ISYE 6414 - HW7

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In [1]:

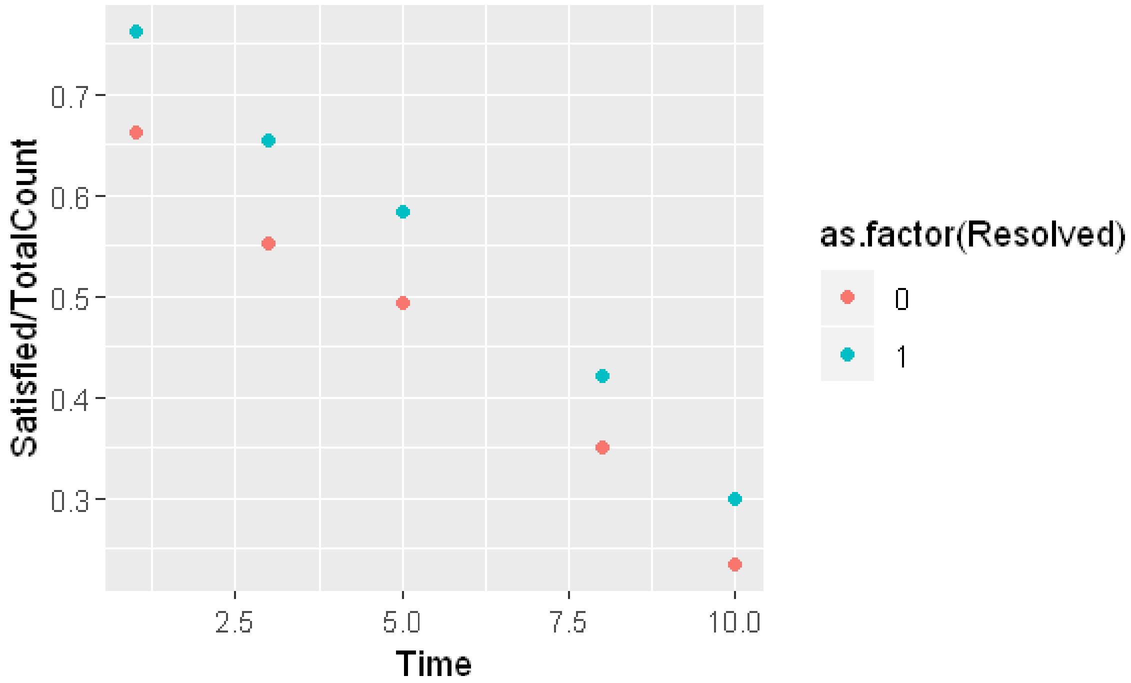
```
library(ggplot2)
data1 = read.csv('homework07data01.csv')
TotalCount = data1[, 1]
Time = data1[, 2]
Resolved = data1[, 3]
Satisfied = data1[, 4]
```

```
Registered S3 methods overwritten by 'ggplot2':
  method         from 
  [.quosures     rlang
  c.quosures     rlang
  print.quosures rlang
```

Q1

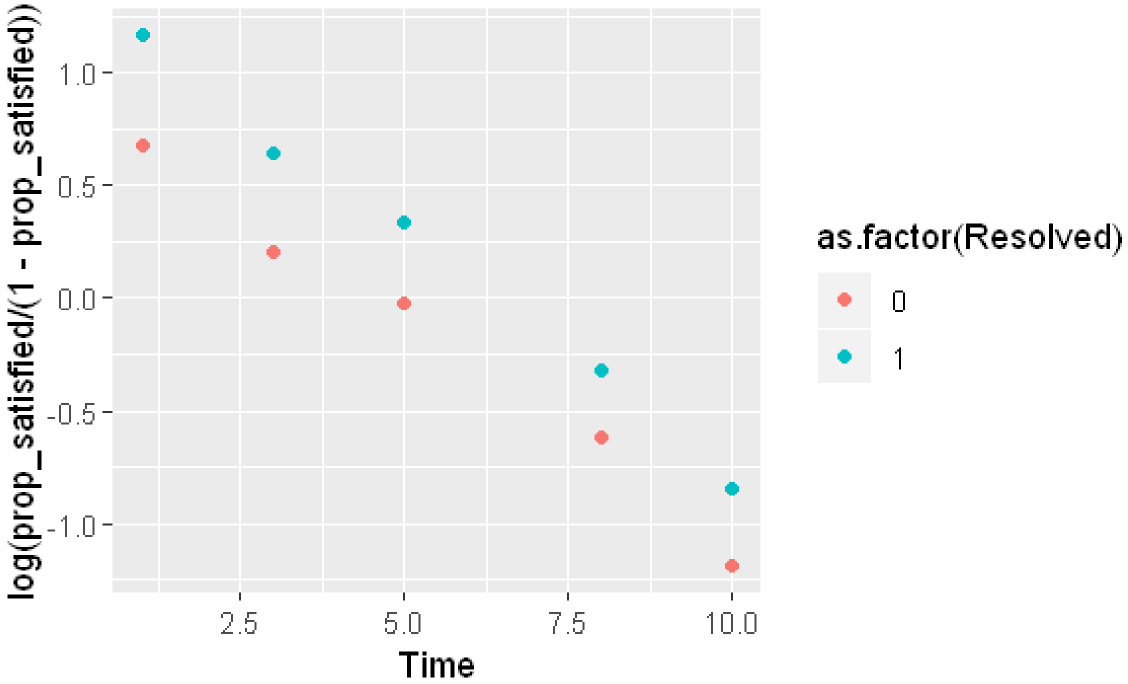
In [2]:

```
options(repr.plot.width = 5, repr.plot.height = 3)
ggplot(data1, aes(x = Time, y = Satisfied/TotalCount, colour
                  = as.factor(Resolved))) + geom_point()
```



In [3]:

```
prop_satisfied = Satisfied/TotalCount
options(repr.plot.width = 5, repr.plot.height = 3)
ggplot(data1, aes(x = Time, y = log(prop_satisfied/(1-prop_satisfied)), colour
                  = as.factor(Resolved))) + geom_point()
```



The trend of the logit function vs time looks almost linear, seems like logistic regression would be a good fit.

Q2

In [4]:

```
model1 = glm(Satisfied/TotalCount ~ Resolved + Time, weights =
             TotalCount, family = binomial)
```

In [5]:

```
summary(model1)
```

```
Call:
glm(formula = Satisfied/TotalCount ~ Resolved + Time, family = binomial,
    weights = TotalCount)

Deviance Residuals:
     Min        1Q    Median        3Q       Max
-0.46754  -0.28478  -0.03754   0.34630   0.44079

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)  0.87899    0.12414   7.080 1.44e-12 ***
Resolved     0.41527    0.12510   3.319 0.000902 ***
Time        -0.19964    0.02327  -8.577  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 95.7521  on 9  degrees of freedom
Residual deviance:  1.0727  on 7  degrees of freedom
AIC: 55.886

Number of Fisher Scoring iterations: 3
```

$\hat{\beta}_0 = 0.87899$

$\hat{\beta}_{Resolved} = 0.41527$

$\hat{\beta}_{Time} = -0.19964$

Q3

The log odds of customer satisfaction seem to increase by 0.41527 for Resolved vs. Not Resolved (odds ratio is $e^{0.41527} = 1.5148$) holding Time fixed. Thus the odds of satisfaction are $51.48\%(= e^{0.41527} - 1)$ higher for customers whose issues were resolved holding Time fixed.

Q4

The log odds of customer satisfaction seem to decrease by -0.19964 for a unit (1 minute) increase in time (odds ratio is $e^{-0.19964} = 0.8190$) holding all other variables fixed. Thus the odds of satisfaction are $18.09\%(= e^{-0.19964} - 1)$ lower for 1 minute increase in time.

Q5

In [6]:

```
new1 = data.frame(Time = 3, Resolved = 1)
new2 = data.frame(Time = 3, Resolved = 0)
```

In [7]:

```
predict(model1, new1)
```

1: 0.695353982258555

In [8]:

```
predict(model1, new2)
```

1: 0.280079539245874

The mean log odds that a customer will be satisfied when their call took 3 minutes and their issue was resolved is 0.6953. Odds ratio is $e^{0.6953} = 2.0043$. The odds that the customer will be satisfied are $100 \times \frac{2.0043}{1+2.0043}\% = 66.71\%$.

The mean log odds that a customer will be satisfied when their call took 3 minutes and their issue was not resolved is 0.2801. Odds ratio is $e^{0.2801} = 1.3233$. The odds that the customer will be satisfied are $100 \times \frac{1.3233}{1+1.3233}\% = 56.96\%$.

In [9]:

```
data2 = read.csv('homework07data02.csv')
PriorWeekPurchase = data2[, 1]
LastWeekCompPurchase = data2[, 2]
Age = data2[, 3]
Region = data2[, 4]
Count = data2[, 5]
Purchase = data2[, 6]
PurchaseCount = data2[, 7]
```

Q6

In [10]:

```
model2 = glm(Purchase/Count ~ PriorWeekPurchase + LastWeekCompPurchase +
             Age + Region, weights = Count, family = binomial)
```

In [11]:

```
summary(model2)
```

```
Call:
glm(formula = Purchase/Count ~ PriorWeekPurchase + LastWeekCompPurchase +
    Age + Region, family = binomial, weights = Count)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-2.6513  -0.8410   0.2038   0.7031   2.2835

Coefficients:
                     Estimate Std. Error z value Pr(>|z|)
(Intercept)          -0.33961    0.12612  -2.693  0.00709 **
PriorWeekPurchase     0.38392    0.08650   4.438 9.06e-06 ***
LastWeekCompPurchase -0.26885    0.08655  -3.106  0.00189 **
Age25 to 34           0.06858    0.12053   0.569  0.56935
Age35 to 44          -0.14704    0.12190  -1.206  0.22773
Age45+               -0.24521    0.12294  -1.995  0.04608 *
Regionb               0.19027    0.12205   1.559  0.11902
Regionc               0.24966    0.12165   2.052  0.04014 *
Regiond               0.26563    0.12273   2.164  0.03044 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 118.705  on 63  degrees of freedom
Residual deviance:  74.991  on 55  degrees of freedom
AIC: 344.32

Number of Fisher Scoring iterations: 4
```

$\hat{\beta}_0 = -0.33961$

$\hat{\beta}_{PriorWeekPurchase} = 0.38392$

$\hat{\beta}_{LastWeekCompPurchase} = -0.26885$

$\hat{\beta}_{Age:25-34} = 0.06858$

$\hat{\beta}_{Age:35-44} = -0.14704$

$\hat{\beta}_{Age:45+} = -0.24521$

$\hat{\beta}_{Region-b} = 0.19027$

$\hat{\beta}_{Region-c} = 0.24966$

$\hat{\beta}_{Region-d} = 0.26563$

Null deviance - Residual Deviance = 43.714

Q7

In [12]:

```
1 - pchisq(model2$null.deviance-model2$deviance, 63-55)
```

6.44101047608814e-07

H_0 : all βs are 0.

H_a : atleast one β is non zero.

P-value = Pr($\mathcal{X}_1^2 > 43.714$) ~ 0

p-value << $\alpha = 0.05 \implies$ we reject $H_0 \implies$ We accept H_a : atleast one β is non zero.

Model is statistically significant.

Q8

In [13]:

```
c(deviance(model2), 1-pchisq(deviance(model2), 55))
```

74.9907728616975   0.0378463616235688

In [14]:

```
pearres2 = residuals(model2,type="pearson")
pearson.tvalue = sum(pearres2^2)
c(pearson.tvalue, 1-pchisq(pearson.tvalue,55))
```

73.5225716343887   0.0483089734645745

H_0 : the logistic model fits the data.

H_a : the logistic model does not fit the data.

With both deviance and Pearson residuals, p-value < $\alpha =$ 0.05, thus, we reject the null hypothesis of good fit with $\alpha = 0.05$.

With both deviance and Pearson residuals, p-value > $\alpha =$ 0.01, thus, we accept the null hypothesis of good fit with $\alpha = 0.01$.

Q9

In [15]:

```
model3 = glm(PurchaseCount ~ PriorWeekPurchase + LastWeekCompPurchase + Age +
             Region + offset(log(Count)), family = "poisson")
```

In [16]:

```
summary(model3)
```

```
Call:
glm(formula = PurchaseCount ~ PriorWeekPurchase + LastWeekCompPurchase +
    Age + Region + offset(log(Count)), family = "poisson")

Deviance Residuals:
     Min        1Q    Median        3Q       Max
-2.48516  -0.65035  -0.06216   0.63731   2.77040

Coefficients:
                     Estimate Std. Error z value Pr(>|z|)
(Intercept)           0.75055    0.04021  18.665  < 2e-16 ***
PriorWeekPurchase     0.39280    0.02678  14.669  < 2e-16 ***
LastWeekCompPurchase -0.21569    0.02663  -8.099 5.54e-16 ***
Age25 to 34           0.06285    0.03586   1.753  0.07967 .
Age35 to 44          -0.04930    0.03686  -1.337  0.18110
Age45+               -0.24960    0.03929  -6.352 2.12e-10 ***
Regionb               0.11974    0.03914   3.060  0.00222 **
Regionc               0.28095    0.03760   7.472 7.92e-14 ***
Regiond               0.17495    0.03876   4.513 6.38e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for poisson family taken to be 1)

    Null deviance: 492.205  on 63  degrees of freedom
Residual deviance:  70.202  on 55  degrees of freedom
AIC: 490.66

Number of Fisher Scoring iterations: 4
```

$\hat{\beta}_0 = 0.75055$

$\hat{\beta}_{PriorWeekPurchase} = 0.39280$

$\hat{\beta}_{LastWeekCompPurchase} = -0.21569$

$\hat{\beta}_{Age:25-34} = 0.06285$

$\hat{\beta}_{Age:35-44} = -0.04930$

$\hat{\beta}_{Age:45+} = -0.24960$

$\hat{\beta}_{Region-b} = 0.11974$

$\hat{\beta}_{Region-c} = 0.28095$

$\hat{\beta}_{Region-d} = 0.17495$

Q10

In [17]:

```
1 - pchisq(model3$null.deviance-model2$deviance, 63-55)
```

0

H_0 : all βs are 0.

H_a : atleast one β is non zero.

P-value = Pr($\mathcal{X}_k^2 > Deviance$) = 0

p-value < $\alpha = 0.05 \implies$ we reject $H_0 \implies$ We accept H_a : atleast one β is non zero.

Model is statistically significant.

Q11

In [18]:

```
c(deviance(model3), 1-pchisq(deviance(model3), 55))
```

70.2020245209093 0.0813134878853016

In [19]:

```
pearres3 = residuals(model3,type="pearson")
pearson.tvalue = sum(pearres3^2)
c(pearson.tvalue, 1-pchisq(pearson.tvalue,55))
```

70.8523032082477 0.0736887015240438

H_0 : the poisson model fits the data.

H_a : the poisson model does not fit the data.

In both cases, p-value > $\alpha =$ 0.05 and 0.01, thus, we accept the null hypothesis of good fit.

Q12

$H_0 : \hat{\beta}_{Age:25-34} = \hat{\beta}_{Age:35-44} = \hat{\beta}_{Age:45+} = 0$

H_a : Atleast one of $\hat{\beta}_{Age:25-34}, \hat{\beta}_{Age:35-44}, \hat{\beta}_{Age:45+} \neq 0$

For the model made in 6, the p-value for $\beta_{Age:45+}$ is < $\alpha = 0.05$. Thus, we reject H_0.

$\implies$ Atleast one of $\hat{\beta}_{Age:25-34}, \hat{\beta}_{Age:35-44}, \hat{\beta}_{Age:45+} \neq 0$

$\implies$ categorical variable 'Age' is statistically significant.

Q13

$H_0 : \hat{\beta}_{Age:25-34} = \hat{\beta}_{Age:35-44} = \hat{\beta}_{Age:45+} = 0$

H_a : Atleast one of $\hat{\beta}_{Age:25-34}, \hat{\beta}_{Age:35-44}, \hat{\beta}_{Age:45+} \neq 0$

For the model made in 9, the p-values for all $\hat{\beta}_{Age:25-34}, \hat{\beta}_{Age:35-44}$ are > $\alpha = 0.05$. But the p-value for $\hat{\beta}_{Age:45+}$ << $\alpha = 0.05$. Thus, we reject H_0.

$\implies$ Atleast one of $\hat{\beta}_{Age:25-34}, \hat{\beta}_{Age:35-44}, \hat{\beta}_{Age:45+} \neq 0$

$\implies$ Coefficients associated with Age cannot be 0.

Q14

$\hat{\beta}_{PriorWeekPurchase} = 0.38392$

The log odds of someone purchasing the new product seem to increase by 0.38392 if the customer bought one of the company's products in the previous week holding all other variables fixed. Odds ratio is $e^{0.38392} = 1.4680$. Thus, the odds of the customer purchasing the new product are $46.80\%(= e^{0.38392} - 1)$ higher for customers who bought one of the company's products in the previous week.

Q15

$\hat{\beta}_{PriorWeekPurchase} = 0.39280$

If the customer bought one of the company's products in the previous week, holding all other variables fixed, the rate ratio of new item purchased would be expected to increase by a factor of $e^{0.39280} = 1.4811$.