CSE 6740 - HW 1

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Collaborators

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References

Pattern Recognition and Machine Learning by Christopher Bishop, Class slides
For debugging and finding different methods: geeksforgeeks.org, stackoverflow.com, towardsdata-science.com

1

1.

$$\begin{aligned} P(heads) &= \sum_{i=1}^{\infty} \frac{1}{2^n} e^{-n} \\ &= \frac{1}{2e}\left(1 + \frac{1}{2e} + \frac{1}{(2e)^2} + ...\right) \\ &= \frac{1}{2e}\left(\frac{1}{1-\frac{1}{2e}}\right) \\ &= \boxed{\frac{1}{2e-1}} \end{aligned}$$

2.

$P(V)$ =Probability of a computer being actually infected with virus V
$P(C)$ =Probability of a computer having corrupted files
$P(V|C)$ =Probability of a computer being actually infected with virus V, given that the computer has corrupted files
$P(C|V)$ =Probability of a computer having corrupted files, given that the computer is actually infected with virus V

$$
\begin{aligned}
P(V|C) &= \frac{P(C|V)P(V)}{P(C)} \\
&= \frac{(0.95)(0.2)(0.15)}{(0.95)(0.2)(0.15)+(0.85)(0.2)(0.1)} \\
&= \boxed{0.6264}
\end{aligned}
$$

3.

(a)

$$
\begin{aligned}
P(w,k) &= \text{Probability of Charlie walking on k-th day} \\
P(b,k) = 1 - P(w,k) &= \text{Probability of Charlie taking the bus on k-th day} \\
P(t(\text{or } l)|w(\text{or } b)) &= \text{Probability of Charlie being on time (or being late),} \\
&\quad \text{given that Charlie walked (or took the bus)}
\end{aligned}
$$

$$
\begin{aligned}
P(w,n) &= P(w,n-1)P(t,w) + P(b,n-1)P(l,b) \\
\text{Let } P(w,k) &= S_k \\
S_n &= S_{n-1}\frac{1}{2} + (1-S_{n-1})\frac{1}{6} \\
S_n &= \frac{S_{n-1}}{3} + \frac{1}{6} \\
||ly, S_{n-1} &= \frac{S_{n-2}}{3} + \frac{1}{6} \\
S_n &= \frac{\frac{S_{n-2}}{3} + \frac{1}{6}}{3} + \frac{1}{6} \\
S_n &= \frac{S_{n-2}}{3^2} + \frac{1}{6.3} + \frac{1}{6} \\
\text{Again, } S_{n-2} &= \frac{S_{n-3}}{3} + \frac{1}{6} \\
S_n &= \frac{\frac{S_{n-3}}{3} + \frac{1}{6}}{3^2} + \frac{1}{6.3} + \frac{1}{6} \\
S_n &= \frac{S_{n-3}}{3^3} + \frac{1}{6.3^2} + \frac{1}{6.3} + \frac{1}{6} \\
&\text{In general} \\
S_n &= \frac{S_{n-k}}{3^k} + \frac{1}{6}\left(1 + \frac{1}{3} + ... + \frac{1}{3^{k-1}}\right) \\
&\text{put } k = n-1 \\
S_n &= \frac{S_{n-(n-1)}}{3^{n-1}} + \frac{1}{6}\left(1 + \frac{1}{3} + ... + \frac{1}{3^{n-2}}\right) \\
S_n &= \frac{S_1}{3^{n-1}} + \frac{1}{6}\left(\frac{1.(1-\frac{1}{3}^{n-1})}{\frac{2}{3}}\right) \\
S_1 &= P(w,1) = p
\end{aligned}
$$

$$S_n = \boxed{P(w,n) = \left(p - \frac{1}{4}\right)\left(\frac{1}{3}\right)^{n-1} + \frac{1}{4}}$$

(b)

$$\begin{aligned} P(\text{Charlie gets late on n-th day}) &= P(w,n)P(l,w) + P(b,n)P(l,b) \\ &= P(w,n)P(l,w) + (1 - P(w,n))P(l,b) \\ &= \left(\left(p - \frac{1}{4}\right)\left(\frac{1}{3}\right)^{n-1} + \frac{1}{4}\right)\frac{1}{2} + \left(1 - \left(p - \frac{1}{4}\right)\left(\frac{1}{3}\right)^{n-1} - \frac{1}{4}\right)\frac{1}{6} \\ &= \boxed{\left(p - \frac{1}{4}\right)\left(\frac{1}{3}\right)^{n} + \frac{1}{4}} \end{aligned}$$

2

(a)

$P(x|\beta) = \frac{1}{\beta}e^{-\frac{x}{\beta}}$

$D = \{x_1, x_2, ...x_n\}$

$$L(\theta|x_1, x_2...x_n) = P(x_1|\beta)P(x_2|\beta)....P(x_n|\beta)$$

$$L(\theta|D) = \left(\frac{1}{\beta}\right)^n exp\left(-\frac{\sum_{i=1}^n x_i}{\beta}\right)$$

Applying ln on both sides

$$\implies \ln(L(\theta|D)) = n\ln\left(\frac{1}{\beta}\right) - \frac{\sum_{i=1}^n x_i}{\beta}$$

Setting derivative of this log function wrt β to 0.

$$\frac{\partial}{\partial\beta}\ln(L(\theta|D)) = \frac{n}{1/\beta}\left(-\frac{1}{\beta^2}\right) + \frac{\sum_{i=1}^n x_i}{\beta^2} = 0$$

$$\frac{\sum_{i=1}^n x_i}{\beta^2} = \frac{n}{\beta}$$

$$\frac{1}{\beta}\left(\frac{\sum_{i=1}^n x_i}{\beta} - n\right) = 0 \implies \boxed{\hat{\beta} = \frac{n}{\sum_{i=1}^n x_i} = \frac{1}{\bar{x}}}$$

(b)

$f(x|x_o, \theta) = \theta x_o^\theta x^{-\theta-1}$ where $x \geq x_o$ and $\theta > 1$

$$L(\theta|x_1, x_2, ...x_n) = f(x_1|x_o, \theta)f(x_2|x_o, \theta)...f(x_n|x_o, \theta)$$
$$= \theta^n x_o^\theta \left(\Pi_{i=1} n x_i^{-\theta-1}\right)$$

Applying ln on both sides

$$\ln(L(\theta|D)) = n\ln\theta + n\theta\ln x_o + \Pi_{i=1}^n \ln x_i^{-\theta-1}$$
$$\ln(L(\theta|D)) = n\ln\theta + n\theta\ln x_o - (1+\theta)\Pi_{i=1}^n \ln x_i$$

Setting derivative of this log function wrt θ to 0.

$$\frac{\partial}{\partial\theta}\ln(L(\theta|D)) = \frac{n}{\theta} + n\ln x_o - \sum_{i=1}^n \ln x_i = 0$$
$$\frac{n}{\theta} + n\ln x_o = \sum_{i=1}^n \ln x_i$$
$$\frac{n}{\theta} = \sum_{i=1}^n \ln x_i - n\ln x_o \implies \boxed{\hat{\theta} = \frac{n}{\sum_{i=1}^n \ln x_i - n\ln x_o}}$$

(c)

$L(\beta, \sigma^2; y, x) = (2\pi\sigma^2)^{-N/2} exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^n (y_i - x_i\beta)^2\right)$

Applying ln on both sides

$\ln(L(\beta, \sigma^2; y, x)) = -\frac{N}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^n (y_i - x_i\beta)^2$

Setting derivative of this log function wrt β to 0.

$$\frac{\partial}{\partial\beta}(\ln(L(\beta, \sigma^2; y, x))) = -\frac{1}{2\sigma^2}\sum_{i=1}^n (y_i - x_i\beta)(-x_i) = 0$$
$$\frac{1}{\sigma^2}\sum_{i=1}^n (x_i y_i - x_i^2\beta) = 0$$

$$\implies \boxed{\hat{\beta}_N = \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2} \equiv (X^T X)^{-1} X^T y}$$

Setting derivative of this log function wrt σ^2 to 0.

$$\frac{\partial}{\partial\sigma^2}(\ln(L(\beta, \sigma^2; y, x))) = -\frac{N2\pi}{2.2\pi.\sigma^2} + \frac{1}{2\sigma^4}\sum_{i=1}^N (y_i - x_i\beta)^2 = 0$$
$$\frac{N}{2\sigma^2} = \frac{1}{2\sigma^2}\sum_{i=1}^N (y_i - x_i\beta)^2$$

$$\implies \boxed{\hat{\sigma}_N^2 = \frac{1}{N}\sum_{i=1}^{N}(y_i - x_1\hat{\beta}_N)}$$

3

1.

$w = \begin{bmatrix}\overrightarrow{w_1} & \overrightarrow{w_2} & . & . & . & \overrightarrow{w_q}\end{bmatrix}$

$$w^T = \begin{bmatrix}\overrightarrow{w_1}\\ \overrightarrow{w_2}\\ .\\ .\\ .\\ \overrightarrow{w_q}\end{bmatrix}$$

$$w^T w = \begin{bmatrix}\overrightarrow{w_1}.\overrightarrow{w_1} & \overrightarrow{w_1}.\overrightarrow{w_2} & \overrightarrow{w_1}.\overrightarrow{w_3} & . & . \\ \overrightarrow{w_2}.\overrightarrow{w_1} & \overrightarrow{w_2}.\overrightarrow{w_2} & . & . & . \\ \overrightarrow{w_3}.\overrightarrow{w_1} & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & \overrightarrow{w_q}.\overrightarrow{w_q}\end{bmatrix}$$

$\overrightarrow{w_1}, ...\overrightarrow{w_q}$ are orthogonal $\implies \overrightarrow{w_i}.\overrightarrow{w_j} = 0$ if $i \neq j$.

$\overrightarrow{w_1}, ...\overrightarrow{w_q}$ are length one $\implies \overrightarrow{w_i}.\overrightarrow{w_j} = 1$ if $i = j$.

$$\therefore w^T w = \begin{bmatrix}1 & 0 & 0 & . & . & . \\ 0 & 1 & 0 & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & 1\end{bmatrix}$$

$\boxed{w^T w = I_q}$

2.

$\mathbf{x} = \begin{bmatrix}\overrightarrow{x_1} & \overrightarrow{x_2} & . & . & . & \overrightarrow{x_n}\end{bmatrix}$ where each x_i is a p-dimensional vector.

$dim(\mathbf{x}) = p$ x n

$\mathbf{w} = \begin{bmatrix}\overrightarrow{w_1} & \overrightarrow{w_2} & . & . & . & \overrightarrow{w_q}\end{bmatrix}$ where each w_i is a p-dimensional vector.

$dim(\mathbf{w}) = p$ x q

$\mathbf{x'} = \left((\mathbf{x}.\mathbf{w})\mathbf{w^T}\right)^T$

$\mathbf{x'} = \left((\mathbf{x^T w})\mathbf{w^T}\right)^T$

$$\mathbf{x}' = \left(\left(\begin{bmatrix} \overrightarrow{x_1}^T \\ \overrightarrow{x_2}^T \\ . \\ . \\ . \\ \overrightarrow{x_n}^T \end{bmatrix}_{n \text{ x } p} . \begin{bmatrix} \overrightarrow{w_1} & \overrightarrow{w_2} & . & . & . & \overrightarrow{w_q} \end{bmatrix}_{p \text{ x } q} \right) \begin{bmatrix} \overrightarrow{w_1} \\ \overrightarrow{w_2} \\ . \\ . \\ . \\ \overrightarrow{w_q} \end{bmatrix} \right)^T$$

$$\mathbf{x}' = \left(\begin{bmatrix} \overrightarrow{x_1}.\overrightarrow{w_1} & \overrightarrow{x_1}.\overrightarrow{w_2} & . & . & . & \overrightarrow{x_1}.\overrightarrow{w_q} \\ \overrightarrow{x_2}.\overrightarrow{w_1} & \overrightarrow{x_2}.\overrightarrow{w_2} & . & . & . & \overrightarrow{x_2}.\overrightarrow{w_q} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \overrightarrow{x_n}.\overrightarrow{w_1} & \overrightarrow{x_n}.\overrightarrow{w_2} & . & . & . & \overrightarrow{x_n}.\overrightarrow{w_q} \end{bmatrix}_{n \text{ x } q} \begin{bmatrix} \overrightarrow{w_1} \\ \overrightarrow{w_2} \\ . \\ . \\ . \\ \overrightarrow{w_q} \end{bmatrix} \right)^T$$

$$\mathbf{x}' = \begin{bmatrix} \sum_{k=1}^{q} (\overrightarrow{x_1}.\overrightarrow{w_k})\overrightarrow{w_k} & \sum_{k=1}^{q} (\overrightarrow{x_2}.\overrightarrow{w_k})\overrightarrow{w_k} & . & . & . & \sum_{k=1}^{q} (\overrightarrow{x_n}.\overrightarrow{w_k})\overrightarrow{w_k} \end{bmatrix}_{\text{p x n}}$$

When $q = 1$, $\mathbf{x}' = \begin{bmatrix} (\overrightarrow{x_1}.\overrightarrow{w_1})\overrightarrow{w_1} & (\overrightarrow{x_2}.\overrightarrow{w_1})\overrightarrow{w_1} & . & . & . & (\overrightarrow{x_n}.\overrightarrow{w_1})\overrightarrow{w_1} \end{bmatrix}_{\text{p x n}}$

That is, $x'_i = (\overrightarrow{x_i}.\overrightarrow{w_1})\overrightarrow{w_1}$

3.

$$\begin{aligned} SSE &= ||\overrightarrow{x_i} - \sum_{k=1}^{q} (\overrightarrow{x_i}.\overrightarrow{w_k})\overrightarrow{w_k}||^2 \\ &= \left(\overrightarrow{x_i} - \sum_{k=1}^{q} (\overrightarrow{x_i}.\overrightarrow{w_k})\overrightarrow{w_k} \right) . \left(\overrightarrow{x_i} - \sum_{k=1}^{q} (\overrightarrow{x_i}.\overrightarrow{w_k})\overrightarrow{w_k} \right) \\ &= (\overrightarrow{x_i}.\overrightarrow{x_i}) - 2\sum_{k=1}^{q} (\overrightarrow{x_i}.\overrightarrow{w_k})(\overrightarrow{x_i}.\overrightarrow{w_k}) + \sum_{k=1}^{q} (\overrightarrow{x_i}.\overrightarrow{w_k})^2 (\overrightarrow{w_k}.\overrightarrow{w_k}) \end{aligned}$$

From 1., $(\overrightarrow{w_k}.\overrightarrow{w_k}) = 1$

$$\begin{aligned} &= ||x_i||^2 - 2\sum_{k=1}^{q} (\overrightarrow{x_i}.\overrightarrow{w_k})^2 + \sum_{k=1}^{q} (\overrightarrow{x_i}.\overrightarrow{w_k})^2 \\ &= ||x_i||^2 - \sum_{k=1}^{q} (\overrightarrow{x_i}.\overrightarrow{w_k})^2 \\ MSE &= \frac{SSE}{n} \\ MSE &= \underbrace{\frac{||x_i||^2}{n}}_{\text{term 1}} - \underbrace{\frac{1}{n} \sum_{k=1}^{q} (\overrightarrow{x_i}.\overrightarrow{w_k})^2}_{\text{term 2}} \end{aligned}$$

Clearly, term 1 only depends on X and term 2 only depends on the projections of X along the q directions.

4.

Minimising projection residuals $\equiv$ minimising MSE

$$MSE = \underbrace{\frac{||x_i||^2}{n}}_{\text{term 1}} - \underbrace{\frac{1}{n}\sum_{k=1}^{q}(\overrightarrow{x_i}.\overrightarrow{w_k})^2}_{\text{term 2}}$$

We know that $Var(\overrightarrow{x_i}.\overrightarrow{w_k}) = \frac{1}{n}\sum_{i=1}^{n}(\overrightarrow{x_i}.\overrightarrow{w_k})^2 - (\frac{1}{n}\sum_{i=1}^{n}\overrightarrow{x_i}.\overrightarrow{w_k})^2$

To minimise MSE, we have to maximise term 2. Consider term 2:

$\frac{1}{n}\sum_{k=1}^{q}(\overrightarrow{x_i}.\overrightarrow{w_k})^2 = \frac{1}{n}(\overrightarrow{x_i}.\overrightarrow{w_1}) + \frac{1}{n}(\overrightarrow{x_i}.\overrightarrow{w_2}) + ... + \frac{1}{n}(\overrightarrow{x_i}.\overrightarrow{w_q})$

Consider one of these terms:

for some $k \leq q$

$\frac{1}{n}\sum_{i=1}^{n}(\overrightarrow{x_i}.\overrightarrow{w_k})^2 = (\frac{1}{n}\sum_{i=1}^{n}\overrightarrow{x_i}\overrightarrow{w_k})^2 + Var(\overrightarrow{x_i}.\overrightarrow{w_k})$

We assume the data is centered, i.e., $\frac{1}{n}\sum_{i=1}^{n}\overrightarrow{x_i} = 0$

$\implies \frac{1}{n}\sum_{i=1}^{n}(\overrightarrow{x_i}.\overrightarrow{w_k})^2 = Var(\overrightarrow{x_i}.\overrightarrow{w_k})\$$

Term 2 of $MSE = \sum_{k=1}^{q} Var(\overrightarrow{x_i}.\overrightarrow{w_k})$

$\therefore$ minimising MSE $\equiv$ maximising term 2 of MSE $\equiv$ maximising sum of variances along the different directions.

4

1.

Let us randomly initialise the 3 centers first:

$\mu_1 = 1, \mu_2 = 4, \mu_3 = 8$

Then, for each point, we allot a cluster based on how close a point is to the cluster center. We get x_1 belongs to cluster 1, x_2 belongs to cluster 2, x_3 could belong to either cluster 2 or 3. Let's pick cluster 3. x_4 belongs to cluster 3.

We now compute the loss function:

$J = \sum_{j=1}^{k}\sum_{i=1}^{n} r_{ij}||x_i - \mu_j||^2$ where

$$r_{ij} = \begin{cases} 1 & \text{point } i \in \text{ cluster } j \\ 0 & \text{point } i \notin \text{ cluster } j \end{cases}$$

We get $J = 5$.

We recompute the new centers as the mean of the points in each clusters.

$\mu_1 = 1, \mu_2 = 4.5, \mu_3 = 7$

Again, we assign points to these centers.

x_1 belongs to cluster 1. x_2 belongs to cluster 2. x_3 belongs to cluster 3. x_4 belongs to cluster 4.

We calculate $J = 3.25$.

Again, we recompute centers for these clusters.

$\mu_1 = 1, \mu_2 = 3, \mu_3 = 7$

Again, we assign points to these centers.

x_1 belongs to cluster 1. x_2 belongs to cluster 2. x_3 belongs to cluster 3. x_4 belongs to cluster 3.

We see no changes in cluster assignment.

We calculate $J = 0.5$

Recomputing centers for these clusters gives us

$\mu_1 = 1, \mu_2 = 3, \mu_3 = 6.5$

Again, we assign points to these centers.

x_1 belongs to cluster 1. x_2 belongs to cluster 2. x_3 belongs to cluster 3. x_4 belongs to cluster 3.

Recomputing centers tells us that the centers haven't changed from last time. We reach the end of our algorithm with the following clusters.

Centers	$\mu_1 = 2$	$\mu_2 = 3$	$\mu_3 = 6.5$
Points	$x_1 = 1$	$x_2 = 3$	$x_3 = 6, x_4 = 7$

2.

Consider the following center initialisation:

$\mu_1 = 1, \mu_2 = 6, \mu_3 = 7$

We get $x_1, x_2 \in C1, x_3 \in C2, x_4 \in C4$.

Cost $J = 4$.

Recomputing centers: $\mu_1 = 2, \mu_2 = 6, \mu_3 = 7$

$x_1, x_2 \in C1, x_3 \in C2, x_4 \in C4$

Cost $J = 2$.

Recomputing centers: $\mu_1 = 2, \mu_2 = 6, \mu_3 = 7$

Centers did not change $\implies$ these are our final centers with cost $J = 2$.

But from **1.**, we know that it is possible to have a set of cluster centers with even lesser cost of $J = 0.5$

Thus, we see that changing the center initialization may give a suboptimal cluster assignment that cannot be further improved.

3.

The cost function in kmeans is defined as $J = \sum_{j=1}^{k} \sum_{i=1}^{n} r_{ij} ||x_i - \mu_j||^2$ where

$$r_{ij} = \begin{cases} 1 & \text{point } i \in \text{ cluster } j \\ 0 & \text{point } i \notin \text{ cluster } j \end{cases}$$

Our goals is to minimise this cost. We perform two steps iteratively in kmeans.

The first one is the Expectation (E) step where data points are allotted to clusters. Here, for each data point, we pick the cluster center that is closest to it (Euclidean distance). The $\sum_{i=1}^{n}$ part of the cost function reduces every time we do this step.

The second one is the Maximisation (M) step where we recompute the cluster centers. To do this, we find the mean of all the data points in a cluster and make that the new cluster center. The $\sum_{j=1}^{k}$ part of the cost function reduces every time we do this step.

As both steps cause the cost function to monotonically decrease, it is clear that the algorithm will converge, although only to a local optimum.

There are only a finite number of possible partitions- $\binom{n}{k}$. In E step, clustering is based only on the previous clustering stage. If the clustering doesn't change in this step, it will never change again. We reach convergence. If the clustering changes, then the new cost will definitely be lower than the old cost. This implies that the kmeans algorithm will take a finite number of steps to converge to a local optimum.

4.

Using Euclidean distance as the distance measure restricts the type of data that can be used to perform kmeans clustering. For instance, we cannot find Euclidean distance for non numerical (say, categorical) data.

Also, Euclidean distance sometimes fails as a good metric to cluster based on the shape of the data. Kmeans works well only when the clusters are nearly spherical. Consider two annular, concentric clusters of data points. The euclidean distance between a point on one ring and a point on another ring might be small enough for them both to be allotted the same cluster, but that is not the clustering we're looking for! We want the two rings to be placed in two different clusters. This cannot be achieved with kmeans.

For data with complicated geometric shapes, kmeans using Euclidean distance measure fails. In these cases, we use kernel methods to help kmeans achieve proper clustering. Algorithms like spectral clustering work well in these cases.

5.

Report follows. Code submitted separately.

CSE 6740 - HW 1: Image Compression Programming Report

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1

For the kmedoids algorithm, ideally, the cluster centroid would be the point with the least sum of distances from all other points in the cluster. This is where I started from. When I used simple for loops to implement this, I realised it takes way too long even for small K values due to the very large size of the data set (pixels in our image). I vectorised this code to use matrix methods, then I ran into memory allocation errors due to the huge matrix size requirements (n*n to store pairwise distances).

I designed a modified method to handle this problem. After initialising cluster centers randomly, I found J (I took J = 10000) points closest to the each center. Then, I computed the distortion function for each of these J points (sum of euclidean distances between point j and all points in cluster). I picked the point with the least distortion value to be the next center for the cluster and repeated this for each cluster.

The entire process is performed iteratively till the cluster centers stop moving. The centers will stop moving when the distortion value can no longer be reduced by changing the cluster centers.

I tried the euclidean and manhattan distance metrics using the scipy function spatial.distance.cdist. I noticed better results with euclidean with only a slight increase in running time. So I decided to stick with euclidean.

If an empty cluster occurs, I call the kmedoids function again, but this time with the next lower value of K. This is done iteratively till we find a value of K for which we do not get empty clusters, and kmedoids is run with this K.

2

Figure 1: My choice of image, dimensions: 320 x 240

3

Kmedoids

(a) k = 2

(b) k = 4

(c) k = 8

(d) k = 16

(e) k = 32

(f) k = 64

(g) k = 128

(h) k = 256

Figure 2: Kmedoids Image Compression

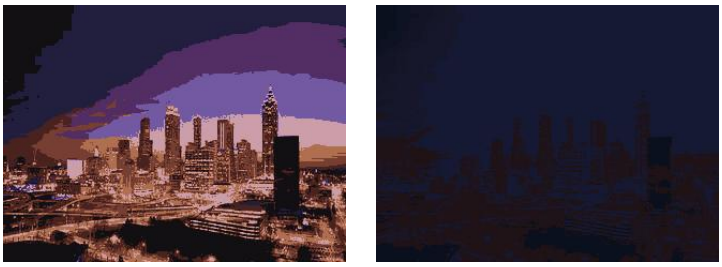
Given K	Used K	No. of Iterations till Convergence	Running time (s)
2	2	8	444.7471
4	4	15	383.2027
8	8	15	182.4632
16	16	15	55.4409
32	32	24	84.0324
64	64	28	112.4823
128	128	13	15.6823
256	172	2	2.5785

In the above table, the 'Used K' field is the final K value (one which does not give rise to empty clusters) used in the kmedoids program. We see that for a high value of K=256, we use K=172 as that is the closest value to 256 that does not give us an empty cluster.

Not much can be said about the relationship between K and required iterations for convergence (or running times). But it seems like as we increase K, due to lesser number of points in each cluster, the running time is following an almost decreasing trend. The compressed image also resembles the original image more and more as the K value increases.

4

Kmedoids Center Initialisation



(a) Randomly initialised centers

(b) Centers initialised from only the first 1000 pixels of the image

Figure 3: Kmedoids Image Compression with K = 8, center initialisation dependence

Initialisation	K	No. of Iterations till Convergence	Running time (s)
Random	8	15	182.4632
First 1000 pixels	8	2	2.3838

Different center initialisations lead to different final results. In figure 3(b), I forced my kmedoids algorithm to pick all the centers from within the first 1000 pixels of the image. As we can see this led to a vastly different, and much worse image.

5

Kmeans

(a) k = 2

(b) k = 4

(c) k = 8

(d) k = 16

(e) k = 32

(f) k = 64

(g) k = 128

(h) k = 256

Figure 4: Kmeans Image Compression

Given K	Used K	No. of Iterations till Convergence	Running time (s)
2	2	18	12.7988
4	4	34	23.4731
8	8	43	29.7732
16	16	118	89.4325
32	32	194	188.5183
64	64	345	342.9209
128	128	325	377.8028
256	172	283	409.8322

In the above table, the 'Used K' field is the final K value (one which does not give rise to empty clusters) used in the kmeans program. We see that for a high value of K=256, we use K=172 as that is the closest value to 256 that does not give us an empty cluster.

We see that as the number of K increases, the number of iterations and the running times also increase. The compressed image also resembles the original image more and more as the K value increases.

Kmeans Center Initialisation

(a) Randomly initialised centers

(b) Centers initialised from only the first 10 pixels of the image

Figure 5: Kmeans Image Compression with K = 8, center initialisation dependence

Initialisation	K	No. of Iterations till Convergence	Running time (s)
Random	8	43	29.7732
First 10 pixels	8	30	22.4709

Different center assignments led to slightly different final results. I forced the initial centers to be picked from the first 10 pixels of the image. This led to a worse final result as we can see from fig. 5.

Although kmedoids is a robust method than kmeans in general, the compromises I made due to the computationally expensive nature of kmedoids (using J closest points instead of all points in the cluster to choose the centroid) led to kmeans being a more robust method in this case.

The output quality is quite close for both methods. For small Ks, kmedoids shows longer running times than kmeans. But for larger Ks, it's the other way around.