June 10, 2015

Abstract

1 General

We deeply thank the referees for their remarks. We tried at best to answer and to modify the text accordingly. Additional corrections have been made. More important corrections include

- changed the wording in the abstract
- corrected the definition of the cold plasma dielectric tensor, to match the convention $\partial_t \to -i\omega$ (it had to be hermitian conjugate; this had been done everywhere, the corresponding corrections had been made in Table 1 and some figures)
- corrected the definitions of α, δ to comply with the definition of the cold plasma dielectric tensor of Stix

  (more precisely, removed terms $\frac{\omega^2}{c^2}$ in front of them)
- corrected the formulation of the lemma 2.3 on the well-posedness and added a remark after the lemma on the sign of ν.
- Section 4.1.1: corrected assumption on smoothness of E_y^ν in the section 'Numerical Experiments'.
- shortened slightly the section on limiting amplitude for a non-resonant case.

Please find below the detail of the modifications/answers.

2 Response to the reviewer 1

A. General issues.

1. We agree with the remarks of the reviewer. We included a short introduction into the physics of the problem, which hopefully will facilitate in reading the article.

- Concerning Sections 4.1.2, 4.3.1: We define more precisely the hybrid resonance now, which in fact requires that α vanishes, and δ does not (while in Section 4.3.1 δ vanishes as well). To give a hint on singularity of equations, we cite an article where a simple model problem is considered.

- The notation: from now on $\varepsilon_\omega^\nu(\mathbf{r})$ denotes the cold plasma dielectric tensor, similarly we denote its parameters. We added explicitely dependence on space in parameters that depend on the spatial variables.

- The case $\omega = \omega_c$: Indeed, $\epsilon_\infty < \infty$ if $\omega_c^2 \neq \omega^2$, but formally one also needs to require the boundedness of ω_p^2. Therefore we added an assumption in the end of the Section Frequency Domain Study on the coefficients of the dielectric tensor, and made a remark on the absence of the cyclotron resonance. The formulation of Lemma is correspondingly corrected.

- The limiting amplitude/absorption principle:

  A brief description of what limiting absorption and limiting amplitude principles had been added in the end of the introduction.

2. 'The reduction to a 1d problem is not clearly justified': indeed, we changed this in the introduction by adding an assumption on N_e, B_0, and changed the wording in the introduction to underline that the 1D case is the one of interest. We also explain that the mathematics of hybrid resonance is understood essentially in 1D, so the reason to concentrate to 1D model. In multiD we know almost nothing concerning the functional spaces that would allow for the study of the hybrid resonance with a limit process $\nu \to 0^+$.

B. Specific remarks:

- p1, abstract: changed

- p2: in the frequency domain the problem is regularized by a choice $|\nu| > 0$, independently of the sign of ν. It is interesting however that for a fixed ω the limits $\lim_{\nu \to 0\pm}$ do not result in the same solution. In the time domain the sign of $\nu > 0$ is indeed of crucial importance for the stability, as well as in Lemma 2.1, provided that λ in the frequency of the antenna is larger than zero. We added a remark on this before Section 2.1

- p2, after (1): L is just a non-negative real number. We found it more convenient to place the point of the isolated hybrid resonance in $x = 0$. For $\nu \neq 0$ the energy decays, we added this in the introduction.

- p3: this should be now fixed in the whole of the article.

- p3 eq (4): the notation had been changed

- p4, top: we changed the notation

- p4, eq. (5) fixed

- p4, eq. (8): changed the notation in eq (8), though a more general result in Lemma 2.1 is formulated with u, v.

- p4: we changed α_ν to β_ν not to confuse with α in the dielectric tensor. The index ν just underlines the dependence of the angle β_ν on ν (indeed, the coercivity result as it is stated, holds for this specific value of β_ν defined in the end of the proof of the lemma).

- p. 5: indeed, we changed this

- p.6: changed

- p 6: Ex changed
- p 8: 'our' code, changed
- p.8: fixed section number
- p.8 eq (12): added $\nu = 0$ + explanation
- p.8: added the Airy equation and pointed out the analyticity of the Airy equation. Additionally added a remark on the behaviour of the Airy function to justify the choice of the right boundary condition $\partial_x E = 0$.
- p.8: the citation had been fixed; we expect E_y to be analytic in this case (since we want it to be equal $\mathrm{Ai}(x)$, this is pointed out right now), and E_x thus is smooth.
- p9, Figure 3: fixed the captions. This had been done for other figures as well. The values $E^c_{x,y}$ had been defined in the caption.
- p9 on top The title of Section 4.1.2 had been changed to Solution of a Frequency-Domain Problem with Resonance
- p9 Equation (15,16): changed
- p9 after (17): it is really necessary to assume that H^2-norm that is bounded, for using the inequality (25). We fixed an assumption on E_y, which was incorrect before.
- p10 before and after Table 1 : fixed
- p 11 Figure 4: fixed
- p11 Figure 5: Agree, fixed. We added a remark in a caption to Figure 4 as well.
- p.12: The h was an error. The $^\nu$ is now for all variables. Note all fields are real functions, this is a time domain simulation. T is a time sufficiently large to that a periodic solution seems to be captured by the scheme.
- p12 Section 4.3: this explanation had been moved to the introduction
- p12 Section 4.3.1

  True, but in this case there is no resonance, since $\delta(x) \equiv 0$. Hence $E_x = \frac{i\delta}{\alpha} E_y \equiv 0$, and E_y solves the Airy equation in the frequency domain. We added a small remark to Section 4.3.1 to underline this.
- p12 Equation (23): it had been removed
- p14 Figures 8 and 9

  Added the captions
- p15 in Conclusion

  Indeed $P1$
- references: corrected

3 Response to Reviewer 2

- p2: We tried to change the wording in the introduction to make it more clear what is a hybrid resonance. The singularity appears for $\nu = 0$, provided that the diagonal part of the dielectric tensor vanishes and the non-diagonal part does not. See definition 1.1.
- 'matricial' had been changed to 'matrix'.
- p6, 3.1.

  We employed the Yee scheme to discretize the problem in the time domain, and the finite element scheme solely for the frequency domain, and considered them independently. We added a remark on that in the very beginning of Section 3.
- p10: mesh multiple times finer. It should be said exactly how many times.

  We added a remark that it was at least two times for very fine meshes, and more for coarser ones.
- p. 12: Typos fixed. Additional references on semi-lagrangian schemes have been included.