

principles

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maths prof in 1952 - 1970

... member of the US National Academy of Sciences, 1981

$f(x) \Rightarrow f: \mathbb{R} \rightarrow \mathbb{R}$

$f: \mathbb{R}^n \rightarrow \mathbb{R}$

integrals, numerical methods -
kings, king, king, king, king
king, king, king, king, king

integration: numerical integration -
numerical integration, numerical integration, numerical integration
(numerical integration, numerical integration, numerical integration)

الله يعلم

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

23) If γ place $\rho'_{\text{cond}} - \gamma^2 / \rho$ $IR^* \subseteq R$

$0 \in \mathbb{R}^*$, $x^* \in \mathbb{R}^*$ \mapsto $\exists i \in \{1, 2, \dots, n\}$ $x_i = x^*$

$$\therefore 0^\alpha = 0$$

$$\frac{|x - x^*|}{|x|}$$

$f(x)$ \approx $2e^{1.5} \approx 3.17$ $e^{1.5}$

うれしいこと
× と どうして そこ が

? なぜかしらべる

$$f(x^*) = f(x'), \quad f(x), \quad x, x^*$$

$$\frac{|x' - x|}{|x'|} |x^* - x| \sim y,$$

$$\left| \frac{f(x^*) - f(x)}{f(x)} \right| = \left| \frac{f(x') - f(x)}{f(x)} \right| =$$

$$\left| \frac{(f(x') - f(x))(x^* - x)}{(x' - x) \cdot f(x)} \right| \leq \frac{|x^* - x|}{|x|}.$$

$$\left| \frac{(f(x') - f(x)) \cdot x}{(x' - x) \cdot f(x)} \right| \approx \left| \frac{x f'(x)}{f(x)} \right| \cdot \frac{|x^* - x|}{|x|}$$

$f \in$ 23rd condition interval

κ (condition number) $x \rightarrow$

$$\text{cond}(f)(x) = \left| \frac{x \cdot f'(x)}{f(x)} \right|$$

$(X, f(x) \neq 0 \text{ and } f'(x) \neq 0)$

$$, f(x) = ax + b \quad \text{for } \text{cond}(f)(x) = \left| \frac{x \cdot a}{ax + b} \right| = \left| 1 - \frac{b}{ax + b} \right|$$

" for 21st problem solution

$$I_n = \int_0^1 \frac{t^6}{t+5} dt \quad \text{from } u/v \text{ rule}$$

$$I_0 = \int_0^1 \frac{dt}{t+5} = \left. \ln(t+5) \right|_0^1 = \ln\left(\frac{6}{5}\right)$$

$$I_{n+1} = \int_0^1 \frac{t^{n+1}}{t+s} dt = \int_0^1 t^n \cdot \frac{t+s-s}{t+s} dt =$$

$$-5 \int_0^1 \frac{t^n}{t+s} dt + \left. \frac{t^{n+1}}{n+1} \right|_0^1 = -5 I_n + \frac{1}{n+1}$$

$\sum_{n=0}^{\infty}$

$$I_n = f_n(I_0) \quad \underline{f_n(x) = (-5)^n x + b_n}$$

$b_n \in \mathbb{R}$ ($\rightarrow \mathbb{C}^1$, vgl.)

$$(\text{and } f_n)(I_0) = \left| \frac{\int_0^1 f_n(t) dt}{I_0} \right| =$$

$$s^n \cdot \left| \frac{I_0}{I_n} \right| \geq s^n$$

$$I_n = \underbrace{I_{n+1} - \frac{1}{n+1}}_{= 5}$$

$$k \gg n$$

$$I_n \approx g_n(I_k) \quad n-k < 0$$

$$g_n^{(k)} = (-5)^{\overbrace{k}^{n-k}} x + c_n$$

$$\text{cond}(g_n)(I_k) = \left| \frac{I_k \cdot (-5)^{n-k}}{\sum_n} \right| =$$

$$5^{n-k} \left(\frac{I_k}{\sum_n} \right) \leq 5^{n-k}$$

a_1

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$$(x_1^*, \dots, x_n^*) \hookrightarrow (x_1, \dots, x_n)$$

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\sim $\| \cdot \| : V \rightarrow \mathbb{R}_{\geq 0}$ $= \{ x \in \mathbb{R} / x \geq 0 \}$

$V = 0 \quad \|V\| = 0 \quad \|V\| = 0 \quad .1$
 $\|av\| = |a| \cdot \|v\| \quad \forall v \in V, a \in \mathbb{R} \quad .2$
 $\forall v \in V \quad .3$

$$\|u+v\| \leq \|u\| + \|v\|$$

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$\mathbb{R} \ni p \geq 1$, $V = \mathbb{R}^d$ $\subset \mathbb{C}^{n \times 1}$

$$\|\langle x_1, \dots, x_d \rangle\|_p = \sqrt[p]{\sum |x_i|^p}$$

/ euclidean norm)

$$\|\langle x_1, \dots, x_d \rangle\|_1 = \sum |x_i|$$

$$\|\langle x_1, \dots, x_d \rangle\|_2 = \sqrt{\sum x_i^2}$$

" $p = \infty$ "

$$\|\langle x_1, \dots, x_d \rangle\|_\infty = \sup \{|x_i|\}$$

, for $\|\cdot\| = \sqrt{\sum x_i^2}$ euclidean norm
 $\vdash \sqrt{\sum x_i^2} \geq \sum |x_i|$

$$d(u, v) = \|u - v\|$$

now (v_i) , $i \sim 30$ $v_i \in V$ is

$$\cdot \|v_i - v\| \rightarrow 0 \quad \forall \epsilon \quad \forall \delta$$

设 γ/Γ_β 为 γ 的 β 次方根

$\|\cdot\|_2$, $\|\cdot\|_1$, $\|\cdot\|_\infty$ 为

某种范数的 β 次方根

$v \in V_{\beta-1} (V_i)$ 为 V_i 中的

$\|v_i - v\|_2 \rightarrow 0$ 则 $\|v_i - v\|_1 \rightarrow 0$

由 $\gamma > 0$ 及 $\gamma^2 < 1$

$\frac{1}{c} \|v\|_1 \leq \|v\|_2 \leq c \|v\|_1$, $\forall v \in V$

\mathbb{R}^d 为 \mathbb{R}^d 上的 β 次方根

由 $\gamma > 1$

\mathbb{R}^d 为 \mathbb{R}^d 上的 β 次方根

$$\frac{\|x^* - x\|}{\|x\|}$$

רעיון $T: V \rightarrow V$ ו- x'

$$\| \cdot \|_V \text{ גודלה } \cup \text{ אוסף } \sim, \omega'$$

$$V \text{ סט } \| \cdot \|_V \text{ -י } V \text{ סט}$$

השאלה היא אם $\|x^*\|_V \leq \|x\|_V$

: רעיון גודלה T כפונקציית

$$\frac{\|Tx^* - Tx\|_V}{\|Tx\|_V} = \frac{\|T(x^* - x)\|_V}{\|T(x)\|_V} =$$

$$\frac{\|T(x^* - x)\|_V}{\|T(x)\|_V} \cdot \frac{\|x^* - x\|_V}{\|x\|_V} \leq \frac{\|T\| \cdot \|x\|_V}{\|T(x)\|_V} \cdot \frac{\|x^* - x\|_V}{\|x\|_V}$$

Recursive algorithm $T: U \rightarrow V$

רְאֵבָבָה, וְנִזְנַתְּנָהָה, וְגַדְעָה, וְכַלְמָה, וְבַשְׂרָבָה.

$$\|T\| = \sup_{x \neq 0} \frac{\|T(x)\|_v}{\|x\|_v} = \sup_{\|x\|_v=1} \|T(x)\|_v$$

$$\text{N}^{\text{H}}\text{O}\text{S} \text{S} \text{N} \text{P} \text{O} \text{S} \text{N} \rightarrow \text{Hom}(\underline{\text{U}}, \text{V})$$

గనిపు ట్రాక్చర్ ను వివరించాలి.

גָּדוֹלָה מְאֻמָּרָה

ת ס 23 נג רוגר ירל

لـ (نـ) x الـ (جـ)

$$\text{cond}(\tau)(x) = \frac{\|\tau\| \cdot \|x\|}{\|\tau(x)\|}$$

$x = T^{-1}(y)$ เมื่อ y เป็นตัว x ของ T

$\text{cond}(\tau) :=$

$$\sup_x \text{cond}(\tau)(x) = \|\tau\| \cdot \|\tau^{-1}\| \quad \text{sic!}$$

zu $\|\tau\|$ kann man nur $\|\tau x\|$

und $\|\tau^{-1}x\|$ für $\tau x = b$

b ist die ursprüngliche Vektoren

? b^* ?

: $(\tau \circ \delta, \tau \circ \gamma)$

$$\tau_n = \begin{pmatrix} 1 & \frac{1}{2} & \dots & \frac{1}{n} \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix}$$

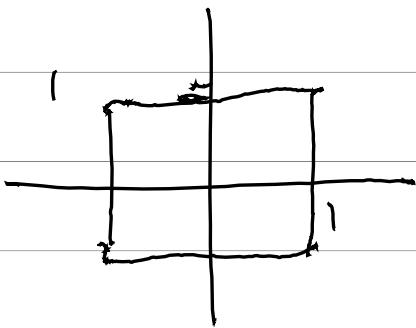
$$\text{Cond}_2 \tau_n = \frac{(V_2 + 1)^{n+4}}{\sum_{k=1}^{n+4} k \cdot \sqrt{\tau_n}}$$

Linear transformation $T: U \rightarrow V$

$$U = \mathbb{R}^n, \quad V = \mathbb{R}^m \quad \| \cdot \| = \| \cdot \|_\infty$$

'?' for and \mathbb{R}^n to \mathbb{R}^m

$$\begin{matrix} (a_{ij})_{1 \leq i \leq n} \\ 1 \leq j \leq m \end{matrix} \Rightarrow \text{3 rows}$$



$$\| T \| = \max_j \sum_{i=1}^n |a_{ij}|$$



$$X = 17$$

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$\text{cond}(f)(x) = \frac{|x| / |f'(x)|}{|f(x)|}$$

$$y = -17 + 8$$

$$2 \cdot 17 = 34$$

$$T: U \rightarrow V \quad . \quad V, W, V$$

$$\|T\| = \sup_{\|u\|=1} \|Tu\|$$

$$\text{cond}(T)(u) = \frac{\|u\| \cdot \|T\|}{\|Tu\|} \leq \|T^{-1}\| \cdot \|T\|$$

$\text{cond}(T)$

$$[1 \cdot 1]$$

$$f: \mathbb{R}^2 \rightarrow \mathbb{R}$$

Übung 3

$$f(x, y) = x + y$$

$$\text{cond}(f)(x, y) = \frac{\|\langle x, y \rangle\| \cdot \|f\|}{|x+y|} = \frac{\max(|x|, |y|) \cdot 2}{|x+y|} \quad \|f\| = \|f\|_\infty$$

$$f: \mathbb{R}^n \rightarrow \mathbb{R}^m . 2$$

$$x^* \in \mathbb{R}^{n^*}, x \in \mathbb{R}^n$$

$$\frac{\|f(x^*) - f(x)\|}{\|f(x)\|} =$$

$$\frac{\|f(x^*) - f(x)\| \cdot \|x\| \cdot \|x^* - x\|}{\|f(x)\| \cdot \|x\| \|x^* - x\|} \approx \varepsilon$$

$$\frac{\|\underline{df(x)}(x^* - x)\| \cdot \|x\| \cdot \varepsilon}{\|f(x)\| \cdot \|x^* - x\|} \leq \frac{\|\underline{df(x)}\| \cdot \|x\| \cdot \varepsilon}{\|f(x)\|}$$

$$\text{cond}(f)(x) = \frac{\|\underline{df(x)}\| \cdot \|x\|}{\|f(x)\|}$$

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$f: \mathbb{R}^n \rightarrow \mathbb{R}^m$ \hookrightarrow $\mathbb{R}^n \times \mathbb{R}^m$

$f_i: \mathbb{R}^n \rightarrow \mathbb{R}$ \hookrightarrow \mathbb{R}^m

For $x \in \mathbb{R}^n$ $\exists i \in \{1, \dots, m\}$ such that

$x_j = 0 \forall j \neq i$ $\Rightarrow f_i(x) = f_i(x_i)$

$(\text{cond}_{x_j}(f_i))_{j \in \mathbb{N}}$

$f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ Ex 3

$$f(x, y) = \left(\underbrace{\frac{1}{x} + \frac{1}{y}}, \underbrace{\frac{1}{x} - \frac{1}{y}} \right)$$

$$df = \begin{pmatrix} -\frac{1}{x^2} & -\frac{1}{y^2} \\ -\frac{1}{x^2} & \frac{1}{y^2} \end{pmatrix}$$

$$\text{cond}(f)(x, y) = \max(|x|, |y|) \cdot \max\left(\frac{1}{x^2}, \frac{1}{y^2}\right)$$

$\max\left(\left|\frac{1}{x} + \frac{1}{y}\right|, \left|\frac{1}{x} - \frac{1}{y}\right|\right)$

$$f_1(x, y) = \frac{1}{x} + \frac{1}{y} \quad f_2(x, y) = \frac{1}{x} - \frac{1}{y}$$

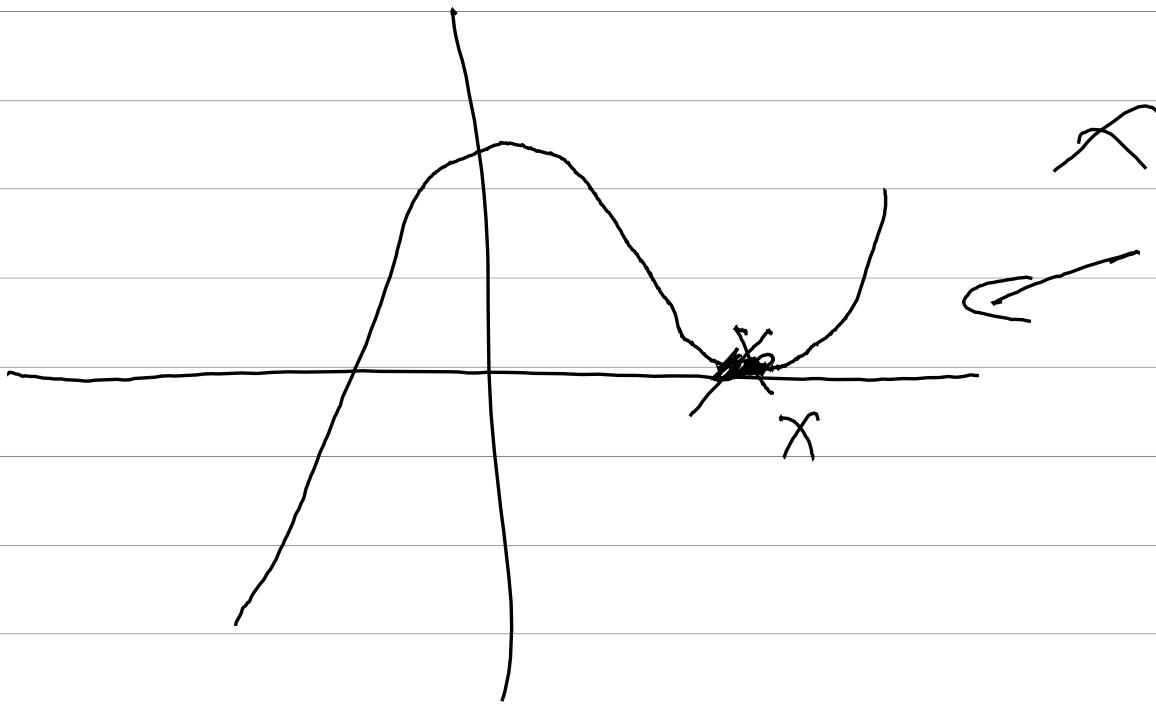
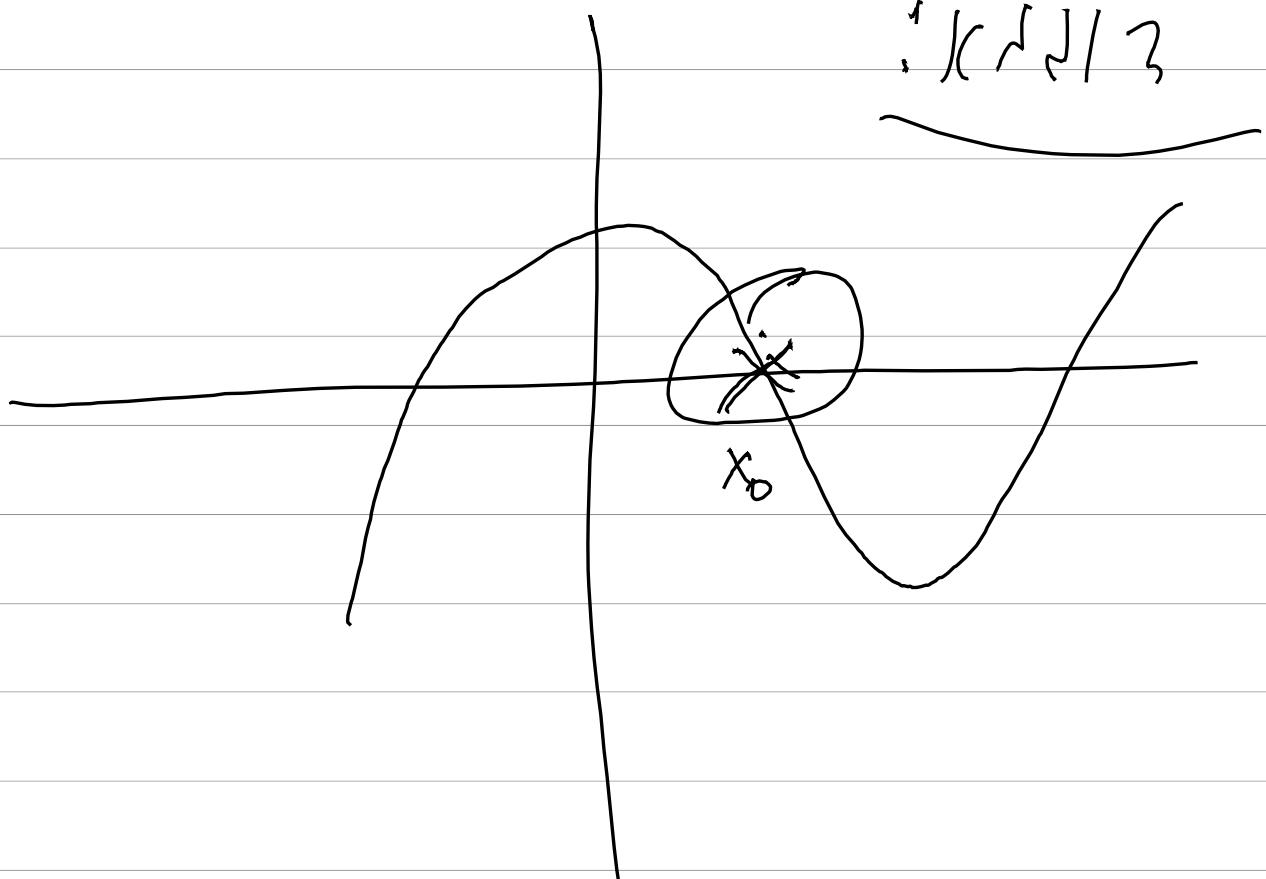
$$\text{cond}_x(f) = \frac{|x| \cdot \frac{1}{x^2}}{\left|\frac{1}{x} + \frac{1}{y}\right|} \quad \frac{|y| \cdot \frac{1}{y^2}}{\left|\frac{1}{x} + \frac{1}{y}\right|}$$

||

$$\frac{|y|}{|x+y|} \quad \frac{|x|}{|x+y|}$$

$$\text{cond}_x(f_2) = \frac{|x| \cdot \frac{1}{x^2}}{\left|\frac{1}{x} - \frac{1}{y}\right|} = \frac{|y|}{|x-y|} \quad \left|\frac{1}{x} - \frac{1}{y}\right| = \frac{|xy|}{|x-y|}$$

$\therefore K \cap J / 3$

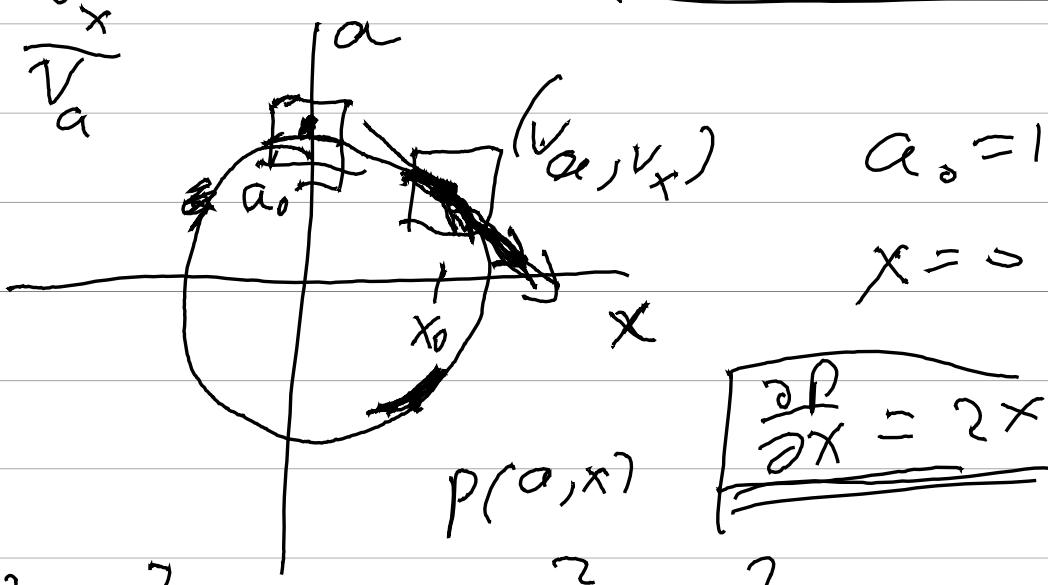


$h \rightarrow \infty ?$ $n' r > 1/p$

$$P_n(\bar{a}, X) = X^n + \sum_{i=0}^{n-1} a_i \cdot X^i$$

$$P_n(\bar{a}_0, \underline{x}_0) = 0 \quad x_0, \bar{a}^0$$

$$\approx F(x, y) = x^2 + y^2$$



$$a^2 + x^2 = 1$$

$$\underbrace{a^2 + x^2 - 1 = 0}_{\text{---}}$$

$$x = x(a)$$

$$x_0 = x(a_0)$$

$$x = \sqrt{1 - a^2}$$

$$P(a, x(a)) = 0$$

$$F(a, x) = 0$$



$$\underline{F(a_0, x_0) = 0}$$

$$\frac{\partial F}{\partial x}(a_0, x_0) \neq 0 \Rightarrow x = X(a)$$



$$\frac{\partial x}{\partial a} = \frac{\partial F}{\partial a} / \frac{\partial F}{\partial x}$$



$$df \cdot \begin{pmatrix} v_a \\ v_x \end{pmatrix} = 0$$

$$\begin{pmatrix} \frac{\partial f}{\partial a} & \frac{\partial f}{\partial x} \end{pmatrix} \begin{pmatrix} v_a \\ v_x \end{pmatrix} = 0$$

$$v_a \frac{\partial f}{\partial a} + v_x \frac{\partial f}{\partial x} = 0$$

$$X = X(\vec{a})$$

$$\text{cond}_{a_i}(x) = \frac{|a_i| \cdot \left| \frac{\partial X}{\partial a_i} \right|}{|x|} = \frac{|a_i| \cdot |f'(x)|}{|x| \cdot |P(x)|}$$

$$\frac{\partial X}{\partial a_i} = - \frac{\partial P / \partial a_i}{\partial P / \partial x} =$$

$$\frac{x^i}{\sum j a_j x^{j-1}} = \frac{x^i}{P'(x)}$$

$$\int_{-\infty}^{\infty} f(x) dx = \sum_{i=1}^n c_i \pi(a_i)$$

$$P(x) = (x-a_1) \dots (x-a_n)$$

인수분해법

$$f^*: \mathbb{R}^x \rightarrow \mathbb{R}^x \quad f: \mathbb{R} \rightarrow \mathbb{R}$$

$$f^*(x^*)$$

$$f(x)$$

$$x' \text{ 가 } x^* \text{ 인데 } f^*(x^*) = f(x') \text{ 인데}$$

$$\frac{|f^*(x^*) - f(x)|}{|f(x)|} = \frac{|f(x') - f(x^*) + f(x^*) - f(x)|}{|f(x)|} \leq$$

$$\frac{|f(x') - f(x^*)|}{|f(x)|} + \frac{|f(x^*) - f(x)|}{|f(x)|}$$
$$\frac{|f(x') - f(x^*)|}{|f(x)|} \approx \frac{|f(x') - f(x^*)|}{|f(x^*)|}$$

$$\underbrace{\text{cond}(f)(x^*)}_{\text{cond}(f)(x)} \cdot \boxed{\frac{|x' - x^*|}{|x^*|}}$$

f^* per i primi 3 m per

$$\text{cond}(f^*)(x^*) := \inf_{f(x') \neq f(x^*)} \frac{|x' - x^*|}{|x^*|}$$

$$\underbrace{\text{cond}(f)(x)}_{\text{cond}(f)(x^*)} \left(\frac{|x - x^*|}{|x|} + \text{cond}(f^*)(x^*) \right)$$

Definition \Rightarrow Definition \Rightarrow $f: X \rightarrow \mathbb{C}$

\Rightarrow $\exists \gamma$ $\exists \rho$ $f: X \rightarrow \mathbb{R}$
 $=$ \mathbb{C}

$\exists \delta$ $\forall n \in \mathbb{N}$ $\exists \rho$ $\forall n \in \mathbb{N}$
 $, \forall x \in X \exists \rho$

For $\forall \exists \gamma \forall n \in \mathbb{N} \exists \rho$ $\forall n \in \mathbb{N} A$

$\text{For } X \text{ "continuous" function }$

$\forall n \in \mathbb{N} \forall \gamma \exists \rho \forall n \in \mathbb{N} \exists \rho \forall n \in \mathbb{N} A$

$\forall \rho \forall \gamma \exists \rho \forall n \in \mathbb{N} A$

$X - \sim \rho \gamma \exists \rho \forall n \in \mathbb{N} A = C(X)$

ר'ב גראן ב' נרנ'ן סט $P \subseteq A$

ר'ב גראן מושג

ר'ב גראן סט $P = \{x \mid f$

ר'ב גראן קבוצת סט $\underline{\underline{P}}$

$P = \{x \mid f(x) \in \text{הצורה}$

ר'ב גראן סט $f(A)$ ר'ב גראן סט

ר'ב גראן סט

$\{0,1\}$ ר'ב גראן סט $x = s'$

. 0,1 סט של סט'ס נון

$$f(x) = e^{2\pi i x} = \cos 2\pi x + i \sin 2\pi x \subseteq \mathbb{C}$$

'3' for each C for $\lambda \lambda \lambda \lambda \lambda \lambda \lambda$.

$$\text{Ansatz: } f(x) = \sin(2\pi n x) - 1 + \cos(2\pi n x)$$

$$\underbrace{\dots}_{(\Delta x \approx r_0)} \quad \text{hence}$$

$$\underbrace{\{e^{2\pi i n x}\}}$$

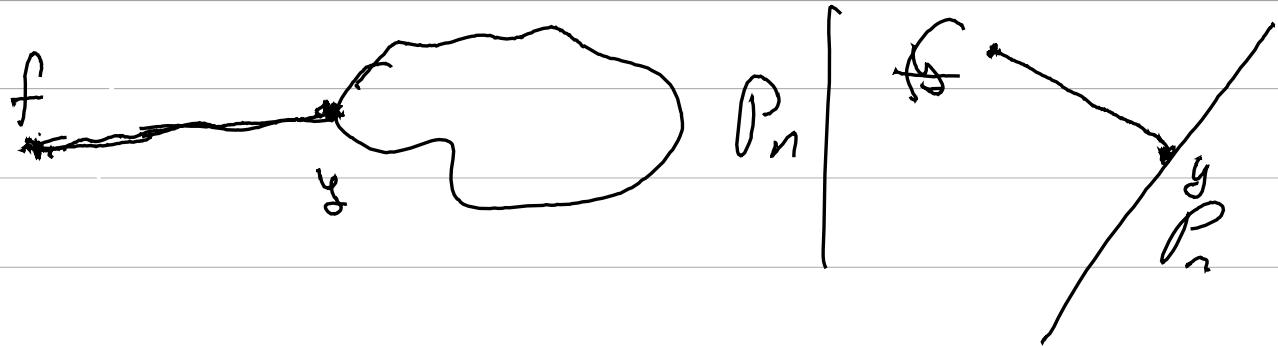
$$P = \bigcup P_i \quad P_0 \subseteq P_1 \subseteq P_2 \dots$$

3.2) 0 in N \Rightarrow f in P_n

$$n \in \mathbb{N} \quad \varepsilon > 0 \quad \text{such that } f \in P_n$$

$$\underbrace{d(f, P_n) < \varepsilon}_{\text{---}} \quad \text{---}$$

$$d(f, P_n) = \inf_{y \in P_n} d(f, y) = \inf_{y \in P_n} \|f - y\|$$



$\exists \delta > 0 \forall \epsilon' \exists C(x) \text{ for}$

$\|f\|_{C(X)} < \infty$

$$\|f\| = \sup_{x \in X} |f(x)| < \infty$$

$\|f\|_{C(X)} \leq \sup_{x \in X} |f(x)| + \|C_0\|_{C(X)}$

$\forall M > 0 \exists \delta > 0 \forall x \in X |f(x)| < M$

($\forall x \in X \exists r_x > 0 \forall y \in B(x, r_x) |f(y)| < M$)

$\rho \in P \quad \exists \delta > 0 \forall x \neq y \in X |x - y| < \delta \Rightarrow$

$|f(x) - f(y)| < \epsilon \quad \rho(x) \neq \rho(y)$

$f(X) \subset \mathbb{R}$ ρ $\in S_C(C(X))$

$\rho(\rho(x)) = \rho(x) \quad \forall x \in X$

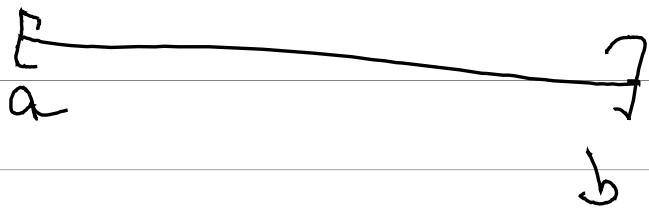
$\rho(\rho(x)) = \rho(x) \quad \forall x \in X$

$C(X), f \in \mathcal{F} : C(X) - \sup_{x \in X} |f(x)| < \delta$

$\exists \rho > 0 \quad \forall x \in X \quad \forall f \in \mathcal{F}$

$$\|f - p\| < \varepsilon$$

$|f(x) - p(x)| < \varepsilon \quad \forall x \in X \quad \forall f \in \mathcal{F}$



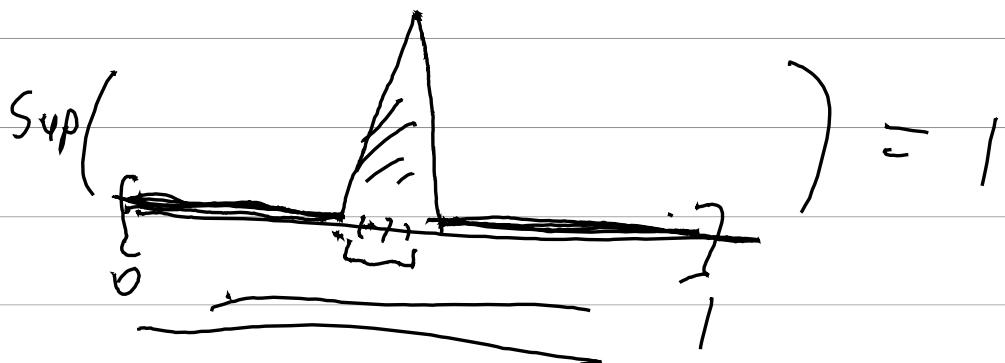
בנוסף לגבולות פונקציונליות, מוגדרים אינטגרלים

הנוגדים למינימום ומקסימום

המוגדרים בסעיפים
הנוגדים למינימום ומקסימום

הנורמליזציה

הו



$\|\cdot\|_p \rightarrow \cup \quad 1 \leq p \leq \infty \quad \int_{\mathbb{R}^n}$

$$\|\bar{x}\|_\infty = \max(|x_i|)$$

$$\|\bar{x}\|_p = \sqrt[p]{\sum |x_i|^p} \quad 1 \leq p < \infty$$

$x \in \mathbb{R}^n$
 $\|\cdot\|_\infty$ $\|\cdot\|_p$

ונורמליזציה

$\mathbb{R}^n \rightarrow \text{הנורמליזציה} \subset \text{הנורמליזציה}$

$\| \cdot \|_2 : C(X) \rightarrow \mathbb{R}$??

$(L_2, \|\cdot\|_2)$?

$$\|f\|_2^2 = \int_X |f|^2$$

$\therefore \rho(F) \approx \underline{X = \{1, \dots, n\}}$ AC

$\rho(F) \approx L_2 \approx \mathbb{R}^n$

$\forall \lambda \in \mathbb{R}$, $\exists \omega \in \mathcal{D}(C)$

$\exists \omega \in \mathcal{D}(C)$ $\omega : X \rightarrow \mathbb{R}$ $\lambda \omega / \in \mathcal{D}(C)$

$$\|f\|_{\omega, 2} = \sqrt{\int_X |f|^2 \cdot \omega}$$

$$\|f\|_2^2 = \int_X |f|^2 \leq \sup_X |f|^2 \cdot \left[\int_X 1 \right] =$$

$$\|f\|_\omega^2$$



$\hookrightarrow \text{Naturale } \text{ und } \text{ reelle } \text{ Zahlen}$

$\sim \text{wegen } \text{ Gitter}$

$X \quad f: X \rightarrow \mathbb{R}$

$$A \subseteq X \quad 1_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \end{cases}$$



نیز پس از اینجا $\sum_{x \in X} f(x)$ را

(\rightarrow زیرا $\sum_{x \in X}$)

$$f: X \rightarrow \mathbb{R}$$

$$\int_X f = \sum_{x \in X} f(x)$$

$$X = \{1, \dots, n\}$$

$$f: X \rightarrow \mathbb{R} \Leftrightarrow \text{دایا}$$

$$\|f\|_p = \sqrt[p]{\sum_{i=1}^n |f_i|^p}$$

لذا $\|\int f\|_p$ را نمایند V را

V را $\lambda[V]$ دانند ، $k = \mathbb{R}$ را

$\langle \cdot, \cdot \rangle: V \times V \rightarrow k$ را $\langle \cdot, \cdot \rangle$ دانند

$v \mapsto \langle v, u \rangle$ 'ər , $u \in V$ ֆ . 1

$\underline{\langle u, v \rangle := \overbrace{\langle v, u \rangle}}$, $u, v \in V$ ֆ . 2

$\langle u, u \rangle \in \mathbb{R}$, $u \in V$ ֆ $S(c)$

$\langle u, u \rangle > 0$ ՏՇ $u \neq 0$ ա՛ւ . 3

V հայելա բազու ՀՇ $\langle \cdot, \cdot \rangle$ ա՛ւ

$v \mapsto \|v\| := \sqrt{\langle v, v \rangle}$ ՀՇ պահանջ ս՛կ

. V հայելա բազու ՀՇ

$u, v \in V$ բազու , հարաց ՀՇ համապատասխան

$\|u+v\|^2 = \langle u+v, u+v \rangle = \langle u, u \rangle + 2\langle u, v \rangle + \langle v, v \rangle$

$\|u\|^2 + \|v\|^2 + \underline{2\langle u, v \rangle}$

הנ'יה יסוד נורמליזציה

מכפלה סקלרית

$$\frac{\|u+v\|^2 - \|u\|^2 - \|v\|^2}{2} \leftarrow (u, v)$$

הנ'יה יסוד נורמליזציה

הנ'יה יסוד נורמליזציה, $\rho = 2$ מילר

הנ'יה $C(x)$ גורם

$$(u, v) \mapsto \int_X u \cdot v$$

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הנ'יה יסוד נורמליזציה
 $\langle u, v \rangle = 0$ מילר u, v

$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$ $\sqrt{\lambda}$

$\sqrt{\lambda}$

$$\left\| \sum a_i v_i \right\|^2 = \sum a_i^2 \|v_i\|^2$$

(Eigenvalues of $A^T A$)

$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$ $\sqrt{\lambda}$

$P = U P' \quad -1, X \text{ for } \sqrt{\lambda} \beta \rightarrow$

Matrix diagonalization, λ β

Orthogonal matrix U , λ β

$$[\sqrt{\lambda_1} | \sqrt{\lambda_2}] \underbrace{P_n}_{P_n - \delta} \{ \pi_i \}$$

$f \in A$

$$T(c_1, \dots, c_m) = \|f - \sum c_i \pi_i\|^2 = \langle f - \sum c_i \pi_i, f - \sum c_i \pi_i \rangle =$$

$$\|f\|^2 - 2 \underbrace{\sum c_i \langle f, \pi_i \rangle}_{\text{underlined}} + \sum c_i c_j \langle \pi_i, \pi_j \rangle$$

Suppose T is a linear operator

such that $\sum c_i \pi_i$ is in the range of T

$$\cdot 0 \quad \int \pi_i c_i \rightarrow \int \pi_i$$

$$0 = \frac{\partial T}{\partial c_k} = -2 \langle f, \pi_k \rangle + 2 \sum c_j \langle \pi_k, \pi_j \rangle$$

$$\sum c_j \langle \pi_k, \pi_j \rangle = \langle f, \pi_k \rangle$$

$\|f\|$ is the norm of f

$$A \tilde{c} = b$$

$$b_i = \langle f, \pi_i \rangle \quad \in \mathbb{C}$$

$$A = (\langle \pi_i, \pi_j \rangle)_{i,j}$$

$$\lambda_1, \lambda_2, \dots, \lambda_n \in C_n^{\perp} \quad \text{and} \quad A$$

$$(x, y) \mapsto \underline{\langle x, Ay \rangle} \leftarrow \begin{matrix} \mathbb{R}^n \times \mathbb{R}^m \\ \text{from } \mathbb{R}^m \end{matrix}$$

$$S \subset \tilde{X} \neq \emptyset \quad \text{and}, \quad \text{w.l.o.g}$$

$$\underline{\tilde{x} \cdot A \tilde{x}} \geq 0$$

$$\tilde{x} \cdot A \tilde{x} = \sum_{i,j} x_i x_j \langle \pi_i, \pi_j \rangle = \underline{\|\sum x_i \pi_i\|^2}$$

$$\tilde{x} \neq 0 \quad \text{and} \quad \{x_i \pi_i \neq 0\} \subset \{\pi_i\}$$

i) $\mu\sigma$ $\Rightarrow \sigma \text{ is } A$, $\exists \omega$
 $\sigma \text{ is not } f(\omega)$

$$(\pi_i)_{i \geq 0}$$

$$A = \left(\begin{matrix} \langle \pi_i, \pi_j \rangle \end{matrix} \right)_{1 \leq i, j \leq n}$$

$$A \bar{c} = b \quad \bar{c} = \langle f, \pi_i \rangle$$

$$[0, 1] \quad \text{for} \quad \pi_i = t^{\frac{i}{n}} \underbrace{[t^n, 1]}$$

$$\langle f, g \rangle = \int_0^1 f \cdot g \, dt$$

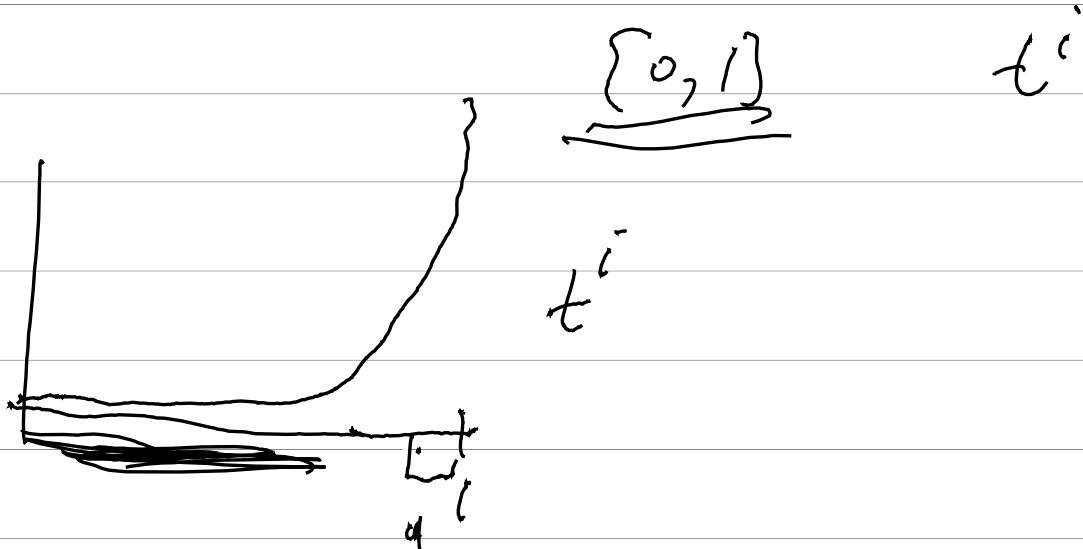
$$\langle \pi_i, \pi_j \rangle = \int_0^{t^{\frac{j}{n}}} t^{\frac{i}{n}} \, dt = \frac{t^{\frac{i+j+1}{n}}}{\frac{i+j+1}{n}} \Big|_0^1 =$$

$$H_3 = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{1} \\ \frac{1}{3} & \frac{1}{1} & \frac{1}{2} \end{pmatrix}$$

$H_n C = I$ \Rightarrow $C^{-1} = H_n$

\Rightarrow $C = H_n^{-1}$ \Rightarrow $C = H_n$

$C = H_n$ \Rightarrow $C^{-1} = H_n^{-1}$



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: ($\sum_{i=1}^n \pi_i / n - 1/n$) $\leq \mu(n) / n$

$$\langle \pi_i, \pi_j \rangle = 0 \quad i \neq j \quad (\langle \pi_i, \pi_i \rangle = 1)$$

$$f = \sum a_i \pi_i$$

$$\underbrace{\langle f, \pi_i \rangle = a_i \langle \pi_i, \pi_i \rangle}$$

π_1, π_2, \dots

$\sum \pi_i \geq 1$

$$\hat{\pi}_i = \pi_i$$

$$\hat{\pi}_{k+1} = \pi_{k+1} - \sum \left(\frac{\hat{\pi}_{k+1}}{\hat{\pi}_i} \right)^{1/2} \hat{\pi}_i \quad \text{for } k < n$$
$$\langle \hat{\pi}_{k+1}, \hat{\pi}_i \rangle = \left(\sum_{i=1}^{k+1} \hat{\pi}_i \right)^{1/2} \langle \hat{\pi}_{k+1}, \hat{\pi}_i \rangle = 0$$

$$P = \bigcup P_i \quad P_0 \subseteq P_1 \subseteq \dots$$

$$P_i = \left\{ i \geq \frac{\epsilon \sqrt{N} \sum S_j}{\delta} \right\}$$

$$\dim(P_i) = i$$

$$\text{Span}(\langle \pi_i \rangle_{i \leq v}) = \text{Span}(\langle \hat{\pi}_i \rangle_{i \leq v})$$

$$\hat{\pi}_i \in P_i \quad | \quad \text{plan } p(j|s)$$

. i > 22

$$\hat{\pi}_{i+1} = t \hat{\pi}_i - \alpha_i \hat{\pi}_i + \sum_{j=0}^{i-1} b_j \hat{\pi}_j =$$

$$(t - \alpha_i) \hat{\pi}_i + \beta_i \cdot \hat{\pi}_{i-1} + \sum_{j=0}^{i-2} b_j \hat{\pi}_j$$

$$\langle \hat{\pi}_{i+1}, \hat{\pi}_i \rangle = \langle (t - \alpha_i) \hat{\pi}_i, \hat{\pi}_i \rangle \Rightarrow$$

$$\alpha_i \cdot \| \hat{\pi}_i \|^2 = \langle t \hat{\pi}_i, \hat{\pi}_i \rangle$$

$$\Rightarrow \alpha_i = \frac{\langle t\hat{\pi}_i, \hat{\pi}_i \rangle}{\|\hat{\pi}_i\|^2}$$

$$0 = \underbrace{\langle (t - \gamma_i) \hat{\pi}_i, \hat{\pi}_{i-1} \rangle}_{\beta_i \cdot \|\hat{\pi}_{i-1}\|^2} +$$

$$\beta_i = - \frac{\langle t\hat{\pi}_i, \hat{\pi}_{i-1} \rangle}{\|\hat{\pi}_{i-1}\|^2} =$$

$$- \frac{\langle \hat{\pi}_i, t\hat{\pi}_{i-1} \rangle}{\|\hat{\pi}_{i-1}\|^2} = - \frac{\|\hat{\pi}_i\|^2}{\|\hat{\pi}_{i-1}\|^2}$$

$$\hat{\pi}_{i+1} = (t - \gamma_i) \hat{\pi}_i + \underbrace{\beta_i \hat{\pi}_{i-1}}$$

For a more rigourous view see

$[-a, a]$ $\rightarrow \mathbb{R}^n$ \rightarrow L^2

$w(t) = w(t, t)$ \wedge $C^1 \circ f(t)$ \rightarrow \mathbb{R}^n , t

$$\left[\langle f, g \rangle = \int_a^b f(t) \bar{g}(t) \underline{w(t)} dt \right]$$

we can write π_k \in \mathcal{S}'

thus we have $\langle f, \pi_k \rangle$

$\forall c = 0$ \Rightarrow $\langle f, \pi_k \rangle$

Real $\{-1, 1\}$ $\rightarrow \mathbb{C}^2$ is closed

$(f(1), f(-1))$ \rightarrow \mathbb{C}^2

$$T_n(t) = \underbrace{\frac{k!}{(2k)!}}_{\text{symmetric}} \frac{d^k}{dt^k} (t^2 - 1)^k$$

$$(\ln/(t - \pi_k) - e^{-\sqrt{t}}) \int \dots$$

从 π_k 到 t^k , 令 t^k 为 π_i 的

$$0 = \langle \pi_k, t^i \rangle = \int_{-1}^1 \frac{d^k}{dt^k} (t^2 - 1)^k \cdot t^i dt =$$

$$\dots = 0$$

$$\pi_0 = 1, \quad \pi_1 = \frac{1}{2}(t^2 - 1)' = t$$

$$\pi_2 = ((t^2 - 1)^2)' \cdot \frac{2}{4!} = \frac{1}{12} \cdot ((t^2 - 1)^2)''$$

$$\pi_k = \underbrace{t^k + \mu_k t^{k-2} + \dots}$$

$$\pi_{k+1} = t \cdot \pi_k + \beta_k \cdot \pi_{k-1} \Rightarrow \boxed{\beta_k = \frac{\pi_{k+1} - t \pi_k}{\pi_{k-1}}}$$

$$\beta_k = \mu_k - \mu_{k+1}$$

$$\mu_k = \frac{k(k-1)}{2(2k-1)} \Rightarrow$$

$$\beta_k = \frac{1}{4-k^2}$$

Wichtigste Ergebnisse

if $\alpha \sim \pi$ then β

$$\underline{f: \mathbb{R} \rightarrow \mathbb{R}} \quad f(f+f) = f(\cancel{f})$$

↓ . t ↗

$$f: [0,1] \rightarrow \mathbb{R} \quad f(0) = f(1)$$

$$(\sin(2\pi t) + \cos(2\pi t)) = \underline{\underline{e^{2\pi i t}}}$$

(=)

$$g: \mathbb{S}' \rightarrow \mathbb{C}$$

$$\mathbb{S}' = \{ z \in \mathbb{C} \mid |z| = 1 \}$$

$$E: [0,1] \rightarrow \mathbb{S}'$$

$$E(t) = e^{2\pi i t}$$

$$g: \mathbb{S}' \rightarrow \mathbb{C} \rightsquigarrow g \circ E \text{ - } \text{rotation}$$

$$\int_{\mathbb{S}'} g := \int_0^1 g \circ E dt$$

$$z, w \in \mathbb{S}' \quad \text{if} \quad z, w \in \mathbb{S}' \quad \text{no!}$$

For formal $a \in \mathbb{S}'$ $\forall \alpha$

$$g_a(z) = g(a \cdot z)$$

$$\int_{\mathbb{S}'} g_a = \int_{\mathbb{S}'} g \quad \text{sic}$$

$g : S' \rightarrow \mathbb{C}^*$ $\rightsquigarrow r'/\mathbb{H}$ \mathbb{H}/\mathbb{C}

$g(z \cdot w) = g(z) \cdot g(w)$ $\rightsquigarrow \mathbb{H}' \supset \mathbb{N}$

z für $g(z) = 1$ \mathbb{H}/\mathbb{C} S/\mathbb{C}

$\int_S g = 1$ S/\mathbb{C}

$\int_S g = 0$ $\mathbb{H}/\mathbb{C} \rightarrow$

→ \mathbb{H}/\mathbb{C}'

$g(a) \neq 1 \quad \text{e. } \quad a \in S' \quad e' \cdot \omega$

$g_a(x) = g(ax) = g(a) g(x)$

$\int_S g = \int_S g_a = \int_S g(a) \cdot g = \underbrace{g(a)}_{\neq 1} \int_S g$ S/\mathbb{C}

$\int_S g = 0$ S/\mathbb{C}

$$g_n(x) \approx x^n \quad \text{for } n \in \mathbb{Z} \quad \text{def}$$

or \approx in $\mathcal{O}(x)$ we have x^n

$$\overline{g_n(x)} = g_{-n}(x) \quad g_n \cdot g_m = g_{n+m}$$

\approx $\mathcal{O}(x)$ \Rightarrow $\mathcal{O}(x)$ \approx $\mathcal{O}(x)$

$$\approx \mathcal{O}(x) \subset \mathcal{O}(x)$$

$$\langle f, g \rangle = \int_{\mathcal{S}'} f, \bar{g}$$

\approx $\mathcal{O}(x)$ \Rightarrow $\mathcal{O}(x)$ \approx $\mathcal{O}(x)$

$$\approx \mathcal{O}(x)$$

\approx $\mathcal{O}(x)$ \approx $\mathcal{O}(x)$ \approx $\mathcal{O}(x)$

$$\langle \mathcal{O}(x), \mathcal{O}(x) \rangle \approx \mathcal{O}(x) \approx \mathcal{O}(x)$$

$\rho' \cap \cup_{j=1}^m \sigma_j$ is a disjoint set

$\rho' \cap C \cap \cup_{j=1}^m \sigma_j$

$$c = \int_0^1 x^n = \int e^{2\pi i n t} dt = \int_{\gamma} e^{2\pi i n t + i \frac{2\pi}{n} t^2} dt$$

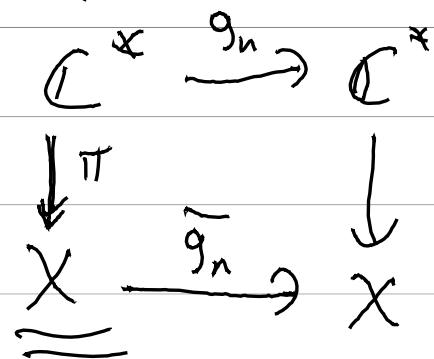
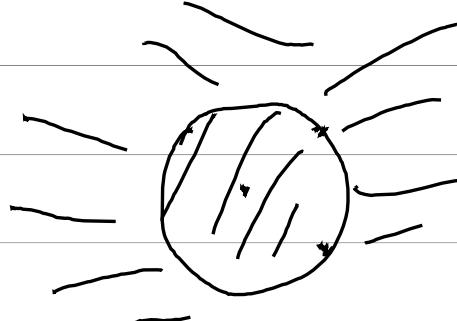
$\therefore f(z) \neq$

$$\mathbb{C}^* = \{z \in \mathbb{C} \mid z \neq 0\}$$

$$f_C: x = y \quad \sim_C \quad * \sim_Y$$

$$X = \mathbb{C}^*/\sim$$

$$g_n(x) = \frac{1}{x^n} \sim x^n \Rightarrow x = \frac{1}{y}$$



$$\pi(x) = x + \frac{1}{x} \in \mathbb{C}$$

$$\pi(x) = \pi\left(\frac{1}{x}\right)$$

$$\begin{aligned} & \text{if } z \in \mathbb{S}' \text{ in } \mathbb{H}_C \\ \pi(z') &= \operatorname{Re}(z) \\ \pi(S') &= [-1, 1] = X_C \end{aligned}$$

$$g_n\left(\frac{x + \frac{1}{x}}{2}\right) = \frac{x^n + \frac{1}{x^n}}{2} = \pi(g_n(x))$$

thus X is in \mathbb{H}_C

$$\star \int_{X_0}^X h = \int_{S'} h \circ \pi = \int_{S'} h\left(\frac{x + \frac{1}{x}}{2}\right)$$

$$\int_0^1 h(\operatorname{Re}(e^{2\pi i t})) dt = \int_0^1 h(\cos(2\pi t)) dt$$

$$y = \cos(2\pi t) \quad dy = 2\pi \sin(2\pi t) dt =$$

$$dy = -2\pi \sqrt{1-y^2} dt$$

$$x = \int_{-1}^1 h(y) \frac{1}{\sqrt{1-y^2}} dy$$

~~\int_0^1~~

Integrals \rightarrow $\widehat{g}_n \rightarrow x$

now we have $\int_{-1}^1 f g \frac{1}{\sqrt{1-y^2}} dy$

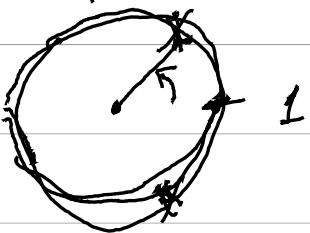
$\int_{-1}^1 f g \frac{1}{\sqrt{1-y^2}} dy$ $\sim \int_{-1}^1 f g dy$

$$\langle f, g \rangle = \int_{-1}^1 f \cdot g \frac{1}{\sqrt{1-y^2}} dy$$

~~\int_{-1}^1~~

$$\widehat{g}_n (\cos 2\pi n t) = \underbrace{\cos 2\pi n t}$$

$$S^1 = \{ z \in \mathbb{C} \mid |z| = 1 \}$$



$\downarrow P$

$$\sin 2\pi t = \sqrt{1 - y^2}$$

$$0 \leq t \leq \frac{1}{2}$$

$$[\leftarrow]_x = [-1, 1] \subset \mathbb{R}$$

X

$$P(z) = \frac{z + \bar{z}}{2} (= \frac{z + \bar{z}}{2} = \operatorname{Re}(z)) \quad S'$$

$$f : X \rightarrow \mathbb{C}$$

$$y = \cos 2\pi t$$

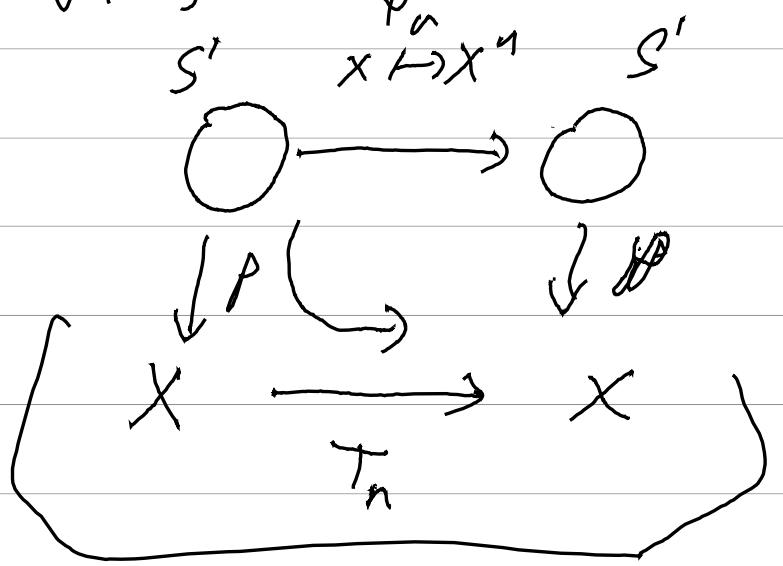
$$dy = -2\pi \sin 2\pi t dt$$

$$\int_X f := \int_{S^1} f \circ P = \int_0^1 f \circ P \circ e^{2\pi i t} dt =$$

$\frac{1}{2}$

$$\int_0^1 f(\cos 2\pi t) dt = 2 \int_0^{\frac{1}{2}} f(\cos 2\pi t) dt =$$

$$2 \int_{-1}^1 f(y) \cdot \left(-\frac{1}{2\pi}\right) \frac{dy}{\sqrt{1-y^2}} =$$



$$\varphi_n(x) = x^n$$

$$T_n\left(\frac{x+\frac{1}{x}}{2}\right) = \underbrace{x^n + \frac{1}{x^n}}_2$$

$$\int_X T_n = \int_{S'} T_n \circ \rho = \int_{S'} \underbrace{x^n + \frac{1}{x^n}}_2 =$$

$$\left\{ \begin{array}{ll} 1 & h=0 \\ 0 & h \neq 0 \end{array} \right. \left\{ \begin{array}{l} \int_{S'} \varphi_n \bar{\varphi}_m = \\ \int \varphi_n \varphi_{n+m} = \int \varphi_{n-m} \end{array} \right.$$

$$(T_n \cdot T_m) \left(\frac{z + \frac{1}{z}}{2} \right) = \underbrace{\left(z^n + \frac{1}{z^n} \right)}_{2} \left(\frac{z^m + \frac{1}{z^m}}{2} \right).$$

$$\frac{1}{2} \underbrace{\left(z^{m+n} + \frac{1}{z^{m+n}} + z^{n-m} + z^{m-n} \right)}_{2} =$$

$$\frac{1}{2} \left(T_{n+m} \left(\frac{z + \frac{1}{z}}{2} \right) + T_{n-m} \left(\frac{z + \frac{1}{z}}{2} \right) \right)$$

$$\int T_n \cdot T_m = \frac{1}{2} \left(\int T_{n+m} + \int T_{n-m} \right) =$$

$$\int \begin{cases} \frac{1}{z} & n = m \neq 0 \\ 1 & n = m = 0 \\ 0 & (n \neq m) \end{cases} \int T_n = T_{-n}$$

$$T_0 = 1 \quad T_0\left(\frac{z + \frac{1}{z}}{2}\right) = 1$$

$$T_1\left(\frac{z + \frac{1}{z}}{2}\right) = z + \frac{1}{z} \quad T_1(z) = z$$

$$T_n \cdot T_1 = \frac{1}{2} (T_{n+1} + T_{n-1}) \Rightarrow$$

$$T_{n+1}(z) = 2z T_n(z) - T_{n-1}(z)$$

$$T_2(z) = 2z^2 - 1$$

$$\cos(2t) = 2(\cos^2 t - 1)$$

$$\cos(\alpha t) = T_n(\cos t)$$

$$\text{For } p \in \mathbb{P}_{2^n}, \quad n \in \mathbb{N} \quad T_n(p) = \sum_{k=0}^{2^n-1} f_k(p) \cdot z^k$$

3) $\lim_{n \rightarrow \infty} f_n(x)$

f 3) $\lim_{n \rightarrow \infty} f_n(x)$

$\lim_{n \rightarrow \infty} f_n(x) = 0$ $\forall x \in \mathbb{R}$

$c_0, \dots, c_n \in [a, b] \subseteq \mathbb{R}$

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$\lim_{n \rightarrow \infty} f_n(x) = 0$ $\forall x \in \mathbb{R}$

c_0, \dots, c_n für $f(x) = \sum_{i=0}^n a_i x^i$

0, 1, ..., n+1 \Rightarrow a_0, a_1, \dots, a_n

$$f_i(c_j) = \begin{cases} 1 & j=i \\ 0 & j \neq i \end{cases}$$

$$P_i(x) = \frac{\prod_{j \neq i} (x - c_j)}{\prod_{j \neq i} (c_i - c_j)} \in P_n = \text{Span } \{1, x, x^2, \dots, x^n\}$$

$$f(c_i) = f_i \Rightarrow$$

$$f \sim \underbrace{\sum f_i l_i}_{=} = \pi_{\tilde{C}}(f)$$

$$\pi_{\tilde{C}} : C[a,b] \rightarrow C[a,b]$$

$$\sup_{\|f\|=1} \|\pi_{\tilde{C}}(f)\| = \sup_{\|f\|=1} \left\| \sum f_i l_i \right\| =$$

$$= \sum_{i=0}^n \|l_i\|$$

$$\lambda_n(x) = \sum_{i=0}^n |l_i(x)|$$

$$f - \underbrace{f - \pi_C(f)}_{\text{orthogonal projection}} \rightarrow f - \hat{P}_n$$

$$\|P_n - \cdot\|$$

$$\|\underbrace{f - \pi_C(f)}_{\text{orthogonal projection}}\| = \|f - \hat{P}_n - \pi_C(f - \hat{P}_n)\|$$

$$\leq \|f - \hat{P}_n\| + \|\pi_C\| \|f - \hat{P}_n\| =$$

$$\underbrace{\left(1 + \|\pi_C\|\right)}_{\text{constant}} \|f - \hat{P}_n\|$$

برهان انتقالی

$$\begin{aligned} & C^{n+1}[a, b] \ni f \\ & (f - \pi_C(f))(x) = \underbrace{\frac{f^{(n+1)}(\xi)}{(n+1)!} \prod_{i=0}^n (x - c_i)}_{\text{orthogonal projection}} \end{aligned}$$

$(x \rightarrow i\beta_n)$
 $\cup / \gamma/\approx \cup \{ e' : \approx \subseteq$

$(f \in C^{n+1}[a, b]) \quad | \cup$

$c_i \neq x \quad x \geq c_i \quad \underline{i \geq n+1}$

$G(t) = \underbrace{f(t) - \pi_{\mathcal{E}}(f)(t)}_{= 0} -$

$$\frac{f(x) - \pi_{\mathcal{E}}(f)(x)}{\prod_{i=0}^n (x - c_i)} = \frac{n}{\prod_{i=0}^n (t - c_i)}$$

マハガル $\Rightarrow G$ の \approx は c_i

$x - 1 \quad i=0, \dots, n \quad c_i$

$G^{(n+1)}$ $- f : \mathbb{R} \rightarrow \text{Gen}$

$\cdot \{ \quad \text{odd } e'$

ו'נ'ג = $n+1$ (∞) $\cap \mathcal{S}_{\rho}$

$$G^{(n+1)}(t) = f^{(n+1)}(t) - (n+1)! \cdot \frac{f(x) - f(t)}{\prod_{i=1}^n (x - c_i)}$$

$t = \zeta$ $\omega_{\lambda} \varphi$

$\zeta \in [a, b]$

$[a, b] \rightarrow \cup_{\lambda} \mathcal{S}_{\rho}$

(x, c_i) \square $\cap \mathcal{S}_{\rho}$

$$\int_{C(3)} \int_{\gamma_0} f \underbrace{d\gamma}_{{}^{\text{def}} \gamma'}$$

$$= \int_{\gamma_0} f \circ \tilde{\gamma} \in \mathbb{C}^{(n)}_{\gamma_0}$$

$$= \int_{\gamma_0} f \circ \tilde{\gamma} = \int_{\gamma_0} f \circ \tilde{\gamma} \circ \tilde{\gamma}'$$

$$\pi_{C^{(n)}}(f) \rightarrow f$$

$$\int_{\gamma_0} f \circ \tilde{\gamma} = \int_{\gamma_0} f$$

$$\pi_{C^{(n)}}(f)$$

$$\|f - \pi_{C^{(n)}}(f)\| \leq \left\| \frac{f^{(n+1)}(\xi)}{(n+1)!} \right\|_{C^{(n)}} \|x - c_i^{(n)}\|$$

$$\leq \frac{m_{n+1}(f)}{(n+1)!} \cdot (b-a)^{n+1}$$

$$\boxed{[a,b] \ni f \text{ has } m_{n+1}(f) \text{ small}}$$

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$$\frac{m_n(f) \cdot (b-a)^n}{n!} \rightarrow 0$$

$c_i \in [a, b]$ γ' 3 מינימום של m_{n+1} ב-

$$\|f - \pi_{\bar{c}}(f)\| \leq \left\| \frac{f^{(n+1)}(\xi)}{(n+1)!} \cdot \prod_{i=1}^n (x-c_i) \right\|$$
$$\leq \underbrace{\frac{m_{n+1} \cdot (b-a)^{n+1}}{(n+1)!}}$$

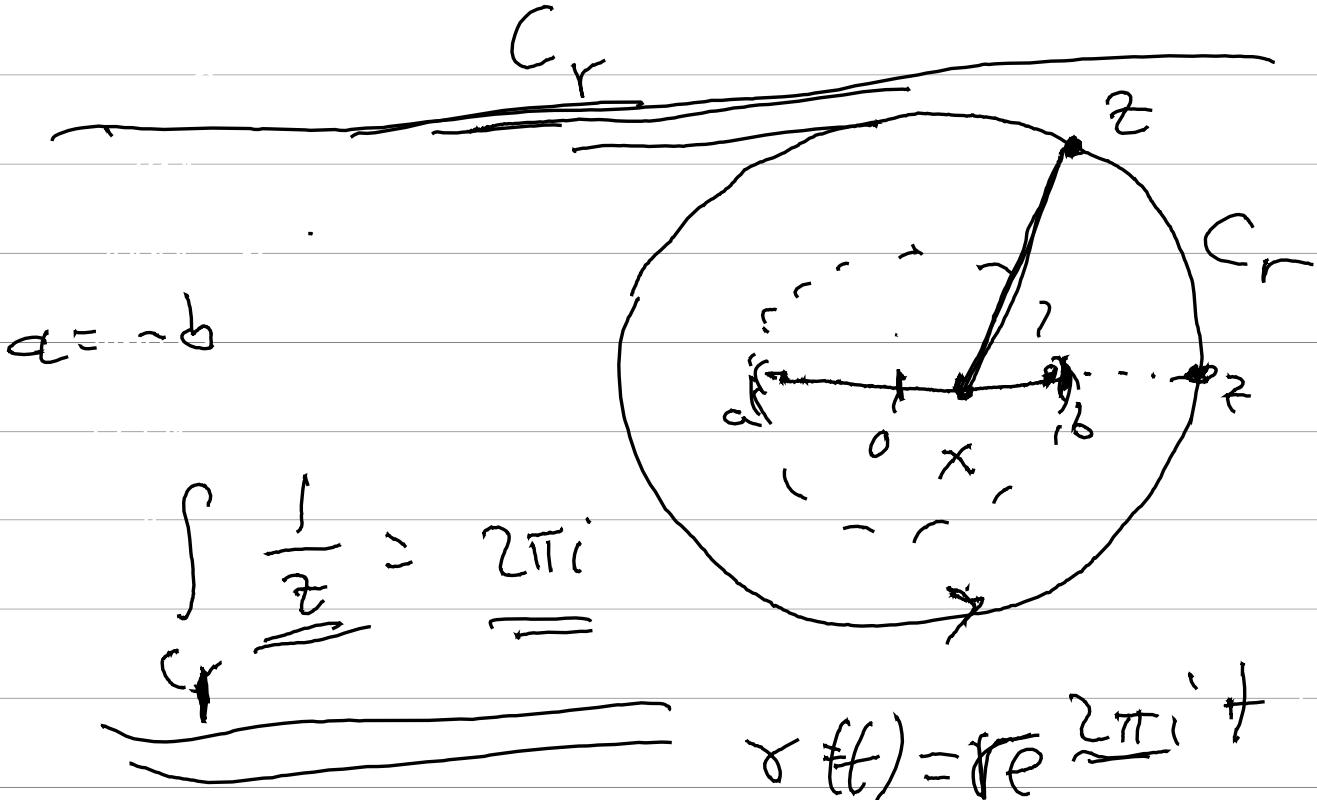
$$m_n = \|f^{(n)}\|_{\infty}$$

ר' סיסיק גאנז לנ' א' f

$\text{Rückgrat f. der : } \ell \rightarrow \text{nach}$

S/ℓ

$$f^{(k)}(x) = \frac{k!}{2\pi i} \oint \frac{f(z)}{(z-x)^{k+1}} dz$$



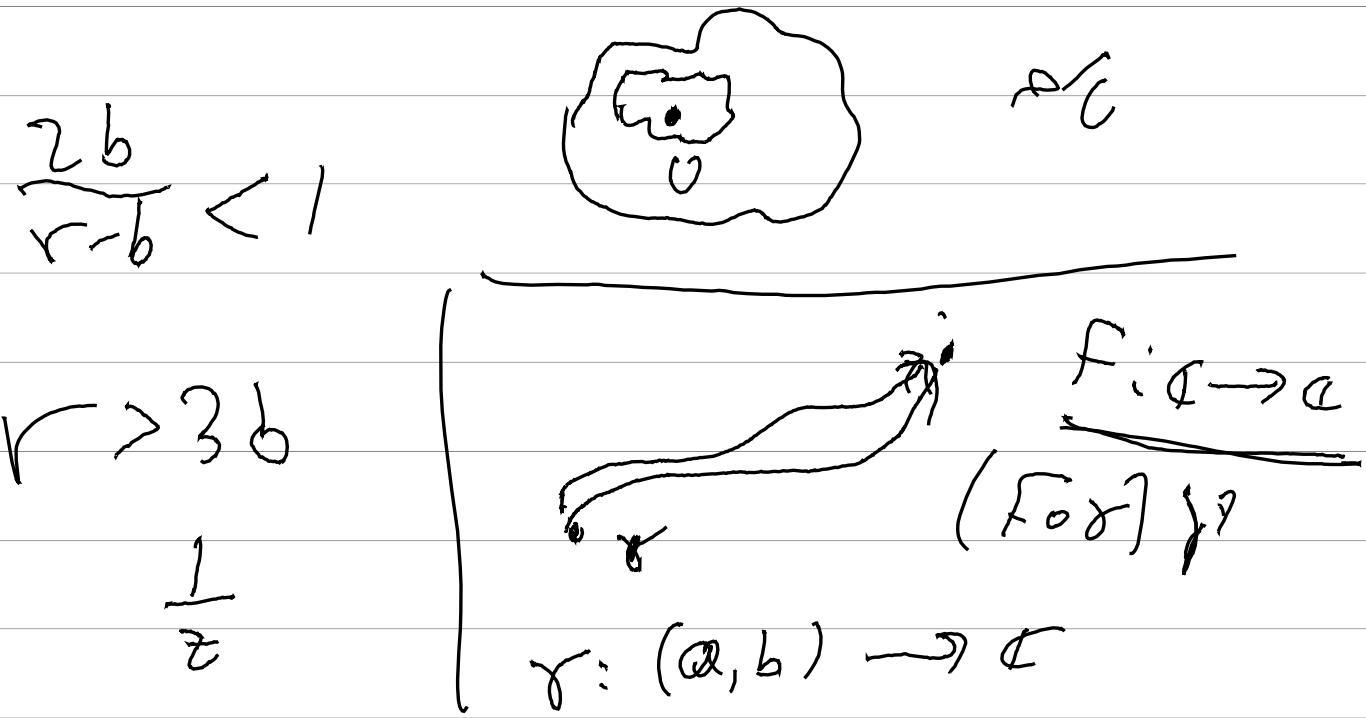
$$\underbrace{\|z-x\|}_{\geq r-b}$$

$$\|f^{(k)}(x)\| \leq \frac{k!}{2\pi} \frac{M_N}{(r-b)^{k+1}} \cdot 2\pi r \Rightarrow$$

$$\frac{M_n \cdot (2b)^n}{n!} \leq \frac{n! \cdot M_0}{(r-b)^{n+1}} \cdot r \cdot (2b)^n =$$

$n!$

$$\frac{M_0 \cdot r}{r-b} \cdot \left(\frac{2b}{r-b}\right)^n \rightarrow 0$$



רְאֵבָנָה | פְּלִימָה אֶלְגַּיְלָה לְאַנְשָׁה
 $([-1, 1] \setminus \{0\})$ || • 160 גְּוֹדָרֶת נ

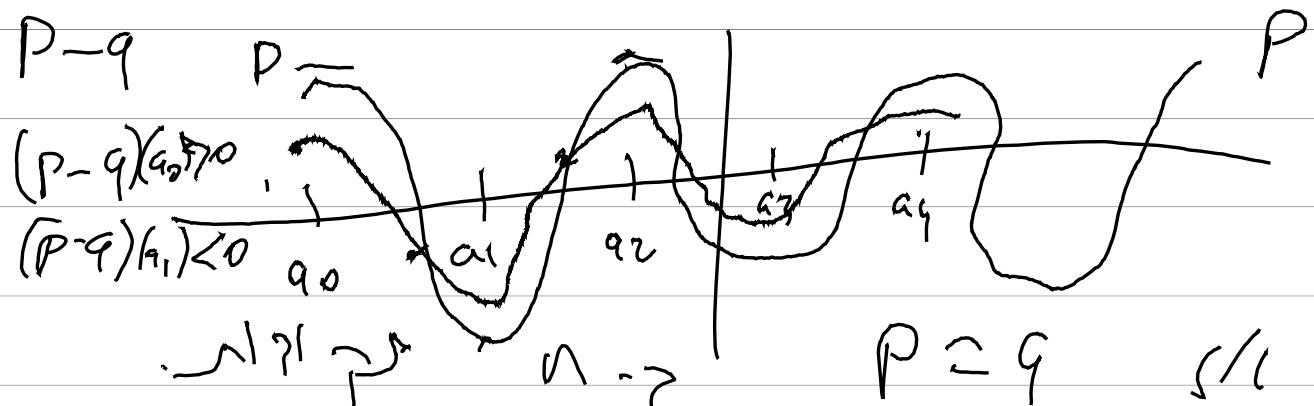
? גְּדֻלָּה;

רְאֵבָנָה | פְּלִימָה אֶלְגַּיְלָה P אֶלְגָּה

ולא $\|P\|_\infty = |P(a_i)|$ -& > n
 $P(a_i) = -P(a_{i+1})$
 $P \quad \text{ס/י} \quad a_i \quad \text{ט/פ/פ} \quad n+1$

רְאֵבָנָה $\|P\|_\infty$ וְאֶלְגַּיְלָה פְּלִימָה

$\|q\| < \|p\| \Rightarrow \text{ט/פ/פ}$



Complex numbers in polar form

input x \mapsto \sim^n

$$T_n(\operatorname{Re} z) = \underbrace{\operatorname{Re}(z^n)}$$

$$\Rightarrow T_n(x) = 0$$

$$\text{nc } z^n$$

\Leftrightarrow

$$x = \cos\left(\pi \frac{2k+1}{2n}\right)$$

$$z^n = \pm i$$

$$y = \cos\left(\pi \frac{k}{n}\right)$$

$$z^n = i \Leftrightarrow$$

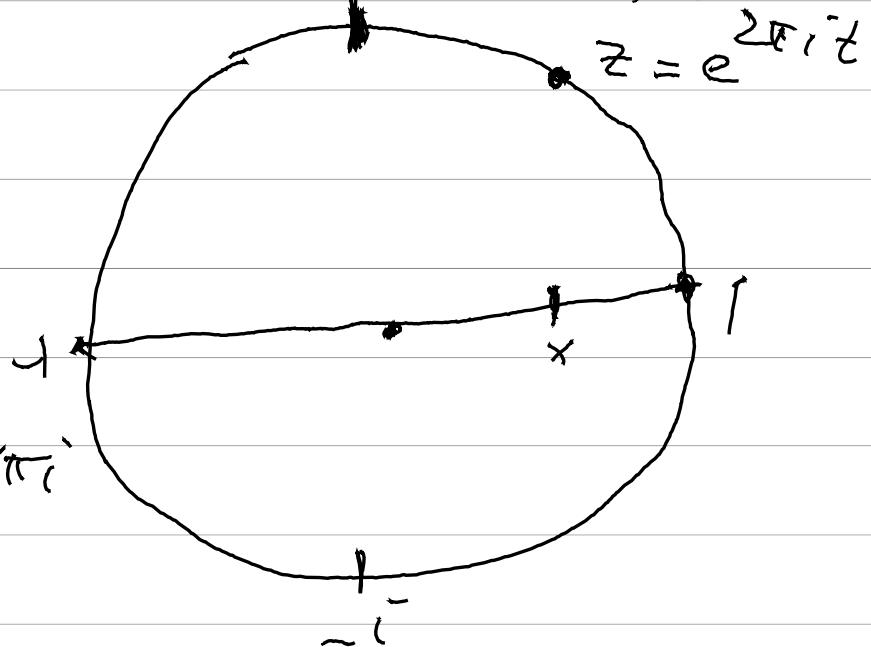
$$k=0, \dots, n$$

$$z = e^{2\pi i \frac{k}{n}}$$

$$e^{2\pi i \operatorname{int}} = i$$

$$2\pi i \operatorname{int} = \frac{\pi i}{2} + k\pi i$$

$$\Rightarrow \operatorname{int} = \frac{2k+1}{4n}$$



Ex $\left| \frac{1}{2} T_n \right| \rightarrow 0$ for $n \rightarrow \infty$

$$2^{n-1} \cdot k \cdot T_n$$

Now $\left| \frac{1}{2} T_n \right| = \frac{1}{2^{n-1}} \cdot T_n$ s/c

Since $\left\| \frac{1}{2} T_n \right\| = \frac{1}{2^{n-1}}$, $n \rightarrow \infty$

$a_i = \cos\left(\frac{\pi i}{n}\right)$ $\rightarrow 1/2$

Given $x'_n = \sum_{i=1}^n a_i \cdot e_i$

for all $n \in \mathbb{N}$ & $\frac{1}{2} T_n$

and $\left\| \sum_{i=1}^n a_i \cdot e_i \right\| \leq \frac{1}{2} T_n$

\rightarrow $\sum_{i=1}^n a_i^2 \leq \frac{1}{4} T_n^2$

$$||f - \Pi_{\tilde{C}^{(n)}}(f)|| \leq \frac{|f^{(n+1)}(\xi)|}{(n+1)!} \cdot \|T_n\|_\infty =$$

$$\frac{|f^{(n+1)}(\xi)|}{(n+1)!} \cdot 2^{n-1}$$

$$(c_0, c_1, \dots) \rightarrow \text{S'Ol'sic 2730}$$

$$P_0, P_1, \dots \quad \deg(P_i) \leq i$$

δ_i

P_i

$$P_{i+1}(x) = P_i(x) + a_{i+1,0}(x-c_0) \dots (x-c_c)$$

$$a_{i+1} (c_{i+1} - c_0) \dots (c_{i+1} - c_i) = f_{i+1} - p_i f_{i+1}$$

$$\underline{a_{i+1}} = \underline{(c_{i+1} - c_0) \dots (c_{i+1} - c_i)} = \frac{f_{i+1} - p_i(f_{i+1})}{f_{i+1} - p_i(f_{i+1})}$$

$$\underline{[c_0, \dots, c_{i+1}]} f$$

$$\underline{[c_0, \dots, c_{i+1}]} f \subseteq \underline{[c_0, \dots, c_i]} f \sim \underline{[c_1, \dots, c_{i+1}]} f$$

$c_{i+1} - c_0$

$$\tilde{C} = (c_0, \dots, \overset{\text{:=} c_i}{c_{i+1}}, \dots, c_{i+1})$$

$$\underline{P_{\tilde{C}}(x)} = P_{\tilde{C}_{i+1}} - \frac{(x - c_0)}{(c_{i+1} - c_0)} (P_{\tilde{C}_{i+1}} - P_{\tilde{C}_0}) =: q(x)$$

$\text{if } 0 < j < i+1 \text{ do } :[n' 33$

$$q(c_j) = f_j = P_{\tilde{C}}(c_j)$$

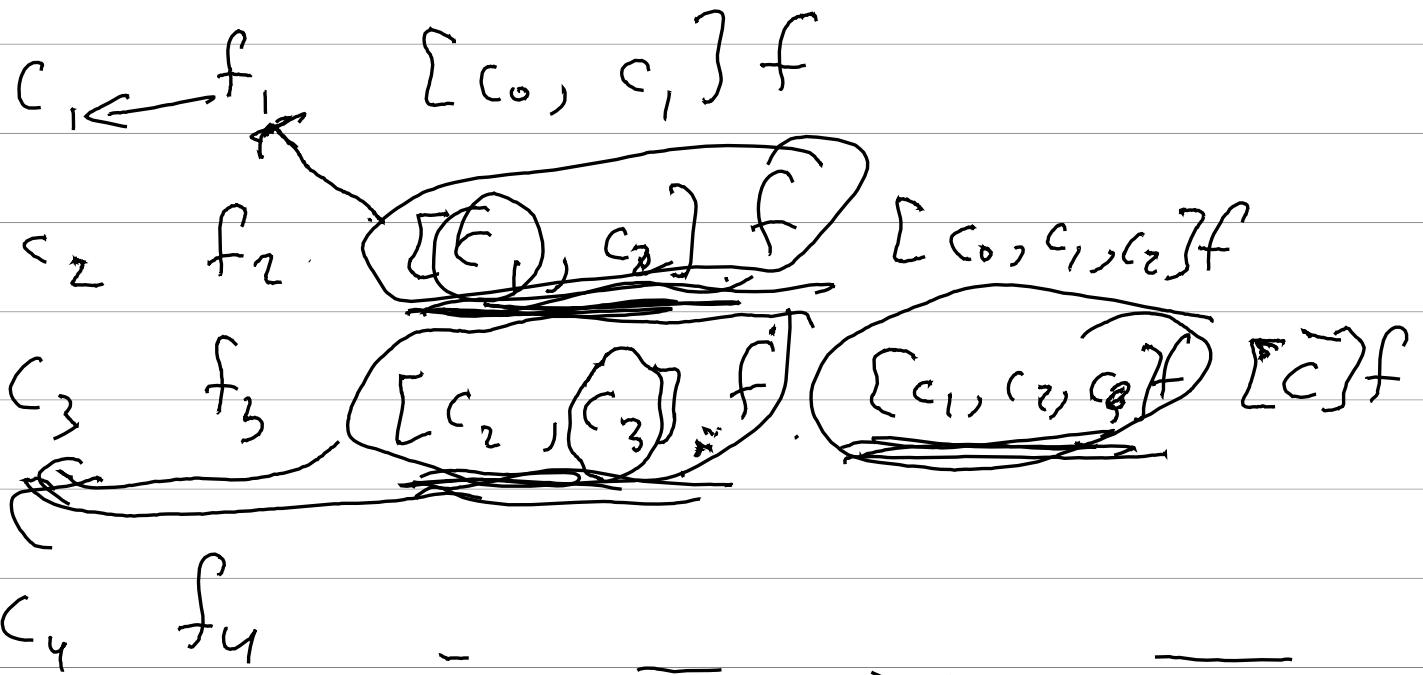
$$q(c_0) = P_{\tilde{C}_{i+1}}(c_0) = f_0 = P_{\tilde{C}}(c_0)$$

$$q(c_{i+1}) = P_{\tilde{C}_{i+1}}(c_{i+1}) - (P_{\tilde{C}_{i+1}}(c_{i+1})) -$$

$$P_{\tilde{C}_0}(c_{i+1}) = P_{\tilde{C}_0}(c_{i+1}) = f_{i+1} = P_C(f_{i+1})$$

c f

c_0 f_0



$$\{c_1, c_2, c_3\}f = \frac{\{c_2, c_3\}f - \{c_1, c_2\}f}{c_3 - c_1}$$

$$\bar{c} = c_0, \dots, c_i$$

$$P_{\bar{c}}(x) = P_{\bar{c}_{\leq i}}(x) + [\bar{c}] f \cdot \prod_{j < i} \pi(x - c_j)$$

$$\|P_{\bar{c}}(x) - P_{\bar{c}_{\leq i}}(x)\| = \|[\bar{c}] f \cdot \prod_{j < i} \pi(x - c_j)\|$$

$$\hookrightarrow \underbrace{\left[P_{\bar{c}}^{(i+1)}(\xi) \cdot \prod_{j < i} \pi(x - c_j) \right]}_{(i+1)}$$

$$\underline{\underline{f}} \left[\begin{array}{c} c_0, c_1, \dots, c_n \\ f_0, f_1, \dots, f_n \end{array} \right]$$

$$P_{\bar{C}}(x) = [\bar{c}] f(x^n) + \dots =$$

$$[\bar{c}] f(x - c_0) \dots (x - c_n) + P_{\sum c_i}$$

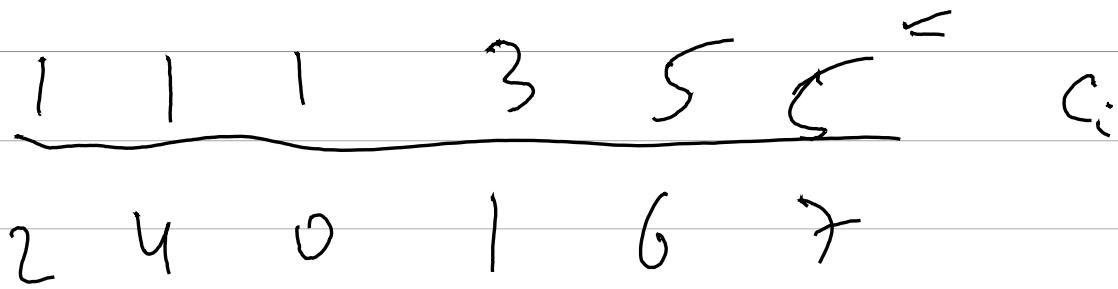
$$\vec{c}_i = \{c_0, \dots, c_n\} \setminus \{c_i\}$$

$$[\bar{c}] f = \frac{[\vec{c}_0] f - [\vec{c}_n] f}{c_n - c_0}$$

c_0, \dots, c_h

f_0, \dots, f_n

$$\bar{C} = 3 \cdot \{ \} + \{ \} + 2 \cdot \{ \}$$



$$P(1)=2, \quad P'(1)=4, \quad P''(1)=0$$

$$P(3)=1, \quad P(5)=6, \quad P'(5)=7$$

$$P_{\bar{C}}(x) = (\bar{C} \bar{f}) \cdot x^4 + \dots$$

$$\hat{c}_i = \bar{c} - \{c_i\}$$

$$\bar{C} = \sum n_i \{c_i\} \quad \deg(\bar{C}) = \sum n_i = n$$

$$\int_C c_i \neq c_j \text{ and } \underline{\text{is simple}}$$

$$\{\tilde{c}\} f = \frac{\sum \tilde{c}_i f - \sum \tilde{c}_j f}{c_j - c_i}$$

$$\tilde{c} = \sum_{k=1}^m a_k [c_k] \quad \underline{\text{closed}}$$

$$n = \deg(\tilde{c}) = \sum a_k$$

$$N(\tilde{c}) = n = \infty \quad \leftarrow \text{not } \beta_1 \beta_2$$

$$n_k > 1 \quad \epsilon' \leq \epsilon \quad n-k > 0 \quad \text{and}$$

$$\tilde{d} = d_\varepsilon = \tilde{c} - \{c_n\} + \{c_n + \varepsilon\}$$

$$\int_C \tilde{d} = \int_C c_n + \varepsilon \quad \varepsilon > 0 \quad \text{and}$$

$$N(\tilde{d}) < N(\tilde{c})$$

$$[\tilde{d}]f = \frac{[\tilde{d}_i]f - [\tilde{d}_j]f}{\tilde{d}_j - \tilde{d}_i}$$

$$\text{Definim } \sum_{i=1}^n \tilde{d}_i f_i = \sum_{i=1}^n c_i f_i$$

$$\text{at } \tilde{c}_0 \rightarrow \infty \quad \tilde{d}_0 f_0 \rightarrow 0$$

$$[(n-1)[\tilde{d}_0] + [c_0 + \varepsilon]]f \xrightarrow{\varepsilon \rightarrow 0} [n[c_0]]f$$

$$[c_0, c_0 + \varepsilon]f = \frac{f(c_0 + \varepsilon) - f(c_0)}{\varepsilon} \xrightarrow{\varepsilon \rightarrow 0} f'(c_0)$$

$$P_n(c_0) = \sum_{k=0}^n \frac{f^{(k)}}{k!} (x - c_0)^k$$

$$c_0 = c_1 = c_2$$

$$c_5 = c_6$$

$$c_0 \quad f_0$$

$$c_1 \quad f_0 = [f]_{c_0, c_1} f = f_1$$

$$c_2 \quad f_0 \cdot f_1 \quad f_2$$

$$c_3 \quad f_3 \quad [c_2, c_3] f = \frac{f_3 - f_0}{c_3 - c_0} \cdot [c_1, c_2, c_3] f = \frac{[c_2, c_3] f - f_1}{c_3 - c_0}$$

$$c_4 \quad f_4$$

$$c_5 \quad f_5$$

$$c_6 \quad f_5 \quad f_6$$

$$c_7 \quad f_0 \quad [c_6, c_7] f = \frac{f_0 - f_5}{c_7 - c_6}$$

c_6^{11}

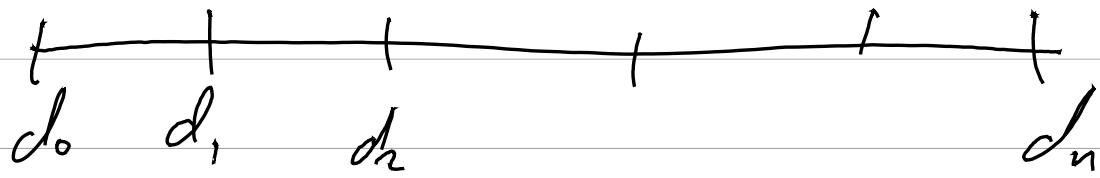
$$[c_5, c_3] f$$

$$\underbrace{p'j's \circ}_{a < b \in R}$$

$$1g/\delta n \quad \sim \gamma / p$$

$$a = d_0 < d_1 < \dots < d_n = b$$

↓

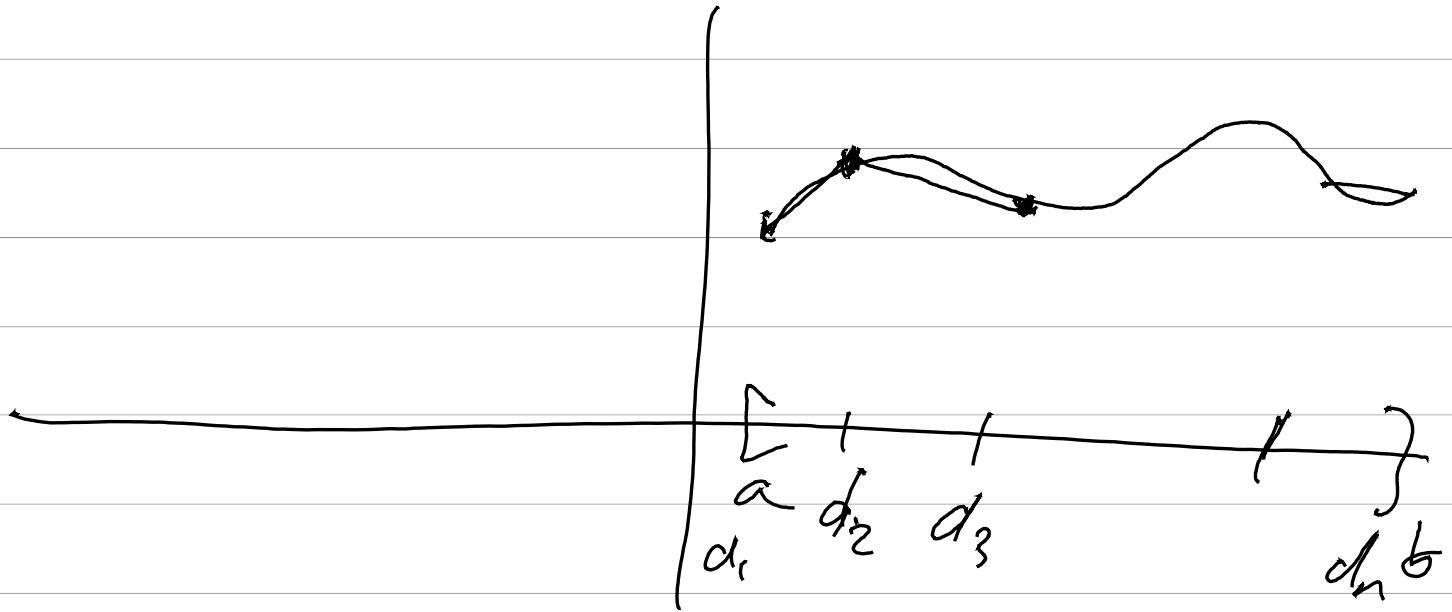


$$S_m^k(D) = \left\{ s \in C^k[a, b] \mid s|_{[d_i, d_i]} \right\}$$

$m \geq 1, 2, \dots, k$

$$k < m$$

$$S_1^{\circ}(\delta) = \int_{\sqrt{d}}^{\sqrt{d+\delta}} \sin^{-1} f(x) dx$$



137277's
f \approx

f \int_a^x \approx $\sum_{i=0}^{n-1}$ $f(c_i) \Delta x$, MV
 $\|f\|_{\infty}$ Δx

$$|f(x) - S(x)| \leq \frac{M}{2} \cdot |(x-d_i)(x-d_{i+1})| \leq$$

$$\frac{M}{8} \cdot (d_{i+1} - d_i)^2$$

$$M = \max_{x \in [d_i, d_{i+1}]} f''(x)$$

(a) \rightarrow λ) $\wedge \lambda$ \rightarrow λ λ

$$\frac{M}{8} \cdot |\Delta|^2$$

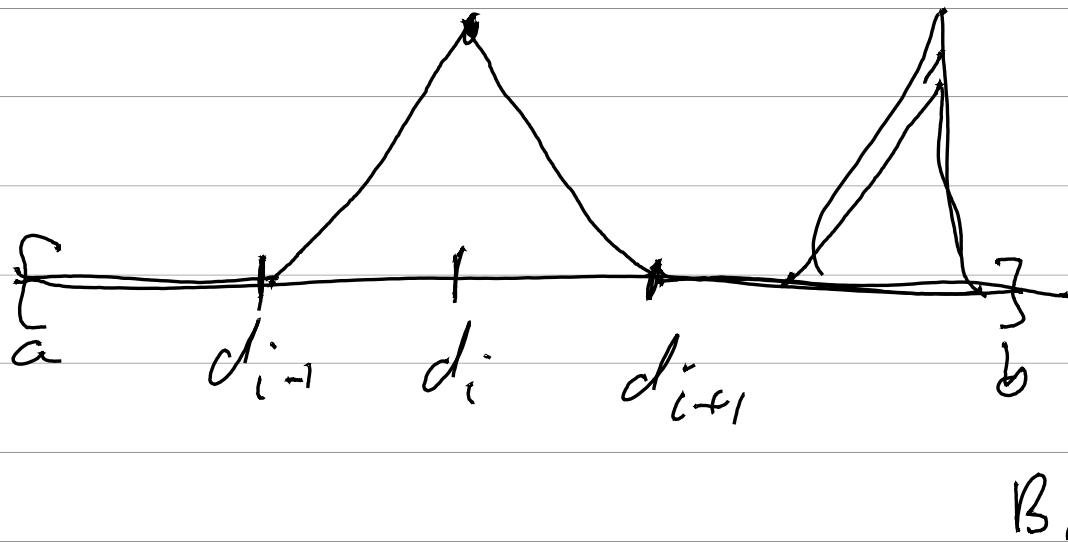
$|\Delta| =$ $\sqrt{\text{area}} \propto \sqrt{T} \propto \sqrt{c}$

$$\|f - \zeta_1\| \leq \underbrace{\|f - \zeta_0(\theta)\|}_{\text{error}}$$

$V =$ ∂V \cap Λ \cap $\zeta_0(\theta)$

1260 0.025 \approx 1000

$R^{\{d_1, \dots, d_n\}}$ μ



$$\langle B_i, B_j \rangle \neq 0$$

~~|i - j| \leq 1 \text{ 且 } i > j~~

~~且有~~

$$\hat{s} = \sum c_i B_i$$

~~且~~ $\sqrt{\sum c_i^2} \leq \sqrt{\int f^2 dx}$

~~所以~~ $\sum c_i^2 \leq \int f^2 dx$

$$T \bar{c} = \bar{d}$$

$$T = \left(\langle B_i, B_j \rangle \right)_{i,j} \in \mathbb{R}^{C \times C}$$

$$d_i := \langle f, B_i \rangle$$

$$T = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

• $\sum_i d_i^2 / C$ "Trace"

$$\|f - \hat{g}\|_\infty \leq 4 \cdot \overbrace{d(f, S_1^\circ(\delta))}^{> 3\gamma/\delta}$$

$\therefore 3 \rightarrow \text{approx} \approx 1520$

دلتا '3' for f the 2121

نحوه . $S_3'(d)$ π

- $c \geq d \leq e^3$ for f

$s(d_i) = f(d_i)$ i لـ f

$s|_{[d_{i-1}, d_i]}$ \geq approx 1812

$s'(d_i) = m_i$

$m_i = \underline{\underline{f'(d_i)}} \cdot l_c$

- c m_i if 1812

$s \in S_3^2(D)$

1) def and lin

. (def) f for $\boxed{f: \mathbb{R} \rightarrow \mathbb{R}}$

for map is f

. (def) or 'f/c' $[a, b]$

numbers 113w in range

$f(x) = 0$ if else

then return if else return x

return $\left(\begin{array}{c} x \\ \vdots \\ x_n \end{array} \right)$ else

$F(t, x, x', \dots, x^{(n)}) = 0$.?

$x(1) = x_1$, $x(0) = x_0$

$f(a) \cdot f(b) \leq 0$ \checkmark C is so,

Now we can see $\exists c' \in]a, b[$ such that

$f(a) \cdot f(c') < 0$ i.e. $a' \in f^{-1}(0)$

$f(a) \cdot f(b) \leq 0$ \checkmark " $c = \frac{a+b}{2}$

$a' < c$, $f(a) \cdot f(c) \leq 0$ \checkmark

$\{a, c\} \cup \{c, b\}$ \Rightarrow $\exists c' \in f^{-1}(0)$

$\overbrace{\dots}^{\text{f}(x)}$

$f(c) \geq 0$
 $\underbrace{\leq 0}_{\text{f}(c)}$

$X_n - a \in \mathbb{Q}$ \checkmark $\exists c' \in f^{-1}(0)$
 $c \in \mathbb{R}$ \checkmark $\exists c' \in f^{-1}(0)$

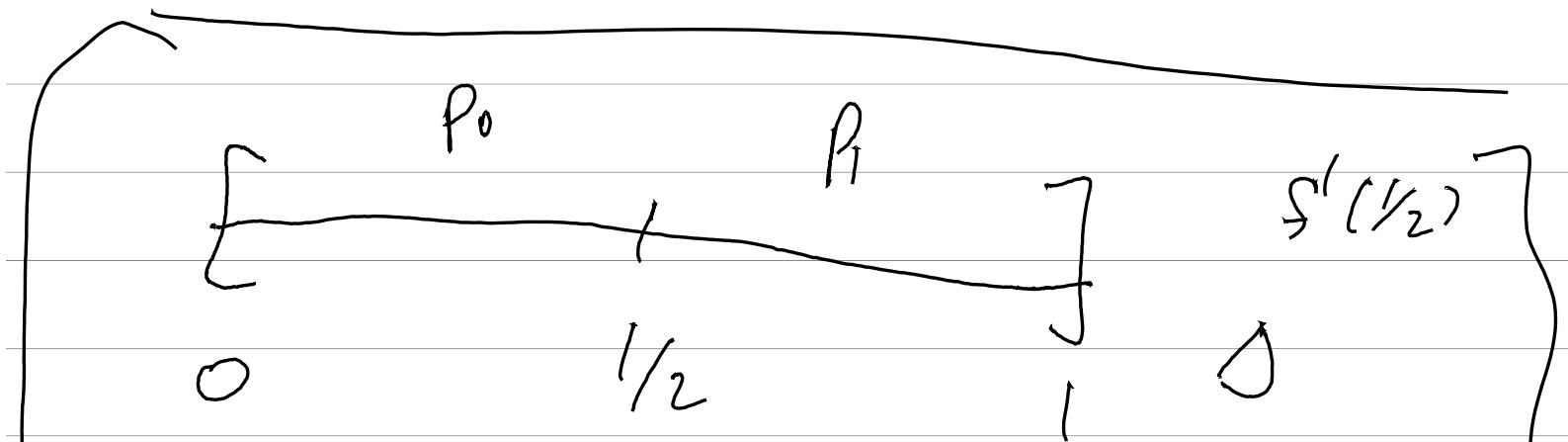
$$|x_n - a| \leq \frac{(b-a)}{2^n} = \varepsilon_n$$

→ $\rho \approx 2$

$$\frac{\varepsilon_{n+1}}{\varepsilon_n} \underset{n \rightarrow \infty}{\sim} \frac{1}{2}$$

\sqrt{c} $P > 30\%$ value

$$\cdot (P > 1 \text{ value}) \quad \overbrace{\varepsilon_{n+1} / \varepsilon_n}^{\rightarrow c > 0} \rightarrow c > 0$$



$$f \quad S \in S_3'(\Delta) \quad P_0'(1/2) = P_1'(1)$$

$$P_0(0) = f(0), \quad P_0(1/2) = f(1/2), \quad P_1(1/2) = f(1/2), \quad P_1(1) = f(1)$$

$$\cdot f(x) = 0 \quad [a, b]$$

$$x_i \rightarrow 0 \quad f(c) = 0$$

$$\left| \frac{x_{i+1}}{x_i} \right| < \varepsilon_i \quad \frac{\varepsilon_{i+1}}{\varepsilon_i} \rightarrow c < 1$$

$$r^l, c^l, \delta \sim \mathcal{O}(r^l)$$

$$p>k$$

$$\frac{\varepsilon_{i+1}}{\varepsilon_i} \rightarrow c > 0$$

p چون λ داشته باشد

1' 3' 1' 2' 3' 0' : 1' 1'
 f_0, \dots, f_n 1' 3'

For $n \in \mathbb{N}$ define $\varphi_{\{a,b\}}$ by
 $\{a,b\} \subset \text{range } \varphi_{\{a,b\}}^n$

$f_{n+1} = -f_n$, $f_{-1} = 0$ /not

(1') f_n a ND range ref)

$r \in \{a,b\}$ So $0 \leq i \leq n$ \exists

$f_{i+1}(r), f_{i-1}(r) \in S \cap f_i(r) = 0$ \rightsquigarrow c

or $\exists c \in \Gamma(x)$ $\begin{cases} \text{no } \\ \text{if } \end{cases}$

$f_0(x), \dots, f_n(x)$ 1' 3' 0' e range

1' (a)-oth b. f_n \in range $\exists c \in$ $\Gamma(x)$

ר \in $\{a, b\}$ \Rightarrow $f(r) \in \underbrace{\{a, b\}}$

ר \in $[c, d]$ \Rightarrow $f(r) \in [a, b]$

$[a, b] \rightarrow$ ר \in $[c, d]$

ר \in $[c, d]$ \Rightarrow $f(r) \in [a, b]$

ר \in $[c, d]$ \Rightarrow $f(r) \in [a, b]$

$\left[\begin{array}{c} f \\ x \end{array} \right] \in \left[\begin{array}{c} x \\ x \end{array} \right] \in \left[\begin{array}{c} x \\ x \end{array} \right] \in \left[\begin{array}{c} x \\ x \end{array} \right]$

ר \in $[a, b]$ \Rightarrow $f(r) \in [a, b]$

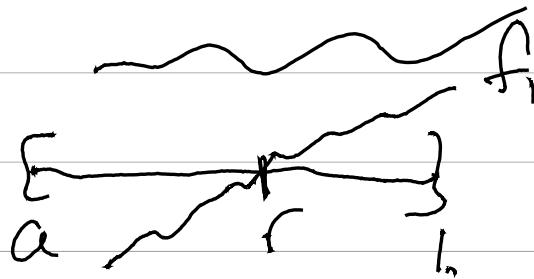
$r \in [a, b] \Rightarrow f(r) \in [f(a), f(b)]$

$n \in \mathbb{N}$ $\exists r \in \mathbb{Q}^+$ $\forall \epsilon > 0$ $\exists N \in \mathbb{N}$ $\forall n \geq N$

$$\checkmark n=0$$

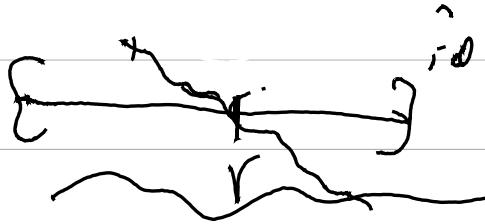
$\exists r \in \mathbb{Q}^+$ $\forall \epsilon > 0 \exists N \in \mathbb{N} : n \geq N \Rightarrow |f_n(x) - f(x)| < \epsilon$

$\rightarrow \text{AP/NH}$



$$\underline{f(a)=0}$$

$$\overline{f(b)=1}$$



$$f_1(r) = 0 \Rightarrow \underbrace{f_0(r) \cdot f_1'(r)}_{< 0} < 0$$

$$f_2'' = -f_1'(r)$$

$$f_0(r) < 0, f_1'(r) < 0$$

$\epsilon' \rightarrow n - 5$ ו L_0 מינימום f'

: מינימום f'

לפ x_1 x'_0 ל x_1 . $f_1(r) \neq 0$. A

רנ"ל יק"ר $f_1, \dots, \underline{f_{n+1}}$ ניגו,)

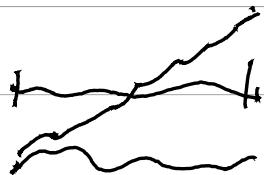
ר'ג'ג'ג' \cup , ר'ג'ג'ג' מ'ג

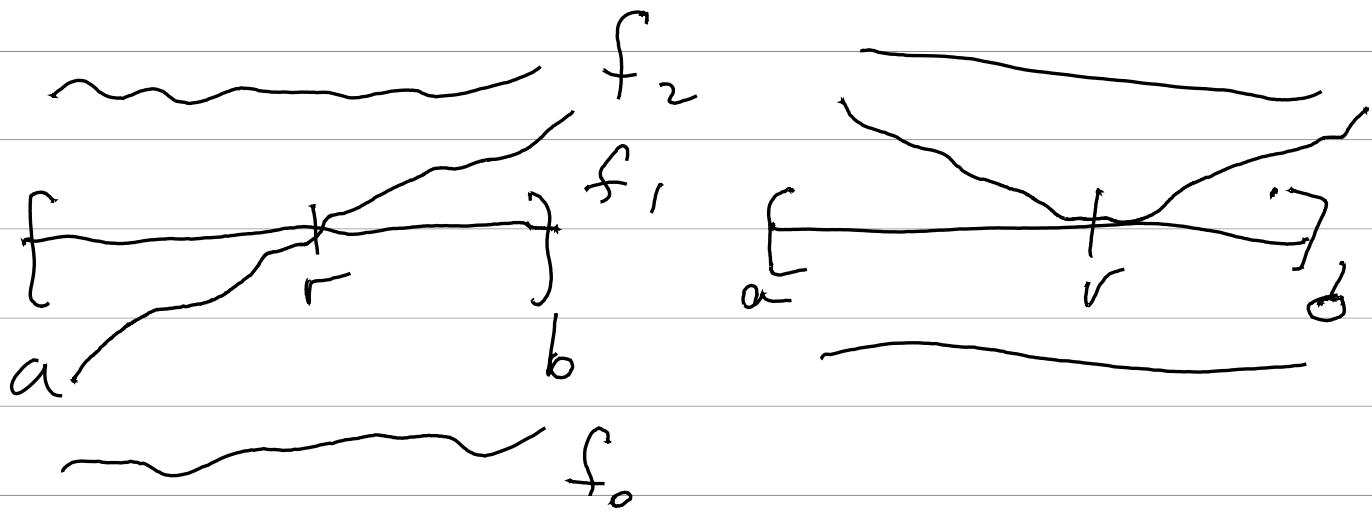
ר'ג'ג'ג' |I| 1-p f(1)

$f_0, f_1 \rightarrow$ C ר'ג'ג'ג' . $f_1(r) = 0$. 2

$f_0(r) \cdot f_1(r) < 0$ ר'ג'ג'ג' ס'

$f_2(r) \neq 0$ ס'ג





$\omega \cup \{f_i\}_{i=0}^n$ \sim $\wedge' \cup \{f_i\}_{i=0}^n$

\sim $\wedge' G/\mathcal{E}_d$ \sim $\wedge' L/\mathcal{E}$ \sim

$$f_{n-1} = p^1, f_n = p \quad \gamma' \text{?} \text{J}$$

~~$f_{k+1} = q_k \cdot f_k - f_{k-1}$~~

$$n \leq k-1 \Rightarrow \deg(f_{kn}) < \deg(f_k) \quad \text{reduces}$$

$$\vdash \forall n \forall j \forall i \forall l \forall m \forall p \forall q \forall r \forall s \forall t \forall u \forall v \forall w \forall x \forall y \forall z \forall k \forall l$$

רנינ גודל פולינום ב/c

בנין גודל פולינום ב/c
 $f_{n+1} \cdot f_{n-1} = -\underbrace{(f')^2}_{\geq 0} \leq 0$

המונומיאים $\{f_i\}$ נקראים מונומיאים של f .
 $\deg(f_i) = i$

המונומיאים $\{f_i\}$ נקראים מונומיאים של f .

$$f_{i+1} = (t-a_i) \cdot f_i - b_i f_{i-1}$$

$$\cdot b_i > 0 \rightarrow$$

$$\text{לפיכך } f_i(b) > 0, b > 0 \text{ ו } c \\ \sigma(b) \leq 0 \text{ ס'ו}$$

$\tau(a) \cap g(a) \subset \omega_1$

numeral i is f_i

ρ' / JC

τ_2' / c $\infty \delta \delta$ and $/ c$ $\rho \delta$

$\rightarrow f_{k+1} \in \omega_1$ $(3n)$

$\tau(a) \subseteq \omega_1$

$$x_{k+1} = \underbrace{x_k + x_{k+1}}_{\infty}, \in \omega_1$$

$\tau(x_{k+1}) \in \omega_1$

$\tau(x_{k+1}) \in \omega_1$

$\tau(x_{k+1}) \in \omega_1$

For $f(x) = 0$ if $\exists \gamma \in \text{sm}$

$\Rightarrow \exists \beta > \gamma$, f , $[a, b]$ $\forall f \geq 0$

$$f(a) \cdot f(b) < 0$$

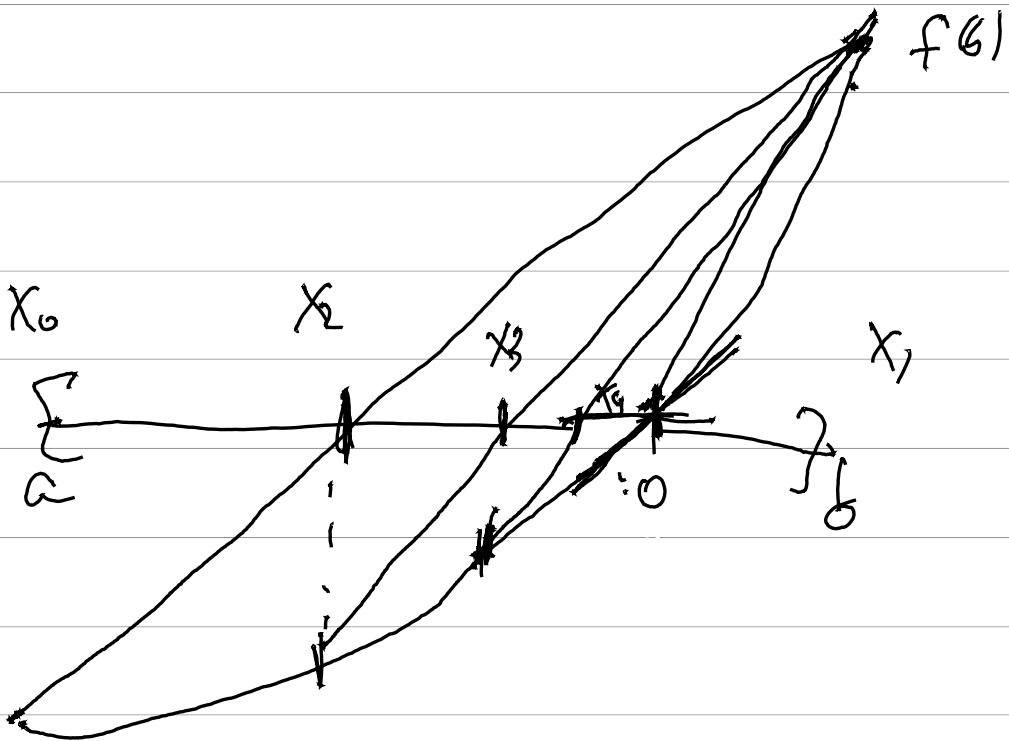


$$\frac{f(b)}{b-a}(x-a) + \frac{f(a)}{a-b}(x-b) = 0$$

\Downarrow

$$f(b)(x-a) - f(a)(x-b) = 0$$

$$x = \frac{f(b)a - f(a)b}{f(b) - f(a)}$$



$$x_{n+1} = \frac{f(x_n) \cdot x_{n-1} - f(x_{n-1}) \cdot x_n}{f(x_n) - f(x_{n-1})}$$

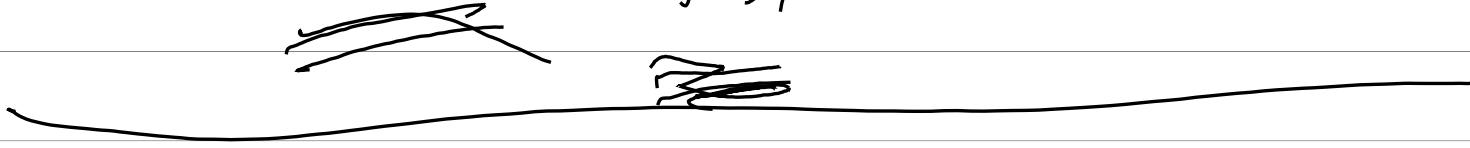
$$\frac{(f(x_n) - f(x_{n-1})) \cdot x_{n-1} + f(x_n) \cdot (x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

$$x_n' = f(x_n) \cdot \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$$

$$\frac{x_{n+1}}{x_n} = 1 - \frac{f(x_n)}{x_n} \cdot \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} =$$

$$1 - \frac{f(x_n)}{x_n} \cdot \frac{x_n - b}{f(x_n) - f(b)} \rightarrow :$$

$$1 - f'(0) \cdot \frac{b}{f(b)} =: c$$



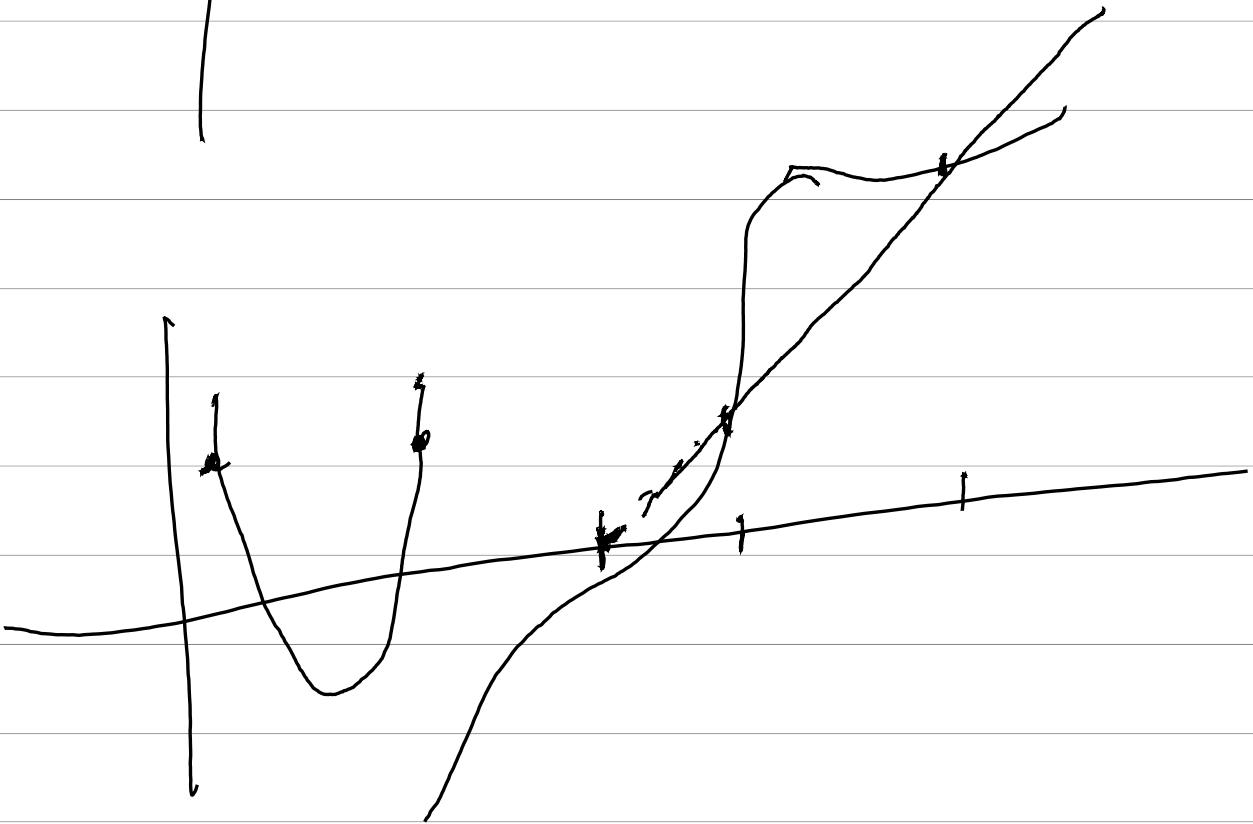
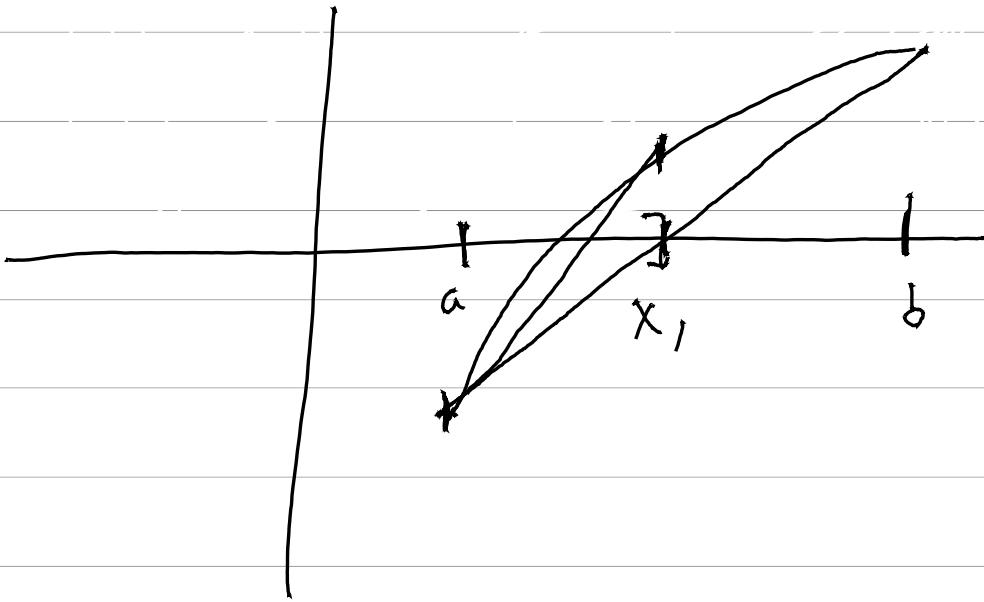
? $\subset \mathbb{N}$, $n+1 - ? \supset \mathbb{N}$

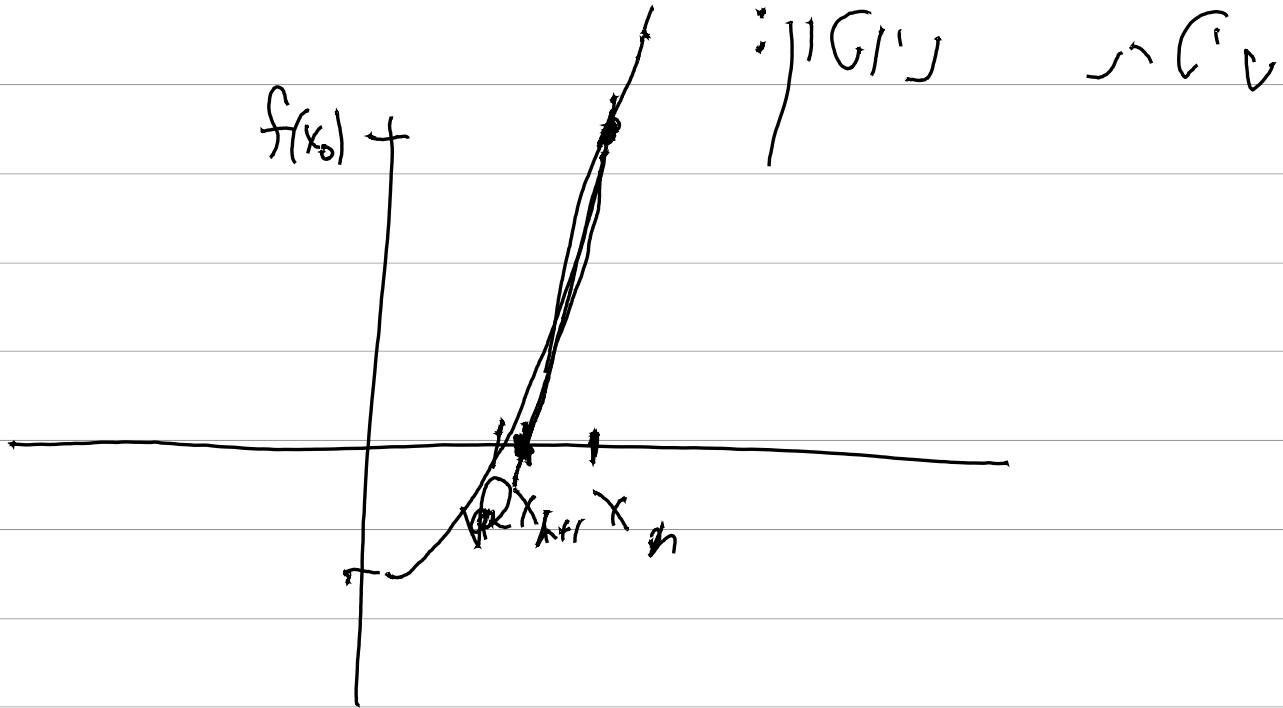
$$d = \frac{f(x_n) \cdot x_{n-1} - f(x_{n-1}) \cdot x_n}{f(x_n) - f(x_{n-1})}$$

$$x_{n+1} = d \quad \text{if } f(d) \cdot f(x_n) \leq 0 \text{ or } \\ x_{n+1} = x_n, \quad x_n = d \quad \text{if } f(d) \cdot f(x_n) > 0 \text{ or }$$

Secant -> $\cup C_1$.1

$| \cup C_1' \cup C_1$.2





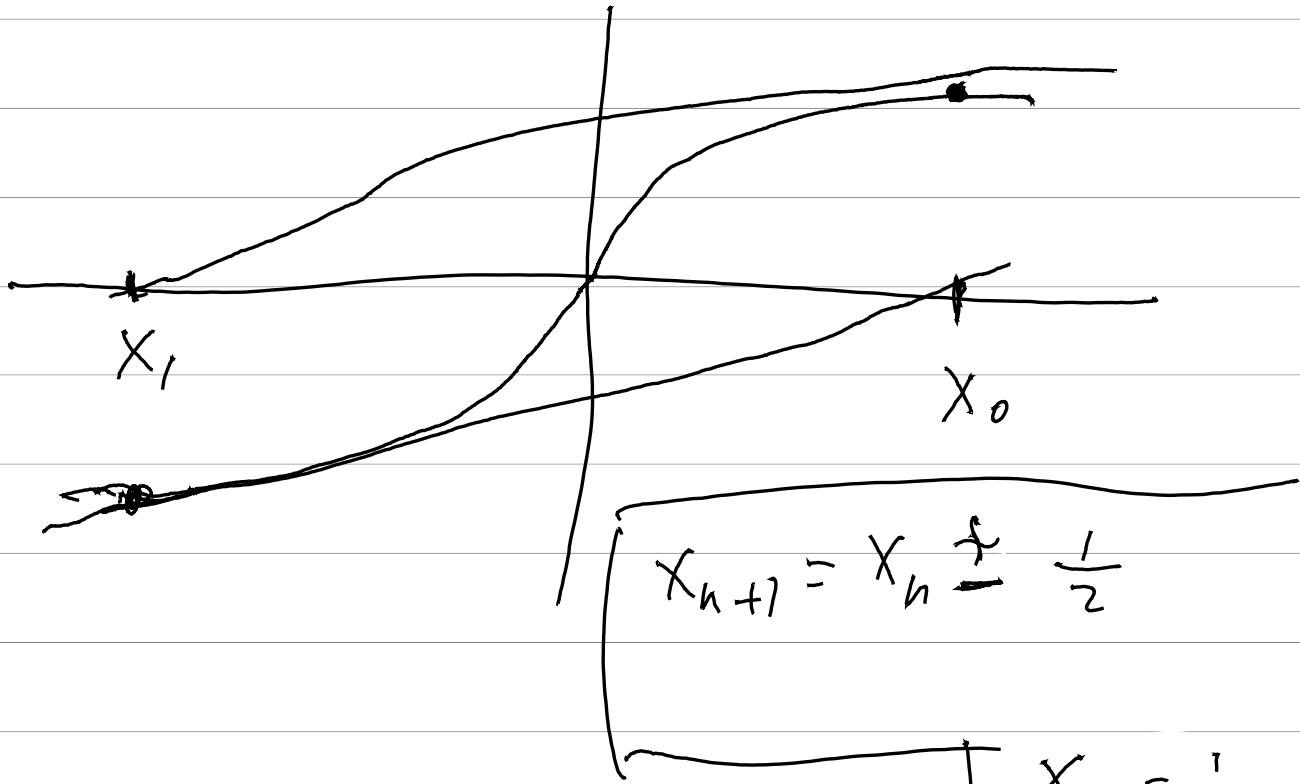
$$\left[\frac{x_{n+1} - x_n}{0 - f(x_n)} = \frac{1}{f'(x_n)} \right] \Rightarrow$$

$$x_{n+1} = x_n \circ \underbrace{\begin{pmatrix} f(x_n) \\ f'(x_n) \end{pmatrix}}_{\text{f(x_n)}}$$

\sqrt{a} src $\overbrace{c \wedge f^{-1} \circ}$ lceil d / 3

$$x^2 - a = 0 \quad (=) \quad a > 0$$

$$x_{n+1} = x_n - \frac{x_n^2 - a}{2x_n} = \frac{1}{2} \left(x_n + \frac{a}{x_n} \right)$$



$$f(x) = \begin{cases} \sqrt{x} & x \geq 0 \\ -\sqrt{-x} & x \leq 0 \end{cases}$$

$$x_0 = \frac{1}{4}$$

$$x_1 = -\frac{1}{4}$$

$$x_2 = \frac{1}{16} = \infty$$

$$f'(x) = \begin{cases} \frac{1}{2\sqrt{x}} & x \geq 0 \\ -\frac{1}{2\sqrt{-x}} & x \leq 0 \end{cases}$$

$$\varphi(x) = x - \frac{f(x)}{f'(x)}$$

$$x_{n+1} = \varphi(x_n)$$

Se f ist
auf \mathbb{R} stetig und
durchl.

$$\varphi(x) = x - \frac{f(x)}{f'(x)}$$

in \mathbb{R} mit
einem
Grenz

$$\text{in } M \text{ mit } (M, d) \text{ mit}$$

$\varphi: M \rightarrow M$ ist
stetig

$$d(\varphi(x), \varphi(y)) \leq c \cdot d(x, y)$$

$c > 0$ mit $x, y \in M$

$X_{n+1} = \varphi(X_n)$, $\forall n \geq 0$, \exists $\lim_{n \rightarrow \infty}$

$\Rightarrow \lim_{n \rightarrow \infty} X_n$

, $i \leq j$ $\Rightarrow \lim_{n \rightarrow \infty} X_i$

$d(X_i, X_j) \leq c^{|i-j|} \cdot d(X_0, X_{j-i})$

$d(X_0, X_k) \leq d(X_0, X_1) + \dots + d(X_{k-1}, X_k) \leq$

$d(X_0, X_k) / (1 + c + c^2 + \dots + c^{k-1}) = \frac{1-c}{1-c} d(X_0, X_k)$

$\Rightarrow d(X_i, X_j) \leq \frac{c^{|i-j|}}{1-c} d(X_0, X_1) \rightarrow 0$

$c' \subset [c] \cap \cup_{i=0}^{\infty} (X_i)_{i \geq 0} \subset$

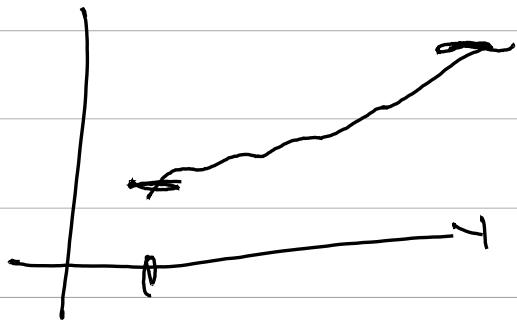
$(r^2, r^3, r^4) \subset \Delta \subset \cup_{i=0}^{\infty} (X_i)_{i \geq 0}$

$\varphi(\alpha) = \varphi(\lim_{i \rightarrow \infty} X_i) = \lim_{i \rightarrow \infty} \varphi(X_i) = \lim_{i \rightarrow \infty} x_i = \alpha$

$\text{Def}(\nu)$
 $\nu \in \mathbb{N}^{\mathbb{N}}$ $M \subseteq \mathbb{R}$ $\nu \in \mathbb{N}^{\mathbb{N}}$

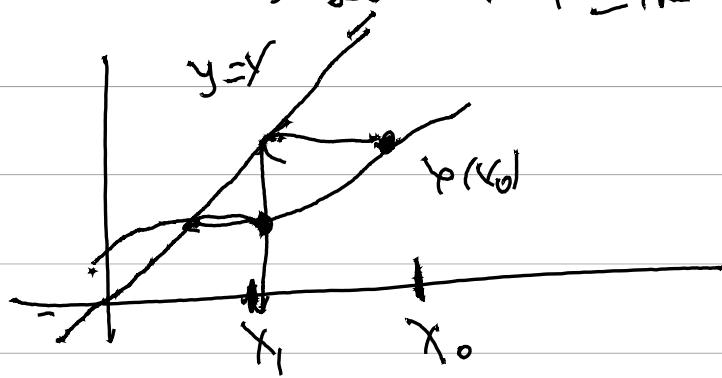
-1, No 3 to 7's & x

all N & if c. x<0 δ < x<c<1



f is surjective $\Rightarrow \forall v \exists n$]]

$\varphi: M \rightarrow N$
 φ is surjective $M \subseteq \mathbb{R}^n$ $\varphi(x)$



$x^p - \alpha^p$ $\neq 0$ $\forall \alpha \in C$

$$\varphi'(\alpha) = \varphi''(\alpha) = \dots = \varphi^{(p-1)}(\alpha) = 0 \quad -1$$

$$(\varphi \in C^p(M)) \cdot \underbrace{\varphi^{(p)}(\alpha) \neq 0}_{\text{---}} \quad -1$$

: p 1. α $\neq 0$ $\forall \alpha \in C$

$$\frac{x_{n+1} - \alpha}{(x_n - \alpha)^p} \rightarrow \frac{\varphi^{(p)}(\alpha)}{p!}$$

$$\varphi(x) = \alpha + \varphi'(\alpha)(x - \alpha) + \frac{\varphi^{(p-1)}(\alpha)}{(p-1)!}(x - \alpha)^{p-1} + \frac{\varphi^{(p)}(\alpha)}{p!}(x - \alpha)^p$$

$$\alpha + \frac{\varphi^{(p)}(u)}{p!}(x - \alpha)^p \Rightarrow u \in \{\alpha, x\}$$

$$x_{n+1} - \alpha = \frac{\varphi^{(p)}(u)}{p!} \cdot (x - \alpha)^p$$

לעסן גראונד

$$\varphi(x) = x - \frac{f(x)}{f'(x)}$$

$$\varphi'(x) = 1 - \frac{f'^2 - f \cdot f''}{f'^2} = \frac{f \cdot f''}{(f')^2}$$

$$\varphi'(x) = 0$$

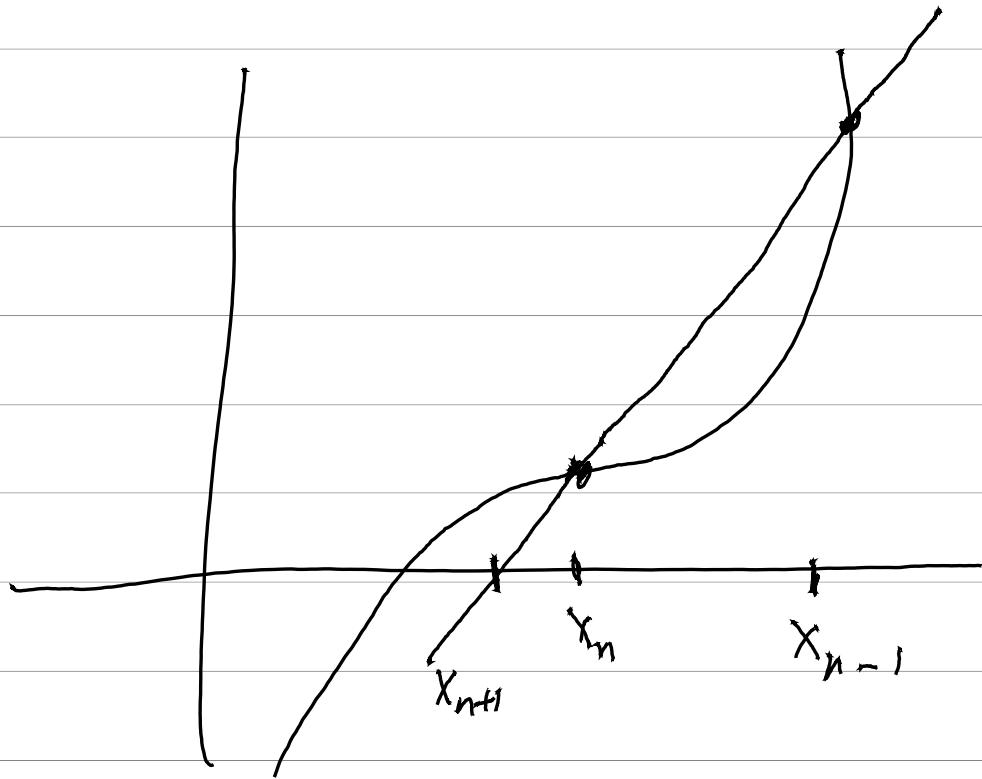
נתקה (ונרמזו) גראונד

ב' גראונד

$$u \text{ sod } \varphi'(u) \leq 0 \Leftrightarrow f'(u) \neq 0$$

ולא נרמזו גראונד

Secant \rightarrow C_V



$$\frac{x_{n+1} - x_n}{-f(x_n)} = \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$$

$$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \cdot f(x_n)$$

α (red) en ℓ' e $\wedge \vee$

$$x_{n+1} - \alpha = x_n - \alpha - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n) =$$

$$(x_n - \alpha) \left(1 - \frac{f(x_n)}{(x_n - \alpha)[x_n, x_{n-1}] f} \right) =$$

$$(x_n - \alpha) \left(1 - \frac{[x_n, \alpha] f}{(x_n, x_{n-1}) f} \right) =$$

$$(x_n - \alpha) \left(\frac{[x_n, x_{n-1}] f - [x_n, \alpha] f}{[x_n, x_{n-1}] f} \right) =$$

$$(x_n - \alpha)(x_{n-1} - \alpha) \cdot \left(\frac{[x_n, x_{n-1}, \alpha] f}{[x_n, x_{n-1}] f} \right)$$

$$(x_n - \alpha)(x_{n-1} - \alpha) \cdot M$$

$$m = \max_{t, s_Q} \left| \frac{f''(s)}{2f'(t)} \right| \rightarrow R_f, \rightarrow$$

∴ $f'(x_0)$

290 | { } 290 ↴

$$U_\alpha = \{x \mid |x - a| < \varepsilon\}$$

$$\text{If } \varepsilon \cdot n \leq 1 - \delta \quad \text{and} \quad x_{n+1}, x_n \in U$$

$\overbrace{\hspace{10em}}$

$$|x_{n+1} - x_n| \leq \varepsilon^2 n \leq \varepsilon \cdot 1 = \varepsilon$$

$$\therefore x_n \rightarrow d \Rightarrow n' \in \mathbb{N} \text{ s.t. } |x_{n'} - d| < \epsilon$$

$$|X_{n+1} - \alpha| \leq |X_n - \alpha| \cdot |X_{n-1} - \alpha| \cdot M \leq$$

$$\varepsilon \cdot |X_{n-1} - \alpha| \leq \varepsilon^2 (|X_n - \alpha|)^2$$

$$\underbrace{(\varepsilon \cdot M)}_1^n \cdot |X_1 - \alpha|$$

$$E_n = \underbrace{|X_n - \alpha|}_1 \cdot M$$

$$\frac{|X_{n+1} - \alpha|}{(|X_n - \alpha|)^P} = \frac{E_{n+1}/M}{\frac{E_n^P}{M}} \leq \phi$$

$$E_{n+1} \leq E_n \cdot E_{n-1}$$

$$\underbrace{P^2 = P + 1}_{||}$$

$$P = \frac{1 + \sqrt{5}}{2}$$

$$E_n \leq E^{P^n}$$

$$E = \max(E_0, E^{\frac{1}{P}})$$

$$E_{n+1} \leq E_n \cdot E_{n-1} \leq E^{P^n} \cdot E^{P^{n-1}} = E^{P^{n-1}(P+1)} = E^{P^{n+1}}$$

$$[x_0, x_1]f \doteq f'(u)$$

$$u \in \{x_0, x_1\}$$

$$[x_0, \dots, x_k] = \frac{f^{(k)}(u)}{k!}$$

$$u \in [a, b]$$

$$\therefore r^{\prime} N \cup S \rightarrow \int \parallel C's \quad \text{sum}$$

$$x_{n+1} = x_n - \underbrace{\frac{f(x_n)}{f'(x_n)}}$$

$$x^n + a \begin{cases} b_n = c_n = 1 \\ b_k = t b_{k+1} + c_k \\ c_k = t c_{k+1} + b_k \end{cases}$$

$$(x-t)(x^{n-1} + b_{n-1}x^{n-2} + \dots + b_1) + b_0 = P(x)$$

$$b_n = 1 \quad b_k = t b_{k+1} + c_k$$

3) 2. C.J.'s

$$f'(c_0)$$

$$c_0, \dots, c_n \quad f \rightsquigarrow P_{\bar{c}, f}$$

$$\underline{P_{\bar{c}, f}(x)} = [\bar{c}] f \cdot \underline{(x - c_0) \cdots (x - c_{n-1})} \dots$$

$$P^1(c_0) = [\bar{c}] f \cdot (c_0 - c_1) \cdots (c_0 - c_{n-1}) +$$

$$[c_0, \dots, c_{n-1}] f \cdot (c_0 - c_1) \cdots (c_0 - c_{n-1}) + \dots$$

$$f(x) = P_{\bar{c}, f}(x) + \underbrace{\frac{f^{(n+1)}(\xi(x))}{(n+1)!} \cdot (x - c_0) \cdots (x - c_n)}$$

$$f'(c_0) = P_{\bar{c}, f}(c_0) + \underbrace{\frac{f^{(n+1)}(\xi(c_0))}{(n+1)!} (c_0 - c_1) \cdots (c_0 - c_n)}$$

$$\text{�}\mathcal{F}_1, \quad h = \max\{|c_0 - c_i|\} \quad \forall c$$

$$h^u \rightarrow \underbrace{\text{near zero}}$$

$$c_1 = c_0 + h, \quad c_0 \quad \text{and} \quad \underbrace{\dots}$$

$$f'(c_0) = \frac{f(c_0+h) - f(c_0)}{h} + h \frac{f''(\cdot)}{2}$$

$$P_{\bar{c}, f}(x) = f(c_0) + \underbrace{(f(c_0) - f(c_1))}_{(c_0 - c_1)} \cdot (x - c_0)$$

$$f(c_1) - f(c_0)$$

$$h = c_1 - c_0$$

$$c_{-1} = c_0 - h, \quad c_1 = c_0 + h \quad -2$$

$$P_{\bar{C}, f}(x) = f(c_0) + [c_0, c_1]f \cdot (x - c_0) +$$

$$\underbrace{[c_0, c_1, c_{-1}]f}_{(x - c_0)(x - c_1)}$$

$$[c_0, c_1]f = \frac{f(c_1) - f(c_0)}{h}$$

$$[c_0, c_1, c_{-1}]f = \underbrace{[c_0, c_1]f - [c_0, c_{-1}]f}_{2h} =$$

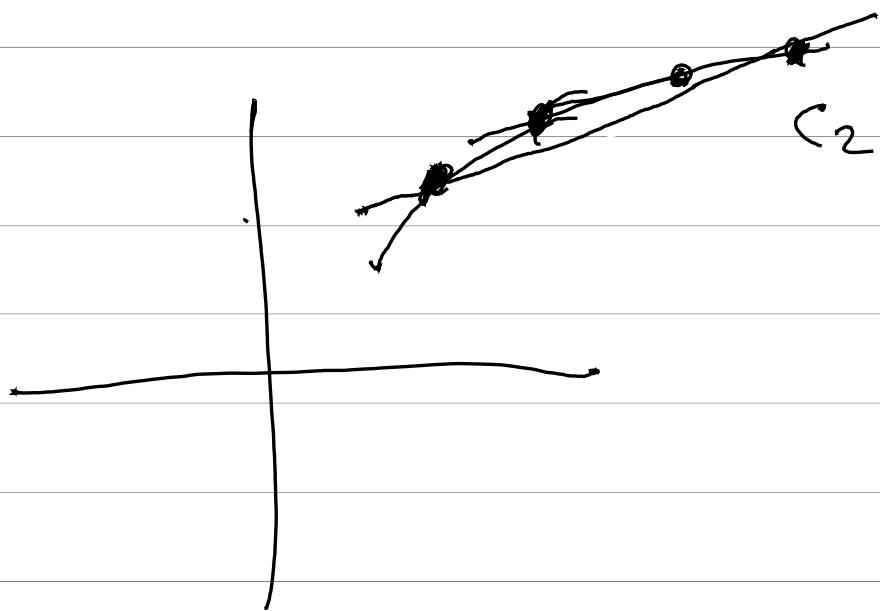
$$f(c_1) - f(c_0) + [f(c_{-1}) - f(c_0)]$$

$$\underbrace{- \frac{2}{2h^2}}_{=}$$

$$\left[\frac{f(c_1) - 2f(c_0) + f(c_{-1})}{2h^2} \right] P = \frac{f(c_1) - f(c_0)}{h} - \frac{f(c_0) - 2f(c_0) + f(c_{-1})}{2h} =$$

$$\boxed{f(c_1) - f(c_{n+1})}$$

2 h



$$e \approx \frac{f'''(\xi)}{6} \cdot h^2$$

$$f_1 = f(c_1) + \varepsilon \quad : \text{plus a small error}$$

$$f_{-1} = f(c_{-1}) + \varepsilon$$

$$f'(c_0) = \frac{f(c_1) - f(c_{-1})}{2h} + \ell_2 =$$

$$\frac{f_1 - f_{-1}}{2h} - \left(\frac{\varepsilon}{h} \right) + \ell_2 =$$

in regression analysis

$$E(h) = \boxed{M \cdot h^2} - \underbrace{\frac{\varepsilon}{h}}$$

$$h_0 \Rightarrow \left(\frac{\varepsilon}{2M} \right)^{1/3} \quad E(h_0) = \frac{3}{2} (2M)^{1/3} \cdot \varepsilon^{2/3}$$

Up now) if $\int \sqrt{f(x)^2} dx$ f no E

$$f'(x_0) = \frac{1}{2\pi i} \oint \frac{f(x)}{(x-x_0)^2} dx$$



$$\underbrace{\approx 3 \gamma \langle C_j \rangle}_k$$

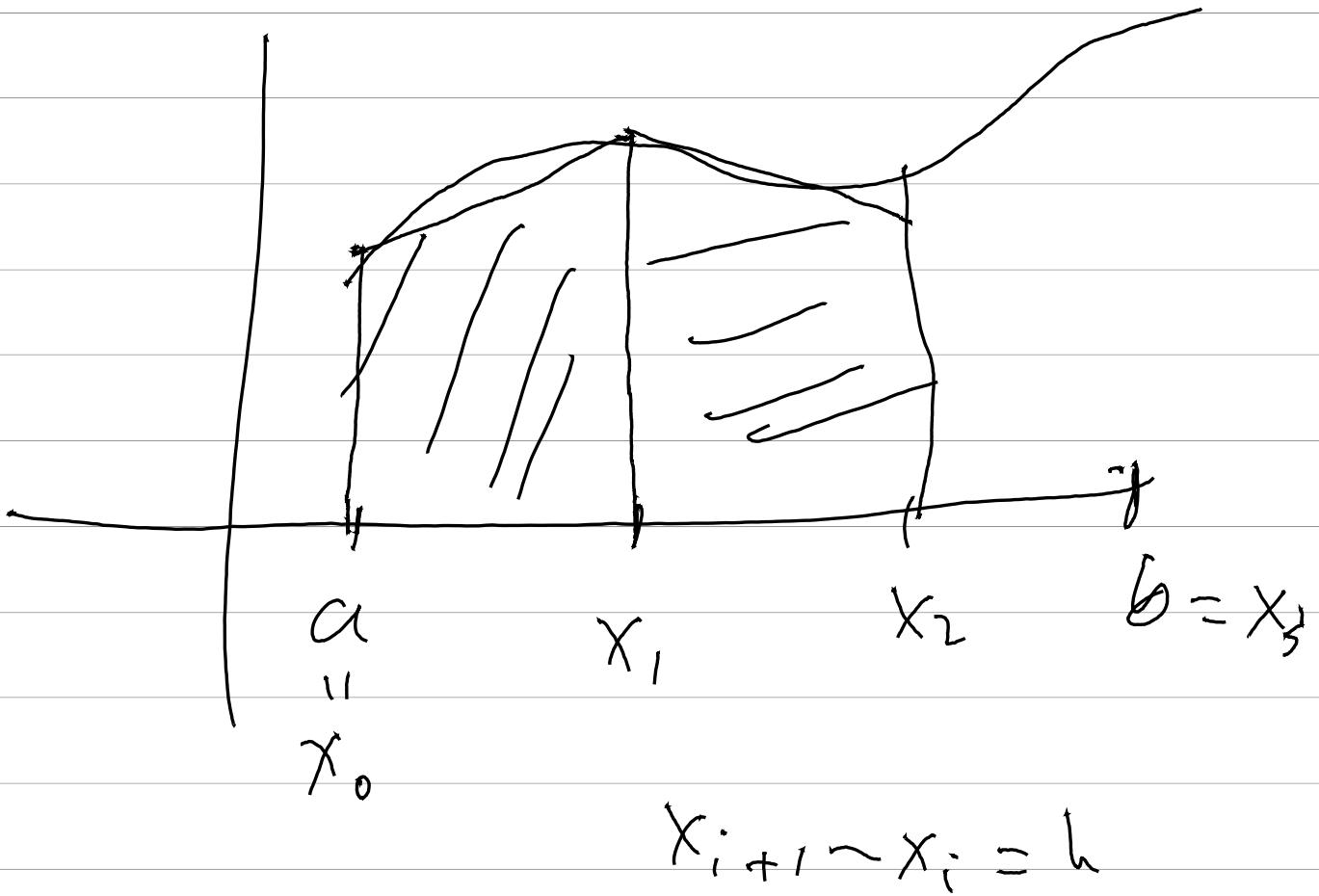
so it pers $\approx 3 \gamma$

$$\int_a^b f$$

$\rho^* \nu(f[a, b])$ so $\approx \int_a^b f$

$$a = x_0 < x_1 < \dots < x_n = b$$

$\approx \int_a^b$



$$\int_{x_i}^{x_{i+1}} f \approx \frac{f(x_{i+1}) + f(x_i)}{2} \cdot (x_{i+1} - x_i)$$

$$R_n(x) = \frac{f''(x_k)}{2} \cdot (x - x_i)(x - x_{i+1})$$

$$\int_{x_i}^{x_{i+1}} R_i(x) = \int_{x_i}^{x_{i+1}} (x - x_i)(x - x_{i+1}) dx.$$

$$\underbrace{\frac{f''(c_i)}{2}}_{2} = \frac{h^3}{12} \cdot f''(c_i)$$

$$\int_a^b f dx = h \cdot \left(\frac{1}{2} \cdot f_0 + f_1 + \dots + \frac{1}{2} f_n \right) +$$

$$\underbrace{-\frac{h^3}{12} \cdot \sum f''(c_i)}_{II}$$

$$E_h^T(f) = -\frac{h^2}{12} \cdot \underbrace{(b-a)}_n \sum f''(c_i) = -\frac{h^2}{12} (b-a) \cdot \frac{1}{n}$$

Numerical Integration

Simpson's rule for f
'y12' \approx '3'

x_{k+2}

$$\int_{x_i}^{x_{k+2}} f(x) dx = \frac{h}{3} (f_k + 4f_{k+1} + f_{k+2}) -$$

$$\overline{\int_a^b} h^S f^{(n)}(c)$$

$$\overline{\int_a^b} f = \frac{h}{3} (f_0 + 4f_1 + 2f_2 + \dots + f_n) +$$
$$E_n^S(f)$$

$$E_n^S(f) = -\frac{1}{180} \cdot (b-a) h^4 \cdot f^{(4)}(c)$$

$$[a, b] = [0, 2\pi]$$

$$E_n^T \cdot (e^{2\pi i kx}) = \int_0^{2\pi} e^{i kx} dx$$



$$\left(\frac{1}{2} e_k(0) + \sum_{i=1}^{n-1} e_k\left(\frac{2\pi i}{n}\right) + \frac{1}{2} e_k(2\pi) \right) \cdot \frac{2\pi}{n}$$

$$= 0 \quad k < n$$

結論 $\rightarrow f \in C$

$$f(x) = \sum_i a_i \cos(i x) + b_i \sin(i x)$$

$$E_F(f) = \sum_i a_i$$

$f \in C^r(\mathbb{R})$

$\forall c: \exists \beta / \gamma$

$$\underbrace{\int_{\text{mod } 0} f(x) dx}_{\approx a_i(f)} \leq c$$

i^{-r} / n^{γ}



$\approx 10^{10} \text{ rad}, \sqrt{5} \approx 2.2$

$\int_{\text{mod } 0} f(x) dx \approx \int_{\text{mod } 0} g(x) dx + \epsilon$

$\lambda \approx 132 \text{ nm}$

b

$$\int_a^b f(t) \underline{w}(t) dt = \sum_{k=1}^n w_k \cdot f(\underline{t}_k) + E_n(f)$$

$A =$

$$\begin{matrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ X & S \end{matrix}$$

A_1, A_{-1}

?

$$T: A \rightarrow A \quad T(f) = f \circ c \quad A,$$

$\underbrace{\mu}_{=1}$

$$T^2 = Id$$

$$\overbrace{T^2 = Id}$$

$$T \text{ 使得 } T \circ f = f$$

$$A_i = \text{span}_i \text{ 为 }$$

$$\begin{cases} T^m = Id \\ T^G = 0 \end{cases}$$

$$x + \frac{1}{x}$$

$$T(x) = \frac{1}{x}$$

$$A^m = \left\{ t \in \mathbb{C}[x, \frac{1}{x}] \mid t(x) = \sum_{i=-m}^m a_i x^i \right\}$$

$$\dim A^m = 2m+1$$

$$\left(x - \frac{1}{x} \right)^k \in A_1^m = A^m \cap A, \quad \begin{matrix} 0 \leq k \leq m \\ (x - \frac{1}{x}) \cdot (x - \frac{1}{x})^k \end{matrix}$$

רְאֵבָנָן רְאֵוֹן 'ז' סֶקְטָרִים 'ק

$$\int_a^b \underline{f(t)} \underline{w(t)} dt = \sum_{i=1}^n (\underline{w_i} f(t_i)) + E_n(f)$$

, ר' ג'ס 'ק) "פְּגַם" - w(t)

$$\int_a^b f(t) w(t) dt, \text{ ג'ס}$$

(f ~) \int_s

ר' ג'ס (ה) ה'גנ'ס (ק) ג'ס

: ה'גנ'ס (ק). t_i \rightarrow \int_s

m \geq \text{ה'גנ'ס} \text{ ס'גנ'ס} E_n(f) = 0

. ר' ג'ס m \rightarrow \rho y

$$1, \text{px} \sqrt{\lambda / 3} p^{1/2} \leq n - V$$

$$\psi(f) = \int_{\mathbb{R}} f(t) w(t) dt$$

WJS 28.07.2017

$\psi: V \rightarrow \mathbb{R}$

$$S_t(f) = (f(t_1), \dots, f(t_n))$$

$$S_T: V \rightarrow \mathbb{R}^n$$

$$\psi: \mathbb{R}^n \rightarrow \mathbb{R}$$

ψ

$\dim P_k = k$

P_k $k > n$ \Rightarrow $\dim P_k = n < k$ \Rightarrow

$$\psi|_{P_k} = \underbrace{\psi \circ S_T}_{\sim} |_{P_k}$$

$P_k \subseteq V$

13.3.

$$S_{\tilde{t}}|_{P_n} : P_n \rightarrow \underline{\mathbb{R}^n}$$

(3) \exists $\delta > 0$ $\forall \epsilon > 0$

$$\varphi = \psi \circ S_{\tilde{t}}^{-1} \quad \text{by } \text{SC}$$

. ($\exists \delta' \forall \epsilon < \delta'$) $\exists r > 0$ $\forall x \in U_r$

- $\exists \delta' \forall \tilde{t}$ $|t - \tilde{t}| < \delta' \Rightarrow$

$$\psi|_{P_{n+1}} = \varphi \circ S_{\tilde{t}}$$

$$\psi(f) = 0 \quad \text{f.f. ker } f \text{ is } \mathbb{R}^n, \text{ CTDR}$$

$\Leftrightarrow \forall x \in \mathbb{R}^{2n} \exists y \in \mathbb{R}^{2n} \quad f(x) = y$

$$S_{\tilde{t}}(f) = 0 \Rightarrow \varphi(f) = 0$$

$$\sum_{t=1}^T f(t) = 0 \Rightarrow \Psi(f) = 0 \quad f \in P_n$$

$$f(t_i) = 0 \quad \forall i \Leftrightarrow f = \pi_n \cdot g \quad g \in P_{n-1}$$

$$\pi_n(t) = (t-t_1) \dots (t-t_n) \quad \text{rec}$$

π_n は n 次の n 次の

$$\Psi(\pi_n \cdot g) = 0 \quad \text{rec}$$

$$l \leq n \quad g \in P_l \quad \text{は } l \text{ 次の}$$

π_n は n 次の

π_n は n 次の n 次の

は π_n は n 次の $n-1$ 次の

$(u, v) \mapsto \underline{\langle u, v \rangle} = \underline{\Psi(u \cdot v)}$ は n 次の

$a = -1, b = 1, \omega(t) \leq 1$.1 $\int_{\gamma} \int_{\Gamma} \int_{\Omega}$

35' $\int_{\Gamma} \int_{\Omega}$ \int_{Γ} $\rho' \partial(\gamma)$ \tilde{t}_n

$$\omega(t) = \frac{1}{\sqrt{1-t^2}}, a = -1, b = 1 - 2$$

π_n \int_{Ω} $\omega \rightarrow C_1$ t_i

$$\omega_i = \int_{(t-t_i) \cdot \pi_n^{-1}(t_i)}^{\pi_n(t)} w(t) dt$$

???

$\omega(t_i)$

$\Rightarrow T_{th}$ fe mehrere \exists -

, $\forall \epsilon / \exists N$

. $\forall \epsilon > 0 \exists N \in \mathbb{Z}$

$\Rightarrow \exists \delta_0 / \forall \text{curve } \rho / \int_{\rho} > \rho_0$

$$w_i = \sum w_i l_i^2(t_i) \stackrel{\text{def}}{=} \underbrace{\int_{t_i}^{t_{i+1}} \omega dt}_{\geq 0} > 0$$

$E_n(f) \xrightarrow[n \rightarrow \infty]{} 0$, f \mathcal{L}^f -

, $p_i \xrightarrow[i \rightarrow \infty]{} f$ $\Rightarrow \exists \delta' \forall i$

$\Rightarrow \exists N \forall i \geq N \quad p_i$

$$|E_n(f)| = |E_n(f - P_{2n-1})| =$$

$$\left| \int_a^b (f - P_{2n-1}) \omega(t) dt - \sum_{i=1}^n w_i \cdot (f(\xi_i) - P(\xi_i)) \right| \leq$$

$$\left| \int_a^b (\underline{f - P}) \omega(t) dt \right| + \sum_{i=1}^n w_i |f(\xi_i) - P(\xi_i)| \leq$$

$$\|f - P\|_\infty \left(\underbrace{\int \omega(t) dt}_{\| \quad \|} + \underbrace{\sum w_i}_{\| \quad \|} \right) =$$

$$\underbrace{\|f - P\|_\infty}_{\| \quad \|} \cdot 2 \cdot M_0$$

17. 1730 \wedge 17. 1730

8. 1730 \wedge P_K \sim 1730

$V_K \subseteq V_{K+1}$ \wedge 1730

$\overbrace{\text{NP} \text{ P12} \text{ 27-} - V}$

$\psi: V \rightarrow \mathbb{R}$

$\theta = \varphi \circ \tilde{s}: V \rightarrow \mathbb{R}$

$E = \psi - \theta$

$P \in P_K$ \wedge $E P = 0$

$f \in C^k((0, 5], \mathbb{R})$

$$f(x) = \sum_{i=0}^k a_i x^i + \frac{1}{k!} \int_0^x (x-t)^k f^{(k+1)}(t) dt$$

$$E(f) = \frac{1}{k!} E \int_0^x (x-t)^k f^{(k+1)}(t) dt =$$

$$\frac{1}{k!} E \int_0^x (x-t)_+^k f^{(k+1)}(t) dt$$

$$\frac{1}{k!} \int_0^x E((x-t)_+^k) f^{(k+1)}(t) dt$$

$$(x-t)_+ = \begin{cases} x-t & t \leq x \\ 0 & t > x \end{cases}$$

$$E(f) = \int_a^b K_k(t) f^{(k+1)}(t) dt \geq$$

$$K_k(t) \geq 0 \quad \forall t$$

$$\geq f^{(k+1)}(z) \cdot \int_0^1 K_k(t) dt$$

Numerical Linear Algebra

Folkmar Bornemann

A linear system of equations

$$A \bar{x} = b$$

row echelon form

row reduction

$$\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$$

$$c y = d_2 \Rightarrow y = \frac{d_2}{c}$$

$$A = \underbrace{L}_{\text{NLS}} U \quad \begin{cases} \text{IS} & \text{IBW} \\ \text{SIS} & \text{U} \end{cases} \quad \text{NLS}$$

این دست روش را می‌دانیم که
 $A = L U$

$$Ax = b \Rightarrow L U x = b \Rightarrow$$

$$Lc = b \quad \text{و} \quad Ux = c$$

از اینجا L -و U -و را می‌توانیم
 حل کرد.

این روش را می‌دانیم که

$$L' U' C' \Rightarrow \begin{cases} L' & \text{is} \\ U' & \text{is} \end{cases} \quad \frac{\text{inv}}{\text{inv}}$$

$$L_1 U_1 = L_2 U_2 \Rightarrow L_2^{-1} L_1 = U_2 U_1^{-1}$$

(P)

$$\underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}_{\text{Knell}} = \begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix} \begin{pmatrix} a & b \\ 0 & x \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix} \begin{pmatrix} a & b \\ 0 & x \end{pmatrix}$$

$[a \neq 0 \rightarrow \text{IJ}, a=0 \rightarrow \text{N}^T \cap \text{W}]$

$$Xa = c \Rightarrow X = \frac{c}{a}$$

$$bx + y = d \Rightarrow y = d - \frac{bx}{a}$$

$$A = L \cup =$$

$n \rightarrow$

$$(X | r_1) = (\alpha_1 | a_1)$$

$$X = \underbrace{\alpha_1}_{\neq 0} \quad \left| \begin{array}{c} 1 \\ 0 \\ \vdots \\ 0 \end{array} \right. \quad \left(\begin{array}{c} \alpha_1 \\ a_1 \end{array} \right)$$

$$\underline{l_1} = \underline{b_1 / \alpha_1}$$

$$B = L' \cup' \quad \text{and} \quad \left(\begin{array}{c} 1 \\ 0 \\ \vdots \\ 0 \end{array} \right) \in L'$$

$$B = D' - l_1 r_1$$

23. Find $\lambda^3 \text{adj } A$, if $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{pmatrix}$

$$-l \quad \} \Rightarrow P$$

$$PA = LU$$

Find the row U such that

1. All off-diagonal entries in
each row $|x| \leq 1$ are positive for

$$L \rightarrow x \rightarrow \text{Gauss Elim}$$

$$\begin{pmatrix} 0 & 1 & 2 \\ 3 & 0 & 1 \\ 5 & 1 & 2 \end{pmatrix} = P_1 \cdot \begin{pmatrix} 5 & 0 & 3 \\ 3 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix} =$$

$$P_1 \begin{pmatrix} 1 & 0 & 0 \\ 3/5 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 5 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 3/5 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -4/5 \\ 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -\gamma/\varepsilon \\ 1 & 2 \end{pmatrix} = P_2 \begin{pmatrix} 1 & 2 \\ 0 & -\gamma/\varepsilon \end{pmatrix}$$

$$P_1 \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & P_2 \end{pmatrix}$$

$\overset{11}{P}$

$$A = \begin{pmatrix} \varepsilon & 1 \\ 0 & 0 \end{pmatrix} \quad 0 < \varepsilon$$

$| \Gamma \rangle$

$$= \begin{pmatrix} 0 & 1 \\ 1/\varepsilon & 1 \end{pmatrix} \underbrace{\begin{pmatrix} \varepsilon & 1 \\ 0 & 1 - 1/\varepsilon \end{pmatrix}}_{\sim} \sim$$

$$\begin{pmatrix} 1 & 0 \\ 1/\varepsilon & 1 \end{pmatrix} \begin{pmatrix} \varepsilon & 1 \\ 0 & 1 - 1/\varepsilon \end{pmatrix} = \begin{pmatrix} \varepsilon & 1 \\ 1 & 0 \end{pmatrix}$$

Cholesky

1.5

: $\lambda^* \lambda^* \lambda / A = A^* - c$ $\lambda^* \lambda \lambda$

$\bar{x}^* A \bar{x} > 0$, $\bar{x} \neq 0$ Es

$(\bar{x}^* A \bar{x})^* = x^* A^* \bar{x} = x^* A \bar{x} \in \mathbb{R}$ $\text{s/c } A = A^* \text{ sym}$

$(\bar{x}, \bar{y}) \mapsto \bar{x}^* A \bar{y}$ symmetric

$\therefore \bar{x}^* \bar{y} = \bar{y}^* \bar{x}$

$\Rightarrow N \text{ sym} \quad \text{and} \quad \text{symmetric}$ s/c
 $\therefore \text{invertible}$

sic nach A $\delta(\cdot)$ ist

positive für $0 < \sqrt{c} < 1$

negativ für $\sqrt{3} < \sqrt{c} < 2$

$\sqrt{2}/h \approx 1.4$

je 2 e' 150 $\approx 3^{\circ} C/f$ s/c

$$A = L \cup$$

$$A = A^* = U^* L^*$$

negative $\sqrt{c} > 2$ ist ~~neg~~
-er $\sqrt{2}/h$ ist positive

$$L^* = U$$

$$(U^* = L \text{ und } f)$$

$$A = L \cdot L^* \quad ? \quad ? \quad ? \quad e' \quad \text{pf}$$

Zurück zu den Lernzetteln

Zur Position für den Kurs

. . .

IN 1, 13th Turnier ~ 2000

, LU PZ

$$\begin{pmatrix} \alpha & a_1 \\ & a_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \quad \begin{pmatrix} \alpha & a_1 \\ b_1 & a_2 \end{pmatrix}$$

ר'ז'נַן אֶ-כְּרִיּוֹן

אָגָרְטָה

$A: K^m \rightarrow K^n$

$K = \mathbb{R}/\mathbb{C}$

ר'ז'נַן אֶ-כְּרִיּוֹן

$A = Q R$

פְּרִזְבֵּת

ר'ז'נַן אֶ-כְּרִיּוֹן Q פְּרִזְבֵּת

ר'ז'נַן אֶ-כְּרִיּוֹן R: $K^m \rightarrow K^{m'}$
ר'ז'נַן אֶ-כְּרִיּוֹן A ינטְגָרְטָה

ר'ז'נַן A* A

$\Rightarrow \text{3IN3 } A^* A \underbrace{\text{ is inv.}}$

$\Rightarrow \|Ax\| = 0 \quad \text{SIC, } \Rightarrow \text{3f}$

$\bullet -x=0 \quad (\Rightarrow Ax=0)$

meric sic . y'n' n A-u n' j j

$A^* A = L \cdot L^*$ meric > 5

np' j . ? / Nen mehr L

$Q = A R^{-1} \quad ?/ ? \text{ , } R = L^*$

SIC. $Q^* Q = \text{Id} \sim \{ \}$

$Q^* Q = (R^{-1})^* A^* A R^{-1} = \underline{L^{-1} \cdot (L \cdot L^*) \cdot R^{-1}}$

$R \cdot R^{-1} = \text{Id.}$

Sei $A = Q, R,$ $\mathcal{S}_C, \mathcal{J}_L, \mathcal{B}_L$

$$A^T A = R_1^T Q_1^T Q_1 R_1 = R_1^T R_1$$

$$\begin{matrix} q' & n' \\ \nearrow & \searrow \\ \text{---} & \end{matrix}$$

Wieder $Q = \{q'_1, q'_2, \dots, q'_n\}$

Wieder $R = \{r_1, r_2, \dots, r_n\}$

Wieder $Q = \{q'_1, q'_2, \dots, q'_n\}$

$$A = Q \cdot R \Leftrightarrow \underbrace{A \cdot R^{-1}}_{= Q} = Q$$

$$R^{-1} = \begin{pmatrix} \alpha & \beta & \cdots \\ r & \ddots & \cdots \end{pmatrix}, \quad A = (a_1, \dots, a_n)$$

$$\alpha = \frac{1}{\|a_1\|}, \quad \beta = a_1 + \gamma a_2$$

$$A = (a_1, \dots, a_n)$$

: $\mathbb{R}^n \rightarrow \mathbb{C}^m$

$A = Q \cdot R = \underbrace{\begin{pmatrix} q_1 & q_2 & \dots & q_m \end{pmatrix}}_{Q} \cdot \underbrace{\begin{pmatrix} r_1 & & & \\ & r_2 & & \\ & & \ddots & \\ & & & r_m \end{pmatrix}}_{R}$

$$Q = (q_1, \dots, q_m)$$

$$A = Q R$$

$$R = \begin{pmatrix} \rho & \bar{r} \\ 0 & R' \end{pmatrix}$$

$$Q^* A = R$$

$$(a_1, \dots, a_n) = (q_1, \dots, q_m) \cdot$$

$$\rho \cdot q_1 = a_1 \Rightarrow \rho = \|a_1\|, q_1 = \underline{a_1 / \rho}$$

$$q_1 \cdot a_i = r_i \quad Q \cdot A = (\rho, \bar{r})$$

$$\therefore \exists g \in Q^* \ni g \text{ "converges" } \rightarrow Q \cdot g = (1, 0)$$

$$A - q_1 \cdot r_i = Q^* \cdot R'$$

数值解法 为 线性方程组

收敛性

$$(A \in \mathbb{R}^{n \times n}) \quad Ax = b$$

$$\text{cond}(A) = k(A)$$

$$k(A) = \|A^{-1}\| \cdot \|A\|$$

b 为系数矩阵 A 的右端项 - b*

$$\frac{\|b^* - b\|}{\|b\|}$$

$$\frac{\|A^{-1}b^* - A^{-1}b\|}{\|A^{-1}b\|} = \frac{\|A^{-1}(b^* - b)\|}{\|A^{-1}b\|} \leq \frac{\|A^{-1}\| \|b^* - b\|}{\|A^{-1}b\|} =$$

$$\frac{\|A^{-1}\| \cdot \|b\|}{\|A^{-1}b\|} \cdot \frac{\|b^* - b\|}{\|b\|} = \frac{\|A^{-1}\| \cdot \|A \cdot A^{-1} \cdot b\|}{\|A^{-1}b\|} \cdot \frac{\|b^* - b\|}{\|b\|} \leq$$

$$\leq \kappa(A) \cdot \frac{\|b^* - b\|}{\|b\|}$$

$$\|Av\| \leq (\|A\| \cdot \|v\|)$$

$$\|A\| = \sup_{v \neq 0} \frac{\|Av\|}{\|v\|} = \sup_{v \in C} \|Av\|$$

$A \in \mathbb{R}^{n \times n}$

$$\frac{\|\tilde{A}^{-1}b - \tilde{A}'^{-1}b\|}{\|\tilde{A}'^{-1}b\|} = \frac{\|\tilde{A}' \cdot \tilde{A} \tilde{A}'^{-1}b - \tilde{A}' \cdot \tilde{A} \tilde{A}'^{-1}b\|}{\|\tilde{A}'^{-1}b\|} =$$

$$\frac{\|A'(\tilde{A} - A)\tilde{A}'^{-1}b\|}{\|\tilde{A}'^{-1}b\|} \leq \|A'\| \|A - \tilde{A}\| \|\tilde{A}'^{-1}b\|$$

$$\frac{\|A'\| \cdot \|A\| \cdot \|\tilde{A} - A\|}{\|A\|} = \kappa(A) \cdot \frac{\|\tilde{A} - A\|}{\|A\|}$$

$$\|ABv\| \leq \|A\| \|Bv\| \leq \|b\| \|B\| \|v\|$$

: (Kahan) NYC

$$\frac{1}{\kappa(A)} = \min \left\{ \frac{\|A - \tilde{A}\|}{\|A\|} \mid \begin{array}{l} \text{for } \tilde{A} \\ \text{such that} \end{array} \right\}$$

, נס'ה ו'ז'ז ל'ז'ז

$$\frac{\|A - \tilde{A}\|}{\|A\|} \geq \frac{1}{\|A\| \cdot \|\tilde{A}^{-1}\|}$$

$\rightarrow \sqrt{3} \approx \sqrt{6}$

$$\|\tilde{A}^{-1}\| \cdot \|A - \tilde{A}\| \geq 1$$

$\delta \omega / \zeta$

$$\|A^{-1}\| \cdot \|A - \tilde{A}\| \geq \|A^{-1}(A - \tilde{A})\| =$$

$$\|I - A^{-1} \tilde{A}\|$$

$$\sim c \quad \sqrt{\text{cond}} \quad | \text{down } \leq$$

$$\|I - A^{-1} \tilde{A}\| \geq 1$$

$\exists c \quad 0 \neq v \in \ker(A) \subset \mathbb{R}^n$

$$\|I - A^{-1} \tilde{A}\| \geq \|(I - A^{-1} \tilde{A})v\| / \|v\| = \|v\| / \|v\|$$

$\Rightarrow \text{down } \text{rcf } \tilde{A} \quad S3 \quad 4$

$$\|A - \tilde{A}\| = \frac{1}{\|A^{-1}\|}$$

$$w = A^{-1}v, \|A^{-1}v\| = \|A^{-1}\| \quad \begin{matrix} \sim c \\ -1 \end{matrix} \quad \begin{matrix} \checkmark \\ \|v\|=1 \end{matrix}$$

per sen V dann nur mehr J

$$\tilde{A} w = 0 \quad \gamma' \beta \beta' \omega - \delta$$

$$A|_U = A|_{U^{-1}}$$

$$\underbrace{\|A - \tilde{A}\|}_{\|w\|} \underbrace{||(\tilde{A} - A) \cdot w||}_{\|w\|} = \frac{\|v\|}{\|A^{-1}v\|}$$

$$\frac{1}{\|A^{-1}\|}$$

$$\sup_{\substack{\|cw+u\|=1 \\ u \in U}} \|(\tilde{A} - A)(cw + u)\| = \sup \|(\tilde{A} - A)cw\| =$$

$$\sup \|Aw\| / \|w\|$$

প্রার্থনা করুন কি হচ্ছে

মনসা'র প্রয়োগ নিয়ে একটি ফ

প্রার্থনা - f

e' এবং , y টাকা মুদ্রা

~~$f(y) = f(y^*)$~~

-e \rightarrow y^*
 $(y \text{ সংজ্ঞা})$

মনসা'র ক্ষেত্রে, কোন কোন ক্ষেত্রে

cond(f)(y) এর ক্ষেত্রে নিম্নলিখিত দুটি ক্ষেত্র

প্রক্রিয়া ও ফর্মাট
কানুন

$a^* b = a \cdot b$ ক্ষেত্র নথি

• $\lambda \circ f \circ g$: $\mathbb{R}^n \rightarrow \mathbb{R}^m$ injektiv

$\gamma \Gamma \vdash P \quad \exists' x \in C \quad \lambda b \geq n$

$\therefore \exists' x' \in C \quad \lambda f \vdash x$

$\exists' x' \in C \quad \lambda f \vdash x' \in C \quad f \geq x$

$$\begin{pmatrix} a \\ b \end{pmatrix} \hat{\cdot} (c d) = \begin{pmatrix} * & * \\ * & * \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} \cdot (c d)$$

$\text{det} \neq 0$

 — — —

$$\underbrace{f = h \circ g}_{\rightarrow \cup} \quad \wedge \quad \wedge' \cup$$

(1) $\lambda x \vdash \exists' y \in C \quad f$

$$\begin{array}{l} \hat{f} = h \circ f \circ g \\ \hline \lambda x \vdash \exists' y \in C \quad f \end{array}$$

$\mathcal{N}(x^*)$ of $\mathcal{M} \cap \mathcal{S}_{\lambda^*}$ if

$\nabla f(x^*)$ is non-zero.

then $f^{-1}(g(\mathcal{S}_{\lambda^*}))$ is non-empty.

$\nabla f(x^*)$ is zero.

Proof of $\mathcal{M} \cap \mathcal{S}_{\lambda^*} \neq \emptyset$

$\|\lambda^*\|_{L^2}$

Suppose x^* is a point in $\mathcal{M} - \mathcal{X}$

$$y^* = \tilde{x}, \quad \hat{f}(y) = \tilde{x}$$

Then $y^* \in \mathcal{S}_{\lambda^*}$

$$\omega(\tilde{x}) = \min \left\{ \frac{\|\tilde{x} - A\|}{\|A\|} \mid \tilde{x} = b \right\} =$$

$$\min \left\{ \frac{\|E\|}{\|A\|} \mid (A+E)\tilde{x} = b \right\}$$

Rigal-Gaches) Dirac

$$w(\tilde{x}) = \frac{\|b - Ax\|}{\|A\| \|x\|}$$

rn/r
 rn/r
 ——————
 25% 50% 100%

$$\frac{\|x - \tilde{x}\|}{\|\tilde{x}\|} = \frac{\|\tilde{A}'b - \tilde{A}'x\|}{\|\tilde{x}\|} \leq$$

$$\|\tilde{A}^{-1}\| \cdot \frac{\|b - Ax\|}{\|\tilde{x}\|} = k(A) \cdot w(\tilde{x})$$

દ્વારા કે ઉપરાંતે એવી વિધિ,

$$f = \underline{\underline{h}} \circ \underline{\underline{g}}$$

જે

$$\underline{\underline{g(A)}} = \begin{pmatrix} \underline{\underline{m}}, \underline{\underline{N}} \end{pmatrix}, \quad m \cdot N = A$$

$h(m, n, b)$ - સંગ્રહ પોંગ

- 1

સિલેક્શન
 $\checkmark - h$

નું જો એવું હોય તો એવી વિધિ ફ

ચેક કરી શકતું હોય -
એવું હોય -

جذب الماء

جذب الماء

$$\frac{\|\tilde{M} \cdot \tilde{N} - M \cdot N\|}{\|M \cdot N\|} =$$

$$\frac{\|\tilde{M} \cdot \tilde{N} - M \tilde{N} + M \tilde{N} - M \cdot N\|}{\|M \cdot N\|} \leq \left(\frac{\|\tilde{M} - M\| \|\tilde{N} - N\|}{\|M\|}, \frac{\|\tilde{M}\|}{\|M\|} \right)$$

$$\|\tilde{M} - M\| \cdot \|\tilde{N}\| + \|M\| \|\tilde{N} - N\|$$

$$\frac{\|\tilde{M} - M\| \cdot \|N\| \|\tilde{N}\| + \|M\| \|\tilde{N}\| \cdot \|\tilde{N} - N\|}{\|M \cdot N\|} = 2 \cdot \frac{\|M\| \|\tilde{N}\| \|\tilde{N} - N\|}{\|M \cdot N\|}$$