ISYE 6414 - HW6

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In [1]:

```
library(car)
```

```
Loading required package: carData
```

In [2]:

```
data = read.csv('HW5ShipmentData.csv')
```

Q1

In [3]:

```
model = lm(Cost ~ Weight + Distance + I(Weight^2) + I(Weight*Distance), data = data)
```

In [4]:

```
summary(model)
```

```
Call:
lm(formula = Cost ~ Weight + Distance + I(Weight^2) + I(Weight *
    Distance), data = data)

Residuals:
    Min      1Q  Median      3Q     Max
-0.7487 -0.2558  0.0532  0.2266  0.9142

Coefficients:
                       Estimate Std. Error t value Pr(>|t|)
(Intercept)           0.4746969  0.4584500   1.035 0.316870
Weight               -0.5781705  0.1706879  -3.387 0.004062 **
Distance              0.0090777  0.0026535   3.421 0.003791 **
I(Weight^2)           0.0867388  0.0193380   4.485 0.000436 ***
I(Weight * Distance)  0.0072587  0.0006176  11.753 5.74e-09 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4346 on 15 degrees of freedom
Multiple R-squared:  0.9937,	Adjusted R-squared:  0.9921
F-statistic: 594.6 on 4 and 15 DF,  p-value: 2.541e-16
```

Here, $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_1 x_2 + \epsilon$ where $x_1 =$ package weight (in pounds) and $x_2 =$ distance shipped (in miles).

With $\alpha = 0.05$, β_1, β_2, β_3 and β_4 are significant as all their $p - values$ are $< \alpha$. β_0 is not significant with $p - value = 0.31 > \alpha$.

Q2

With $x_1 = 5$, y changes by $\beta_2 + \beta_4 x_1$ when x_2 increases by one mile.

$\delta y = \beta_2 + \beta_4 x_1 = \beta_2 + 5\beta_4 = 0.0090777 + 5(0.0072587) = 0.0453712$

The cost of shipment increases by $\$0.0453712$ when the weight is held constant at 5 pounds and distance inceases by a mile.

Q3

In [5]:

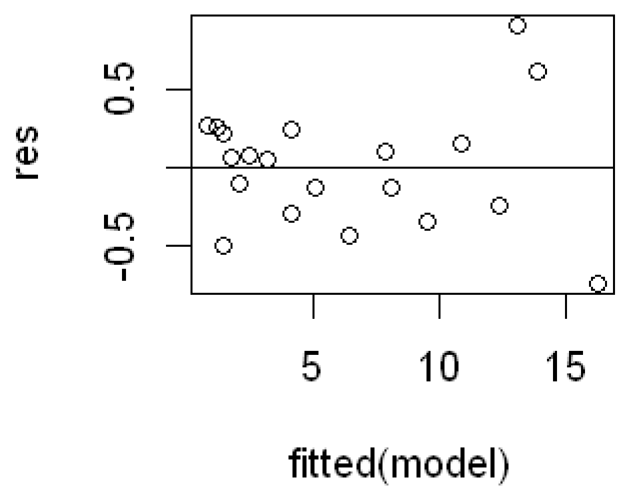
```
res = resid(model)
mean(res)
```

7.97633985770429e-18

Expectation (or in this case, mean) of the residuals is nearly zero.

In [6]:

```
options(repr.plot.width = 3, repr.plot.height = 3)
plot(fitted(model), res)
abline(0,0)
```

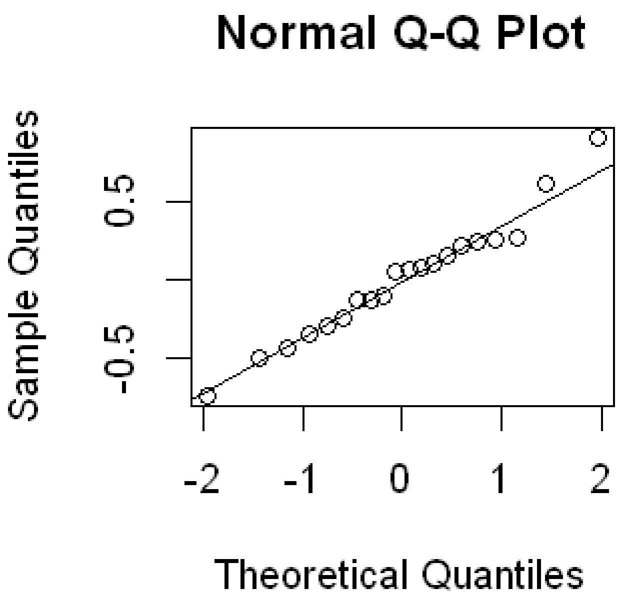


The variance of the residuals seems to increase at higher fitted values $\Longrightarrow$ they are not of constant variance.

The identical distribution assumption is violated.

In [7]:

```
options(repr.plot.width = 3, repr.plot.height = 3)
qqnorm(res)
qqline(res)
```



The normality assumption seems fine from the QQ plot.

Q4

In both models, the $E(\epsilon_i) = 0$ holds.

The normality assumption also holds for both, but seems to hold better here.

The identical distribution assumption is less violated here than in the reduced model as there is no clear pattern in the complete model unlike the bowl shape we noticed in the reduced model.

Q5

H_0 : The error terms are not autocorrelated.

H_a: The error terms are positively or negatively correlated.

$n = 24$

$k = 3$

$\alpha/2 = 0.05$

Durbin–Watson test statistic: $d = 0.829$

From the tables,

$d_{L,0.05} = 1.101$ and $d_{U,0.05} = 1.656$.

Here, $d < d_{L,\alpha/2}$. Hence we reject H_0. $\implies$ The error terms are positively or negatively correlated.

In [8]:

```
data1 = read.csv('homework04Hospital.csv')
```

In [9]:

```
model1 = lm(Hours~.,data = data1)
```

Q6

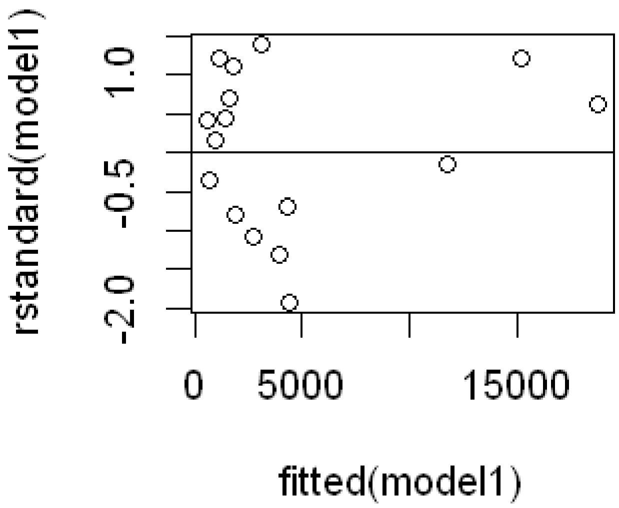
In [10]:

```
rstandard(model1)
```

1
-0.346041819889165
2
0.418441654281339
3
0.167655812250159
4
1.20758417481276
5
0.440226122495145
6
-0.807723376747137
7
0.692457807577267
8
1.10570456119942
9
-1.07103004215539
10
-1.3134905000325
11
1.39663117874807
12
-1.92935514424747
13
-0.700788004173128
14
-0.143453503317171
15
1.22487518080154
16
0.613304129045166

In [11]:

```
options(repr.plot.width = 3, repr.plot.height = 3)
plot(fitted(model1), rstandard(model1))
abline(0,0)
```



Two of the observations seem unusual with very high fitted values.

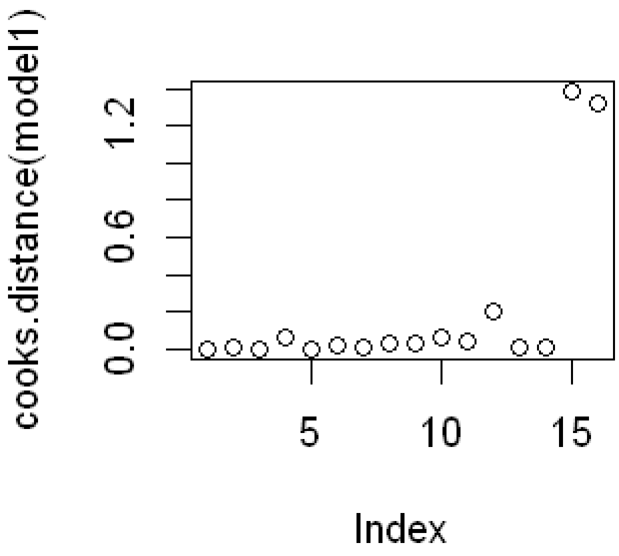
In [12]:

```
cooks.distance(model1)
```

1
0.00411135048941892
2
0.0134505843509641
3
0.00104706984405207
4
0.0688030121000577
5
0.00460986921074952
6
0.0210699915091699
7
0.0112886488737494
8
0.0278596031784254
9
0.0275416478680978
10
0.0673307576843639
11
0.0443350889334346
12
0.201515927768729
13
0.00872236772938958
14
0.0128713680372215
15
1.3838052434306
16
1.31699432115184

In [13]:

```
options(repr.plot.width = 3, repr.plot.height = 3)
plot(cooks.distance(model1))
```



D_{15} and D_{16} are $> 4/n = 0.25$ and > 1. Thus, the 15th and 16th data points are unusual observations/outliers.

Q7

In [14]:

```
summary(model1)
```

```
Call:
lm(formula = Hours ~ ., data = data1)

Residuals:
    Min      1Q  Median      3Q     Max 
-677.23 -270.19   60.93  228.32  517.70 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) 1946.80204  504.18193   3.861  0.00226 ** 
Xray           0.03858    0.01304   2.958  0.01197 *  
BedDays        1.03939    0.06756  15.386 2.91e-09 ***
Length      -413.75780   98.59828  -4.196  0.00124 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 387.2 on 12 degrees of freedom
Multiple R-squared:  0.9961,	Adjusted R-squared:  0.9952 
F-statistic:  1028 on 3 and 12 DF,  p-value: 9.919e-15
```

The predictor β_{Length} being negative is counterintuitive. The average length of stay and required labor hours cannot be inversely related.

This could be due to multicollinearity.

Q8

In [15]:

```
vif(model1)
```

Xray
7.82831925495341
BedDays
11.3961947324676
Length
2.51955937074732

$R^2_{model} = 0.9961$

$\frac{1}{1-R^2_{model}} = 256.4103$

$VIF_{BedDays} \geq 10$ but $< \frac{1}{1-R^2_{model}}$

VIF_{Xray} and VIF_{Length} are both < 10 and $< \frac{1}{1-R^2_{model}}$.

Thus, high multicollinearity is not detected.