Report 1

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Exercise 1 (Holm's multiple testing procedure)

By using simulations, we will draw the curves of the average power and the Family Wise Error Rate (for different numbers of non-zero entries in the mean vector and different covariance between each two random variables). We will compare our results from Holm's procedure with Bonferroni's procedure.

Firstly, let us introduce some notations.
- As a null hypothesis we understand: $H_{0,j} : \theta_j = 0 \; \forall j = 1, ..., n$.
- Power is the probability of rejecting the null hypothesis when, in fact, it is false.
- Family Wise Error Rate: FWER $= Pr(card(FP) > 0)$, where card(FP) is a number of null hypotheses correctly not rejected (false positives).

Legend to the plots:
- non_zero: number of non-zero elements in the mean vector,
- cov: covariance of each two variables (variance will be always equal to 1).

I) Power

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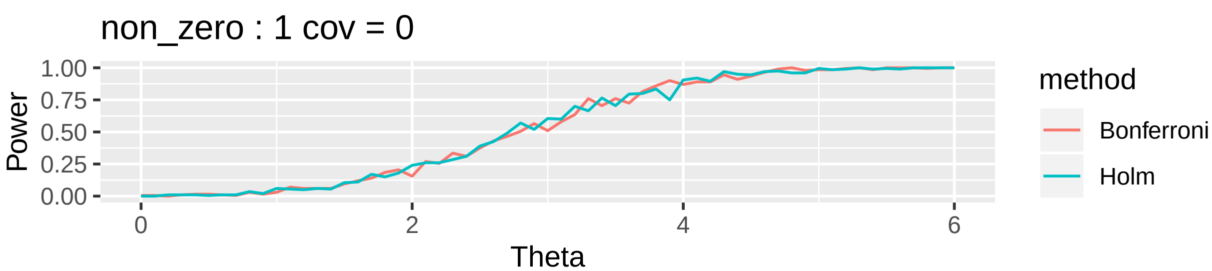
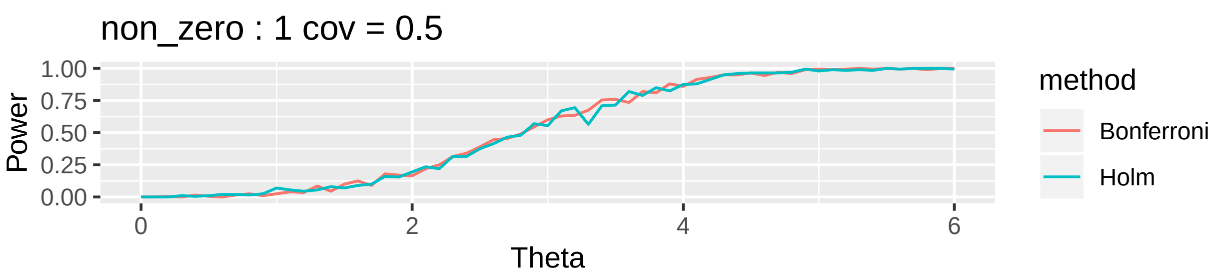
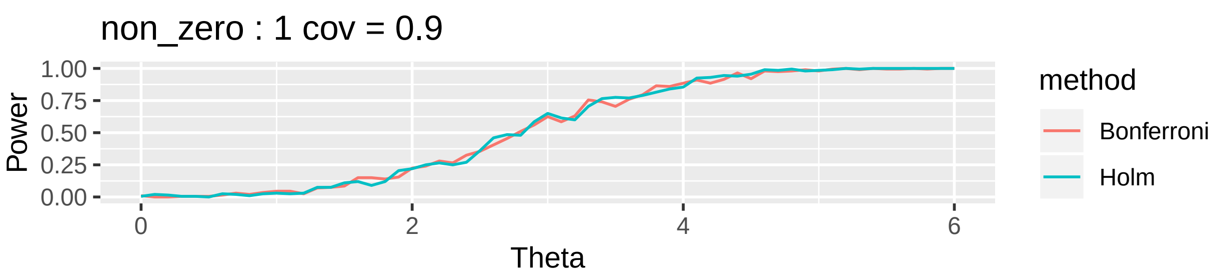
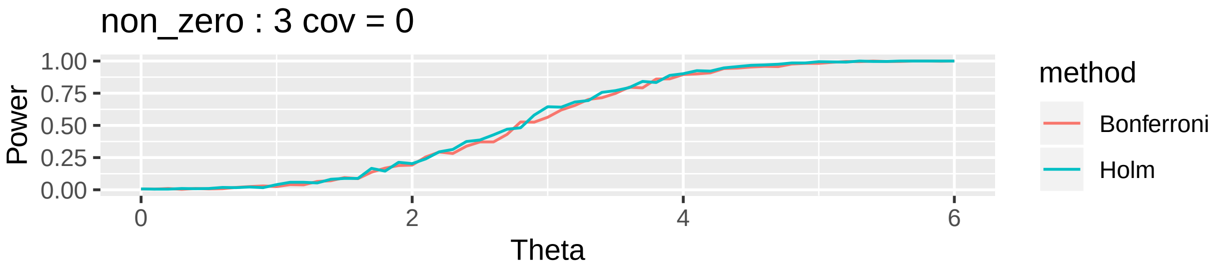
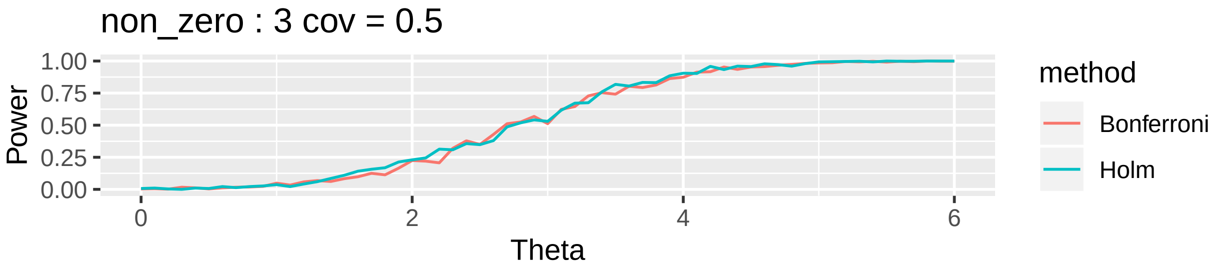


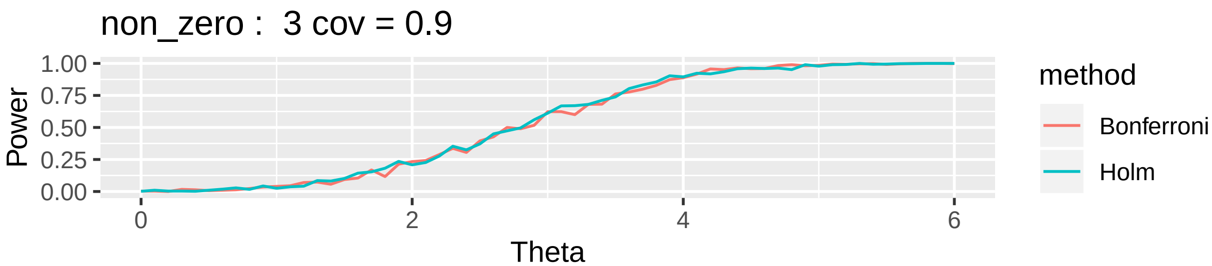




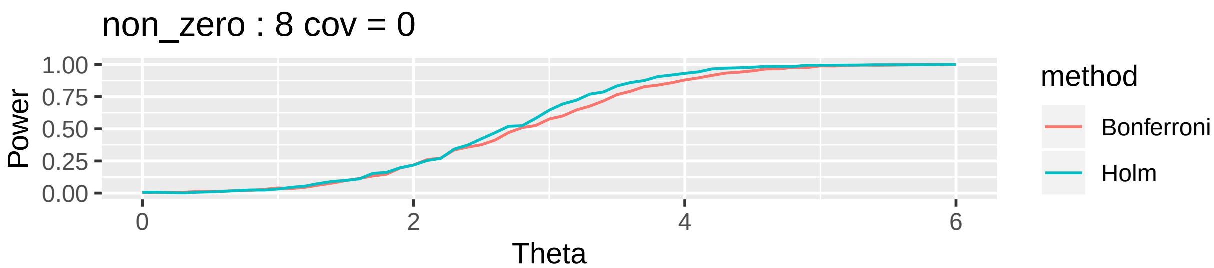
In example with only one non-zero entry of vector theta, there aren't any big differences between the two methods. Both of them work similarly. Also changing the value of the covariance doesn't have an influence on the average power. It increases mostly between theta equal to 2 and 4.

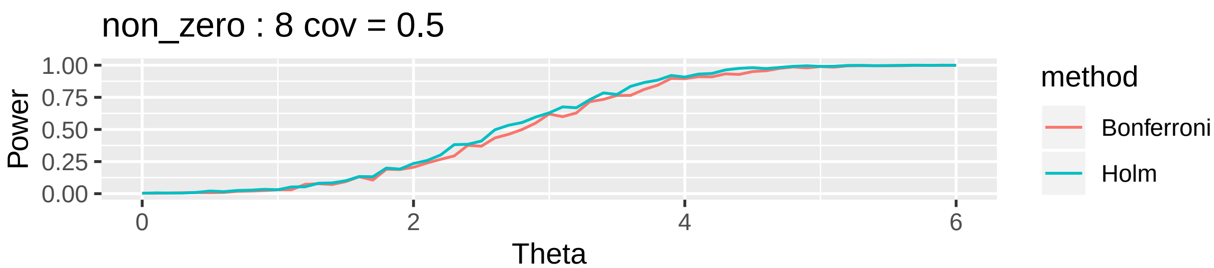


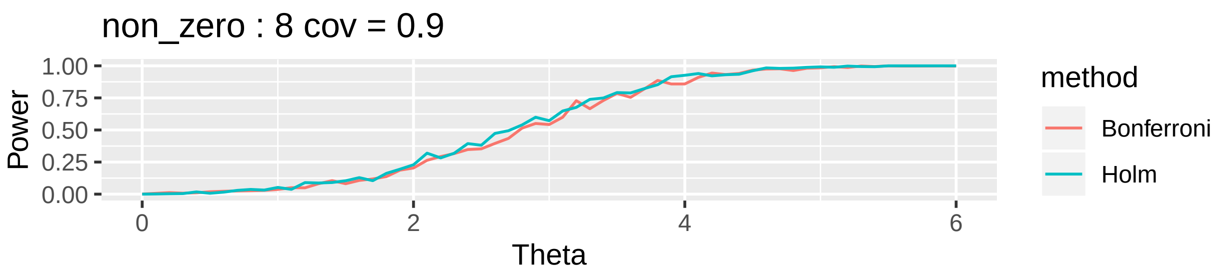




After changing the number of non-zero entries to 3, we don't notice big differences. Still, both procedures work similarly in terms of power (they are maybe covering better each other this time - mainly on the tails). What about influence of covariance? Once again, we don't see any changes after increasing it.





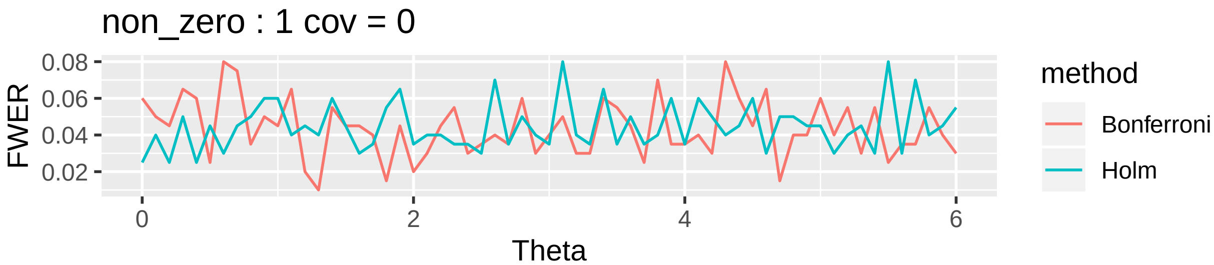


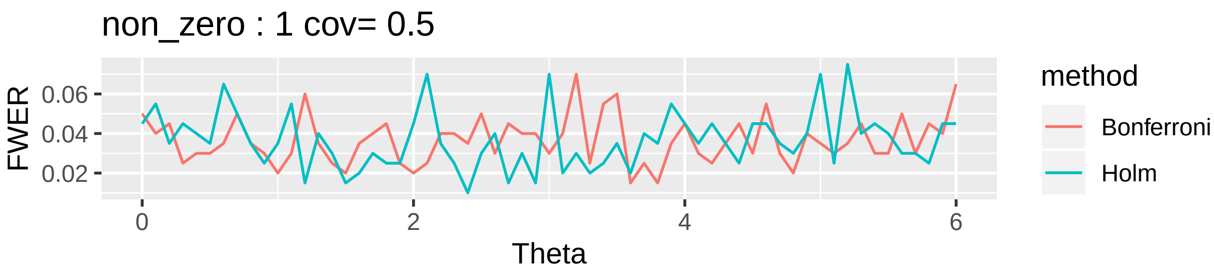
When almost every value is non-zero (8 from 10), we may notice that the curves of power for Holm's procedure are higher. It's changing after increasing the covariance, but still, differences are easier to see than in the previous examples. In this case, the average power of the Holm's procedure is larger than the average power of the Bonferroni's procedure.

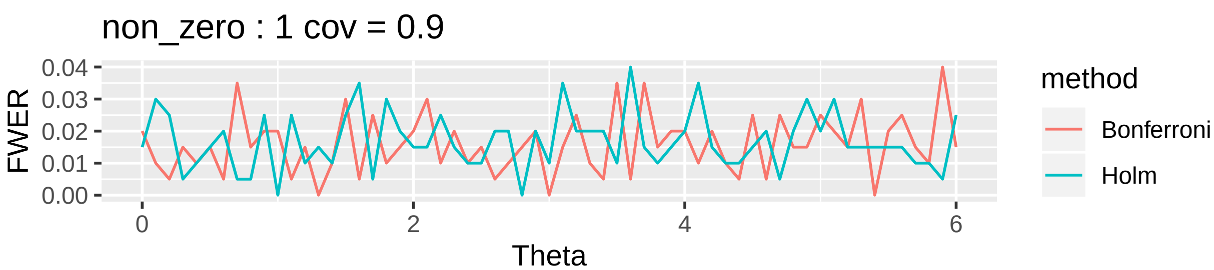
Conclusions

In terms of power there aren't significant differences between the two procedures.

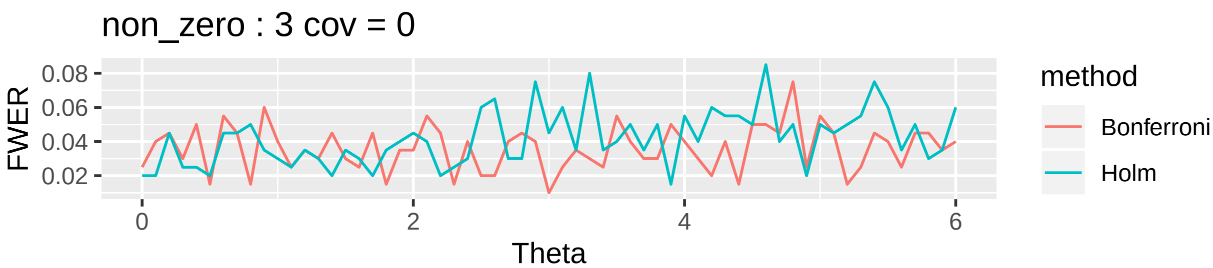
II) FWER

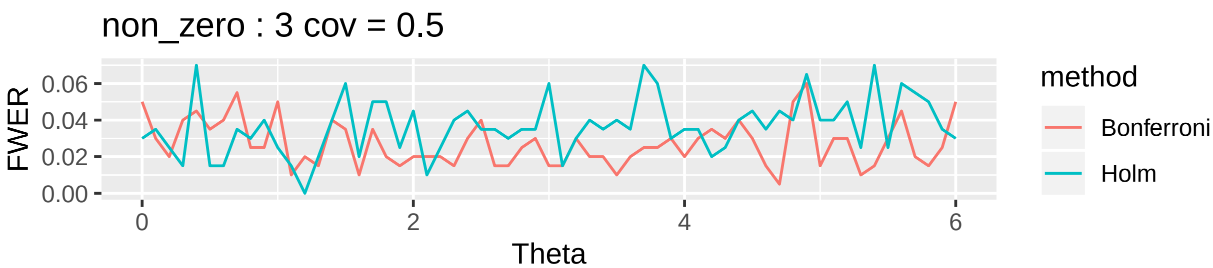


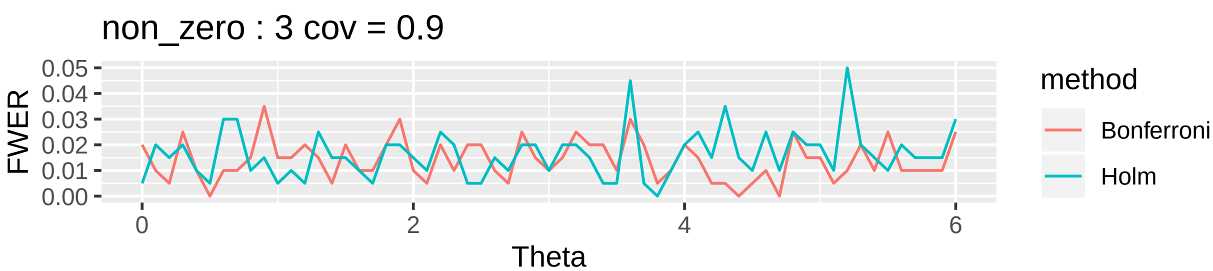




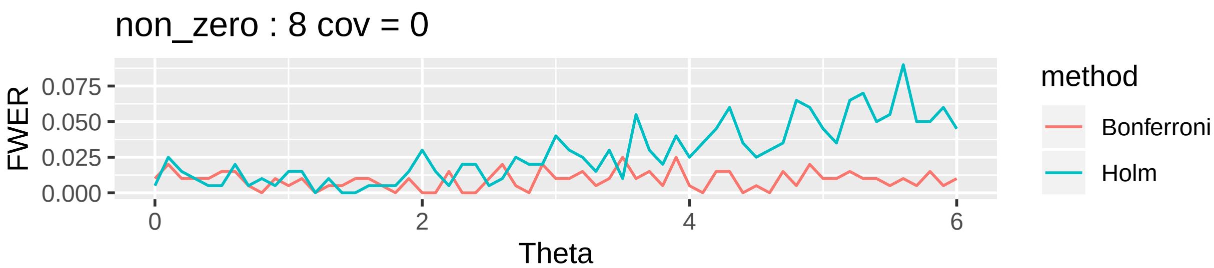
Curves of FWER don't cover that much. But once again, we're able to say that both procedures don't differ really. Curves are more jagged this time. Honestly, it's hard to say which procedure works better. The biggest rate is, when covariance is equal to 0 - when covariance increases, the FWER decreases.

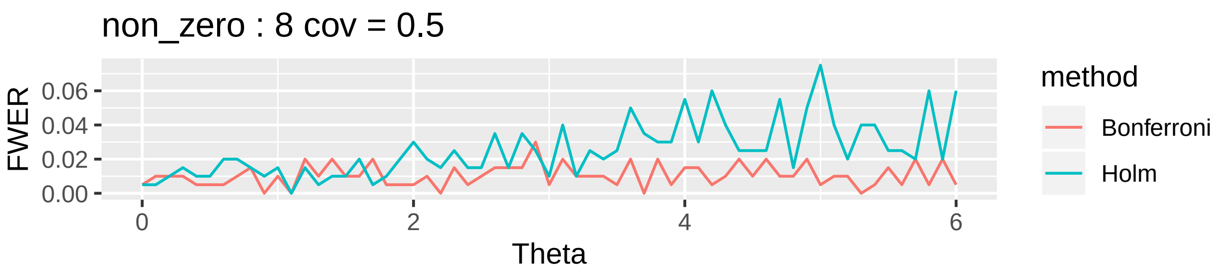


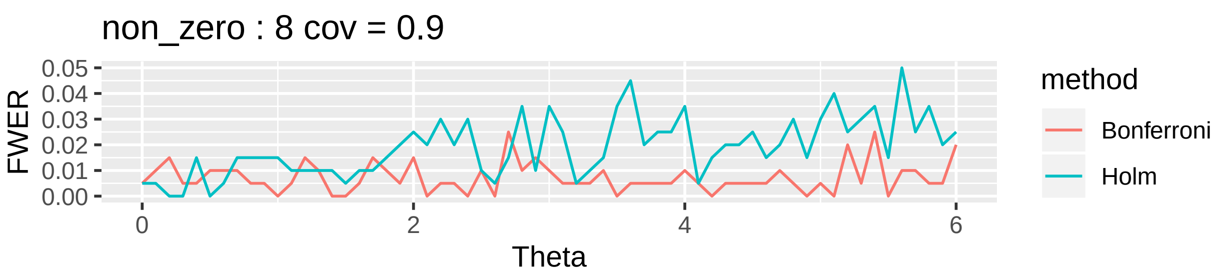




When we increase the number of non-zero entries to 3, the curves are beginning to look a little different (apart from example with covariance equal to 0.9 - they are looking very similar in this case). It looks like Bonferroni behaves better for bigger values of theta, and Holm for smaller one.







When most of entries are non-zero, the curves for the two procedures differ much. It's easy to notice that Bonferroni's procedure works better in this case (for every covariance) - the FWER equals almost 0. The FWER for Holm's procedure increases much for bigger entries of theta.

Conclusions

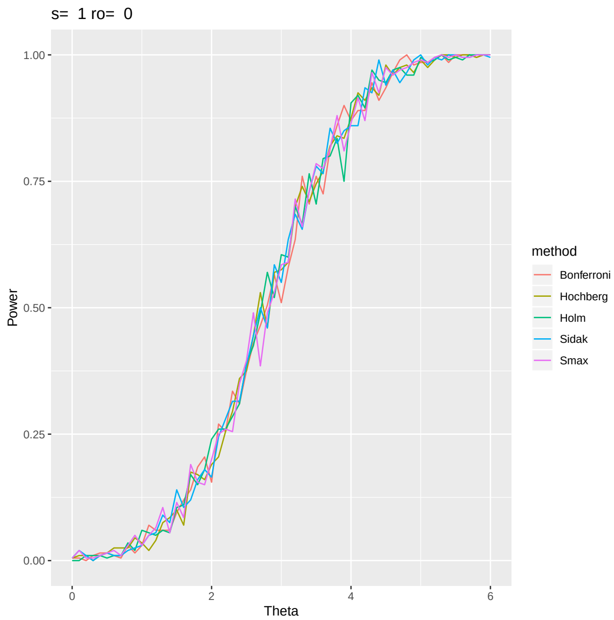
The biggest differences between the two procedures (in terms of FWER) can be noticed when most of entries of the mean vector are non-zero. Bonferroni's procedure behaves better then - the FWER is much smaller.

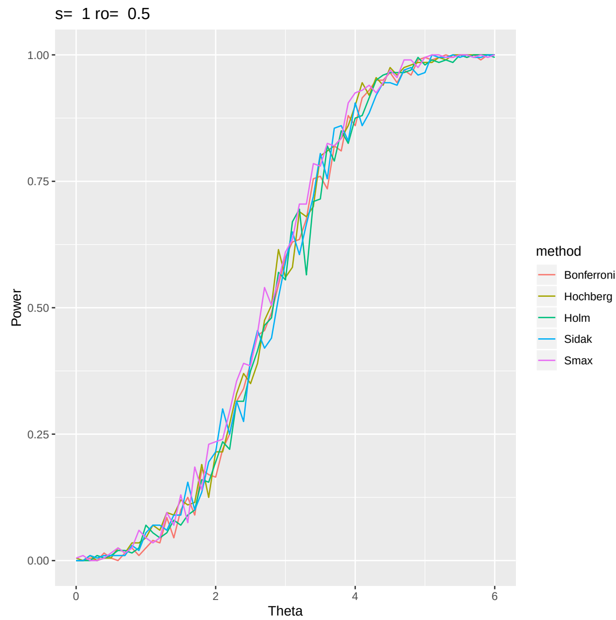
Exercise 2 (Comparison of procedures)

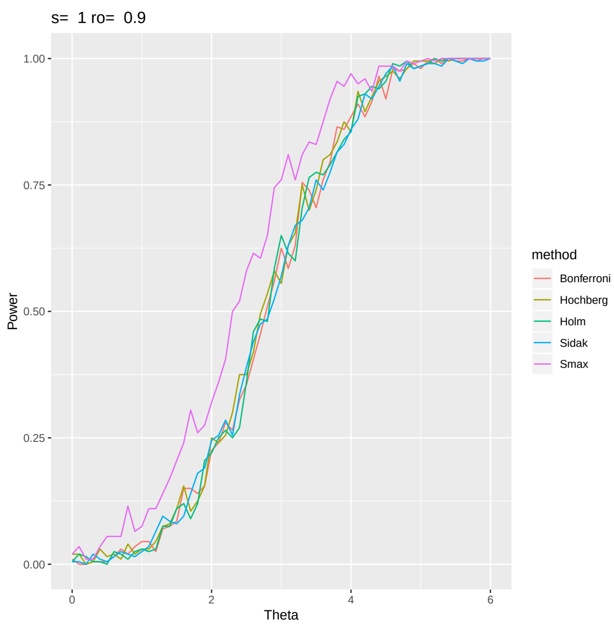
In the second task, we will compare 5 procedures: Bonferroni, Hochberg, Holm, Sidak and Smax. Similarly to the first exercise, we will draw the curves of power and Family Wise Error Rate.

Number of non-zero entries: 1

I) Power

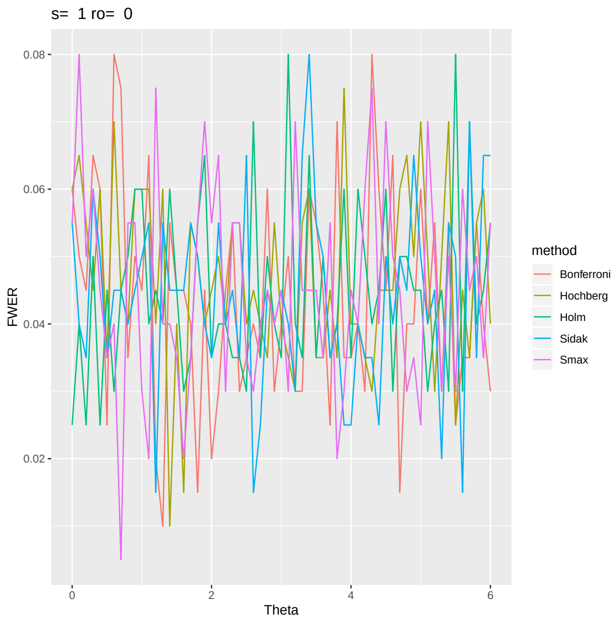


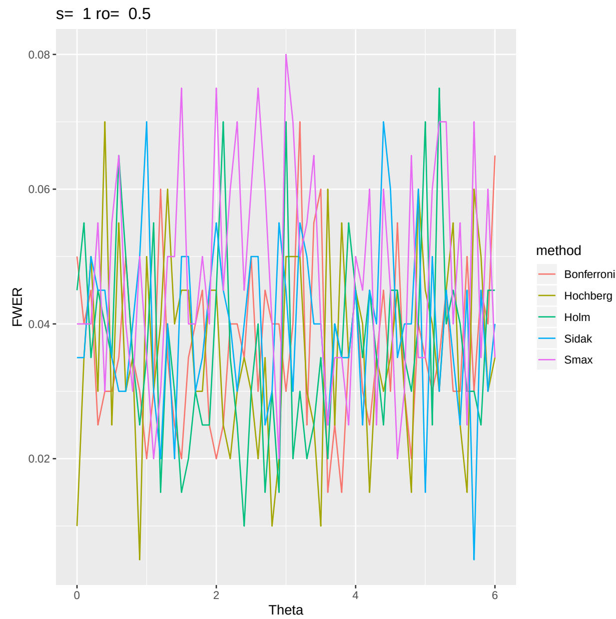


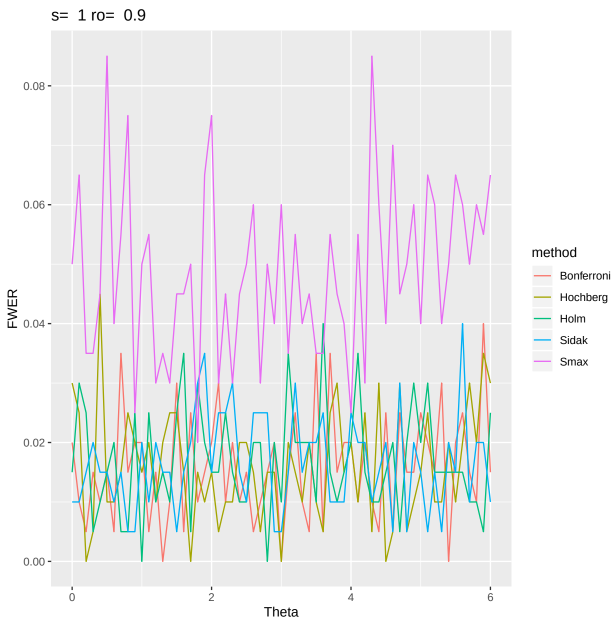


The only significant difference can be noticed when covariance is equal to 0.9. Then the highest power has the Smax procedure - it's clearly visible.

II) FWER



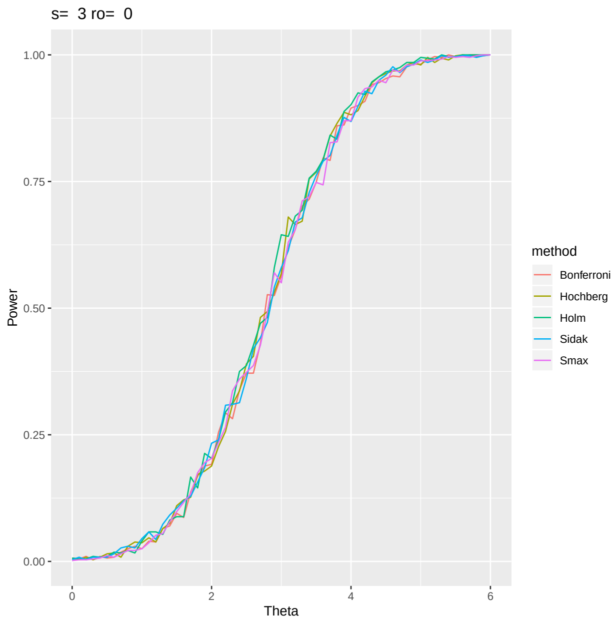


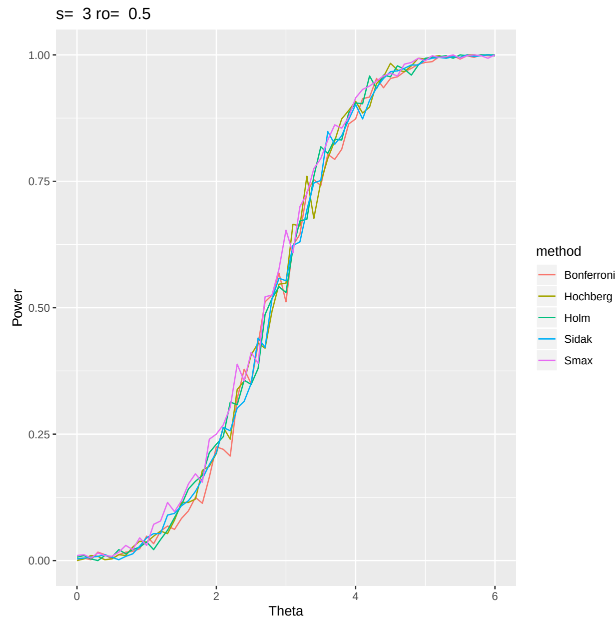


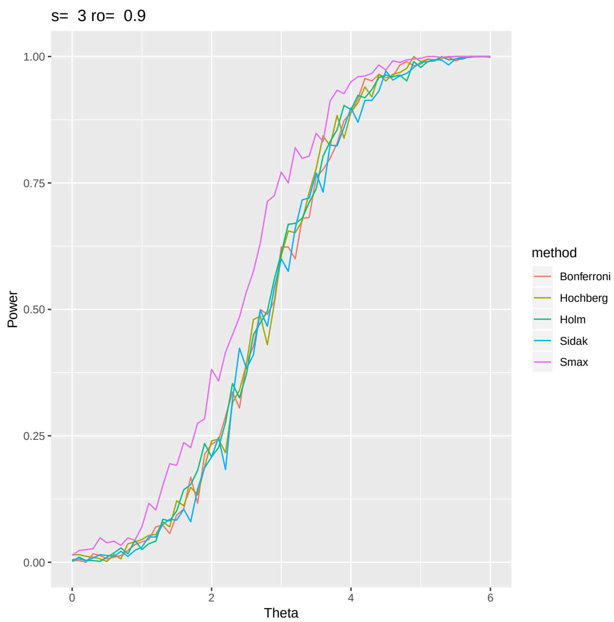
It isn't hard to see that each time the highest values of FWER are for Smax procedure. Especially when covariance is equal to 0.9 - the curve stays on the same level when the others move down. That's because the power for this procedure is also high.

Number of non-zero entries: 3

I) Power

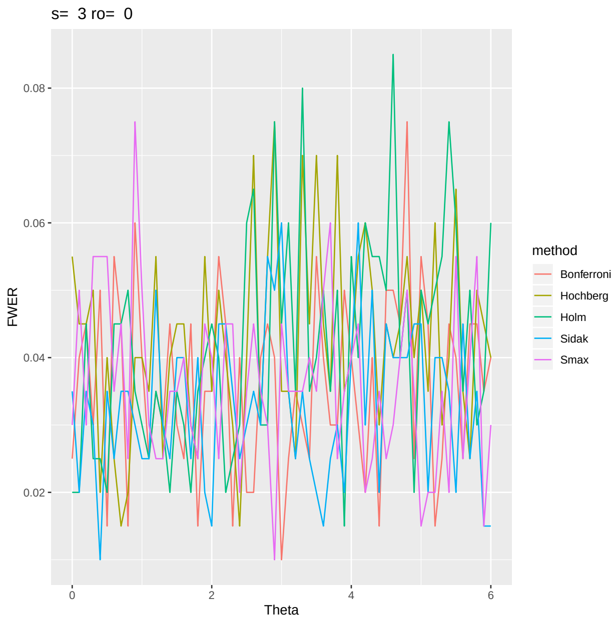


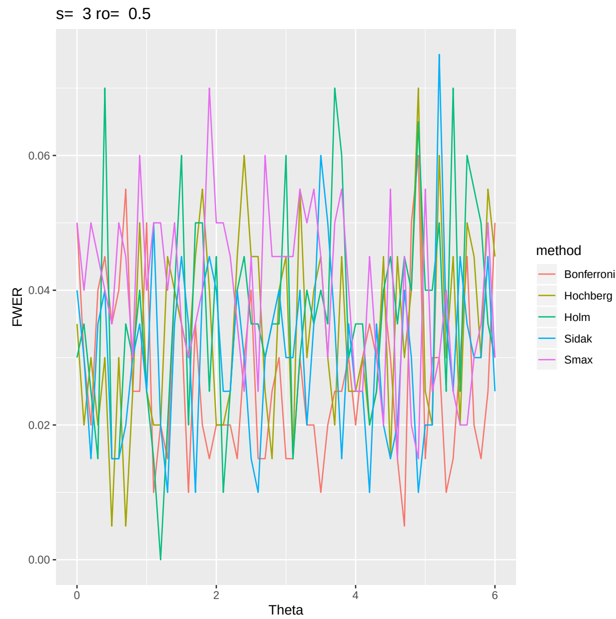


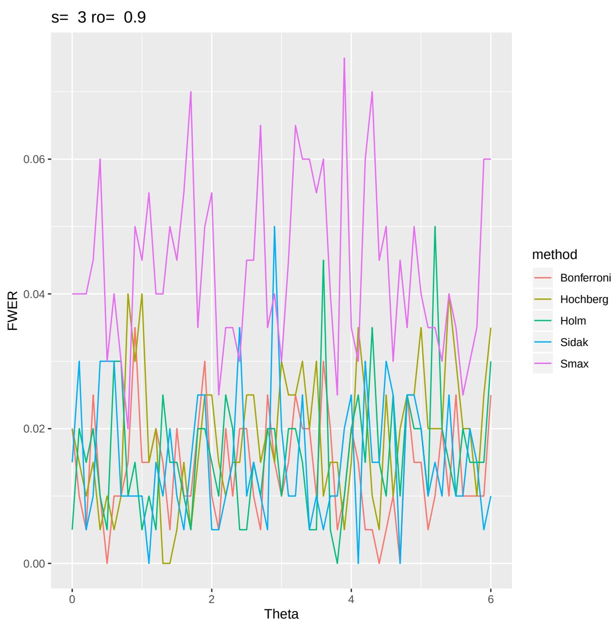


When we increase the number of non-zero entries, the power for covariance equal to 0 is almost the same for every procedure. When we increase covariance, the curves are moving away a little. And once again, when covariance is equal to 0.9, the highest power has the Smax procedure.

II) FWER



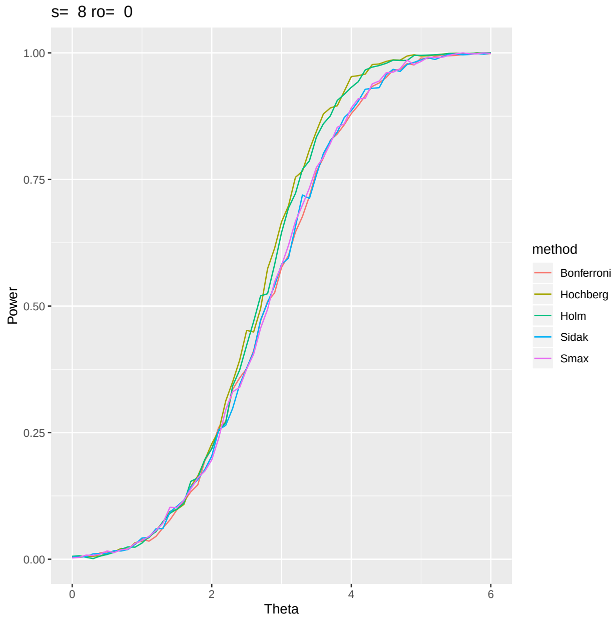


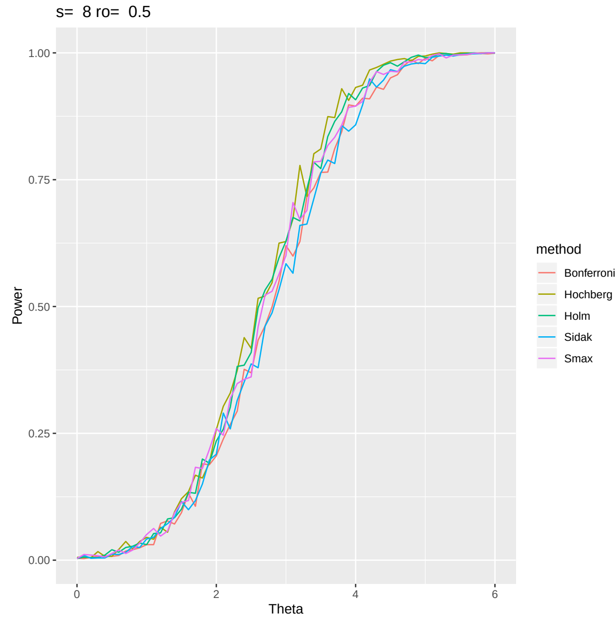


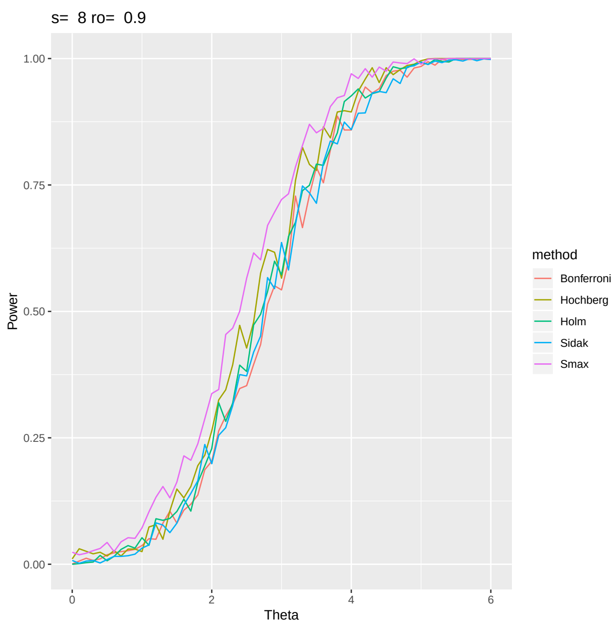
This time, for covariance equal to 0 and 0.5 it's not clear which procedure has the highest FWER. The results repeat when covariance is equal to 0.9 - the Smax procedure clearly has the highest FWER.

Number of non-zero entries: 8

I) Power

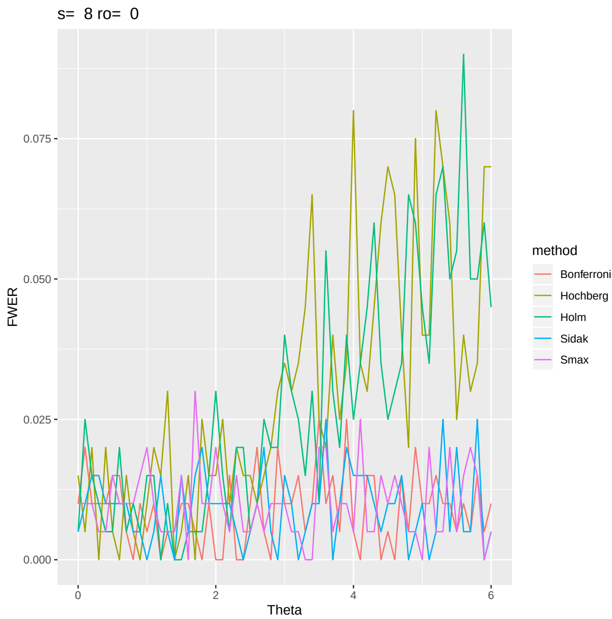


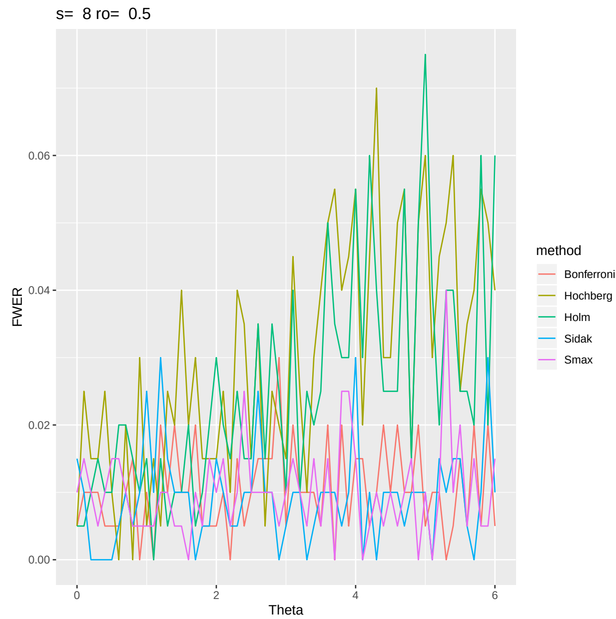


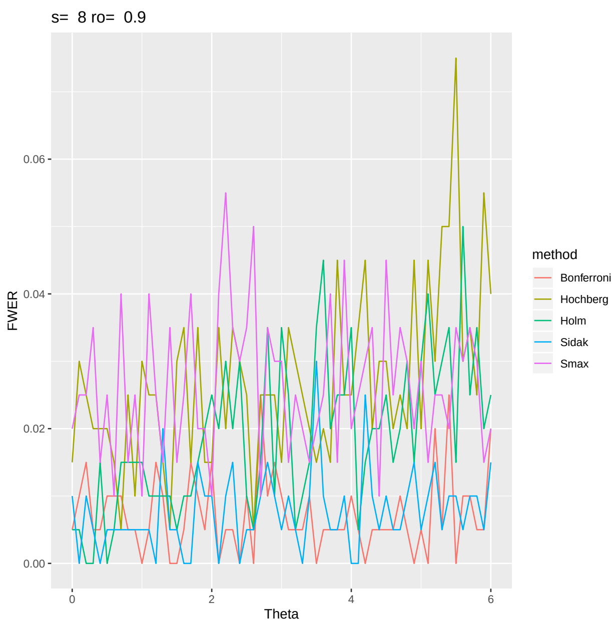


In the last case, the curves differ the most. When covariance is equal to 0, Hochberg and Holm work the best - they give us the biggest power. To my mind, when covariance is equal to 0.5, the highest average power has Hochberg's procedure and the lowest Sidak's (but it's not that clear). Finally, when covariance is 0.9, once again wins Smax, but this time the difference is smaller than earlier.

II) FWER







When most of the entries are non-zero the FWER curves have the most differences. When covariance is zero, the highest FWER has Hochberg and Holm (as we could imagine from the curves for power). Situation is the same when covariance increases. And when covariance is equal to 0.9, the most visible procedure is Smax (the FWER is the highest). We can also notice that Sidak and Bonferroni reach the lowest values then. When values of non-zero theta are high, the FWER for Hochberg procedure is also high and the FWER for Smax procedure decrease then - that's interesting.