Hamiltonian Embedding Paper Tests

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Target Matrix

Eq. (2.6) from paper has $A = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$

Thus we can see that $M_{adj} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ with graph: $0 \leftrightarrow 1 \leftrightarrow 2 \leftrightarrow 3 \leftrightarrow 4$

as target Hamiltonian to simulate, with the various embedding schemes given. For these schemes, $H_{embedding} = gH^{pen} + Q$.
For each embedding method, $g \in \{1e2, 1e4, 1e6, 1e9\}$ is tested on $t \in [0, 100]$ in steps of 0.1, where the y-axis is $||e^{-i\hat{H}t}|_S - e^{-iAt}||$, where $||\cdot||$ is the Frobenius norm (fidelity).

Unary Embedding (this looks the same as the rest but isn't right)

$$Q = -\hat{n}_1 + \hat{n}_4 + \sum_{j=1}^{4} X_j, \quad H^{pen} = -\sum_{j=1}^{3} (Z_{j+1} Z_j + Z_1 - Z_4)$$

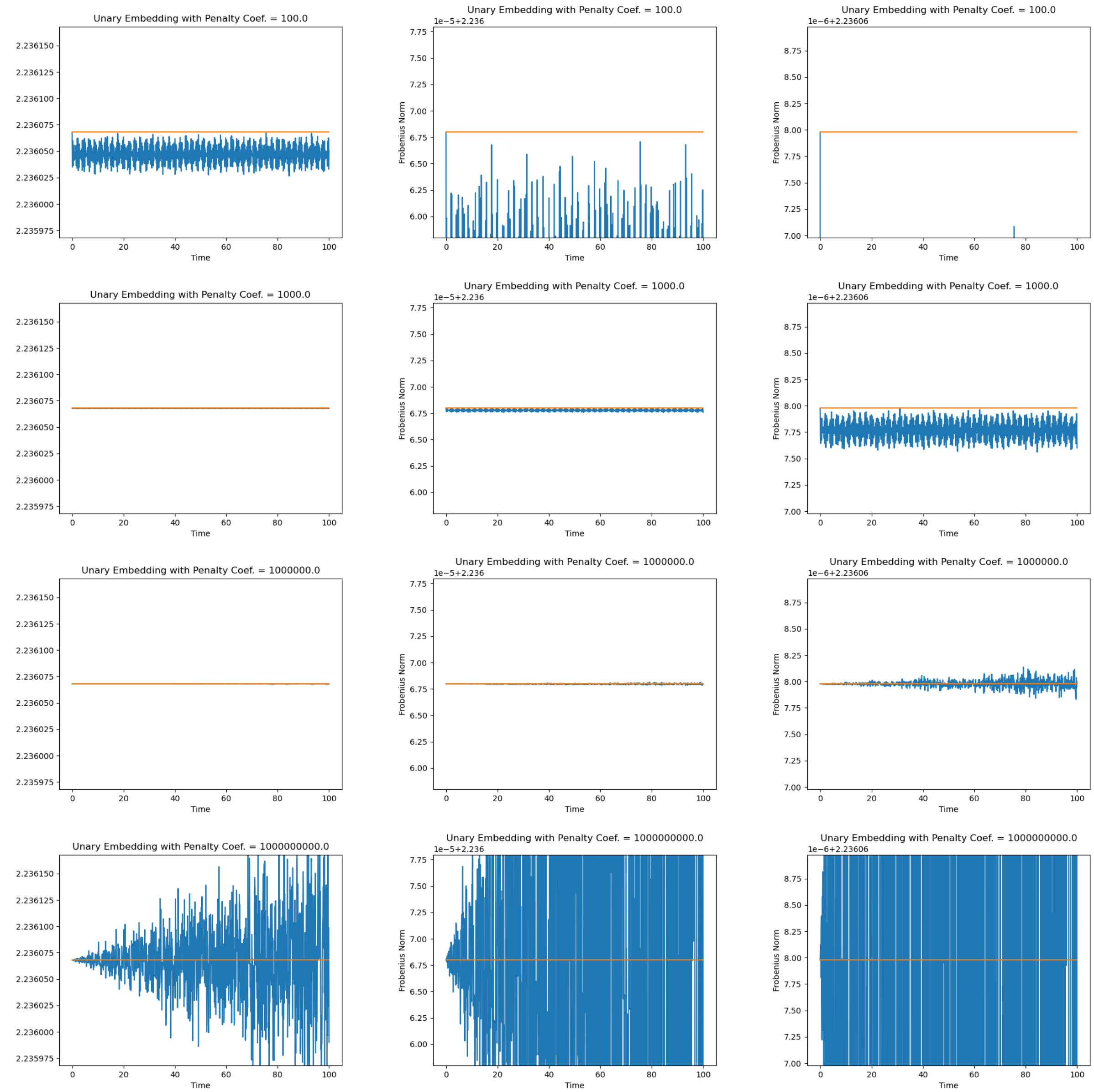

Figure 1: Unary Embedding

Antiferromagnetic Embedding

$$Q = -\hat{n}_1 - \hat{n}_4 + \sum_{j=1}^{4} X_j, \quad H^{pen} = \sum_{j=1}^{3} (Z_{j+1} Z_j + Z_1 + Z_4)$$

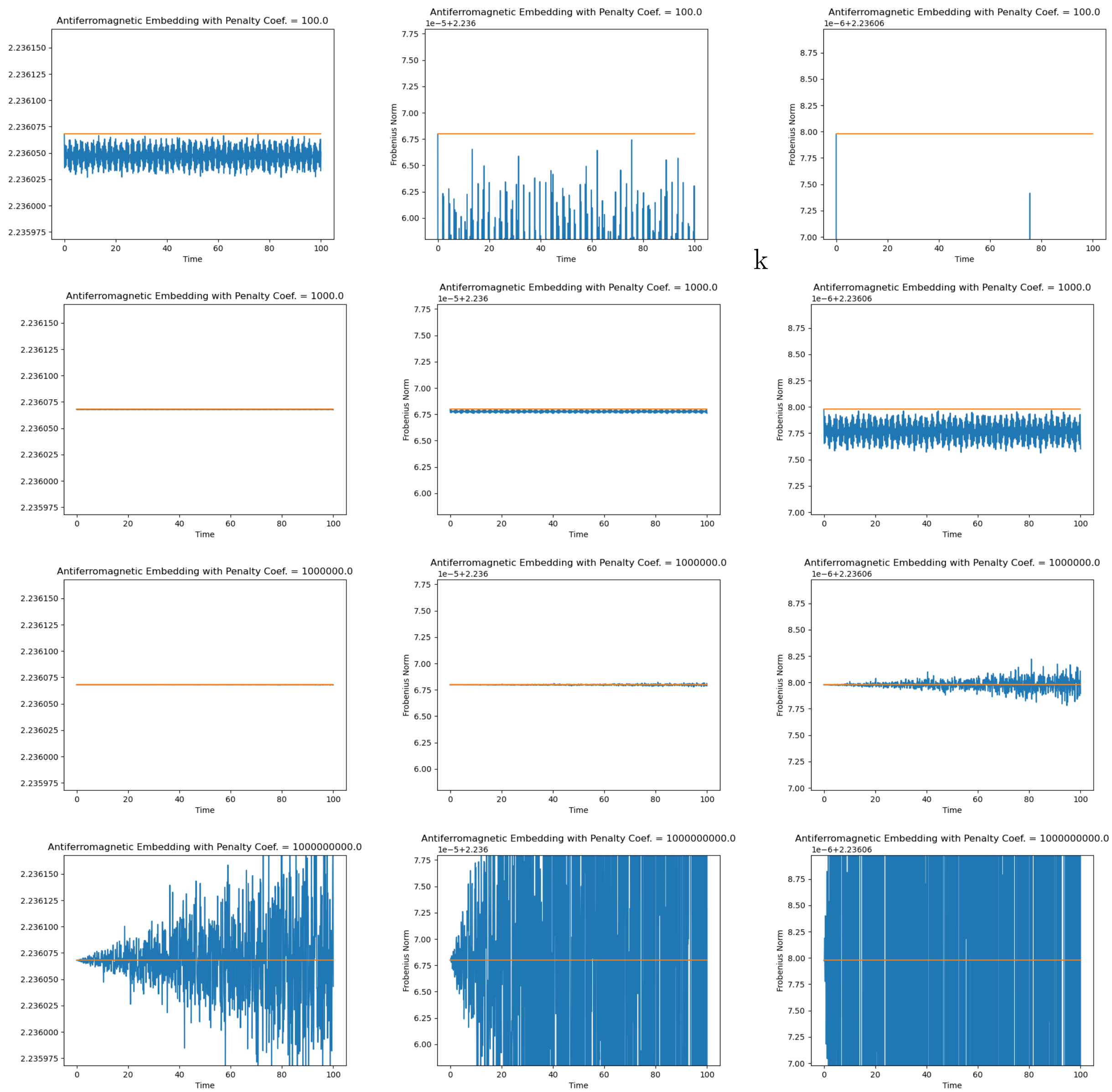

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Figure 2: Antiferromagnetic Embedding

One-hot Embedding

$$Q = \left(-\hat{n}_1 - \hat{n}_5 - 2\sum_{j=2}^{4} \hat{n}_j\right) + \sum_{j=1}^{4} X_{j+1}X_j, \quad H^{pen} = \left(\sum_{j=1}^{5} \hat{n}_j - 1\right)^2$$

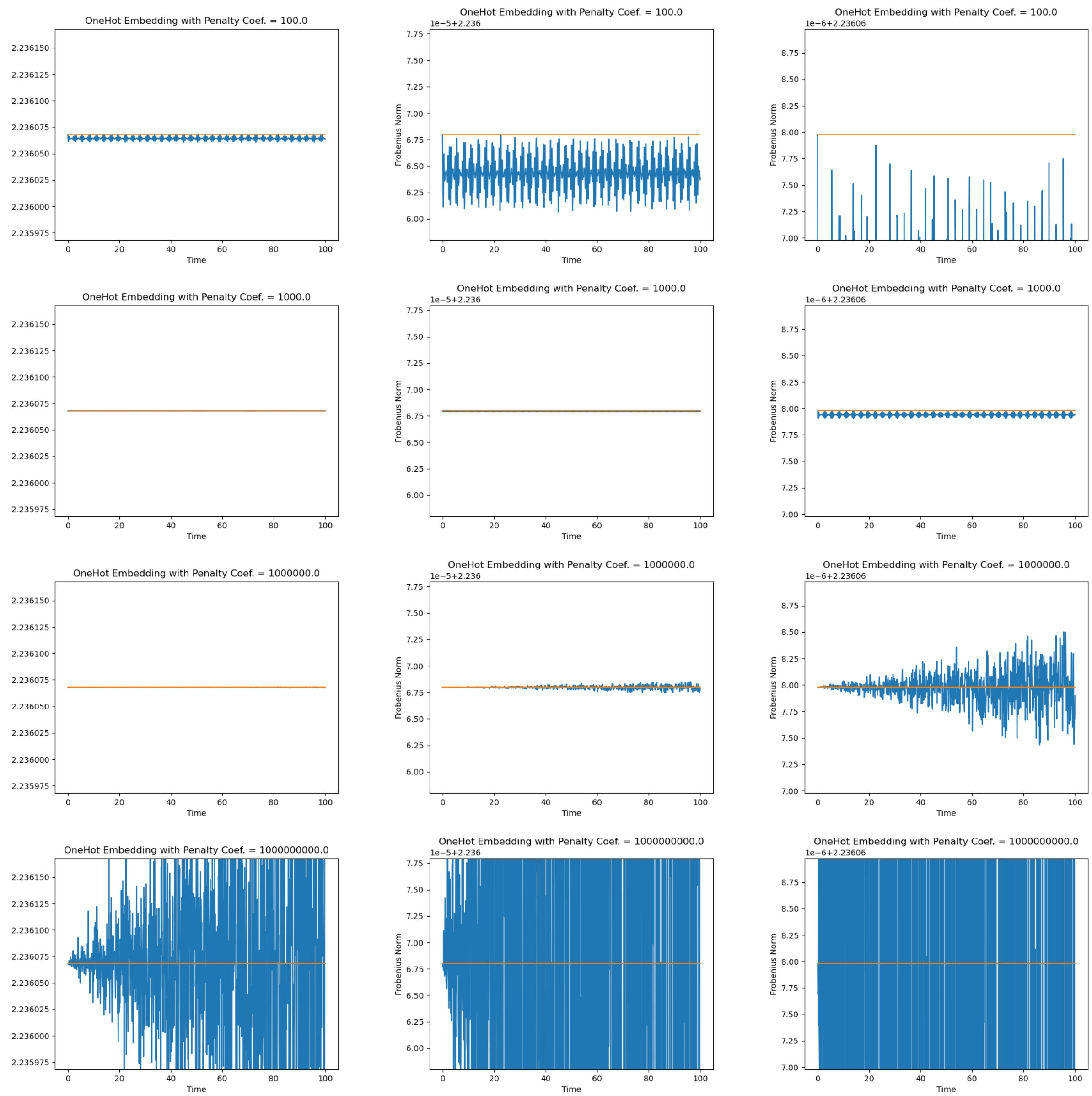

Figure 3: One-Hot Embedding

Penalty-free One-hot Embedding

$$Q = (-\hat{n}_1 - \hat{n}_5 - 2\sum_{j=2}^{4} \hat{n}_j) + \sum_{j=1}^{4}(X_{j+1}X_j + Y_{j+1}Y_j), \quad H^{pen} = 0$$

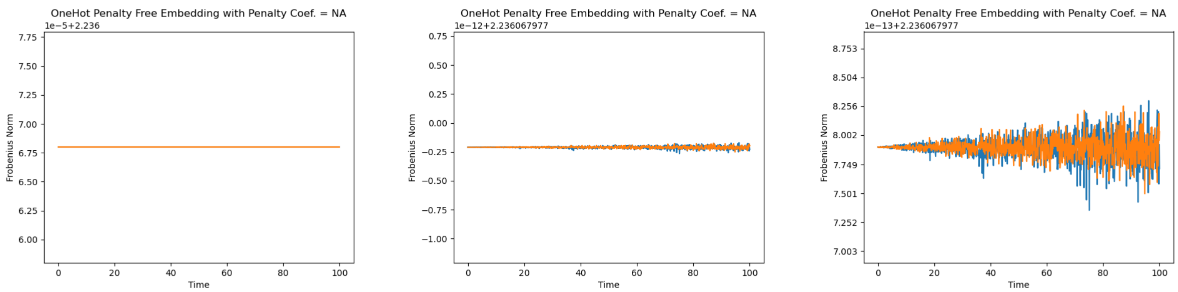

Figure 4: One-Hot Penalty Free Embedding