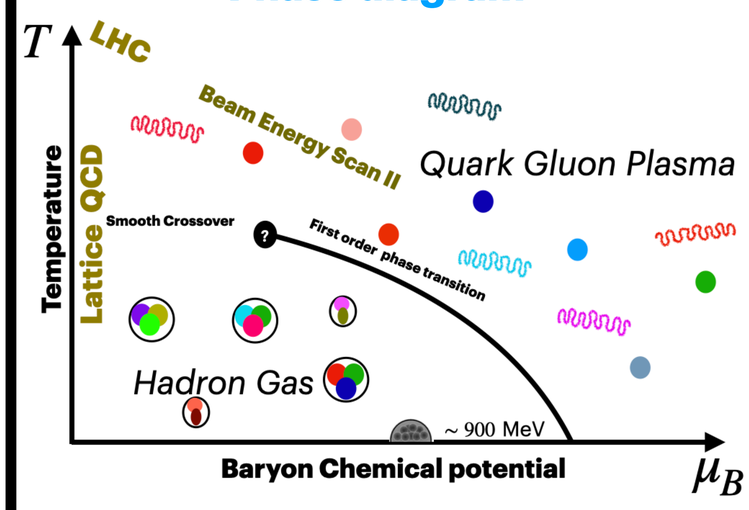
Finite density QCD equation of state: critical point and lattice-based T'-expansion (Ising-TExS)

Micheal Kahangirwe[1], Stefan A. Bass[2], Elena Bratkovskaya[3], Johannes Jahan[1], Pierre Moreau[2], Paolo Parotto[4,5], Damien Price[1], Claudia Ratti[1], Olga Soloveva[3] and Mikhail Stephanov[6].

[1]University of Houston, [2]Duke University, [3]Goethe-Universität, [4]Pennsylvania State University, [5]Università di Torino and INFN Torino, [6]University of Illinois at Chicago

Phase diagram



At $\mu_B = 0$ Quantum chromodynamics (QCD) phase diagram is well established.

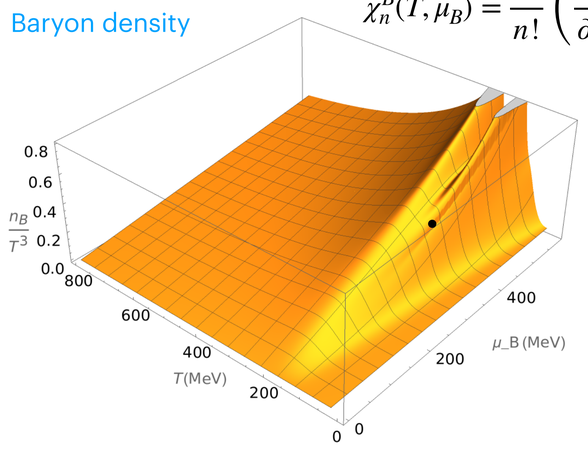
At finite μ_B a critical point is expected but has not yet been observed.

Taylor Expansion

$$n_B(T,\mu_B) = T^3 \sum_{n=1}^{2} \frac{1}{(2n-1)!} \chi_{2n}^{\text{non-Ising}}(T) \left(\frac{\mu_B}{T}\right)^{2n-1} + \frac{T_C^4}{T} n_B^{\text{Ising}}(T,\mu_B)$$

$$\chi_n^B(T,\mu_B) = \frac{1}{n!} \left(\frac{\partial}{\partial(\mu_B/T)}\right)^n (P/T^4)$$

Baryon density



$\mu_{BC} = 350\ MeV,$

For $\mu_B > 450$ MeV, thermodynamics observables have unphysical oscillations due to limitations of truncated Taylor expansion, hindering critical point studies [1]

Motivation

The available Equation of State (EoS) with a critical point has limited coverage in baryon chemical potential μ_B due to the truncation of the Taylor expansion. [1]

Goal: To build an Equation of State with a critical point from a 3D Ising model that captures a large part of the phase diagram and matches lattice QCD results at low chemical potential μ_B.

Tools

- T'-expansion scheme [2]
- 3D-Ising Model

T'- Expansion Scheme

As a solution, the Wuppertal-Budapest lattice collaboration [2] developed a T'- expansion scheme that exhibits smooth behavior at high μ_B and copes well with the QCD transition temperature.

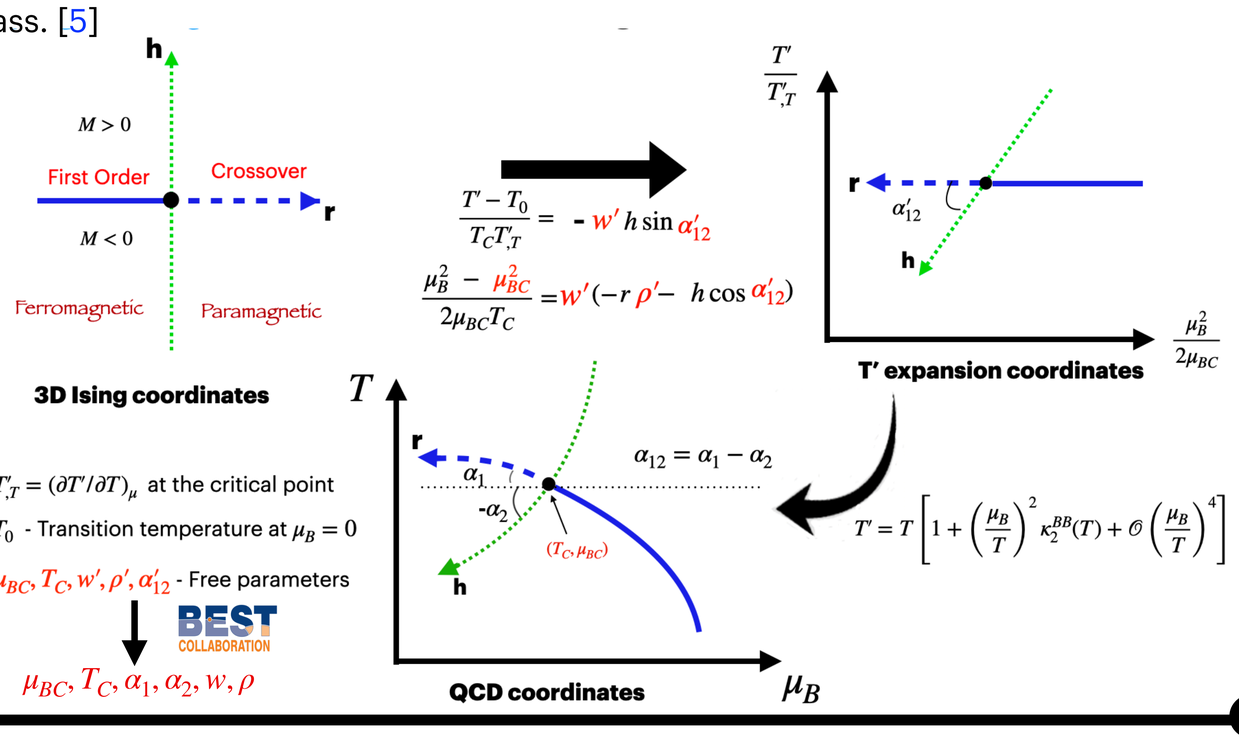
$$T\frac{\chi_1^B(T,\mu_B)}{\mu_B} = \chi_2^B(T',0)$$

$$T'(T,\mu_B) = T\left[1 + \kappa_2^{BB}(T)\left(\frac{\mu_B}{T}\right)^2 + \kappa_4^{BB}(T)\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}\left(\frac{\mu_B}{T}\right)^6\right]$$

From the T'-expansion scheme, as long as $T\chi_1^B/\mu_B$ is smooth, then finite density physics, such as the critical point can be encoded in T'.

Mapping 3D-Ising to QCD

If the critical point in QCD exists, then it must be in the 3D-Ising model universality class. [5]



Merging 3D-Ising with T'-Expansion

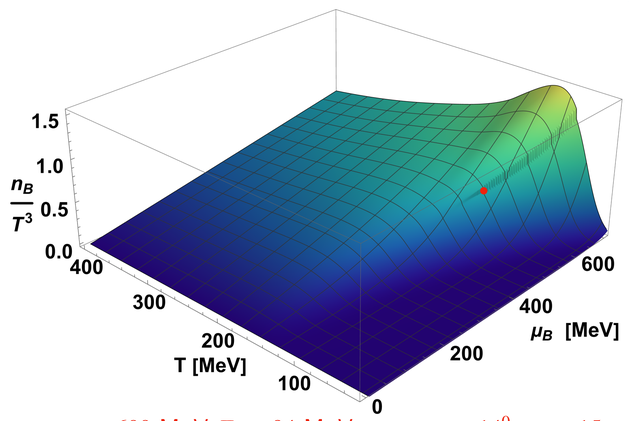
$$\frac{n_B(T,\mu_B)}{T^3} = \chi_1^B(T,\mu_B) = \left(\frac{\mu_B}{T}\right)\chi_{2,lat}^B(T',0)$$

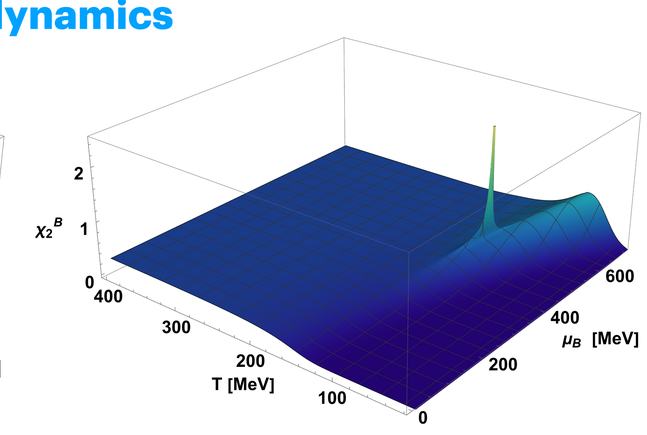
We introduce the critical point in T' by separating into the critical part T'_{crit} and the non-critical parts [4,5]

$$T' = \underbrace{T'_{lat}(T,\mu_B)}_{\text{lower order in } (\frac{\mu_B}{T})} + \underbrace{T'_{crit}(T,\mu_B) - Taylor[T'_{crit}(T,\mu_B)]}_{\text{higher orders in } (\frac{\mu_B}{T})}$$

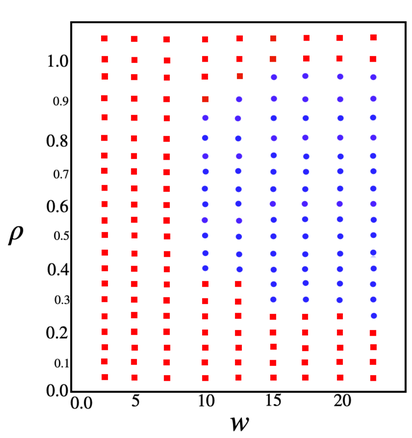
$$T'_{crit}(T,\mu_B) \approx \left(\frac{\partial\chi_{2,lat}^B(T,0)}{\partial T}\bigg|_{T=T_0}\right)^{-1} \frac{n_B^{crit}(T,\mu_B)/T^3}{(\mu_B/T)}$$

Thermodynamics





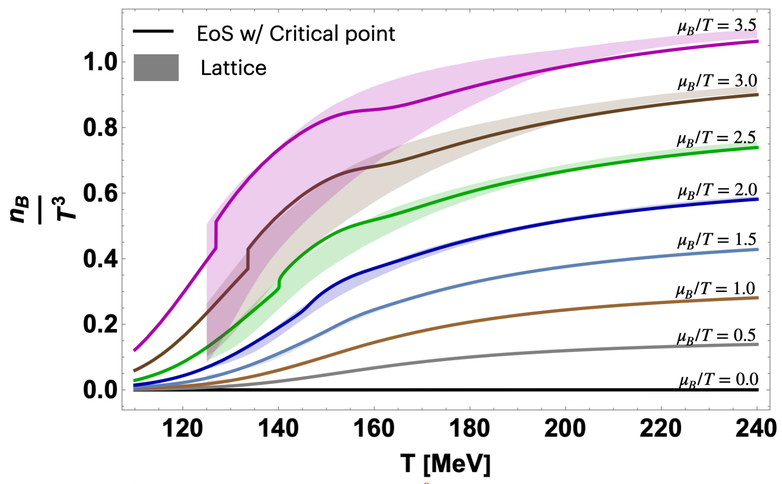
$\mu_{BC} = 600$ MeV, $T_C = 94$ MeV $\alpha_{12} = \alpha_1 = 14^0$, $w = 15$, $\rho = 0.3$



Known constraints on the EoS

- Lattice QCD: disfavors $\mu_{BC} < 300$ MeV
- Transition line: Choosing μ_{BC} fixes T_C and α_1 [3]
- Physical quark masses: $\alpha_{12} = \alpha_1$
- Stability and causality: fix w and ρ

Results



$\mu_{BC} = 350$ MeV, $T_C = 140$ MeV $\alpha_{12} = 90^0$, $w = 2$, $\rho = 2$

We check that we match lattice QCD results at $\mu_B = 0$, and our EoS with a critical point is within the error band of extrapolated lattice QCD results for certain parameter choices. [4,5]

Summary

Disclaimer! : We don't predict the location of the critical point

We provide an **enhanced coverage** for family of EoS with a 3D Ising critical point up to $\mu_B = 700$ MeV and matching lattice at low μ_B with adjustable parameters. [4,5]

References

[1] Parotto, P. *et al.*, **Phys.Rev.C** 101 034901 (2020)

[2] Borsányi, S *et al.*, **Phys.Rev.L** 108(1), 101.034901 (2021)

[3] Pradeep, M. S. & Stephanov, M., **Phys.Rev.D** 100(5), 056003 (2019)

[4] The code for this work is found at: DOI=10.5281/zenodo.10652327

[5] Kahangirwe M., Johannes J. *et al.*, **Phys.Rev.D** 109 094046(2024)

Acknowledgements

This material is based upon work supported by the National Science Foundation under grants No.PHY-2208724,PHY-1654219 and PHY-2116686, within the framework of the MUSES collaboration, under grant number No.OAC-2103680 and by the National Aeronautics and Space Agency(NASA) under Award Number 80NSSC24K0767.This material is also based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear 11 Physics, under Awards Number DE-SC0022023, DE-FG02-05ER41367 and DE-FG0201ER41195. O.S. and E.B. acknowledge support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through the grant CRC-TR 211 'Strong-interaction matter under extreme conditions' - Project number 315477589 - TRR 211. This work is supported by the European Union's Horizon 2020 research and innovation program under grant agreement No 824093 (STRONG-2020)