Math 342 Tutorial

July 16, 2025

Question 1. Show the following: **(a)** If n is an Euler pseudo prime to the bases a and b, then n is an Euler pseudoprime to the base ab. **(b)** If n is an Euler pseudoprime to the base b, then n is also an Euler pseudoprime to the base $n - b$. **(c)** If $n \equiv 5 \pmod 8$ and n is an Euler pseudoprime to the base 2, then n is a strong pseudoprime to the base 2. **(d)** If $n \equiv 5 \pmod{12}$ and n is an Euler pseudoprime to the base 3, then n is a strong pseudoprime to the base 3.

Question 2. Show that if $n = p_1 \cdots p_k$ is square-free, and if each $(p_i - 1) \mid (n - 1)$, then n is a Carmichael number.

Question 3. Show the following: **(a)** Show that if n is a pseudoprime to the bases a and b, then n is a pseudoprime to the base ab. **(b)** Suppose that $(n, a) = 1$. If n is a pseudoprime to the base a, then n is a pseudoprime to the base $\overline{a}$ where $\overline{a}$ is the inverse of a modulo n.

Question 4. Show that if $n = (a^{2p} - 1)/(a^2 - 1)$, where $a > 1$ is an integer, and p an odd prime not dividing $a(a^2 - 1)$, then n is a pseudoprime to the base a. Conclude there are infinitely many pseudoprimes to any any base. [Hint: To establish that $a^{n-1} \equiv 1 \pmod n$, show that $2p \mid n - 1$, and demonstrate that $a^{2p} \equiv 1 \pmod n$.]

Question 5. Show that if n is a Carmichael number, then n is square free.