

## Thermal expansion calculations:

### For a mirror:

I assume the optical axis is  $\hat{z}$ , for transversal axis I choose  $y$ .

For a mirror, we have from Jeremy's paper:

$$R_{\text{th}}^{-1} = -N \frac{\alpha P}{\kappa w^2}$$

And

$$\left[ \frac{1}{\text{m}} = \frac{\frac{1}{\cancel{K}} \cdot \frac{1}{\cancel{s}}}{\frac{\cancel{s} \cdot \text{m}}{\cancel{m^2} \cdot \cancel{K}} \cancel{m^2}} = \frac{1}{\text{m}} \right] \checkmark$$

$$R^{-1} = R_0^{-1} + R_{\text{th}}^{-1}$$

### For a lens:

I will calculate everything for every facet of the lens independently.

Let us start by calculating the relation between the optical length  $\xi(y, z)$ , [m] and  $R, n$  of one facet of the lens:

The facet of the lens is defined be:

$$z^2 + y^2 = R^2 \setminus -y^2$$

$$z^2 = R^2 - y^2 \setminus \sqrt{\phantom{x}}$$

$$z = \sqrt{R^2 - y^2} \setminus \text{Simplify}$$

$$z = R \sqrt{1 - \frac{y^2}{R^2}}$$

To leading order this equals:

$$z = R \left( 1 - \frac{y^2}{2R^2} \right)$$

Multiplying it by  $n$  we get (all is up to an additive constant):

$$\xi = n \cdot z(y) = nR \left( 1 - \frac{y^2}{2R^2} \right)$$

And also it is useful to write:

$$\frac{d^2 \xi}{dy^2} = \frac{d^2}{dy^2} \left[ nR \left( 1 - \frac{y^2}{2R^2} \right) \right]$$

$$\frac{d^2 \xi}{dy^2} = \frac{n \cancel{R}}{\cancel{R^2}} = \frac{n}{R}$$

$$\left[ \frac{1}{\text{m}} = \frac{1}{\text{m}} \right] \checkmark$$

An important note is that  $\frac{d^2\xi}{dy^2} \propto n, R^{-1}$ , and so if we later find that  $\frac{d^2\xi}{dy^2}$  should be increased by  $a$  we can either transform  $n \rightarrow n + aR$  (such that the new  $\frac{d^2\xi}{dy^2}$  is

$$\frac{d^2\xi_{\text{new}}}{dy^2} = \frac{n + aR}{R} = \frac{n}{R} + a \quad (1)$$

) or transform  $R \rightarrow R' = \frac{nR}{n+aR}$  (so that the new

$$\frac{d^2\xi_{\text{new}}}{dy^2} = \frac{n}{R'} = \frac{\cancel{n}'}{\frac{\cancel{n}R}{n+aR}} = \frac{n + aR}{R} = \frac{n}{R} + a \checkmark \quad (2)$$

)

more generally, the total optical length is (up to a constant):

$$\xi(y) = \int_0^{z(y)} n(y, z) dz$$

Where  $z(y)$  is like before, the location of the end of the lens.

And so  $\xi$  can be modified by changing  $n(y, z)$  or by changing  $z(y)$ .

As said before, this is the change due to the thermal effects only in one side of the lens:

**The change  $\xi$  due to the change in  $n$ :** This size can be expressed as:

$$\Delta\xi_n = \int_0^{z(y)} \Delta n(y, z) dz = \int_0^{z(y)} \frac{\delta n(y, z)}{\delta T} \cdot \delta T(y, z) dz$$

For an approximately constant  $\frac{\delta n}{\delta T}$  we can write:

$$\Delta\xi_n = \frac{\delta n}{\delta T} \int_0^{z(y)} \delta T(y, z) dz$$

**The change in  $\xi$  due to the change in the thickness of the lens** We start by writing an expression for how much the thickness  $z(y)$  changes due to the heating:

$$\Delta z(y) = \alpha \cdot \frac{1 + \nu}{1 - \nu} \cdot \int_0^{z(y)} \delta T(y, z) dz \quad (3)$$

$$\left[ m = \frac{1}{K} \cdot 1 \cdot K \cdot m = m \right] \checkmark$$

$$\Delta\xi_d = n \cdot \Delta z(y) = n \cdot \alpha \cdot \frac{1 + \nu}{1 - \nu} \cdot \int_0^{z(y)} \delta T(y, z) dz$$

Assuming:

$$\int_0^{z(y)} \delta T(y, z) dz \propto (T_{\max} - T_0) \cdot w \cdot F(y, \sigma = w) \quad (4)$$

For some dimensionless symmetrical function  $F$  with typical width of  $w$ , mean of 0, and maximum of  $w$  (like  $w e^{-\frac{y^2}{w^2}}$ ). The typical width of  $w$  is there because the wider the beam is, the wider the heating profile, and the typical height of  $w$  is because the wider it is, the deeper the heat will go into the lens before dissipating.

Moreover,  $(T_{\max} - T_0)$  should go like

$$(T_{\max} - T_0) \propto \frac{\beta P_{\text{Laser}}}{\kappa w_{\text{on lens}}} \Rightarrow$$

$$(T_{\max} - T_0) = C \cdot \frac{\beta P_{\text{Laser}}}{\kappa w_{\text{on lens}}}$$

$$\left[ K = 1 \cdot \frac{1 \cdot \frac{J}{s}}{\frac{\frac{J}{s} \cdot m}{m^2 \cdot K} \cdot m} = K \right] \checkmark$$

Where  $C$  is some dimensionless constant,  $\beta$  is the absorption coefficient on the face of the lens  $[\beta] = 1$ ,  $\kappa$  is the heat conductivity  $[\kappa] = \frac{\frac{J}{s} \cdot m}{m^2 \cdot K}$ ,  $w_{\text{on lens}}$  is the width of the laser  $[w_{\text{on lens}}] = m$  and  $P_{\text{laser}}$  is the laser power  $[P_{\text{laser}}] = \frac{J}{s}$ .

Plugging it back to (4):

$$\int_0^\infty \delta T(y, z) dz \approx C \cdot \frac{\beta P_{\text{Laser}}}{\kappa w_{\text{on lens}}} \cdot \cancel{w_{\text{on lens}}} \cdot F(y) \quad (4)$$

$$[K \cdot m = K \cdot m] \checkmark$$

And so

$$\frac{d^2 \Delta \xi}{dy^2} = \frac{d^2 \Delta \xi_n}{dy^2} + \frac{d^2 \Delta \xi_d}{dy^2} \approx \frac{C \cdot \frac{\beta P_{\text{Laser}}}{\kappa}}{w_{\text{on lens}}^2} \left( \underbrace{\frac{\delta n}{\delta T}}_{\text{change in refractive index term}} + \underbrace{n \cdot \alpha \cdot \frac{1+\nu}{1-\nu}}_{\text{change in thickness term}} \right) = C \cdot \frac{\beta P_{\text{Laser}}}{\kappa w_{\text{on lens}}^2} \left( \frac{\delta n}{\delta T} + n \cdot \alpha \cdot \frac{1+\nu}{1-\nu} \right)$$

$$\left[ \frac{1}{m} = \frac{m}{m^2} = 1 \cdot \frac{1 \cdot \frac{J}{s}}{\frac{\frac{J}{s} \cdot m}{m^2 \cdot K} \cdot m} \cdot \left( \frac{1}{K} + \frac{1}{K} \right) = \frac{1}{m} \right] \checkmark$$

Where  $\frac{\delta n}{\delta T}$ ,  $\nu$ ,  $\alpha$ ,  $n_0$ ,  $k$ ,  $P_{\text{laser}}$ ,  $\kappa$ ,  $w_{\text{on lens}}$ ,  $\beta$  are constants of the system.

Having this, we can substitute back to (1) and (2):

$$n \rightarrow n' = n + aR = n + C \cdot \frac{\beta P_{\text{Laser}}}{\kappa w_{\text{on lens}}^2} \cdot \frac{\delta n}{\delta T} \cdot R$$

$$R \rightarrow R' = \frac{nR}{n + aR} = \frac{nR}{n + \left( C \cdot \frac{\beta P_{\text{Laser}}}{\kappa w_{\text{on lens}}^2} n \cdot \alpha \cdot \frac{1+\nu}{1-\nu} \right) R}$$

The new origin of the sphere will be:

$$z'(0) = R'$$

where  $z'(0)$  is the new position of the edge of the lens, which is

$$z'(0) \stackrel{(3)}{=} z(0) + C \cdot \frac{\beta P_{\text{Laser}}}{\kappa w_{\text{on lens}}^2} \alpha \cdot \frac{1 + \nu}{1 - \nu}$$