1. Discretization

1.1. Discretization of the Frequency Domain Problem

Let $\alpha(x),\ \delta(x)$ satisfy

Then, for all $\nu_* > 0$ and for all $\nu > \nu_*$, the error between the numerical solution $\mathbf{E}^{h,\nu}$ and the solution $\mathbf{E}^{\nu}$

$$\|\mathbf{E}^{\nu} - \mathbf{E}^{h,\nu}\|_V \leq$$

Proof. The Céa's lemma states

$$\|\mathbf{E}^{\nu} - \mathbf{E}^{h,\nu}\| \leq \frac{C_c}{C_i} \min_{\mathbf{v} \in V_h} \|\mathbf{E} - \mathbf{v}\|_V,$$

where C_c is the continuity and C_i is the coercivity constants. Indeed,

$$\min_{\mathbf{v} \in V_h} \|\mathbf{E} - \mathbf{v}\|_V \leq \min_{\mathbf{v} \in V_h} \|\mathbf{E} - I^h \mathbf{v}\|,$$

where $I^h \mathbf{v}$ is an interpolation operator. The estimates from [**?**, Chapter 0] give us

$$\|E_1 - I^h E_1\|_{L_2(\Omega)} \leq C h^2 |E_1''|_{L_2(\Omega)},$$
$$\|E_2 - I^h E_2\|_{H_1(\Omega)} \leq C h |E_2''|_{H_1(\Omega)}.$$

Let us first estimate

$$|E_1''|^2_{L_2(\Omega)} = \int_{-L}^{H}$$

□

1.2. Discretization of the Time Domain Problem

2. Numerical Experiments

2.1. Frequency Domain

2.2. Time Domain