$R_x = \frac{2 \times 3}{2+2+3} = \frac{6}{7} = 0.86\,\Omega$

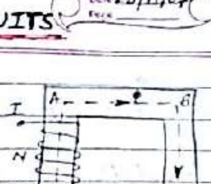
$R_y = \frac{2 \times 3}{2+2+3} = \frac{6}{7} = 0.86\,\Omega$

$R_z = \frac{2 \times 2}{2+2+3} = \frac{4}{7} = 0.57\,\Omega$

UNIT – III

Date: 28/11/07

MAGNETIC CIRCUITS



Magnetic Circuit

Magnetic Circuits :- The closed path followed by magnetic flux is called a magnetic circuit. Practically all electrical power machinery depend for their operation upon the magnetism produced by the magnetic circuits. There is a lot of similarities between the magnetic & electric circuits.

Terminology

1) **Magnetic Field** :- Magnetic field is the region around a magnet within which the influence of the magnet is felt. The space around a compass needle, around the earth and around a permanent magnet are examples of magnetic field.

2) **Magnetic Flux** :- A magnetic field over a region in space is represented graphically by magnetic lines of forces, these lines of forces are called flux. The total lines of magnetic force in any particular magnetic field passing through a surface is called magnetic flux. It is symbolized by ϕ measured in (wb) weber.

$\phi = \frac{MMF}{S}$

3) **Flux Density** :- It is defined as the magnetic flux passing per unit area through any material

through a plane at right angles to the direction of magnetic flux. It is measured in Tesla (Wb/m^2).

4) **MMF** :- MMF of the magnetic circuit is defined as the magnetic potential that drives or tends to drive flux around the magnetic circuit and is analogous to the emf in an electric circuit. It measured in ampere turns (AT).

5) **Magnetic Field Intensity** :- MMF per unit length (along the path of magnetic flux) is called the magnetic field intensity (H) and is given as

$$\boxed{H = \frac{MMF}{length} = \frac{NI}{l}\ AT/m}$$

6) **Reluctance** :- It is the name given to that property of material which opposes the creation of magnetic flux in it. It, in fact, measures the resistance offered to the passage of magnetic flux through a material and is analogous to a resistance in an electric circuit. It is measured in ampere turn/wb. [AT/Wb].

$$\boxed{S = \frac{l}{\mu a}\ AT/Wb}$$

~~7) Permeability :- It is the name given to that property of material which opposes~~

7) **Permeability** :- It is a measure of receptiveness of the material of having magnetic flux developed in it.

Every substances possesses a certain power of conducting magnetic flux. For Eg iron is a better conductor for magnetic flux than air. Permeability of material (μ) is thus its conducting power for magnetic flux. It is equal to the ratio of flux density B & the magnetic field intensity H.

i.e. $\mu = \frac{B}{H}$.

8) **Relative Permeability** :- Relative permeability of a material is defined as the ratio of the no. of lines of magnetic flux per unit area in the given medium to the no. of lines of magnetic flux per unit area, when medium is replaced by vacuum.

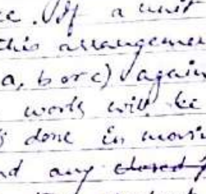
Relative permittivity of free space is unity & of magnetic material may range upto thousands.

9) **Electromagnet** :- A magnetic material which obtains the magnetic properties by providing a current carrying solenoid around it is called an electromagnet.

~~NOAA~~

Work Law :- This law relates to work done in a closed magnetic ~~f~~ path (ie magnetic circuit). Let there be N straight conductors, each carrying a current of I amperes. ~~or~~ This arrangement will set up magnetic lines of force. If a unit N-pole is moved around this arrangement (in any one of the path a, b or c) against the force exerted on it, work will be done. The amount of work done in moving a unit N-pole once round any ~~closed~~ complete path will be equal to the product of current & no. of conductors enclosed in that path. This is known as work law.

Statement :- It can be stated as the work done by or on a unit N-pole in moving once round a complete path is equal to the product of current & no. of turns (or conductors) enclosed by that path.



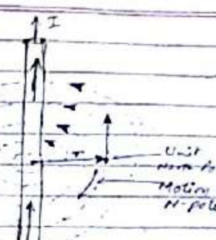
Mathematically ; $\oint H\,ds = NI$

Where, H = the magnetising force at distance r

$\oint$ = the sign which shows the integral is round a complete path.

Application :- Let ~~there~~ a single long straight conductor carrying current of I amperes.

Consider a point P at a radius of r metres. Let field intensity at that point be H N/wb.



It means, a force of H newton is exerted on a unit N-pole when placed at that point which acts tangential to the circular line of force passing through it.

Now, work done in moving a unit N-pole once round the conductor in a circular path

= Force × distance = $H \times 2\pi r$ joules

According to work law this must be equal to the ~~no.~~ product of current & no. of conductors enclosed by the circular path,

$$2\pi r H = I \times 1$$

$$\boxed{H = \frac{I}{2\pi r}}$$

For N conductors :

$$\boxed{H = \frac{NI}{2\pi r}}$$

This shows that the magnetic field strength increases as we go nearer the conductor, but the value of $2\pi r H$ remains the same for all enclosed path.

~~Magnetic Circuits~~

Leakage Flux & Fringing :-

The magnetic flux which does not follow the intended path in a magnetic circuit is called leakage flux.

When a current is passed through a solenoid a magnetic flux is produced by it. Most of this flux is set up in the magnetic core

& passed through air gap (an intended path). This flux is known as useful flux ϕ_u. However, some of the flux is just set-up around the coil & is not utilized for any works. This flux is called leakage flux ϕ_l.

Total flux produced by solenoid,

$$\phi = \phi_u + \phi_l$$

Leakage co-efficient or leakage factor :- The ratio of total flux (ϕ) ~~flux~~ to the useful flux set up in the air gap is known as leakage co-efficient. It is generally represented by letter 'λ'.

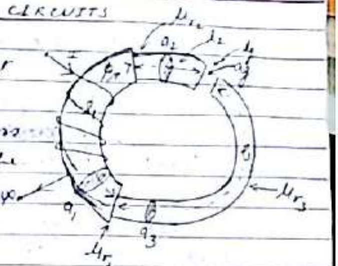
$\therefore$ leakage coefficient, $\boxed{\lambda = \phi/\phi_u}$

Fringing :- The useful flux when sets up in the air-gap, it tends to bulge outwards. This increases the effective area in the air gap & decreases the flux density. This effect is known as fringing. The fringing is directly proportional to the length of air gap.

ANALYSES OF MAGNETIC CIRCUITS

SERIES CIRCUITS

A magnetic circuit that has a no. of parts of different ~~intenss~~ dimensions & ~~may~~ material carrying the same magnetic ~~field~~ flux is called a series magnetic circuit. The circuit shown is called series magnetic circuit.



Now, calculating total reluctance of the given magnetic circuit.

$$S = S_1 + S_2 + S_3 + S_g$$

$$= \frac{l_1}{a_1 \mu_1} + \frac{l_2}{a_2 \mu_2} + \frac{l_3}{a_3 \mu_3} + \frac{l_g}{a_g \mu_g}$$

$$= \frac{l_1}{a_1 \mu_0 \mu_{r_1}} + \frac{l_2}{a_2 \mu_0 \mu_{r_2}} + \frac{l_3}{a_3 \mu_0 \mu_{r_3}} + \frac{l_g}{a_g \mu_0}$$

Total mmf $= \phi \cdot S$

$$= \phi \left[\frac{l_1}{a_1 \mu_0 \mu_{r_1}} + \frac{l_2}{a_2 \mu_0 \mu_{r_2}} + \frac{l_3}{a_3 \mu_0 \mu_{r_3}} + \frac{l_g}{a_g \mu_0} \right]$$

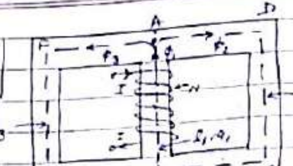
$$= \left[\frac{B_1 l_1}{\mu_0 \mu_{r_1}} + \frac{B_2 l_2}{\mu_0 \mu_{r_2}} + \frac{B_3 l_3}{\mu_0 \mu_{r_3}} + \frac{B_g l_g}{\mu_0} \right]$$

$$\left[\because B = \phi/a \right]$$

$$\boxed{\text{Total mmf} = H_1 l_1 + H_2 l_2 + H_3 l_3 + H_g l_g}$$

$$\left[\because H = B/\mu_0 \mu_r \right]$$

PARALLEL CIRCUITS



A magnetic circuit which has two or more than two paths for the magnetic ~~circuit~~ flux is called a parallel magnetic circuits.

Let a parallel circuit shown in the figure. A current carrying coil is wound on the central limb AB. This coil sets up a magnetic flux ϕ_1 in the central limb. ~~which~~ The flux ϕ_1 is now divided into two paths i) ADCB carrying flux ϕ_2 & ii) AFEB carrying flux ϕ_3.

$$\therefore \phi_1 = \phi_2 + \phi_3$$

The magnetic paths ADCB & AFEB are in parallel. The ATs required for this parallel circuits is equal to the ATs required for any one of the paths. [MMF]

Now,

S_1, Reluctance of Path BA $= \dfrac{l_1}{\mu_0 \mu_{r_1} a_1}$

S_2, Reluctance of Path ADCB $= \dfrac{l_2}{\mu_0 \mu_{r_2} a_2}$

S_3, Reluctance of Path AFEB $= \dfrac{l_3}{\mu_0 \mu_{r_3} a_3}$

$\therefore$ Total mmf required = mmf for BA + mmf for ADCB or AFEB

$$\therefore \text{Total mmf or AT} = \phi_1 S_1 + \phi_2 S_2$$
$$= \phi_1 S_1 + \phi_3 S_3.$$

Series-Parallel Magnetic Circuits.

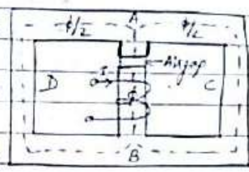


Fig shows, Parallel magnetic circuits ACB & ADB connected across the common magnetic path AB which contains an airgap of length l_g. As usual, the flux ϕ in common core is divided equally at pt. A between two parallel paths which have equal reluctance. The reluctance of the path AB consists of

i) Air gap reluctance &

ii) The reluctance of the central core which is comparatively negligible.

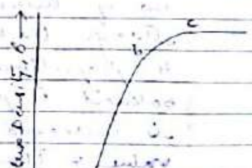
Hence, the reluctance of the central core AB equals only the air gap reluctance across which are connected two equal parallel reluctances. Hence mmf required for this circuit would be the sum of

i) that required for the airgap and

ii) that required for either of two paths.

$$\text{Total mmf} = \phi_1 S_g + \phi/2\, S_2$$
$$= \phi_1 S_g + \phi/2\, S_3$$

Magnetisation or BH curve. -



The curve drawn giving relationship between flux or induction density B & magnetising force H is known as BH curve or magnetising curve.

It has four distinct regions oa, ab, bc & the region beyond c. During region oa the increase in flux density is very small, in region ab the flux density increases almost linearly with the magnetising force H. In region bc the increase in flux density is again small & in region beyond point c, the flux density is almost constant (flat curve).

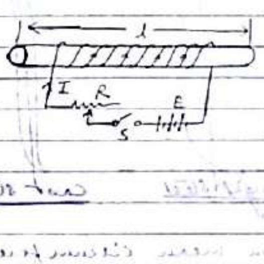
The flat part of BH curve corresponds to magnetic saturation of the material. The non-linearity of the curve indicates that the relative permeability μ_r (ie $\frac{B}{\mu_0 H}$) of a magnetic material is not constant but depends very largely upon the flux density B.

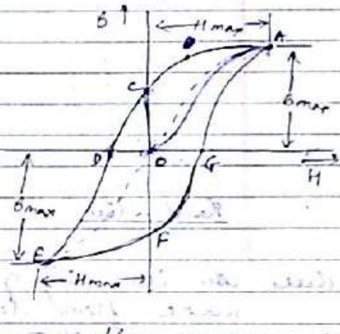
Magnetic Hysteresis

If a magnetic substance is magnetised in a strong magnetic field, it retains a considerable portion of magnetism after the magnetic force has been withdrawn. The phenomenon of lagging of magnetisation or induction flux density behind the magnetising force is known as magnetic hysteresis.

Let a core of iron be wound with a no. of turns of a wire & current be passed through the solenoid. A magnetic field of intensity H proportional to the current flowing through the solenoid is produced. Let magnetising force H is increased from zero to a certain maximum value & then gradually reduced to zero. If the values of flux density B in the core corresponding to various values of magnetising force H are determined and BH curves are drawn for increasing & decreasing values of magnetising forces H then it will be observed that BH curve obtained for decreasing values of H lies above that obtained for increasing values of H.

While decreasing the magnetising force H, when H is brought to zero the induction density B, is represented by OC & is called as residual magnetism. The power of retaining the residual magnetism is called the retentivity of the material.

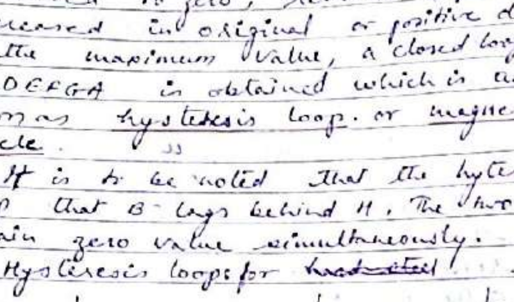




Now if the direction of flow of current is reversed; the magnetising force H is reversed & let the current be increased in the negative direction until the induction density B becomes zero. At this instant i.e. when B = 0, the demagnetising force H = OD which is required to neutralize the residual magnetism, and is known as coercive force. If the demagnetising force H is further increased to the previous maximum value & again gradually decreased to zero, reversed & further increased in original or positive direction to the maximum value, a closed loop ACDEFGA is obtained which is usually known as hysteresis loop. or magnetic cycle.

It is to be noted that the hysteresis loop that B lags behind H. The two never attain zero value simultaneously.

Hysteresis loops for ~~hard steel~~.



Hard steel   Wrought steel   Cast steel

Ques An iron ring of 400 cm mean circumference is made from round iron of cross section 20 cm². Its permeability is 500. If it is wound with 400 turns, what current would be required to produce a flux of 0.001 wb?

Soln:- Length of path, $l = 400\text{ cm} = 4\text{ m}$.

Area of cross section of iron ring, $a = 20\text{ cm}^2 = 20 \times 10^{-4}\text{ m}^2$

~~Absolute permittivity~~

Absolute permeability $\mu_0 = 4\pi \times 10^{-7}$

Now, mmf = flux × reluctance

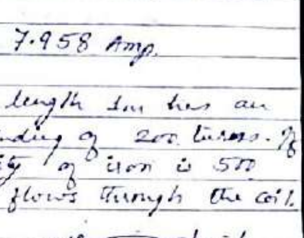
$$NI = \phi \times \frac{l}{a\mu_0\mu_r}$$

$$400 \times I = 0.001 \times \frac{4}{20 \times 10^{-4} \times 4\pi \times 10^{-7} \times 500}$$

$$I = \frac{1}{4\pi} \quad 7.958\text{ Amp.}$$

Ques An iron ring of mean length 1m has an air gap of 1 mm & a winding of 200 turns. If the relative permeability of iron is 500 when a current of 1 A flows through the coil. Find the flux density.

Soln:- mmf = flux × reluctance



$$NI = \phi\left[\frac{l}{a\mu_0\mu_r} + \frac{l_g}{a\mu_0}\right]$$

$$\cancel{NI = B\left[\frac{l}{\mu_0\mu_r} + \frac{l_g}{\mu_0}\right]}$$

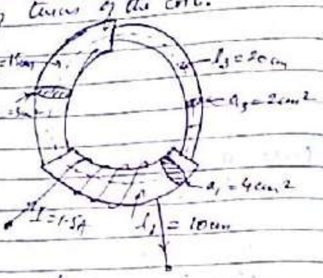
$$NI = B\left[\frac{l}{\mu_0\mu_r} + \frac{l_g}{\mu_0}\right]$$

$$B = \frac{200 \times 1}{\left[\frac{0.999}{4\pi \times 10^{-7} \times 500} + \frac{0.0001}{4\pi \times 10^{-7}}\right]}$$

$$B = \frac{200}{2386.73} = 0.0838\text{ wb/m}^2$$

Ques The ring shaped core is made of a material having a relative permeability of 1000. The flux density in the smallest core area of cross section is 2T. If the current through the coil is not to exceed

1.5A, compute the no. of turns of the coil.



Sol^n :- $\phi = BA$

$= 2 \times 2 \times 10^{-4}$

$= 4 \times 10^{-4}$ Wb

Total reluctance of the magnetic path

$S = S_1 + S_2 + S_3$

$= \frac{l_1}{a_1 \mu_0 \mu_r} + \frac{l_2}{a_2 \mu_0 \mu_r} + \frac{l_3}{a_3 \mu_0 \mu_r}$

$= \frac{1}{\mu_0 \mu_r}\left[\frac{l_1}{a_1} + \frac{l_2}{a_2} + \frac{l_3}{a_3}\right]$

$= \frac{1}{4\pi \times 10^{-7} \times 1000}\left[\frac{0.1}{4 \times 10^{-4}} + \frac{0.15}{3 \times 10^{-4}} + \frac{0.2}{2 \times 10^{-4}}\right]$

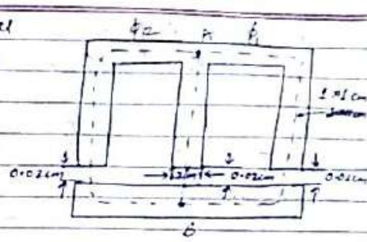
$= 13.926 \times 10^{15}$ AT/Wb

Total mmf required, $NI = \phi S$

$N \times 1.5 = 4 \times 10^{-4} \times 13.926 \times 10^{5}$

$N = 371.36$ turns

Ques The magnetic circuit of has cast steel core with dimensions as shown:-
Mean length A to B through either outer limb = 0.5 m
Mean length A to B through central limb = 0.2 m
In magnetic ckt shown it is required to establish a flux of 0.75 m.wb. in the airgap of the central limb. Determine the mmf of the exciting coil if for the core material relative permeability $\mu_r = 5000$. Neglect Fringing.



Sol^n :- It is a series parallel magnetic ckt. There are four sections to be considered for total mmf. Two section from central limb & one from any one outer limb.

For Central limb

i) Steel section :-

$l_s = ~~0.2m = 0.02m~~ \; 0.2m$

$a_s = 2 \times 1\,cm^2 = 2 \times 10^{-4}\,m^2$

$\phi_s = 0.75$ m Wb $= 0.75 \times 10^{-3}$ Wb.

$mmf_{(s)} = \phi_s \times S_{(s)}$

$= \phi_s \times \frac{l_s}{a_s \times \mu_0 \mu_r}$

$= \frac{0.75 \times 10^{-3} \times 0.2 \times 10^{0}}{2 \times 10^{-4} \times 4\pi \times 10^{-7} \times 5000}$

$= 119.36$ AT

ii) In Air gap :- $l_g = 0.02\,cm = 2 \times 10^{-4}\,m$

$a_g = 2 \times 10^{-4}\,m$

$\phi_g = 0.75$ m Wb $= 0.75 \times 10^{-3}$ Wb.

$\therefore mmf_g = \phi_g S_g = \phi_g \frac{l_g}{a_g \mu_0}$

$= \frac{0.75 \times 10^{-3} \times 2 \times 10^{-4}}{2 \times 10^{-4} \times 4\pi \times 10^{-7}}$

$= 596.83$ AT

mmf of central limb $= 119.36 + 596.83$

$mmf_c = 716.19$ AT

For Outer limb :-

i) In steel section :- $l_s = 0.5\,m$

$a_s = 1 \times 1\,cm^2 = 1 \times 10^{-4}\,m^2$

$\phi_s = \frac{1}{2}\phi_c$

$= \frac{1}{2}(0.75) \times 10^{-3}\,wb$

$= 0.375 \times 10^{-3}\,wb$

$\therefore\ mmf_s = \phi_s \times S_s = \phi_s \frac{l_s}{a_s \mu_0 \mu_r}$

$= \frac{0.375 \times 10^{-3} \times 0.5}{1 \times 10^{-4} \times 4\pi \times 10^{-7} \times 5000}$

$= 298.4\ AT$

ii) In Air gap :- $l_g = 0.02\,cm = 2 \times 10^{-4}\,m$

$a_g = 1 \times 10^{-4}\,m$

$\phi_g = 0.375 \times 10^{-3}\,wb.$

$mmf_g = \phi_g S_g = \frac{0.375 \times 10^{-3} \times 2 \times 10^{-4}}{1 \times 10^{-4} \times 4\pi \times 10^{-7}}$

$= 596.8\ AT$

Hence MMF for outer limb = $mmf_s + mmf_g$

$= 298.4 + 596.8$

$MMF_o = 895.23\ AT.$

Total mmf = $MMF_c + MMF_o$

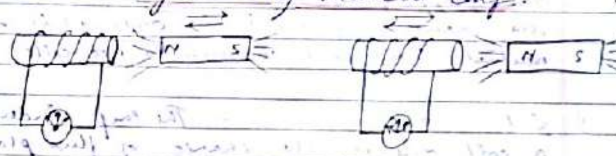
$= 716.19 + 895.23$

$= 1611.42\ ATs.$

AC EXCITATION IN MAGNETIC CIRCUIT

Emf is induced in any coil is due to the change in the flux linkage in the coil. This change of flux linkages can be obtain by two ways:-

1) By either moving the conductor & keeping magnetic field stationary or moving the magnetic field and keeping the conductor stationary, in such a way that conductor cuts across the magnetic field. The emf induced in this way is called dynamically induced emf.



2) By changing the flux linkage in coil without moving either coil or field. However the change of flux produced by the field system linking with the coil is obtained by changing the current in the field system (solenoid) as in transformer. The emf induced in this way is called statically induced emf.

Electromagnetic Induction :- The phenomenon by which an emf is induced in a ~~coil~~ circuit (and hence current flows when the circuit is closed) when magnetic flux linking with it changes is called electromagnetic induction.

Statically ~~Induced~~ induced emf have two types :-

1) Self - Induced emf.

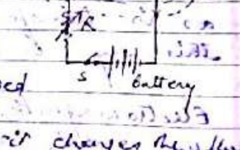
2) Mutually Induced emf.

~~1) Self Induced Emf :-~~

The property of a coil due to which it opposes the change of current flowing through itself is called self inductance or inductance of the coil.

The property of one coil due to which it opposes the change of current in the other coil is called mutual inductance between the two coils.

1) Self induced Emf :- The emf induced in a coil due to the change of flux produced by it linking with its own turns is called Self induced emf.

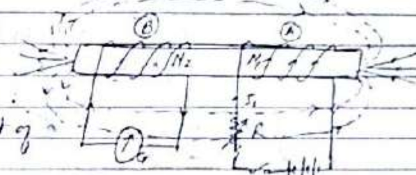


When current I flows through the coil, it produces flux ϕ which also links with its own turns. If the current flowing through coil is changed by changing the value of variable resistance R, it changes the flux linking with the coil & hence, an emf is induced in the coil. This is called self induced emf.

$$e \propto \frac{dI}{dt} \quad \text{or} \quad \boxed{e = L\frac{dI}{dt}}$$

where L is inductance of coil.

2) Mutually Induced Emf :- The emf induced in a coil due to the change of flux produced by another (neighbouring) coil, linking with it is called mutually induced emf.



When current I_1 flows through coil A it produces flux ϕ_1. Out of this flux most of the flux say ϕ_{12} links with the other coil B.

If the current flowing through the coil A is changed by changing the value of variable resistor R, it changes flux linking with the other coil B & hence an emf is induced in it. This is called as mutually induced emf.

Since, the rate of change of flux linking with coil B depends upon the rate of change of current in coil A. Therefore, the magnitude of mutually induced emf is directly proportional to the rate of change of current in coil A i.e.

$$e_m \propto \frac{dI_1}{dt} \quad \text{or} \quad e_m = M\frac{dI_1}{dt}$$

where, M is mutual inductance of coil.

Expressions of Self Inductance & Mutual Inductance

1) $\boxed{L = \frac{e}{dI/dt}}$ e = induced emf. dI/dt = rate of changing current

2) $\boxed{L = \frac{N\phi}{I}}$ N = No. of turns. ϕ = flux. I = current

3) $L = \frac{N\phi}{I}$ ∵ $\phi = \frac{mmf}{reluctance} = \frac{NI}{l/a\mu_0\mu_r}$

$$L = \frac{N}{I} \times \frac{NI}{(l/a\mu_0\mu_r)}$$

$$= \frac{N^2}{S} \qquad \left[\because S = \frac{l}{a(\mu_0\mu_r)}\right]$$

$$\boxed{L = \frac{N^2}{S}} \qquad \& \ [S = \text{reluctance}]$$

Mutual Inductance

1) $\boxed{M = \frac{e_m}{dI_1/dt}}$, e_m = mutually induced emf, dI_1/dt = rate of change of current in the neighbouring coil

2) $\boxed{M = \frac{N_2 \phi_{12}}{I_1}}$

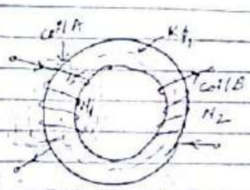
3) $\boxed{M = \frac{N_1 N_2}{S}}$

Coefficient of Coupling :- The fraction of magnetic flux produced by the current in one coil that links with the other is known as Co-efficient of coupling (K) between the two coils.

If the flux produced by one coil completely links with the other, then the value of "k" is one and the coils are said to be magnetically tightly coupled ; whereas, if the flux produced by one coil does not link at all with the other, then the value of "k" is zero & the coils are said to be magnetically isolated.

Mathematical Expression :-

considering coil 1 carrying current I_1



$$L_1 = \frac{N_1\phi_1}{I_1}$$

$$\& \ M = \frac{N_2\phi_{12}}{I_1} = \frac{N_2 K\phi_1}{I_1} \quad \text{—(1)} \qquad [\because \phi_{12} = K\phi_1]$$

Now considering coil 2 carrying current I_2

$$L_2 = \frac{N_2\phi_2}{I_2}$$

$$\& \ M = \frac{N_1\phi_{21}}{I_2} = \frac{N_1 K\phi_2}{I_2} \quad \text{—(2)} \qquad [\because \phi_{21} = K\phi_2]$$

Multiplying eqn (1) & (2) we get

$$M \times M = \frac{N_2 K\phi_1}{I_1} \times \frac{N_1 K\phi_2}{I_2}$$

$$M^2 = K^2 \frac{N_1\phi_1}{I_1} \times \frac{N_2\phi_2}{I_2} = K^2 L_1 L_2$$

$$M = K\sqrt{L_1 L_2}$$

Inductance in series & parallel :

In series,

When fluxes are in same direction

Total Inductance, $L_T = (L_1 + M) + (L_2 + M)$

$$\boxed{L_T = L_1 + L_2 + 2M}$$

When fluxes are in opposite direction

$$L_T = (L_1 - M) + (L_2 - M)$$

$$\boxed{L_T = L_1 + L_2 - 2M}$$

switch is opened, the magnetic field collapses and the stored energy is released and returned to the circuit. This energy is dissipated in the form of heat in the coil resistance.

<u>Mathematical Expression:-</u>

Let at any instant, the current flowing through the coil is i & is increasing at the rate of di/dt. The self induced emf in the coil.

$$e = L\frac{di}{dt}$$

Instantaneous Power, $P = ei = Li\frac{di}{dt}$

Energy stored in the magnetic field or energy supplied to the coil during a short interval of time dt.

$$dw = p\,dt = Li\frac{di}{dt}\,dt$$

$$= Li\,di$$

Total energy stored in the magnetic field or supplied to the coil when current rises from 0 to I (its final value),

$$\int dw = \int_0^I Li\,di$$

$$\boxed{w = \frac{1}{2}LI^2 \text{ joules.}}$$

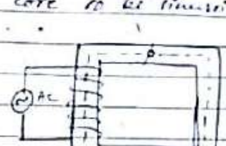
<u>Emf Equation</u> :- In ac electric machines as well as many other applications, the voltages & fluxes vary sinusoidally with time. Consider the coil core assembly; ~~of figure shown~~ having N-turn iron core coil with ac excitation the coil is assumed to be ideal with zero resistance. The induced emf in the coil must be sinusoidal for it to balance the ac applied voltage according to KVL. This constrains the flux in the core to be sinusoidal.

Let $\phi(t) = \phi_{max}\sin\omega t$

where, ϕ_{max} = maxi. core flux

$\omega = 2\pi f$ rad/s, angular velocity

f = frequency



From Faraday's Law, the voltage induced in N turn coil is

$$e(t) = -N\frac{d\phi}{dt} = -N\phi_m\frac{d(\sin\omega t)}{dt}$$

$$= -N\phi_m\omega\cos\omega t = -N\phi_m\omega\sin(\pi/2 - \omega t)$$

$$= N\phi_m\omega\sin(\omega t - \pi/2)$$

$$= E_{max}\,\cancel{\cos\omega t}\,\sin(\omega t - \pi/2)$$

$$\because E_{rms} = \frac{E_{max}}{\sqrt{2}} = \frac{2\pi f\phi_{max}N}{\sqrt{2}}$$

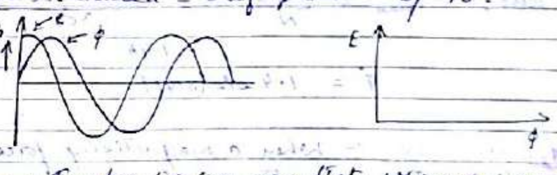
$$E_{rms} = 4.44Nf\phi_{max}$$

$$= 4.44Nf A_c B_{max}.$$

$[\because A_c$ = Area of core cross-section$]$

B_{max} = maximum flux density

It is seen from above that flux phasor lags the induced emf phasor by 90°.



Therefore we can say that whenever any ac excitation is applied in any magnetic circuit then opposing emf will induced in the circuit & it will reduce the exciting current of magnetic circuit.

Ques A 1 ϕ 220 V, 50 Hz supply is connected to the coil, the coil has 300 turns & the parameters of the core are as follows.

Length of core = 120 cm

Cross sectional Area of core = 25 cm^2

Relative permeability of core = 3000

a) Obtain an expression for flux density in the core

b) —— " —— current —— " ——

Sol^n; a) As we know,

$E = 4.44 f N A B_{max}$

$= 4.44 f N \phi_{max}$

$\phi_{max} = \dfrac{220}{4.44 \times 300 \times 50}$ wb

$= 3.3$ mwb

$B_{max} = \dfrac{3.3 \text{ mwb}}{25 \times 10^{-4} \text{ m}^2} = 1.321$ Tesla.

$\therefore \; B = 1.32 \sin(2\pi 50 t)$

$= 1.32 \sin(314t)$.

b) $H_{max} = \dfrac{B_{max}}{\mu_0 \mu_r} = \dfrac{1.32}{4\pi \times 10^{-7} \times 3000} = 350.5$ At/m

also; $I_{max} = \dfrac{H_{max}\, l}{N} = \dfrac{350.5 \times 120 \times 10^{-2}}{300}$

$= 1.4$A

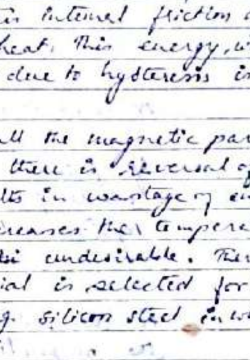
$i = 1.4 \sin(314t)$

Hysteresis Loss :- When a magnetising force is applied, the magnetic material is magnetised and the molecular magnets are lined up in a particular direction. However when the magnetising force in a magnetic material is reversed, the internal friction of the molecular magnets opposes the reversal of magnetism, resulting in hysteresis. To overcome this internal friction of the molecular magnets (or to wipe off the residual magnetism) a part of the magnetising force is used. The work done by the magnetising force against this internal friction of molecular magnets produces heat. This energy, which is wasted in the form of heat due to hysteresis is called hysteresis loss.

Hysteresis loss occurs in all the magnetic parts of electrical machines where there is reversal of magnetisation. This loss results in wastage of energy in the form of heat. It increases the temperature of the machine which is undesirable. Therefore a suitable magnetic material is selected for the construction of such parts eg. silicon steel in which hysteresis loss is minimum.

Mathematical Expression :-

Consider of strip of small thickness dB on the loop hysteresis loop.



for any value of current I, the corresponding value of flux is

$\phi = B \times A$ wb

for a small change ($d\phi$), the workdone

$dW =$ Ampere turns $\times$ change of flux

$= NI \times (dB \times A)$ Joules

$= N\left(\dfrac{Hl}{N}\right) dB \times A$

$dW = Hl\, dB \times A = H(Al)\, dB$ joule

Integrating both the sides, we get $\int H$

work done during a complete cycle

i.e. $W = \int H(Al)\, dB$

$= Al \int H\, dB$ joules.

[where, $\int H dB$ represents area of hysteresis loop]

$\therefore W = Al \times$ (Area of hysteresis loop) joules.

or work done/unit volume, W/m^3 = Area of hysteresis loop in joules.

If f is the no. of cycles (of magnetisation) made per second, then

Hysteresis loss/m3 = area of one hysteresis loop $\times f$ joules/second or watts.

For the sinusoidal flux, the hysteresis loss in the magnetic material per unit volume is expressed as

$$\frac{P_h}{m^3} = \eta B_{max}^{1.6} f \text{ watts}$$

or $\boxed{P_h = \eta B_{max}^{1.6} f \times V \text{ watts}}$

where, P_h = hysteresis loss in watts

η = hysteresis constant or Steinmetz's constant in J/m^3

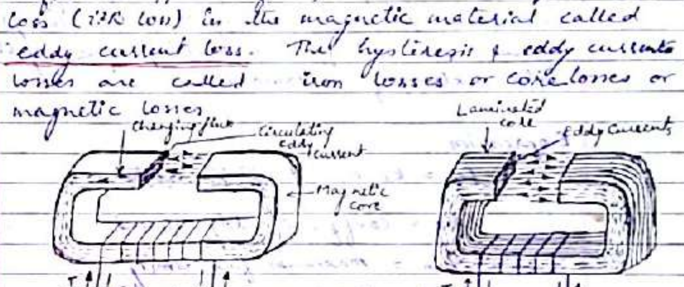
B_{max} = Maximum flux density

f = no. of cycle of magnetisation per second.

V = Volume of magnetic material.

Eddy Current Loss :- When a magnetic material is subjected to a changing (or alternating) magnetic field, an emf is induced in the magnetic material itself according to Faraday's laws of Electromagnetic induction. Since the magnetic material is also a conducting material, these emfs circulate currents within the body of the material. These circulating currents are known as eddy currents.

As these currents are not used for doing any useful work, therefore, these currents produce a loss (i^2R loss) in the magnetic material called eddy current loss. The hysteresis & eddy currents losses are called iron losses or core losses or magnetic losses.



Eddy current $= i^2R$.

As the core is continuous iron block of large cross section, the magnitude of i will be very large and hence greater eddy current loss will result.

To reduce eddy current loss, the obvious method is to reduce magnitude of eddy currents. This can be achieved by splitting the solid core into thin sheets (called lamination) in the parallel to the magnetic field. Each lamination is insulated from the other by a fine thin layer of insulation (varnish or oxide film).

Eddy currents are used to do some useful work e.g. induction heating. In this case an iron shaft is placed as a core of an inductive coil. When high frequency current is passed through the coil, a large amount of heat is produced at the outermost periphery of the shaft by eddy currents. The amount of heat reduces considerably when we move towards the centre of the shaft. This is because outer periphery of the shaft offers low resistance path to the eddy currents. This process is used for surface hardening of heavy shafts like axles of automobiles.

Expression

$$P_e = K_e B_m^2 t^2 f^2 V \text{ watts}$$

K_e = coeff. of eddy current

B_m = maximum flux density

t = thickness of lamination

f = frequency

V = volume of magnetic material.

Analogy between electric & magnetic circuits

Electric Circuit	Magnetic Circuit
Path traced by the current is known as electric current.	Path traced by the magnetic flux is called as magnetic circuit.
EMF is the driving force in the electric circuit. The unit is Volts.	MMF is the driving force in the magnetic circuit. The unit is ampere turns.
There is a current I in the electric circuit which is measured in amperes.	There is flux φ in the magnetic circuit which is measured in the weber.
The flow of electrons decides the current in conductor.	The number of magnetic lines of force decides the flux.

Resistance (R) oppose the flow of the current. The unit is Ohm	Reluctance (S) is opposed by magnetic path to the flux. The Unit is ampere turn/weber.
$R = \rho.\ l/a$. Directly proportional to l. Inversely proportional to a. Depends on nature of material.	$S = l/\ (\mu_o \mu_r a)$. Directly proportional to l. Inversely proportional to $\mu = \mu_o \mu_r$. Inversely proportional to a
The current I = EMF/ Resistance	The Flux = MMF/ Reluctance
The current density	The flux density
Kirchhoff current law and voltage law is applicable to the electric circuit.	Kirchhoff mmf law and flux law is applicable to the magnetic flux.