frac{1}{y} dx}$

$$= e^{\log y} = y$$

$$zy = \int 2y\,dy + c$$

$$zy = 2\frac{y^2}{2} + c_2$$

$$zy - y^2 = c_2$$

$$\phi(c_1, c_2) = \phi\left(\frac{y^2}{2} + \frac{x^2}{2},\ zy - y^2\right)$$

Ans

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Q.2 Solve $(D^2 - DD' - 2D'^2) z = (y-1) e^x$

The auxiliary eq^n is

put $D = m$

$D' = 1$

$(m^2 - m - 2) = 0$

$m^2 - 2m + m = 0$

$m(m-2) + 1(m-2) = 0$

$(m+1)(m-2) = 0$

$m = -1, 2$

$CF = f_1(y + m_1 x) + f_2(y + m_2 x)$

$CF = f_1(y - x) + f_2(y + 2x)$

$PI = \frac{1}{DD'} x^4 y^m$

$= \frac{1}{D^2 - DD' - 2D'^2} (y-1) e^x$

$= \frac{1}{(D - 2D')(D + D')} (y-1) e^x$

$\frac{1}{D + D'} \left[\frac{1}{D - 2D} (y-1) e^x \right]$

Since

$D - 2D' = 0$

$m - 2 = 0$

$m = 2$

Put $y = c - mx$

$$y = c - 2x$$

$$\frac{1}{D+D'}\left[(c-2x-1)e^{x}dx\right.$$

$$\frac{1}{D+D'}\left[(c-2x-1)e^{x}+2e^{x}\right]$$

$$\frac{1}{D+D'}\left[(y-1)e^{x}+2e^{x}\right]\quad [c = y-2x]$$

$$PI = \frac{1}{(D+D')}(y+1)e^{x}$$

$$m = -1$$

Put $y = c - mx$

$$\int (c+x+1)e^{x}dx$$

$$\int (c+x+1)e^{x} - e^{x}$$

$$(y+1)^{ex} - e^{x}$$

$$PI = ye^{x}.$$

Hence the Complete solution

$$z = CF + PI$$

Ans $$z = f_1(y-x) + f_2(y+2x) + (y+1)e^{x}e^{x}$$

Q. 3 $$\frac{\partial^3 z}{\partial x^3} - 7\frac{\partial^3 z}{\partial x \partial y^2} - 6\frac{\partial^3 z}{\partial y^3} = \sin(x+2y)$$

The given eq. is symbolic form can be written as

$$(D^3 - 7DD'^2 - 6D'^3)z = \sin(x+2y$$

$$PI = \left[\frac{1}{D^3 - 7DD'^2 - 6D'^3}\right]\sin(x+2y)$$

$$= \left(\frac{1}{-D + 7D \times 2^2 + 6 \times 2^2 D'}\right)\sin(x+2y)$$

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$$\left(\frac{1}{27D+24D'}\right) \sin(x+2y)$$

$$\left[\frac{27D'-24D'}{27^2D^2-24^2D'^2}\right] \sin(x+2y)$$

$$\left[\frac{27D-24D'}{-729+576\times 2^2}\right] \sin(x+2y)$$

$$\frac{27\cos(x+2y)-48\cos(x+2y)}{1575}$$

$$\frac{-21\cos(x+2y)}{1575}$$

$$-\frac{1}{75}\cos(x+2y) \quad \text{Ans}$$

Q.4. Solve P.D.E $-(p^2+q^2)y = qz$

Here $(p^2+q^2)y - qz = 0$

let $f = (p^2+q^2)y - qz$ — 1

Partially diff eq^n 1. w.r.to x, y, z, p & q

$$\frac{\partial f}{\partial x} = 0 \quad \frac{\partial f}{\partial y} = p^2+q^2, \ \frac{\partial f}{\partial z} = -q, \ \frac{\partial f}{\partial p} = 2py$$

$$\frac{\partial f}{\partial q} = 2qy.z$$

The charpit auxilary eq^n

$$\frac{dp}{\frac{\partial f}{\partial x}+p\frac{\partial f}{\partial z}} = \frac{-\partial q}{\frac{\partial f}{\partial y}+\frac{\partial f}{\partial z}} = \frac{\partial z}{-p\frac{\partial f}{\partial p}-q\frac{\partial f}{\partial q}}$$

$$= \frac{\partial x}{-\frac{\partial f}{\partial p}} = \frac{\partial y}{-\frac{\partial f}{\partial q}} \quad \text{— (2)}$$

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$$\frac{\partial p}{p+p(-q)} = \frac{dq}{p^2+q^2-q^2} = \frac{dz}{-p(2py)-q(2qy-z)}$$

1^{st} & 2^{nd}

$$= \frac{dx}{-1py} = \frac{dy}{-2qy+z}$$

$$\frac{dp}{-pq} = \frac{dq}{p^2} = pdp = -qdq$$

Int^n $\rightarrow \frac{p^2}{2} = \frac{-q^2}{2} + \frac{a^2}{2}$

$$p^2+q^2 = a^2$$

from eq^n (1) & (3) we get

$$a^2y = qz$$

$$\left[q = \frac{a^2y}{z}\right]$$

from eq^n (3) $\rightarrow p^2 + \frac{a^4y}{z^2} = a^2$

$$p^2 = a^2 - \frac{a^4y}{z^2}$$

$$p = \sqrt{\frac{a^2z^2 - a^4y^2}{z^2}} = \left[p = \frac{a}{z}\sqrt{z^2-a^2y^2}\right]$$

The general eq^n $dz = pdx + qdy$

$$dz = \frac{a}{z}\sqrt{z^2-a^2y^2}\,dx + a^2y\,dy$$

$$zdz = a\sqrt{z^2-a^2y^2}\,dx + a^2y\,dy$$

$$zdz - a^2y\,dy = a\sqrt{z^2-a^2y^2}\,dx$$

$$\frac{zdz - a^2y\,dy}{\sqrt{z^2-a^2y^2}} = adx$$

Let $t = \sqrt{z^2-a^2y^2}$

$$dt = 2zdz - 2a^2y\,dy$$

$$\frac{dt}{2} = zdz - a^2y\,dy$$

$$\frac{1}{2}\cdot t^{1/2} = ax+b$$

$$(z^2-a^2y)^{1/2} = ax+b$$

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$$(z^2 - a^2y^2) = (ax+b)^2$$

$$\left[z^2 = a^2y^2 + (ax+b)^2 \right]$$ Ans

Q. 5 Solve $x^2p + y^2q = z^2$

Here the Lagrange's subsidiary eq^n are

$$\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{z^2} \quad \text{— (1)}$$

Taking 1st two function of (1) we get

$$\frac{dx}{x^2} = \frac{dy}{y^2}$$

$Int^n \rightarrow \frac{-1}{x} = \frac{-1}{y}\ c_1 \Rightarrow \left[c_1 = \frac{1}{x} \frac{-1}{y} \right]$

Taking 1 & 3 members of eq^n 1.

$$\frac{dx}{\cancel{d}x^2} = \frac{dz}{z^2}$$

$Int^n \rightarrow \frac{-1}{x} = \frac{-1}{z} - c_2$

$$\left[c_2 = \frac{1}{x} - \frac{1}{z} \right]$$

Hence the required general sol^n of the given eq^n is

$$* \ \phi\left(\frac{1}{x} , \frac{-1}{y} \cdot \frac{1}{x} \frac{-1}{z} \right) = 0$$

where ϕ is an arbitiary function.