ISYE 6414 - HW 5

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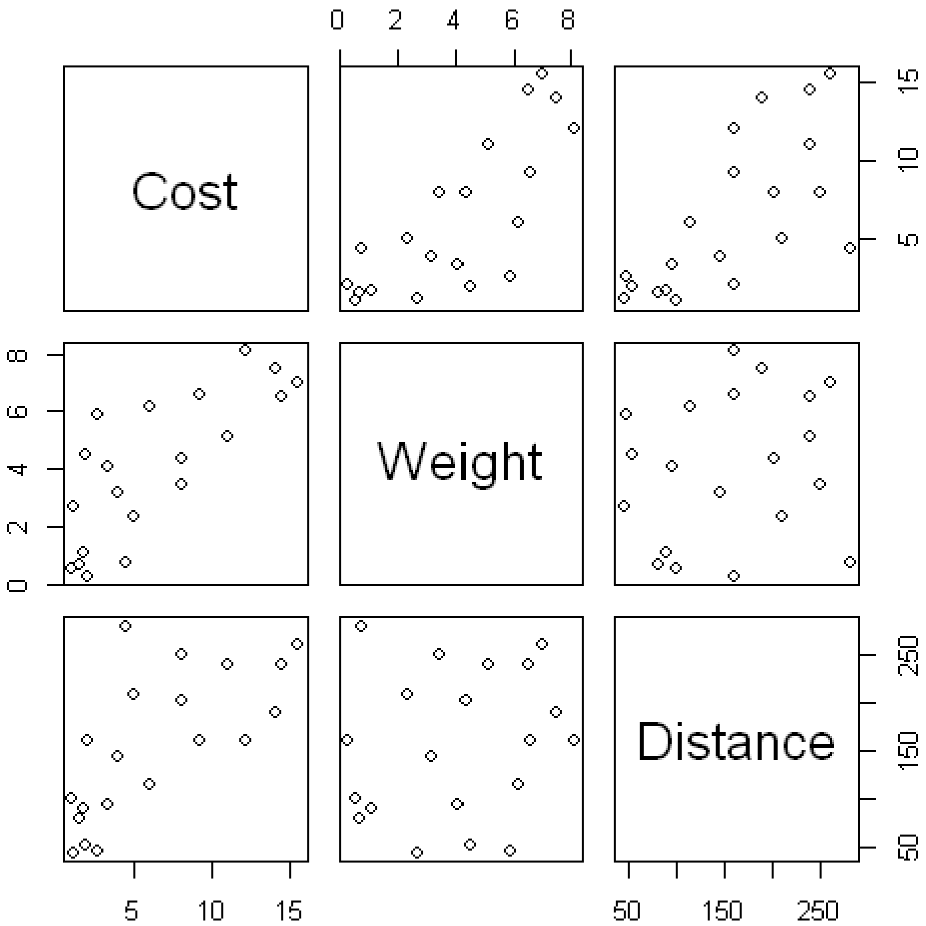
In [2]:

```
data = read.csv('HW5ShipmentData.csv')
```

Q1

In [3]:

```
options(repr.plot.width = 4.5, repr.plot.height = 4.5)
plot(data)
```



It looks like Cost and Weight, and Cost and Distance have a vaguely linear trend. No visible trend between Weight and Distance.

Q2

In [4]:

```
Cost = data[,1]
Weight = data[,2]
Distance = data[,3]
```

In [5]:

```
model = lm(Cost ~ Weight + Distance, data = data)
```

In [6]:

```
summary(model)
```

```
Call:
lm(formula = Cost ~ Weight + Distance, data = data)

Residuals:
   Min     1Q Median     3Q    Max 
-2.239 -1.101 -0.129  1.283  2.313 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) -4.672757   0.891147  -5.244 6.60e-05 ***
Weight       1.292414   0.137842   9.376 3.95e-08 ***
Distance     0.036936   0.004602   8.026 3.49e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.493 on 17 degrees of freedom
Multiple R-squared:  0.9162,	Adjusted R-squared:  0.9063 
F-statistic: 92.89 on 2 and 17 DF,  p-value: 7.066e-10
```

First order model: $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$

From the model summary, we get

$$\beta_0 = -4.673, \beta_1 = \beta_{Weight} = 1.292, \beta_2 = \beta_{Distance} = 0.037$$

In [7]:

```
modelaov = aov(Cost ~ ., data=data)
```

In [8]:

```
summary(modelaov)
```

```
            Df Sum Sq Mean Sq F value   Pr(>F)    
Weight       1  270.6  270.55  121.35 3.68e-09 ***
Distance     1  143.6  143.63   64.42 3.49e-07 ***
Residuals   17   37.9    2.23                     
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

From linear model, $F_0 = 92.89$

From table, $F_{\alpha,k-1,N-k-1} = F_{0.05,1,17} = 4.451$

As $F_0 >> F_{\alpha,k-1,N-k-1}$, the model as a whole is significant.

The p-values for all 3 βs is $<< \alpha = 0.05 \implies$ all βs are significant.

Q3

From ANOVA

$$R^2 = SSR/SST = (270.6 + 143.6)/(270.6 + 143.6 + 37.9) = 414.2/452.1 = 0.91616$$

In [9]:

```
summary(model)$r.squared #from linear model
```

0.916163885197602

R^2 from our calculation using ANOVA table quantities matches the R^2 from model summary.

Q4

In [24]:

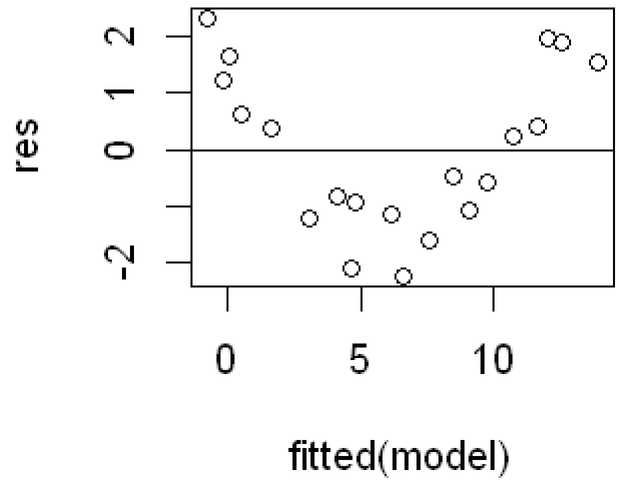
```
res = resid(model)
mean(res)
```

3.61147743654922e-17

Expectation (or in this case, mean) of the residuals is nearly zero.

In [37]:

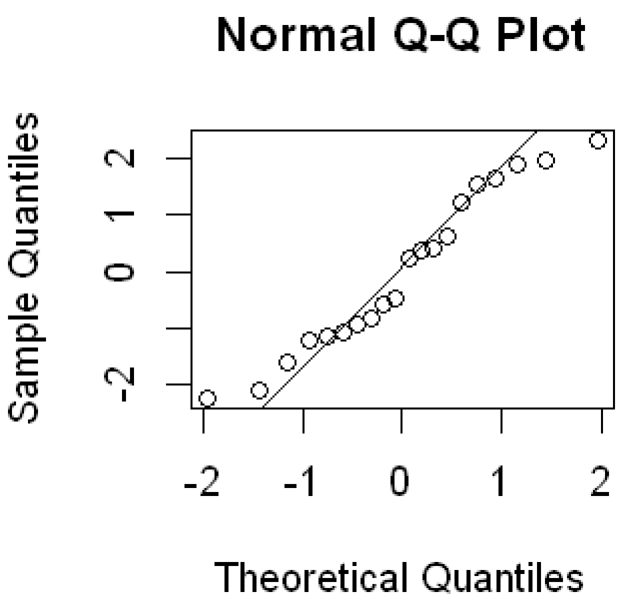
```
options(repr.plot.width = 3, repr.plot.height = 3)
plot(fitted(model), res)
abline(0,0)
```



The residuals follow a 'V' like trend $\Longrightarrow$ they are not of constant variance.

In [26]:

```
options(repr.plot.width = 3, repr.plot.height = 3)
qqnorm(res)
qqline(res)
```



Residuals stray away from the straight line towards the ends $\Longrightarrow$ they are not normally distributed.

Q5

In [27]:

```
new = data.frame(Weight = 6, Distance = 150)
predict.lm(model, new, interval = "confidence", level = 0.95)
```

fit	lwr	upr
8.622054	7.710552	9.533557

We can say with 95% confidence that the estimated value of the cost of shipment lies between $\$7.711$ and $\$9.534$ for package weight of 6 pounds and when distance shipped is 150 miles.

Q6

In [28]:

```
predict.lm(model, new, interval = "prediction", level = 0.95)
```

fit	lwr	upr
8.622054	5.34258	11.90153

We can say with 95% confidence that the predicted value of the cost of shipment lies between $\$5.343$ and $\$11.9015$ for package weight of 6 pounds and when distance shipped is 150 miles.

Q7

In [29]:

```
fullmodel = lm(Cost ~ polym(Weight, Distance, degree = 2))
```

In [30]:

```
anova(model, fullmodel)
```

Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
17	37.901092	NA	NA	NA	NA
14	2.744738	3	35.15635	59.77363	3.176466e-08

H_0 : All coefficients missing in the reduced model are 0.

H_a : Atleast one of the coefficients missing in the reduced model is non zero.

From the ANOVA table, the F-statistic is 59.7763 and the p-value is nearly 0.

From the tables, $F_{0.05,17,14} = 2.4282 << F - statistic$.

Clearly, H_0 can be rejected $\implies$ the full model is better than the reduced model.

In [31]:

```
data1 = read.csv('HW5StateCostData.csv')
COST = data1[,1]
STATE = data1[,2]
```

Q8

In [32]:

```
model1 = lm(COST ~ STATE, data = data1)
```

In [33]:

```
summary(model1)
```

```
Call:
lm(formula = COST ~ STATE, data = data1)

Residuals:
    Min      1Q  Median      3Q     Max 
-299.80  -95.83  -37.90  153.32  295.20 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)     279.60      53.43   5.233 1.63e-05 ***
STATEKentucky    80.30      75.56   1.063   0.2973    
STATETexas      198.20      75.56   2.623   0.0141 *  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 168.9 on 27 degrees of freedom
Multiple R-squared:  0.205,	Adjusted R-squared:  0.1462 
F-statistic: 3.482 on 2 and 27 DF,  p-value: 0.04515
```

The base level is the state of Kansas. With p-value of 0.04515 < $\alpha = 0.05$, this linear regression model with qualitative variable 'State' is significant.

In [34]:

```
aovcat = aov(COST ~ STATE, data = data1)
summary(aovcat)
```

```
            Df Sum Sq Mean Sq F value Pr(>F)  
STATE        2 198772   99386   3.482 0.0452 *
Residuals   27 770671   28543                 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

H_0 : STATE is significant.

H_a : STATE is not significant.

From the ANOVA table, the F-statistic is 3.482 and the p-value is nearly $0.0452 < \alpha = 0.5$.

From the tables, $F_{0.05,2,27} = 3.354 < F - statistic$.

H_0 can be rejected $\implies$ the qualitative variable State with three categories is significant.

Q9

In [35]:

```
contrasts(COST)
```

	Kentucky	Texas
Kansas	0	0
Kentucky	1	0
Texas	0	1

The above table shows the dummy variables used in the model and their values for the states.

Model: $y = \beta_0 + \beta_{Kentucky} + \beta_{Texas} + \epsilon$

$y = 279.60 + 80.30x_1 + 198.20x_2 + \epsilon$ where x_1 and x_2 are described by the columns Kentucky and Texas in the table above.

$\beta_0 = 279.60$ corresponds to mean annual maintenance costs accrued by the system users in Kansas.

Q10

From the model summary in Q8, with $p - value = 0.0141 < \alpha = 0.05$, we see that there is a significant difference in the mean annual maintenance costs accrued by the system users in Kansas and Texas. With $p - value = 0.2973 > \alpha = 0.05$, we see that there isn't a significant difference in the mean annual maintenance costs accrued by the system users in Kansas and Kentucky. This is further validated by the tukey table below.

In [36]:

```
TukeyHSD(aovcat)
```

```
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = COST ~ STATE, data = data1)

$STATE
                 diff       lwr      upr     p adj
Kentucky-Kansas  80.3 -107.0343 267.6343 0.5447834
Texas-Kansas    198.2   10.8657 385.5343 0.0365147
Texas-Kentucky  117.9  -69.4343 305.2343 0.2797413
```

With an adjusted $p - value = 0.0365 < \alpha = 0.05$, we see that the difference between expected costs in Texas and Kansas is significant, while the difference between Kentucky and Texas or Kentucky and Kansas is not significant.