Thermal expansion calculations:

I assume the optical axis is $\hat{z}$, for transversal axis I choose y.

Quadratic approximation for the surface of a sphere:

I will calculate everything for every facet of the lens independently.
Let us start by calculating the relation between the optical length $\xi\left(y,z\right)$, [m] and R, n of one facet of the lens:
The facet of the lens is defined be:

$$z^2 + y^2 = R^2 \quad \backslash - y^2$$

$$z^2 = R^2 - y^2 \quad \backslash\sqrt{}$$

$$z = \sqrt{R^2 - y^2} \quad \backslash\text{Simplify}$$

$$z = R\sqrt{1 - \frac{y^2}{R^2}}$$

To leading order this equals:

$$z\left(y\right) \approx R\left(1 - \frac{y^2}{2R^2}\right) \tag{1}$$

For a lens:

Multiplying (1) by n we get (all is up to an additive constant) the optical length ξ to be:

$$\xi = n \cdot z\left(y\right) = nR\left(1 - \frac{y^2}{2R^2}\right)$$

And also it is useful to write:

$$\frac{d^2\xi}{dy^2} = \frac{d^2}{dy^2}\left[nR\left(1 - \frac{y^2}{2R^2}\right)\right]$$

$$\frac{d^2\xi}{dy^2} = -\frac{n\cancel{R}}{R^{\cancel{2}}} = -\frac{n}{R}$$

$$\left[\frac{1}{\text{m}} = \frac{1}{\text{m}}\right] \checkmark$$

An important note is that $\frac{d^2\xi}{dy^2} = \propto n, R^{-1}$, and so if we later find that $\frac{d^2\xi}{dy^2}$ should be increased by a we can either transform $n \to n - aR$ (such that the new $\frac{d^2\xi}{dy^2}$ is

$$\frac{d^2\xi_{\text{new}}}{dy^2} = -\frac{n - aR}{R} = -\frac{n}{R} + a \tag{2}$$

) or transform $R \to R' = \frac{nR}{n-aR}$ (so that the new

$$\frac{d^2\xi_{\text{new}}}{dy^2} = -\frac{n}{R'} = -\frac{\cancel{n}}{\frac{\cancel{n}R}{n-aR}} = -\frac{n - aR}{R} = -\frac{n}{R} + a\checkmark \tag{3}$$

)

more generally, the total optical length is (up to a constant):

$$\xi(y) = \int_{0}^{z(y)} n(y,z)\, dz$$

Where $z(y)$ is like before, the location of the end of the lens.
And so ξ can be modified by changing $n(y,z)$ or by changing $z(y)$.
As said before, this is the change due to the thermal effects only in one side of the lens:

The change ξ due to the change in n: This size can be expressed as:

$$\Delta\xi_n(y) = \int_{0}^{z(y)} \Delta n(y,z)\, dz = \int_{0}^{z(y)} \frac{\delta n(y,z)}{\delta T} \cdot \delta T(y,z)\, dz$$

For an approximately constant $\frac{\delta n}{\delta T}$ we can write:

$$\Delta\xi_n = \frac{\delta n}{\delta T} \int_{0}^{z(y)} \delta T(y,z)\, dz \tag{4}$$

Assuming:

$$\int_{0}^{z(y)} \delta T(y,z)\, dz \propto (T_{\text{max}} - T_0) \cdot w \cdot F(y, \sigma = w) \tag{5}$$

For some dimensionless symmetrical function F with typical width of w, mean of 0, and maximum of 1 (like $e^{-\frac{y^2}{w^2}}$). The typical width of w is there because the wider the beam is, the wider the heating profile, and the typical height of w is because the wider it is, the deeper the heat will go into the lens before dissipating.
Moreover, $(T_{\text{max}} - T_0)$ should go like

$$(T_{\text{max}} - T_0) \propto \frac{\beta P_{\text{Laser}}}{\kappa w_{\text{on lens}}} \Rightarrow$$

$$(T_{\text{max}} - T_0) = C \cdot \frac{\beta P_{\text{Laser}}}{\kappa w_{\text{on lens}}}$$

$$\left[\mathrm{K} = 1 \cdot \frac{1 \cdot \cancel{\frac{\mathrm{J}}{\mathrm{s}}}}{\frac{\cancel{\frac{\mathrm{J}}{\mathrm{s}}} \cdot \cancel{m}}{\cancel{m^2} \cdot \mathrm{K}} \cdot \cancel{m}} = \mathrm{K} \right] \checkmark$$

Where C is some dimensionless constant, β is the absorption coefficient on the face of the lens $[\beta] = 1$, κ is the heat conductivity $[\kappa] = \frac{\frac{\mathrm{J}}{\mathrm{s}}}{\mathrm{m} \cdot \mathrm{K}}$, $w_{\text{on lens}}$ is the width of the laser $[w_{\text{on lens}}] = \mathrm{m}$ and P_{laser} is the laser power $[P_{\text{laser}}] = \frac{\mathrm{J}}{\mathrm{s}}$.
Plugging it back to (5):

$$\int_{0}^{\infty} \delta T(y,z)\, dz \approx C \cdot \frac{\beta P_{\text{Laser}}}{\kappa \cancel{w_{\text{on lens}}}} \cdot \cancel{w_{\text{on lens}}} \cdot F(y) \tag{5}$$

$$\boxed{\int_0^\infty \delta T\left(y,z\right)dz \approx C\cdot\frac{\beta P_{\text{Laser}}}{\kappa}F\left(y\right)} \tag{6}$$

$$\left[\mathrm{K}\cdot\mathrm{m} = \frac{1\cdot \cancel{W}}{\cancel{W}/\mathrm{m}\cdot\mathrm{K}} = \mathrm{m}\cdot\mathrm{K}\right]\checkmark$$

Plugging back to (4) we get:

$$\Delta\xi_n = C\cdot\frac{\delta n}{\delta T}\frac{\beta P_{\text{Laser}}}{\kappa}F\left(y\right)$$

$$\frac{d^2\xi_n}{dy^2}\approx -C\cdot\frac{\delta n}{\delta T}\frac{\beta P_{\text{Laser}}}{\kappa w^2}$$

Where we assumed $F\left(y\right)$ adds to the curvature a term which goes like w^2

The change in ξ due to the change in refractive index due to volumetric heating Treating only half of the lens, the optical path length will be the:

$$\begin{aligned}\Delta\xi_v &= \frac{\delta n}{\delta T}\int_0^{z(y)}\delta T_{\text{volumetric}}\left(y,z\right)dz\\ &\approx \frac{\delta n}{\delta T}\left(T_{\max}-T_0\right)\cdot z\left(t\right)\cdot F\left(y\right)\\ &\approx \frac{\delta n}{\delta T}C_2\cdot\frac{\alpha_{\text{vol}}P_{\text{Laser}}}{\kappa}\cdot z\left(y\right)F\left(y\right)\end{aligned}$$

Units check:

$$\Delta T = C_2\cdot\frac{\alpha_{\text{vol}}P_{\text{Laser}}}{\kappa}\Rightarrow\left[\mathrm{K} = 1\cdot\frac{\frac{1}{\mathrm{m}}\cdot\cancel{J}/\cancel{s}}{\frac{\cancel{J}/\cancel{s}\cdot\cancel{m}}{\cancel{m^2}\cdot\mathrm{K}}}\cdot\cancel{m} = \mathrm{K}\right]\checkmark$$

The curvature at $y=0$ (set $y=0$ at the optical axis of the beam) will be:

$$\frac{d^2}{dy^2}\left(z\left(y\right)F\left(y\right)\right)_{y=0} = z''F+2z'F'+zF''\Big|_{y=0}\approx -\frac{1}{R}+\cancel{2\cdot 0\cdot 0}-\frac{z\left(0\right)}{w^2} =$$

Let us see if any of those two terms is negligible in the existing system:

$$\begin{aligned}R &\approx 10^{-2}\\ z\left(0\right) &\approx 5\cdot 10^{-4}\\ w^2 &\approx \left(2\cdot 10^{-3}\right)^2 = 4\cdot 10^{-6}\end{aligned}$$

$$\frac{1}{R} \approx 10^2$$
$$\frac{z(0)}{w^2} \approx \frac{10^{-2}}{10^{-6}} = 10^4$$

And so they are comparable, **and we can not neglect any of them:**

And so in total we have:

$$\frac{d^2\Delta\xi_V}{dy^2} = -C_2 \frac{\delta n}{\delta T} \frac{\alpha_{\text{vol}} P_{\text{Laser}}}{\kappa} \left(\cancel{\frac{1}{R}} + \frac{z(0)}{w^2} \right)$$

The change in ξ due to the change in the thickness of the lens We start by writing an expression for how much the thickness $z(y)$ changes due to the heating:

$$\Delta z(y) = \alpha \cdot \frac{1+\nu}{1-\nu} \cdot \int_0^{z(y)} \delta T(y,z)\, dz \tag{7}$$
$$\overset{(6)}{=} \alpha \cdot \frac{1+\nu}{1-\nu} \cdot C \cdot \frac{\beta P_{\text{Laser}}}{\kappa} F(y)$$

$$\left[\mathrm{m} = \cancel{\frac{1}{K}} \cdot 1 \cdot \cancel{K} \cdot \mathrm{m} = \mathrm{m} \right] \checkmark$$

$$\Delta\xi_R = n \cdot \Delta z(y) \quad \backslash(7)$$

$$\boxed{\Delta\xi_R = C \cdot \frac{1+\nu}{1-\nu} \cdot \frac{n\alpha\beta P_{\text{Laser}}}{\kappa} F(y)} \tag{8}$$

And so

$$\frac{d^2\Delta\xi}{dy^2} = \frac{d^2\Delta\xi_n}{dy^2} + \frac{d^2\Delta\xi_R}{dy^2} + \frac{d^2\Delta\xi_V}{dy^2}$$
$$\approx -\frac{P_{\text{Laser}}}{\kappa} \left(\underbrace{\frac{1}{w^2} C\beta \frac{\delta n}{\delta T}}_{\text{change in refractive index term}} + \underbrace{C_2 \alpha_{\text{vol}} \frac{\delta n}{\delta T} \left(\cancel{\frac{1}{R}} + \frac{z(0)}{w^2} \right)}_{\substack{\text{change in refractive index} \\ \text{from volumetric absorbtion}}} + \underbrace{\frac{1}{w^2} Cn \cdot \alpha \cdot \beta \cdot \frac{1+\nu}{1-\nu}}_{\text{change in thickness term}} \right)$$
$$= -C \cdot \frac{\beta P_{\text{Laser}}}{\kappa w^2_{\text{on lens}}} \left(\frac{\delta n}{\delta T} + n \cdot \alpha \cdot \frac{1+\nu}{1-\nu} \right)$$

$$\frac{d^2\Delta\xi}{dy^2} = \frac{d^2\Delta\xi_n}{dy^2} + \frac{d^2\Delta\xi_R}{dy^2} \approx -\frac{P_{\text{Laser}}}{\kappa} \left(\frac{1}{w^2} C\beta \frac{\delta n}{\delta T} + C_2 \alpha_{\text{vol}} \frac{\delta n}{\delta T} \left(\frac{1}{R} + \frac{z(0)}{w^2} \right) + \frac{1}{w^2} Cn \cdot \alpha \cdot \beta \cdot \frac{1+\nu}{1-\nu} \right)$$

$$\left[\frac{1}{\mathrm{m}} = \frac{\mathrm{m}}{\mathrm{m}^2} = 1 \cdot \frac{1 \cdot \cancel{\frac{J}{s}}}{\frac{\cancel{\frac{J}{s}} \cdot \mathrm{m}}{\cancel{m^2} \cdot \cancel{K}} \cdot \cancel{m^2}} \cdot \cancel{\left(\frac{1}{K} + \frac{1}{K}\right)} = \frac{1}{\mathrm{m}}\right] \checkmark$$

Where $\frac{\delta n}{\delta T}, \nu, \alpha, n_0, k, P_{\text{laser}}, \kappa, w_{\text{on lens}}, \beta$ are constants of the system.
Having this, we can substitute back to (2) and (3):

$$n \to n' = n - aR = n - \left(-C \cdot \frac{\beta P_{\text{Laser}}}{\kappa w_{\text{on lens}}^2} \cdot \frac{\delta n}{\delta T}\right) \cdot R$$

$$R \to R' = \frac{nR}{n - aR} = \frac{nR}{n - \left(-C \cdot \frac{\beta P_{\text{Laser}}}{\kappa w_{\text{on lens}}^2} n \cdot \alpha \cdot \frac{1+\nu}{1-\nu}\right) R}$$

The new origin of the sphere will be:

$$z'(0) - R'$$

where $z'(0)$ is the new position of the edge of the lens, which is

$$\begin{aligned} z'(0) &\equiv z(0) + \Delta z \\ &\overset{(7)}{=} z(0) + \alpha \cdot \frac{1+\nu}{1-\nu} \cdot \int_0^{z(y)} \delta T(y, z)\, dz \\ &\overset{(6)}{=} z(0) + \alpha \cdot \frac{1+\nu}{1-\nu} \cdot C \cdot \frac{\beta P_{\text{Laser}}}{\kappa} \end{aligned} \tag{9}$$

For a mirror:

Taking the result from (9) we have:

$$\Delta z \overset{(9)}{=} \alpha \cdot \frac{1+\nu}{1-\nu} \cdot C \cdot \frac{\beta P_{\text{Laser}}}{\kappa}$$

A buldging of the form $\Delta z \cdot F(y, \sigma = w)$ with F with width of w, mean of 0 and maximum of 1like $e^{-\frac{x^2}{w^2}}$ will subtract a $\frac{\Delta z}{w^2}$ from the curvature:

$$\begin{aligned} R_{\text{new}}^{-1} &= R_0^{-1} - \frac{\Delta z}{w^2} \\ &= R_0^{-1} - \underbrace{C \cdot \frac{\alpha \beta P_{\text{Laser}}}{\kappa w^2} \cdot \frac{1+\nu}{1-\nu}}_{R_{\text{th}}^{-1}} \backslash^{-1} \end{aligned}$$

$$R_{\text{new}} = \frac{1}{R_0^{-1} + R_{\text{th}}^{-1}}$$