

principles

Walter Gautschi

maths prof in 1952 - 1970

... member of the ICM, 1970

$f(x) \Rightarrow f: \mathbb{R} \rightarrow \mathbb{R}$

$f: \mathbb{R}^n \rightarrow \mathbb{R}$

integrals, numerical methods -  
books, etc., about numerical methods  
including numerical analysis, 1970

numerical methods: numerical integration -  
numerical differentiation, numerical solution of differential equations  
(numerical methods)

Wicca. Wicca

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

23) If  $\gamma$  place  $\rho'_{\text{cond}} - \gamma^2 / \rho$   $IR^* \subseteq R$

$0 \in \mathbb{R}^*$ ,  $x^* \in \mathbb{R}^*$   $\mapsto$   $|x| - 15$  e,  $x \in \mathbb{R}$

$$\therefore 0^{\alpha} = 0$$

$$\frac{|x - x^*|}{|x|}$$

$f(x)$   $\approx$   $\frac{1}{2}x^2 + \frac{1}{3}x^3 + \dots$

うれしいこと  
× と どうして そこ が

? なぜか なぜか

$$f(x^*) = f(x'), \quad f(x), \quad x, x^*$$

$$\frac{|x' - x|}{|x'|} |x - x'| \sim y,$$

$$\left| \frac{f(x^*) - f(x)}{f(x)} \right| = \left| \frac{f(x') - f(x)}{f(x)} \right| =$$

$$\left| \frac{(f(x') - f(x))(x' - x)}{(x' - x) \cdot f(x)} \right| \leq \frac{|x^* - x|}{|x|}.$$

$$\left| \frac{(f(x') - f(x)) \cdot x}{(x' - x) \cdot f(x)} \right| \approx \left| \frac{x f'(x)}{f(x)} \right| \cdot \frac{|x^* - x|}{|x|}$$

$f \in$  23rd condition interval

$\kappa$  (condition number)  $x \rightarrow$

$$\text{cond}(f)(x) = \left| \frac{x \cdot f'(x)}{f(x)} \right|$$

$(x, f(x) > 0 \rightarrow \text{interval } \text{near})$

,  $f(x) = ax+b$  case 1

$$\text{cond}(f)(x) = \left| \frac{x \cdot a}{ax+b} \right| = \left| 1 - \frac{b}{ax+b} \right|$$

" for 2nd part case 2

$$I_n = \int_0^1 \frac{t^6}{t+5} dt$$

$$I_0 = \int_0^1 \frac{dt}{t+5} = \left[ \ln(t+5) \right]_0^1 = \ln\left(\frac{6}{5}\right)$$

$$I_{n+1} = \int_0^1 \frac{t^{n+1}}{t+s} dt = \int_0^1 t^n \cdot \frac{t+s-s}{t+s} dt =$$

$$-5 \int_0^1 \frac{t^n}{t+s} dt + \left. \frac{t^{n+1}}{n+1} \right|_0^1 = -5 I_n + \frac{1}{n+1}$$

$\sum_{n=0}^{\infty}$

$$I_n = f_n(I_0) \quad \underline{f_n(x) = (-5)^n x + b_n}$$

$b_n \in \mathbb{R}$  ( $\rightarrow \mathbb{C}^1$ ,  $\text{vgl.}$ )

$$(\text{and } f_n)(I_0) = \left| \frac{\int_0^1 f_n(t) dt}{I_0} \right| =$$

$$s^n \cdot \left| \frac{I_0}{I_n} \right| \geq s^n$$

$$I_n = \underbrace{I_{n+1} - \frac{1}{n+1}}_{= 5}$$

$$k \gg n$$

$$I_n \approx g_n(I_k) \quad n-k < 0$$

$$g_n^{(k)} = (-5)^{\overbrace{k}^{n-k}} x + c_n$$

$$\text{cond}(g_n)(I_k) = \left| \frac{I_k \cdot (-5)^{n-k}}{\sum_n} \right| =$$

$$5^{n-k} \left( \frac{I_k}{\sum_n} \right) \leq 5^{n-k}$$

$a_1$

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$$(x_1^*, \dots, x_n^*) \hookrightarrow (x_1, \dots, x_n)$$

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$\sim$   $\| \cdot \| : V \rightarrow \mathbb{R}_{\geq 0}$   $= \{ x \in \mathbb{R} / x \geq 0 \}$

$V = 0 \quad \|V\| = 0 \quad \|V\| = 0 \quad .1$   
 $\|av\| = |a| \cdot \|v\| \quad \forall v \in V, a \in \mathbb{R} \quad .2$   
 $\forall v \in V \quad .3$

$$\|u+v\| \leq \|u\| + \|v\|$$

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$\mathbb{R} \ni p \geq 1$ ,  $V = \mathbb{R}^d$  if  $p \geq 1$

$$\| \langle x_1, \dots, x_d \rangle \|_p = \sqrt[p]{\sum |x_i|^p}$$

/ euclidean norm)

$$\| \langle x_1, \dots, x_d \rangle \|_1 = \sum |x_i|$$

$$\| \langle x_1, \dots, x_d \rangle \|_2 = \sqrt{\sum x_i^2}$$

" $p = \infty$ "

$$\| \langle x_1, \dots, x_d \rangle \|_\infty = \sup \{|x_i|\}$$

, for  $\| \cdot \|$  euclidean norm  $\Rightarrow$  distance

$$d(u, v) = \| u - v \|$$

now  $(v_i)$ ,  $i \sim 30$   $v_i \in V$  is

$$\cdot \| v_i - v \| \rightarrow 0 \quad \forall \epsilon \quad \forall \delta$$

设  $\gamma/\Gamma_\beta$  为  $\gamma$  的  $\beta$  次方根

$\|\cdot\|_2$ ,  $\|\cdot\|_1$ ,  $\|\cdot\|_\infty$  为

某种范数的  $\beta$  次方根

$v \in V_{\beta-1} (V_i)$  为  $V_i$  中的

$\|v_i - v\|_2 \rightarrow 0$  则  $\|v_i - v\|_1 \rightarrow 0$

由  $\gamma > 0$  及  $\gamma^2 < 1$

$\frac{1}{c} \|v\|_1 \leq \|v\|_2 \leq c \|v\|_1$ ,  $\forall v \in V$

$\mathbb{R}^d$  为  $\mathbb{R}^d$  上的  $\beta$  次方根

由  $\gamma > 1$

$\mathbb{R}^d$  为  $\mathbb{R}^d$  上的  $\beta$  次方根

$$\frac{\|x^* - x\|}{\|x\|}$$

רעיון  $T: V \rightarrow V$  ו-  $x'$

$$\| \cdot \|_V \text{ גודלה } \cup \text{ אוסף } \sim, \omega'$$

$$V \text{ סט } \| \cdot \|_V \text{ -י } V \text{ סט}$$

השאלה היא אם  $\|x^*\|_V \leq \|x\|_V$

: רעיון גודלה  $T$  כפונקציית

$$\frac{\|Tx^* - Tx\|_V}{\|Tx\|_V} = \frac{\|T(x^* - x)\|_V}{\|T(x)\|_V} =$$

$$\frac{\|T(x^* - x)\|_V}{\|T(x)\|_V} \cdot \frac{\|x^* - x\|_V}{\|x\|_V} \leq \frac{\|T\| \cdot \|x\|_V}{\|T(x)\|_V} \cdot \frac{\|x^* - x\|_V}{\|x\|_V}$$

יעי גורן  $T: U \rightarrow V$

רְאֵבָבָה, נַנְנָה 'מִקְמָה' יְכָלָה

$$\|T\| = \sup_{x \neq 0} \frac{\|T(x)\|_v}{\|x\|_v} = \sup_{\{x | \|x\|_v = 1\}} \|T(x)\|_v$$

$$\text{N}^{\text{H}}\text{O}\text{S} \text{S} \text{N} \text{P} \text{O} \text{S} \text{N} \rightarrow \text{Hom}(\underline{\text{U}}, \text{V})$$

$\gamma \circ \gamma^{-1}(T) \mapsto ||T||$  .  $\gamma \circ \gamma^{-1}(T)$

גָּדוֹלָה מְאֻמָּרָה

†  $\approx$  23m soon 1'2J

لـ (نـ) X الـ (جـ)

$$\text{cond}(\tau)(x) = \frac{\|\tau\| \cdot \|x\|}{\|\tau(x)\|}$$

$x = T^{-1}(y)$  เมื่อ  $y$  เป็นค่าของ  $T$  และ

$\text{cond}(\tau) :=$

$$\sup_x \text{cond}(\tau)(x) = \|\tau\| \cdot \|\tau^{-1}\| \quad \text{sic!}$$

zu  $\|\tau\|$  kann man nur  $\|\tau x\|$

und  $\|\tau^{-1}x\|$  für  $\tau x = b$

$b$  ist die ursprüngliche Vektoren

?  $b^*$  ?

:  $(\tau \circ \delta, \tau \circ \gamma)$

$$\tau_n = \begin{pmatrix} 1 & \frac{1}{2} & \dots & \frac{1}{n} \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix}$$

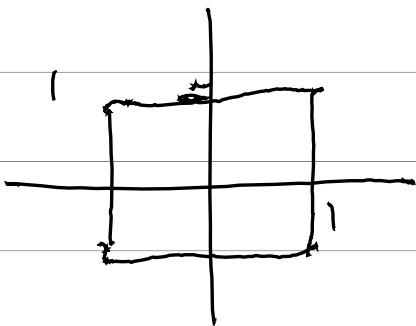
$$\text{Cond}_2 \tau_n = \frac{(V_2 + 1)^{n+4}}{\sum_{k=1}^{n+4} k \cdot \sqrt{\tau_n}}$$

Linear transformation  $T: U \rightarrow V$

$$U = \mathbb{R}^n, \quad V = \mathbb{R}^m \quad \| \cdot \| = \| \cdot \|_\infty$$

'?' for and  $\mathbb{R}^n$  to  $\mathbb{R}^m$

$$\begin{matrix} (a_{ij})_{1 \leq i \leq n} \\ 1 \leq j \leq m \end{matrix} \Rightarrow \text{3 rows}$$



$$\| T \| = \max_j \sum_{i=1}^n |a_{ij}|$$



$$X = 17$$

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$\text{cond}(f)(x) = \frac{|x| / |f'(x)|}{|f(x)|}$$

$$y = -17 + 8$$

$$2 \cdot 17 = 34$$

$$T: U \rightarrow V \quad . \quad V, W, V$$

$$\|T\| = \sup_{\|u\|=1} \|Tu\|$$

$$\text{cond}(T)(u) = \frac{\|u\| \cdot \|T\|}{\|Tu\|} \leq \|T^{-1}\| \cdot \|T\|$$

$\text{cond}(T)$

$$[1 \cdot 1]$$

$$f: \mathbb{R}^2 \rightarrow \mathbb{R}$$

Übung 3

$$f(x, y) = x + y$$

$$\text{cond}(f)(x, y) = \frac{\|\langle x, y \rangle\| \cdot \|f\|}{|x+y|} = \frac{\max(|x|, |y|) \cdot 2}{|x+y|} \quad \|f\| = \|f\|_\infty$$

$$f: \mathbb{R}^n \rightarrow \mathbb{R}^m . 2$$

$$x^* \in \mathbb{R}^{n^*}, x \in \mathbb{R}^n$$

$$\frac{\|f(x^*) - f(x)\|}{\|f(x)\|} =$$

$$\frac{\|f(x^*) - f(x)\| \cdot \|x\| \cdot \|x^* - x\|}{\|f(x)\| \cdot \|x\| \|x^* - x\|} \approx \varepsilon$$

$$\frac{\|\underline{df(x)}(x^* - x)\| \cdot \|x\| \cdot \varepsilon}{\|f(x)\| \cdot \|x^* - x\|} \leq \frac{\|\underline{df(x)}\| \cdot \|x\| \cdot \varepsilon}{\|f(x)\|}$$

$$\text{cond}(f)(x) = \frac{\|\underline{df(x)}\| \cdot \|x\|}{\|f(x)\|}$$

1278

$f: \mathbb{R}^n \rightarrow \mathbb{R}^m$   $\hookrightarrow$   $\mathbb{R}^n \times \mathbb{R}^m$

$f_i: \mathbb{R}^n \rightarrow \mathbb{R}$   $\hookrightarrow$   $\mathbb{R}^m$

For  $x \in \mathbb{R}^n$   $\exists i \in \{1, \dots, m\}$  such that

$x_j = 0 \forall j \neq i$   $\Rightarrow f_i(x) = f_i(x_i)$

$(\text{cond}_{x_j}(f_i))_{j \in \mathbb{N}}$

$f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  Ex 3

$$f(x, y) = \left( \underbrace{\frac{1}{x} + \frac{1}{y}}, \underbrace{\frac{1}{x} - \frac{1}{y}} \right)$$

$$df = \begin{pmatrix} -\frac{1}{x^2} & -\frac{1}{y^2} \\ -\frac{1}{x^2} & \frac{1}{y^2} \end{pmatrix}$$

$$\text{cond}(f)(x, y) = \max(|x|, |y|) \cdot \max\left(\frac{1}{x^2}, \frac{1}{y^2}\right)$$

$\max\left(\left|\frac{1}{x} + \frac{1}{y}\right|, \left|\frac{1}{x} - \frac{1}{y}\right|\right)$

$$f_1(x, y) = \frac{1}{x} + \frac{1}{y} \quad f_2(x, y) = \frac{1}{x} - \frac{1}{y}$$

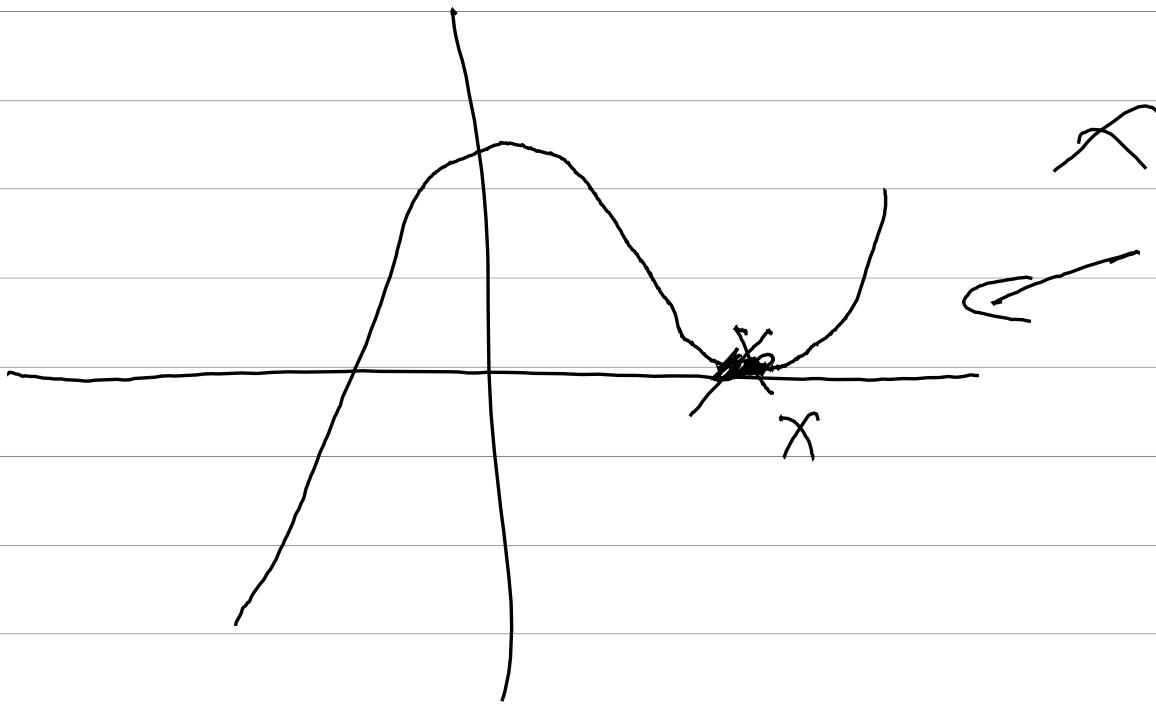
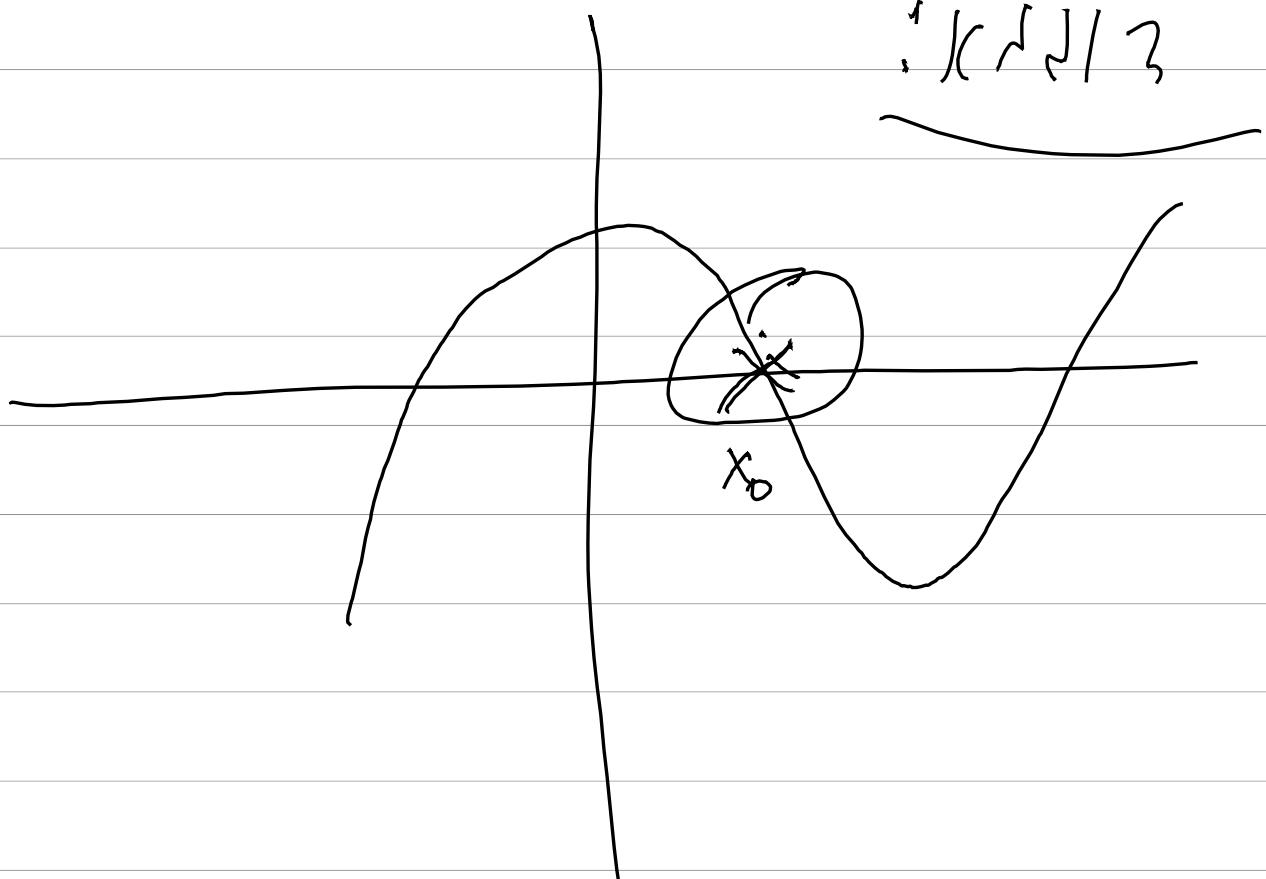
$$\text{cond}_x(f) = \frac{|x| \cdot \frac{1}{x^2}}{\left|\frac{1}{x} + \frac{1}{y}\right|} \quad \frac{|y| \cdot \frac{1}{y^2}}{\left|\frac{1}{x} + \frac{1}{y}\right|}$$

||

$$\frac{|y|}{|x+y|} \quad \frac{|x|}{|x+y|}$$

$$\text{cond}_x(f_2) = \frac{|x| \cdot \frac{1}{x^2}}{\left|\frac{1}{x} - \frac{1}{y}\right|} = \frac{|y|}{|x-y|} \quad \left|\frac{1}{x} - \frac{1}{y}\right| = \frac{|xy|}{|x-y|}$$

$\therefore K \cap J / 3$

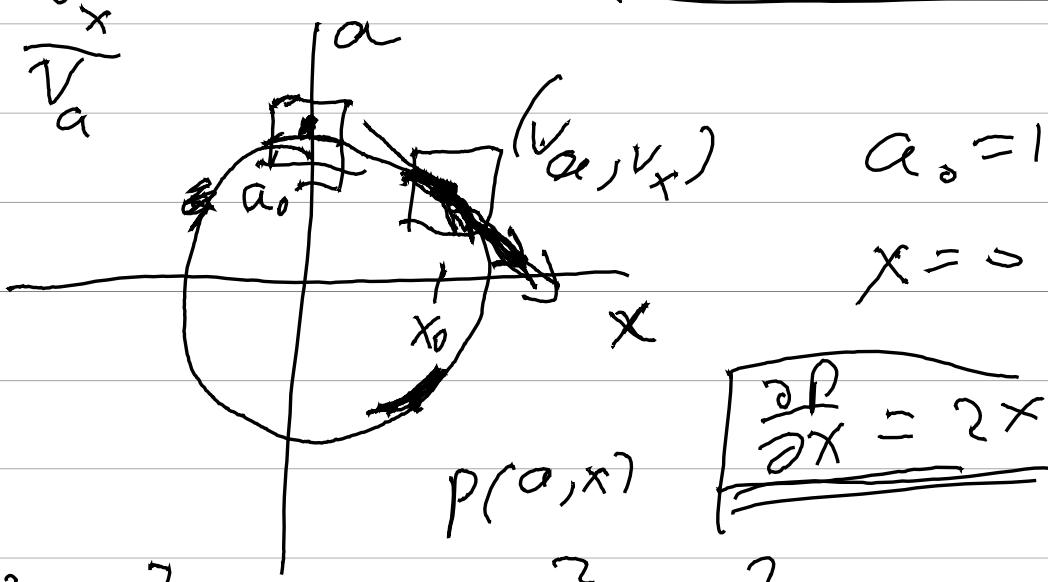


$h \rightarrow \infty ?$   $n' r > 1/p$

$$P_n(\bar{a}, X) = X^n + \sum_{i=0}^{n-1} a_i \cdot X^i$$

$$P_n(\bar{a}_0, \underline{x}_0) = 0 \quad x_0, \bar{a}^0$$

$$\approx F(x, y) = x^2 + y^2$$



$$a^2 + x^2 = 1$$

$$\underbrace{a^2 + x^2 - 1 = 0}_{\text{---}}$$

$$x = x(a)$$

$$x_0 = x(a_0)$$

$$x = \sqrt{1 - a^2}$$

$$P(a, x(a)) = 0$$

$$F(a, x) = 0$$



$$\underline{F(a_0, x_0) = 0}$$



$$\frac{\partial F}{\partial x}(a_0, x_0) \neq 0 \Rightarrow x = X(a)$$



$$\frac{\partial x}{\partial a} = \frac{\partial F}{\partial a} / \frac{\partial F}{\partial x}$$



$$df \cdot \begin{pmatrix} v_a \\ v_x \end{pmatrix} = 0$$

$$\begin{pmatrix} \frac{\partial f}{\partial a} & \frac{\partial f}{\partial x} \end{pmatrix} \begin{pmatrix} v_a \\ v_x \end{pmatrix} = 0$$

$$v_a \frac{\partial f}{\partial a} + v_x \frac{\partial f}{\partial x} = 0$$

$$X = X(\vec{a})$$

$$\text{cond}_{a_i}(x) = \frac{|a_i| \cdot \left| \frac{\partial X}{\partial a_i} \right|}{|x|} = \frac{|a_i| \cdot |f'(x)|}{|x| \cdot |P(x)|}$$

$$\frac{\partial X}{\partial a_i} = - \frac{\partial P / \partial a_i}{\partial P / \partial x} =$$

$$\frac{x^i}{\sum j a_j x^{j-1}} = \frac{x^i}{P'(x)}$$

$$\int_{-\infty}^{\infty} f(x) dx = \sum_{i=1}^n c_i \pi(a_i)$$

$$P(x) = (x-a_1) \dots (x-a_n)$$

인수분해법

$$f^*: \mathbb{R}^x \rightarrow \mathbb{R}^x \quad f: \mathbb{R} \rightarrow \mathbb{R}$$

$$f^*(x^*)$$

$$f(x)$$

$$x' \text{ 가 } x^* \text{ 인데 } f^*(x^*) = f(x') \text{ 인데}$$

$$\frac{|f^*(x^*) - f(x)|}{|f(x)|} = \frac{|f(x') - f(x^*) + f(x^*) - f(x)|}{|f(x)|} \leq$$

$$\frac{|f(x') - f(x^*)|}{|f(x)|} + \frac{|f(x^*) - f(x)|}{|f(x)|}$$
$$\frac{|f(x') - f(x^*)|}{|f(x)|} \approx \frac{|f(x') - f(x^*)|}{|f(x^*)|}$$

$$\underbrace{\text{cond}(f)(x^*)}_{\text{cond}(f)(x)} \cdot \boxed{\frac{|x' - x^*|}{|x^*|}}$$

$f^*$  per i primi 3 m per

$$\text{cond}(f^*)(x^*) := \inf_{f(x') \neq f(x^*)} \frac{|x' - x^*|}{|x^*|}$$

$$\underbrace{\text{cond}(f)(x)}_{\text{cond}(f)(x^*)} \left( \frac{|x - x^*|}{|x|} + \text{cond}(f^*)(x^*) \right)$$

Definition  $\Rightarrow$  Definition  $\Rightarrow$   $f: X \rightarrow \mathbb{C}$

$\Rightarrow$   $\exists \gamma$   $\exists \rho$   $f: X \rightarrow \mathbb{R}$   
 $=$   $\mathbb{C}$

$\exists \delta$   $\forall n \in \mathbb{N}$   $\exists \rho$   $\forall n \in \mathbb{N}$   
 $, \forall x \in X \exists \rho$

For  $\forall \exists \gamma \forall n \in \mathbb{N} \exists \rho$   $\forall n \in \mathbb{N} A$

$\text{For } X \text{ "continuous" function }$

$\forall n \in \mathbb{N} \forall \gamma \exists \rho \forall n \in \mathbb{N} \exists \rho \forall n \in \mathbb{N} A$

$\forall \rho \forall \gamma \exists \rho \forall n \in \mathbb{N} A$

$X - \sim \rho \gamma \exists \rho \forall n \in \mathbb{N} A = C(X)$

ר'ב גראן ב' נרנ'ן סט  $P \subseteq A$

ר'ב גראן מושג

ר'ב גראן סט  $P = \{x \mid f$

ר'ב גראן קבוצת סט  $\underline{\underline{P}}$

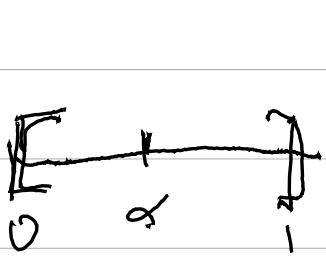
$P = \{x \mid f(x) \in \text{הצורה}$

ר'ב גראן סט  $f(A)$  ר'ב גראן סט

ר'ב גראן סט

$\{0,1\}$  ר'ב גראן סט  $x = s'$

ר'ב גראן סט  $\{0,1\}$  ר'ב גראן סט



$$f(x) = e^{2\pi i x} = \cos 2\pi x + i \sin 2\pi x \subseteq \mathbb{C}$$

'3' for each  $C$  for  $\lambda \lambda \lambda \lambda \lambda \lambda \lambda$ .

$$\text{Ansatz: } f(x) = \sin(2\pi n x) - 1 + \cos(2\pi n x)$$

$$\underbrace{\dots}_{(\Delta x \approx r_0)} \quad \text{hence}$$

$$\underbrace{\{e^{2\pi i n x}\}}$$

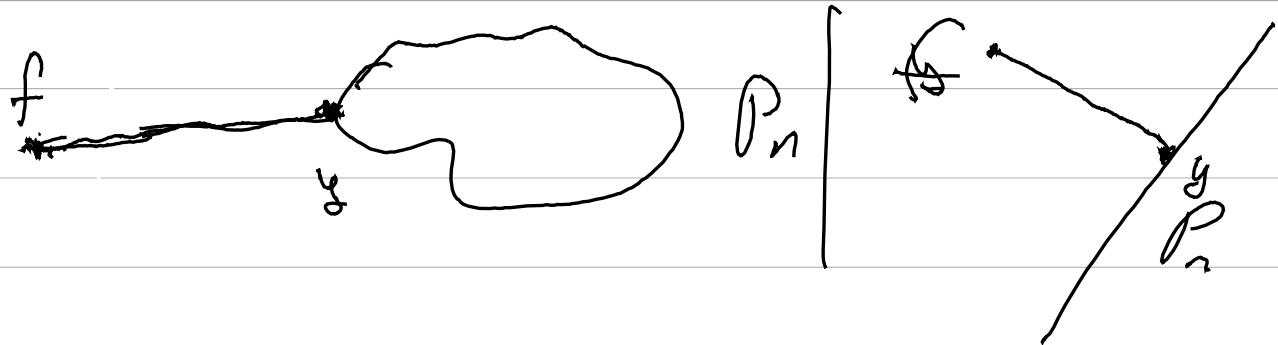
$$P = \bigcup P_i \quad P_0 \subseteq P_1 \subseteq P_2 \dots$$

3.2) 0 in  $N$   $\approx$   $\approx$   $\approx$   $\approx$

$n$   $\epsilon$   $\epsilon > 0$   $\exists f \in F$   $\|f - g\| < \epsilon$

$$\underbrace{d(f, P_n) < \epsilon}_{\text{--}} \quad \text{--}$$

$$d(f, P_n) = \inf_{y \in P_n} d(f, y) = \inf_{y \in P_n} \|f - y\|$$



$\exists \delta > 0 \forall \epsilon' \exists C(x) \text{ for}$

$\|f\|_{C(X)} < \infty$

$$\|f\| = \sup_{x \in X} |f(x)| < \infty$$

$\|f\|_{C(X)} = \inf \left\{ C_0 : \forall x \in X \quad |f(x)| \leq C_0 \right\}$

$\forall M > 0 \exists C > 0 \text{ such that } \|f\|_{C(X)} \leq M$

( $\forall x, y \in X \quad |f(x) - f(y)| \leq M$ ,  $M = \frac{1}{2}C$ )

$f \in P \quad \forall x, y \in X \quad |f(x) - f(y)| \leq M$

$\exists \delta > 0 \forall x, y \in X \quad |x - y| < \delta \Rightarrow |f(x) - f(y)| \leq M$

$\underline{\text{Definition of }} P \quad \text{SIC} \quad (\text{Definition})$

$\rho, A, S \models P \quad \neg I \quad X = [a, b] \quad \text{for } \underline{f \in C(X)}$

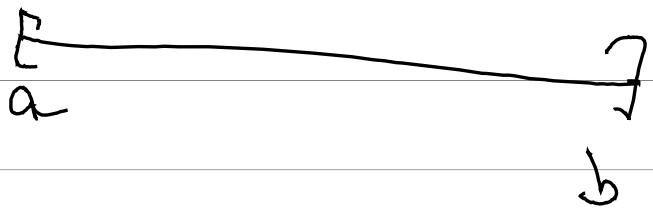
$\neg I \quad P \in C(X) \quad \text{SIC} \quad \exists \delta > 0 \quad \forall x, y \in X \quad |x - y| < \delta \Rightarrow |f(x) - f(y)| \leq M$

$C(x) \ni f \in \mathcal{D} \Leftrightarrow C(x) - \mathcal{D} \ni e^f \in \mathcal{P}$

→ ) P(P) e' {> 0 5 f)

$$\|f - p\| < \varepsilon$$

If  $|f(x) - p(x)| < \epsilon$  for all  $x$  in  $I$ , then  $f$  is continuous at  $x$ .



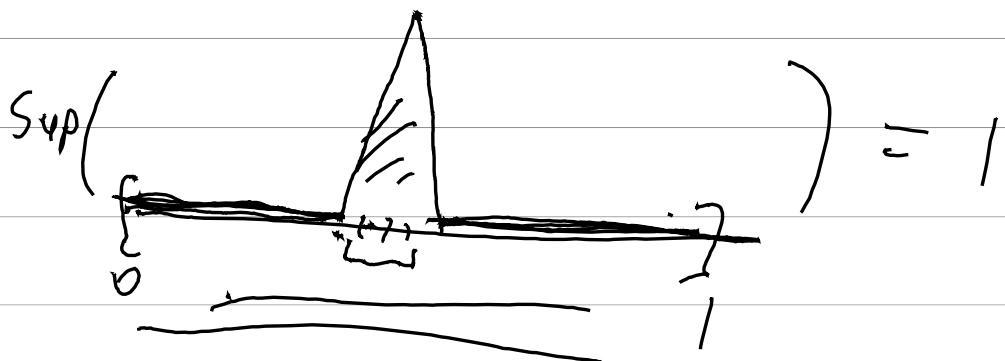
הנתקן נושא פוליטי, מושג של מילון ומשמעותו

ପ୍ରାଚୀ ମହିଳା କାନ୍ଦିଲ

22' 3" 22' 3" 11' 5" : 1cr d / 3  
2' 4" 22' 5" 20' 1" for n, 1

הנורמליזציה

הו



$\|\cdot\|_p \rightarrow \text{ну}$   $1 \leq p \leq \infty \int_{\mathbb{R}^n}$

$$\|\bar{x}\|_\infty = \max(|x_i|)$$

$$\|\bar{x}\|_p = \sqrt[p]{\sum |x_i|^p} \quad 1 \leq p < \infty$$

$x \in \mathbb{R}^n$   
 $\|\cdot\|_\infty$   $\|\cdot\|_p$

ונורמליזציה  $x \rightarrow \bar{x}$

$\mathbb{R}^n \rightarrow \text{ну}$   $\bar{x} \rightarrow \frac{\bar{x}}{\|\bar{x}\|_p}$

$\| \cdot \|_2 : C(X) \rightarrow \mathbb{R}$  ??

$(L_2, \|\cdot\|_2)$  ?

$$\|f\|_2^2 = \int_X |f|^2$$

~~X~~

$\therefore \rho(F) \approx X = \{1, 2, \dots, n\} \quad \text{and}$

$\rho(F) \approx L_2 \approx \mathbb{R}^n \cap \mathbb{R}^n$

$\forall f \in \text{real } C, \exists \omega \in \mathbb{R}^n \cap \mathbb{R}^n$

$\exists \omega \in \omega : X \rightarrow \mathbb{R} \quad \lambda_{\omega} / \lambda_{\omega} \quad \lambda_{\omega} / \lambda_{\omega}$

$$\|f\|_{\omega, 2} = \sqrt{\int_X |f|^2 \cdot \omega}$$

~~X~~

$$\|f\|_2^2 = \int_X |f|^2 \leq \sup_X |f|^2 \cdot \left[ \int_X 1 \right] =$$

$$\|f\|_a^2$$



$\hookrightarrow \text{Naturale } \text{ und } \text{ reelle } \text{ Zahlen}$

$\sim \text{wegen } \text{ Gitter}$

$X \quad f: X \rightarrow \mathbb{R}$

$$A \subseteq X \quad 1_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \end{cases}$$



نیز پس از اینجا  $\sum_{x \in X} f(x)$  را

( $\rightarrow$  زیرا  $\sum_{x \in X}$ )

$$f: X \rightarrow \mathbb{R}$$

$$\underbrace{\int_X f}_{\sum_{x \in X} f(x)}$$

$$X = \{1, \dots, n\}$$

$$f: X \rightarrow \mathbb{R} \Leftrightarrow \text{دایا}$$

$$\|f\|_p = \sqrt[p]{\sum_{i=1}^n |f_i|^p}$$

لذا  $\|\int f\|$  را نمایند  $V$  را

$V$  بر  $\underbrace{\mathcal{P}(V)}$  را فرموده‌اند،  $k = \mathbb{R}$  را

$\langle \cdot, \cdot \rangle: V \times V \rightarrow k$  را  $\langle \cdot, \cdot \rangle$  نمایند

$v \mapsto \langle v, u \rangle$  'ər ,  $u \in V$  ֆ . 1

$\underline{\langle u, v \rangle := \overbrace{\langle v, u \rangle}}$  ,  $u, v \in V$  ֆ . 2

$\langle u, u \rangle \in \mathbb{R}$  ,  $u \in V$  ֆ  $S(c)$

$\langle u, u \rangle > 0$  ՏՇ  $u \neq 0$  ա՛ւ . 3

$V$  հայելա բազու ՀՇ  $\langle \cdot, \cdot \rangle$  ա՛ւ

$v \mapsto \|v\| := \sqrt{\langle v, v \rangle}$  ՀՇ պահանջ ս՛կ

.  $V$  հայելա բազու ՀՇ

$u, v \in V$  բազու , հարաց ՀՇ համապատասխան

$\|u+v\|^2 = \langle u+v, u+v \rangle = \langle u, u \rangle + 2\langle u, v \rangle + \langle v, v \rangle$

$\|u\|^2 + \|v\|^2 + \underline{2\langle u, v \rangle}$

הנ'יה יסוד נורמליזציה

מכפלה סקלרית

$$\frac{\|u+v\|^2 - \|u\|^2 - \|v\|^2}{2} \leftarrow (u, v)$$

הנ'יה יסוד נורמליזציה

הנ'יה יסוד נורמליזציה,  $\rho = 2$  מילר

הנ'יה  $C(x)$  גורם

$$(u, v) \mapsto \int_X u \cdot v$$

הנ'יה יסוד נורמליזציה

הנ'יה יסוד נורמליזציה  
 $\langle u, v \rangle = 0$  מילר  $u, v$

$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$   $\sqrt{\lambda}$

$\sqrt{\lambda}$

$$\left\| \sum a_i v_i \right\|^2 = \sum a_i^2 \|v_i\|^2$$

(Eigenvalues of  $A^T A$ )

$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$   $\sqrt{\lambda}$

$P = U P' \quad -1, X \text{ for } \sqrt{\lambda} \beta \rightarrow$

Matrix diagonalization,  $\lambda$   $\beta$

Orthogonal matrix  $U$ ,  $\lambda$   $\beta$

$$[\sqrt{\lambda_1} | \sqrt{\lambda_2}] \underbrace{P_n}_{P_n - \delta} \{ \pi_i \}$$

$f \in A$

$$T(c_1, \dots, c_m) = \|f - \sum c_i \pi_i\|^2 = \langle f - \sum c_i \pi_i, f - \sum c_i \pi_i \rangle =$$

$$\|f\|^2 - 2 \underbrace{\sum c_i \langle f, \pi_i \rangle}_{\text{underlined}} + \sum c_i c_j \langle \pi_i, \pi_j \rangle$$

Suppose  $T$  is a linear operator

such that  $\sum c_i \pi_i$  is in the range of  $T$

$$\cdot 0 \quad \int \pi_i c_i \rightarrow \int \pi_i$$

$$0 = \frac{\partial T}{\partial c_k} = -2 \langle f, \pi_k \rangle + 2 \sum c_j \langle \pi_k, \pi_j \rangle$$

$$\sum c_j \langle \pi_k, \pi_j \rangle = \langle f, \pi_k \rangle$$

$\|f\|$  is the norm of  $f$

$$A \tilde{c} = b$$

$$b_i = \langle f, \pi_i \rangle \quad \in \mathbb{C}$$

$$A = (\langle \pi_i, \pi_j \rangle)_{i,j}$$

$$\lambda_1, \lambda_2, \dots, \lambda_n \in C_n^{\perp} \quad \text{and} \quad A$$

$$(x, y) \mapsto \underline{\langle x, Ay \rangle} \leftarrow \begin{matrix} \mathbb{R}^n \times \mathbb{R}^m \\ \text{from } \mathbb{R}^m \end{matrix}$$

$$S \subset \tilde{X} \neq \emptyset \quad \text{and}, \quad \text{with}$$

$$\underline{\tilde{x} \cdot A \tilde{x}} \geq 0$$

$$\tilde{x} \cdot A \tilde{x} = \sum_{i,j} x_i x_j \langle \pi_i, \pi_j \rangle = \underline{\|\sum x_i \pi_i\|^2}$$

$$\tilde{x} \neq 0 \quad \text{if} \quad \{x_i \pi_i\} \neq \{0\} \quad \text{and} \quad \{\pi_i\}$$

i)  $\mu\sigma$   $\Rightarrow \sigma \text{ is } A$ ,  $\exists x$   
 $\forall x \forall y$

$$(\pi_i)_{i \geq 0}$$

$$A = \left( \begin{array}{c} \langle \pi_i, \pi_j \rangle \\ \hline \end{array} \right)_{1 \leq i, j \leq n}$$

$$A \bar{c} = b \quad \bar{c} = \langle f, \pi_i \rangle$$

$$[0, 1] \quad \text{for} \quad \pi_i = t^{\frac{i}{n}} \underbrace{[t^n, 1]}$$

$$\langle f, g \rangle = \int_0^1 f \cdot g \, dt$$

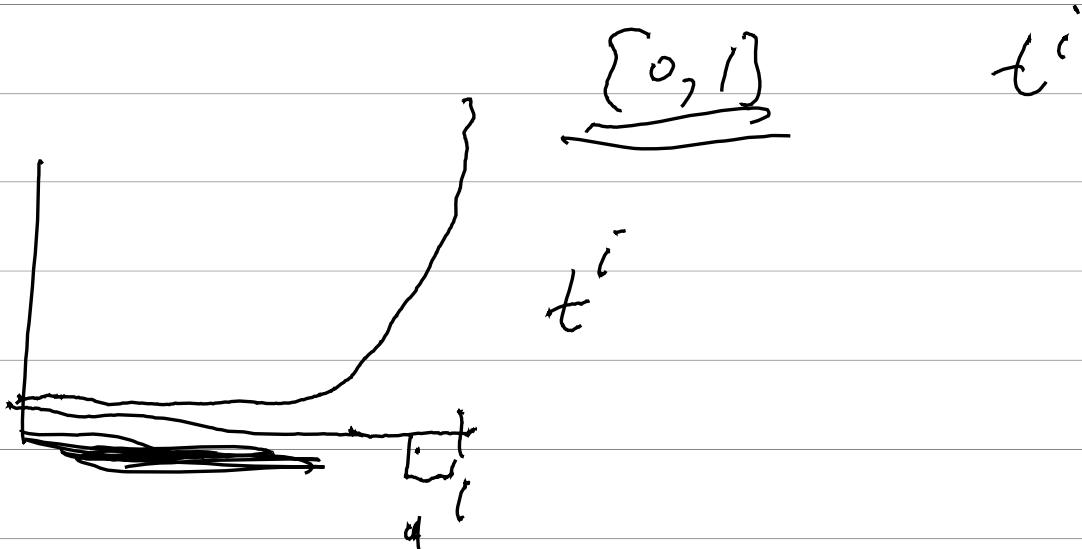
$$\langle \pi_i, \pi_j \rangle = \int_0^{t^{\frac{j}{n}}} t^{\frac{i}{n}} \, dt = \frac{t^{\frac{i+j+1}{n}}}{\frac{i+j+1}{n}} \Big|_0^1 =$$

$$H_3 = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{1} \\ \frac{1}{3} & \frac{1}{1} & \frac{1}{2} \end{pmatrix}$$

$H_n C = I$   $\Rightarrow$   $C^{-1} = H_n$

$\Rightarrow$   $C = H_n^{-1}$   $\Rightarrow$   $C = H_n$

$C = H_n$   $\Rightarrow$   $C^{-1} = H_n^{-1}$



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: ( $\sum_{i=1}^n \pi_i / n - 1/n$ )  $\leq \mu(n) / n$

$$\langle \pi_i, \pi_j \rangle = 0 \quad i \neq j \quad (\langle \pi_i, \pi_i \rangle = 1)$$

$$f = \sum a_i \pi_i$$

$$\underbrace{\langle f, \pi_i \rangle = a_i \langle \pi_i, \pi_i \rangle}$$

$\pi_1, \pi_2, \dots$

$\sum \pi_i \geq 1$

$$\hat{\pi}_i = \pi_i$$

$$\hat{\pi}_{k+1} = \pi_{k+1} - \sum \left( \frac{\hat{\pi}_{k+1}}{\hat{\pi}_i} \right)^{1/2} \hat{\pi}_i \quad \text{for } k < n$$
$$\langle \hat{\pi}_{k+1}, \hat{\pi}_i \rangle = \left( \sum_{i=1}^{k+1} \hat{\pi}_i \right)^{1/2} \langle \hat{\pi}_{k+1}, \hat{\pi}_i \rangle = 0$$

$$P = \bigcup P_i \quad P_0 \subseteq P_1 \subseteq \dots$$

$$P_i = \left\{ i \geq \frac{\epsilon \sqrt{N} \sum S_j}{\delta} \right\}$$

$$\dim(P_i) = i$$

$$\text{Span}(\langle \pi_i \rangle_{i \leq v}) = \text{Span}(\langle \hat{\pi}_i \rangle_{i \leq v})$$

$$\hat{\pi}_i \in P_i \quad | \quad \text{planar } \rho/J^2$$

. . . . .

$$\hat{\pi}_{i+1} = t \hat{\pi}_i - \alpha_i \hat{\pi}_i + \sum_{j=0}^{i-1} b_j \hat{\pi}_j =$$

$$(t - \alpha_i) \hat{\pi}_i + \beta_i \cdot \hat{\pi}_{i-1} + \sum_{j=0}^{i-2} b_j \hat{\pi}_j$$

$$\langle \hat{\pi}_{i+1}, \hat{\pi}_i \rangle = \langle (t - \alpha_i) \hat{\pi}_i, \hat{\pi}_i \rangle \Rightarrow$$

$$\alpha_i \cdot \| \hat{\pi}_i \|^2 = \langle t \hat{\pi}_i, \hat{\pi}_i \rangle$$

$$\Rightarrow \alpha_i = \frac{\langle t\hat{\pi}_i, \hat{\pi}_i \rangle}{\|\hat{\pi}_i\|^2}$$

$$0 = \underbrace{\langle (t - \gamma_i) \hat{\pi}_i, \hat{\pi}_{i-1} \rangle}_{\beta_i \cdot \|\hat{\pi}_{i-1}\|^2} +$$

$$\beta_i = - \frac{\langle t\hat{\pi}_i, \hat{\pi}_{i-1} \rangle}{\|\hat{\pi}_{i-1}\|^2} =$$

$$- \frac{\langle \hat{\pi}_i, t\hat{\pi}_{i-1} \rangle}{\|\hat{\pi}_{i-1}\|^2} = - \frac{\|\hat{\pi}_i\|^2}{\|\hat{\pi}_{i-1}\|^2}$$

$$\hat{\pi}_{i+1} = (t - \gamma_i) \hat{\pi}_i + \underbrace{\beta_i \hat{\pi}_{i-1}}$$

For a more rigourous view see

$[-a, a]$   $\rightarrow \mathbb{R}^n$   $\rightarrow$   $L^2$

$w(t) = w(t, t)$   $\wedge$   $\mathcal{C}^0$   $\cap L^2$   $\rightarrow$   $\mathcal{H}$

$$\left[ \langle f, g \rangle = \int_a^b f(t) \overline{g(t)} w(t) dt \right]$$

we have  $\pi_k$   $\in$   $\mathcal{H}$

thus  $u-f$   $\rightarrow$   $\|u-f\|_k$   $\rightarrow 0$

$\forall \epsilon > 0$   $\exists$   $\delta$   $\forall k$   $\|u-f\|_k < \epsilon$

so  $\forall k \in \{-1, 1\}$   $\|u-f\|_k$   $\leq \epsilon$

$(f_k)_k \in \mathcal{H}$   $\rightarrow$   $f$

$$T_n(t) = \underbrace{\frac{k!}{(2k)!}}_{=} \frac{d^k}{dt^k} (t^2 - 1)^k$$

$$(\ln/(t - \pi_k) - e^{-\sqrt{c_2} \int \gamma})$$

从  $\pi_k$  到  $c_k$ ,  $t^i$  为  $\pi_i$  的

$$0 = \langle \pi_k, t^i \rangle = \int_{-1}^k \frac{d^k}{dt^k} (t^2 - 1)^k \cdot t^i dt =$$

$$\dots = 0$$

$$\pi_0 = 1, \quad \pi_1 = \frac{1}{2}(t^2 - 1)' = t$$

$$\pi_2 = ((t^2 - 1)^2)^{1/2} \cdot \frac{2}{4!} = \frac{1}{12} \cdot ((t^2 - 1)^2)''$$

$$\pi_k = t^k + \underbrace{\mu_k t^{k-2}}_{\dots} + \dots$$

$$\pi_{k+1} = t \cdot \pi_k + \beta_k \cdot \pi_{k-1} \Rightarrow \boxed{\beta_k = \frac{\pi_{k+1} - t \pi_k}{\pi_{k-1}}}$$

$$\beta_k = \mu_k - \mu_{k+1}$$

$$\mu_k = \frac{k(k-1)}{2(2k-1)} \Rightarrow$$

$$\beta_k = \frac{1}{4-k^2}$$

Wichtigste Ergebnisse

if  $\alpha \sim \pi$  then  $\beta$

$$\underline{f: \mathbb{R} \rightarrow \mathbb{R}} \quad f(f+f) = f(\cancel{f})$$

↓ . t ↗

$$f: [0,1] \rightarrow \mathbb{R} \quad f(0) = f(1)$$

$$(\sin(2\pi t) + \cos(2\pi t)) = \underline{\underline{e^{2\pi i t}}}$$

(=)

$$g: \mathbb{S}' \rightarrow \mathbb{C}$$

$$\mathbb{S}' = \{ z \in \mathbb{C} \mid |z| = 1 \}$$

$$E: [0,1] \rightarrow \mathbb{S}'$$

$$E(t) = e^{2\pi i t}$$

$$g: \mathbb{S}' \rightarrow \mathbb{C} \rightsquigarrow g \circ E \text{ - } \text{rotation}$$

$$\int_{\mathbb{S}'} g := \int_0^1 g \circ E dt$$

$$z, w \in \mathbb{S}' \quad \text{if} \quad z, w \in \mathbb{S}' \quad \text{no!}$$

For formal  $a \in \mathbb{S}'$   $\forall \alpha$

$$g_a(z) = g(a \cdot z)$$

$$\int_{\mathbb{S}'} g_a = \int_{\mathbb{S}'} g \quad \text{sic}$$

$g : S' \rightarrow \mathbb{C}^*$   $\rightsquigarrow r'/\mathbb{H}$   $\mathbb{H}/\mathbb{C}$

$g(z \cdot w) = g(z) \cdot g(w)$   $\rightsquigarrow \mathbb{H}' \supset \mathbb{N}$

$z$  für  $g(z) = 1$   $\mathbb{H}/\mathbb{C}$   $S/\mathbb{C}$

$\int_S g = 1$   $S/\mathbb{C}$

$\int_S g = 0$   $\mathbb{H}/\mathbb{C} \rightarrow$

→  $\mathbb{H}/\mathbb{C}'$

$g(a) \neq 1 \quad \text{e. } \quad a \in S' \quad e' \cdot \omega$

$g_a(x) = g(ax) = g(a) g(x)$

$\int_S g = \int_S g_a = \int_S g(a) \cdot g = \underbrace{g(a)}_{\neq 1} \int_S g$   $S/\mathbb{C}$

$\int_S g = 0$   $S/\mathbb{C}$

$$g_n(x) \approx x^n \quad \text{for } n \in \mathbb{Z} \quad \text{def}$$

or  $\approx$  in  $\mathcal{O}(x)$  we have  $x^n$

$$\overline{g_n(x)} = g_{-n}(x) \quad g_n \cdot g_m = g_{n+m}$$

$\approx$   $\mathcal{O}(x)$   $\Rightarrow$   $\mathcal{O}(x)$   $\approx$   $\mathcal{O}(x)$

$$\approx \mathcal{O}(s) \quad \square$$

$$\langle f, g \rangle = \int_{S^1} f, \bar{g}$$

$\approx$   $\mathcal{O}(s)$   $\Rightarrow$   $\mathcal{O}(s)$

$$\approx \mathcal{O}(s)$$

$\approx$   $\mathcal{O}(s)$   $\approx$   $\mathcal{O}(s)$   $\approx$   $\mathcal{O}(s)$

$$\langle \mathcal{O}(s), \mathcal{O}(s) \rangle \approx \mathcal{O}(s) \quad \text{def}$$

$\rho' \cap \cup_{j=1}^m C_j$  is a disjoint set

$\rho' \cap C \cap \cup_{j=1}^m C_j$

$$c = \int_0^1 x^n = \int e^{2\pi i n t} dt = \int_{\gamma} e^{2\pi i n t + i \frac{2\pi}{n} t^2}$$

$\gamma$

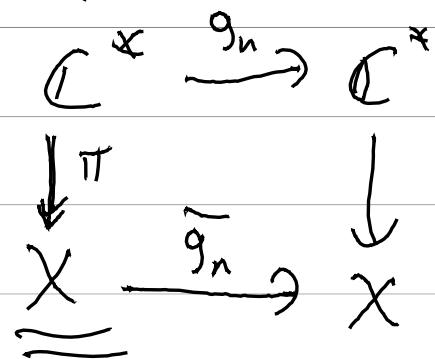
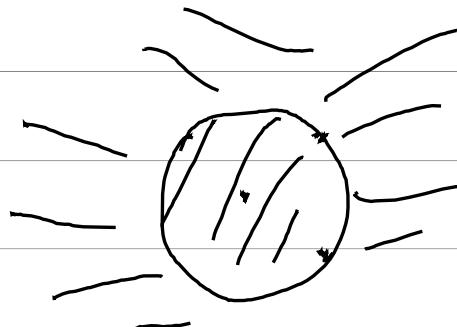
$\therefore \exists f(z) \neq$

$$\mathbb{C}^* = \{z \in \mathbb{C} \mid z \neq 0\}$$

If  $x=y$   $\Rightarrow f(x) \approx y$

$$x = \mathbb{C}^*$$

$$g_n(x) = \frac{1}{x^n} \approx x^n \Rightarrow x = \frac{1}{g_n(x)}$$



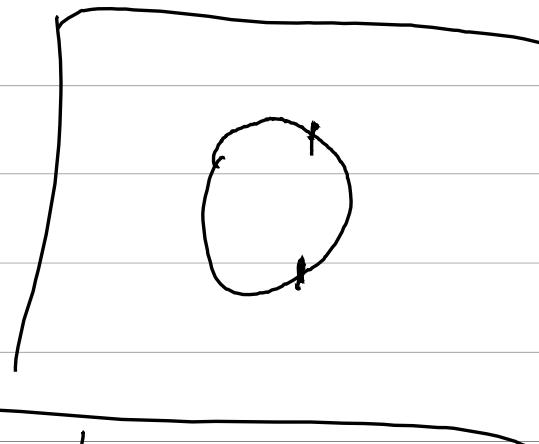
$$\pi(x) = x + \frac{1}{x} \in \mathbb{C}$$

$$\pi(x) = \pi\left(\frac{1}{x}\right)$$

$$\begin{aligned} & \text{if } z \in \mathbb{S}' \text{ in } \mathbb{H}_C \\ \pi(z') &= \operatorname{Re}(z) \\ \pi(S') &= [-1, 1] = X_C \end{aligned}$$

$$g_n\left(\frac{x + \frac{1}{x}}{2}\right) = \frac{x^n + \frac{1}{x^n}}{2} = \pi(g_n(x))$$

thus  $X$  is in  $\mathbb{H}_C$

$$\star \int_{X_0}^X h = \int_{S'} h \circ \pi = \int_{S'} h\left(\frac{x + \frac{1}{x}}{2}\right)$$


$$\int_0^1 h(\operatorname{Re}(e^{2\pi i t})) dt = \int_0^1 h(\cos(2\pi t)) dt$$

$$y = \cos(2\pi t) \quad dy = 2\pi \sin(2\pi t) dt =$$

$$dy = -2\pi \sqrt{1-y^2} dt$$

$$x = \int_{-1}^1 h(y) \frac{1}{\sqrt{1-y^2}} dy$$

~~$\int_0^1$~~

Integrals  $\rightarrow$   $\widehat{g}_n \rightarrow x$

now we have  $\int_{-1}^1 f g \frac{1}{\sqrt{1-y^2}} dy$

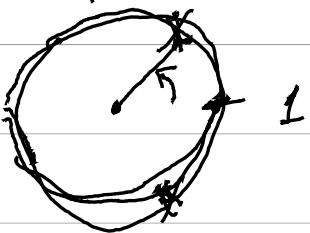
$\int_{-1}^1 f g \frac{1}{\sqrt{1-y^2}} dy$   $\sim \int_{-1}^1 f g dy$

$$\langle f, g \rangle = \int_{-1}^1 f \cdot g \frac{1}{\sqrt{1-y^2}} dy$$

~~$\int_{-1}^1$~~

$$\widehat{g}_n (\cos 2\pi n t) = \underbrace{\cos 2\pi n t}$$

$$S^1 = \{ z \in \mathbb{C} \mid |z| = 1 \}$$



$\downarrow P$

$$\sin 2\pi t = \sqrt{1 - y^2}$$

$$0 \leq t \leq \frac{1}{2}$$

$$[ \leftarrow ]_x = [-1, 1] \subset \mathbb{R}$$

$X$

$$P(z) = \frac{z + \bar{z}}{2} (= \frac{z + \bar{z}}{2} = \operatorname{Re}(z)) \quad S'$$

$$f : X \rightarrow \mathbb{C}$$

$$y = \cos 2\pi t$$

$$dy = -2\pi \sin 2\pi t dt$$

$$\int_X f := \int_{S^1} f \circ P = \int_0^1 f \circ P \circ e^{2\pi i t} dt =$$

$\frac{1}{2}$

$$\int_0^1 f(\cos 2\pi t) dt = 2 \int_0^{\frac{1}{2}} f(\cos 2\pi t) dt =$$

$$2 \int_{-1}^1 f(y) \cdot \left(-\frac{1}{2\pi}\right) \frac{dy}{\sqrt{1-y^2}} =$$

$S' \xrightarrow{\varphi_n} S'$

$\varphi_n(x) = x^n$

$$T_n\left(\frac{x+\frac{1}{x}}{2}\right) = \underbrace{x^n + \frac{1}{x^n}}_2$$

$$\int_X T_n = \int_{S'} T_n \circ \varphi = \int_{S'} \underbrace{x^n + \frac{1}{x^n}}_2 =$$

~~$\int_X T_n = \int_{S'} T_n \circ \varphi = \int_{S'} x^n + \frac{1}{x^n} =$~~

$$\begin{cases} 1 & h=0 \\ 0 & h \neq 0 \end{cases} \quad \begin{cases} \int_{S'} \varphi_n \bar{\varphi}_m = \\ \int \varphi_n \varphi_{n+m} = \int \varphi_{n-m} \end{cases}$$

$$(T_n \cdot T_m) \left( \frac{z + \frac{1}{z}}{2} \right) = \underbrace{\left( z^n + \frac{1}{z^n} \right)}_{2} \left( \frac{z^m + \frac{1}{z^m}}{2} \right).$$

$$\frac{1}{2} \underbrace{\left( z^{m+n} + \frac{1}{z^{m+n}} + z^{n-m} + z^{m-n} \right)}_{2} =$$

$$\frac{1}{2} \left( T_{n+m} \left( \frac{z + \frac{1}{z}}{2} \right) + T_{n-m} \left( \frac{z + \frac{1}{z}}{2} \right) \right)$$

$$\int T_n \cdot T_m = \frac{1}{2} \left( \int T_{n+m} + \int T_{n-m} \right) =$$

$$\int \begin{cases} \frac{1}{z} & n = m \neq 0 \\ 1 & n = m = 0 \\ 0 & (n \neq m) \end{cases} \int T_n = T_{-n}$$

$$T_0 = 1 \quad T_0\left(\frac{z + \frac{1}{z}}{2}\right) = 1$$

$$T_1\left(\frac{z + \frac{1}{z}}{2}\right) = z + \frac{1}{z} \quad T_1(z) = z$$

$$T_n \cdot T_1 = \frac{1}{2} (T_{n+1} + T_{n-1}) \Rightarrow$$

$$T_{n+1}(z) = 2z T_n(z) - T_{n-1}(z)$$

$$T_2(z) = 2z^2 - 1$$

$$\cos(2t) = 2(\cos^2 t - 1)$$

$$\cos(\alpha t) = T_n(\cos t)$$

$$\text{For } p \in \mathbb{P}_{2^n}, \quad n \in \mathbb{N} \quad T_n(p) = \sum_{k=0}^{2^n-1} f_k(p) \cdot z^k$$

3.5) چهارم

f 3)  $\lim_{n \rightarrow \infty} f_n(x)$

نیز  $\int_0^1 f_n(x) dx$

$c_0, \dots, c_n \in [a, b] \subseteq \mathbb{R}$

3)  $\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i) \Delta x$

$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i) \Delta x$

$c_0, \dots, c_n$  برای مینیموم و مکالم

0.02 نیز،  $a+1$  بین

$$f_i(c_j) = \begin{cases} 1 & j=i \\ 0 & j \neq i \end{cases}$$

$$P_i(x) = \frac{\prod_{j \neq i} (x - c_j)}{\prod_{j \neq i} (c_i - c_j)} \in P_n = \int_a^b f(x) dx$$

$$f(c_i) = f_i \Rightarrow$$

$$f \sim \underbrace{\sum f_i l_i}_{=} = \pi_{\tilde{C}}(f)$$

$$\pi_{\tilde{C}} : C[a,b] \rightarrow C[a,b]$$

$$\sup_{\|f\|=1} \|\pi_{\tilde{C}}(f)\| = \sup_{\|f\|=1} \left\| \sum f_i l_i \right\| =$$

$$= \sum_{i=0}^n \|l_i\|$$

$$\lambda_n(x) = \sum_{i=0}^n |l_i(x)|$$

$$f - \underbrace{f - \pi_C(f)}_{\text{orthogonal projection}} \rightarrow f - \hat{P}_n$$

$$\|P_n - \cdot\|$$

$$\|\underbrace{f - \pi_C(f)}_{\text{orthogonal projection}}\| = \|f - \hat{P}_n - \pi_C(f - \hat{P}_n)\|$$

$$\leq \|f - \hat{P}_n\| + \|\pi_C\| \|f - \hat{P}_n\| =$$

$$\underbrace{\left(1 + \|\pi_C\|\right)}_{\text{constant}} \|f - \hat{P}_n\|$$

برهان انتقالی

$$(f - \pi_C(f))(x) = \sum_{i=0}^{n+1} \frac{f^{(i+1)}(c_i)}{(i+1)!} (x - c_i)^{i+1}$$

$(x \rightarrow i\beta_n)$   
 $\cup / \gamma/\approx \cup \{ e' : \approx \subseteq$

$(f \in C^{n+1}[a, b]) \quad | \cup$

$c_i \neq x \quad x \geq c_i \quad \underline{i \geq n+1}$

$G(t) = \underbrace{f(t) - \pi_{\mathcal{E}}(f)(t)}_{= 0} -$

$$\frac{f(x) - \pi_{\mathcal{E}}(f)(x)}{\prod_{i=0}^n (x - c_i)} = \frac{n}{\prod_{i=0}^n (t - c_i)}$$

マトリクス  $\Rightarrow G$  の  $n+1$  次

$x - 1 \quad i=0, \dots, n \quad c_i$

$G^{(n+1)} \quad -f: \mathbb{R} \rightarrow \mathbb{C}$

$\cdot \{ \quad \text{odd } e'$

ו'נ'ג =  $n+1$  ( $\infty$ )  $\cap \mathcal{S}_{\rho}$

$$G^{(n+1)}(t) = f^{(n+1)}(t) - (n+1)! \cdot \frac{f(x) - f(t)}{\prod_{i=1}^n (x - c_i)}$$

$t = \zeta$   $\omega_{\lambda} \varphi$

$\zeta \in [a, b]$

$[a, b] \rightarrow \cup_{\lambda} \mathcal{S}_{\rho}$

$(x, c_i)$   $\square$   $\cap \mathcal{S}_{\rho}$

$$\int_{C(3)} \int_{\gamma_0} f \underbrace{d\gamma}_{{}^{\text{def}} \gamma'}$$

$$= \int_{\gamma_0} f \circ \tilde{\gamma} \in \mathbb{C}^{(n)}_{\gamma_0}$$

$$= \int_{\gamma_0} f \circ \tilde{\gamma} = \int_{\gamma_0} f \circ \tilde{\gamma} \circ \tilde{\gamma}'$$

$$\pi_{C^{(n)}}(f) \rightarrow f$$

$$\int_{\gamma_0} f \circ \tilde{\gamma} = \int_{\gamma_0} f$$

$$\pi_{C^{(n)}}(f)$$

$$\|f - \pi_{C^{(n)}}(f)\| \leq \left\| \frac{f^{(n+1)}(\xi)}{(n+1)!} \right\|_{C^{(n)}} \|x - c_i^{(n)}\|$$

$$\leq \frac{m_{n+1}(f)}{(n+1)!} \cdot (b-a)^{n+1}$$

$$\boxed{[a,b] \ni f \text{ has } m_{n+1}(f) \text{ small}}$$

-c ρ' 31n sic

$$\frac{m_n(f) \cdot (b-a)^n}{n!} \rightarrow 0$$

$c_i \in [a, b]$  γ' 3 מינימום של  $m_{n+1}$  ב-

$$\|f - \pi_{\bar{c}}(f)\| \leq \left\| \frac{f^{(n+1)}(\xi)}{(n+1)!} \cdot \prod_{i=1}^n (x-c_i) \right\|$$
$$\leq \underbrace{\frac{m_{n+1} \cdot (b-a)^{n+1}}{(n+1)!}}$$

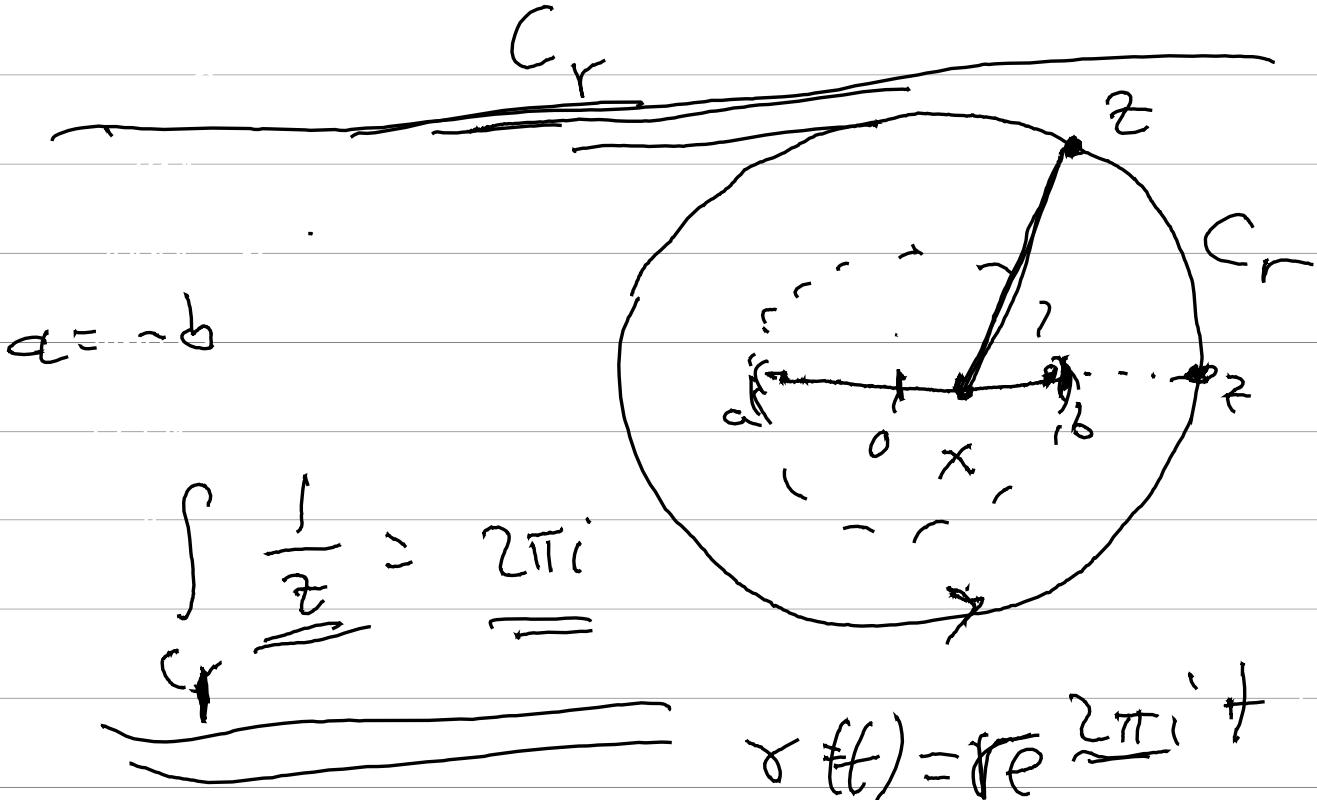
$$m_n = \|f^{(n)}\|_{\infty}$$

ר' סיסיק גאנז לנ' א' f

$\text{Rückgrat f. der : } \ell \rightarrow \text{nach}$

$S/\ell$

$$f^{(k)}(x) = \frac{k!}{2\pi i} \oint \frac{f(z)}{(z-x)^{k+1}} dz$$



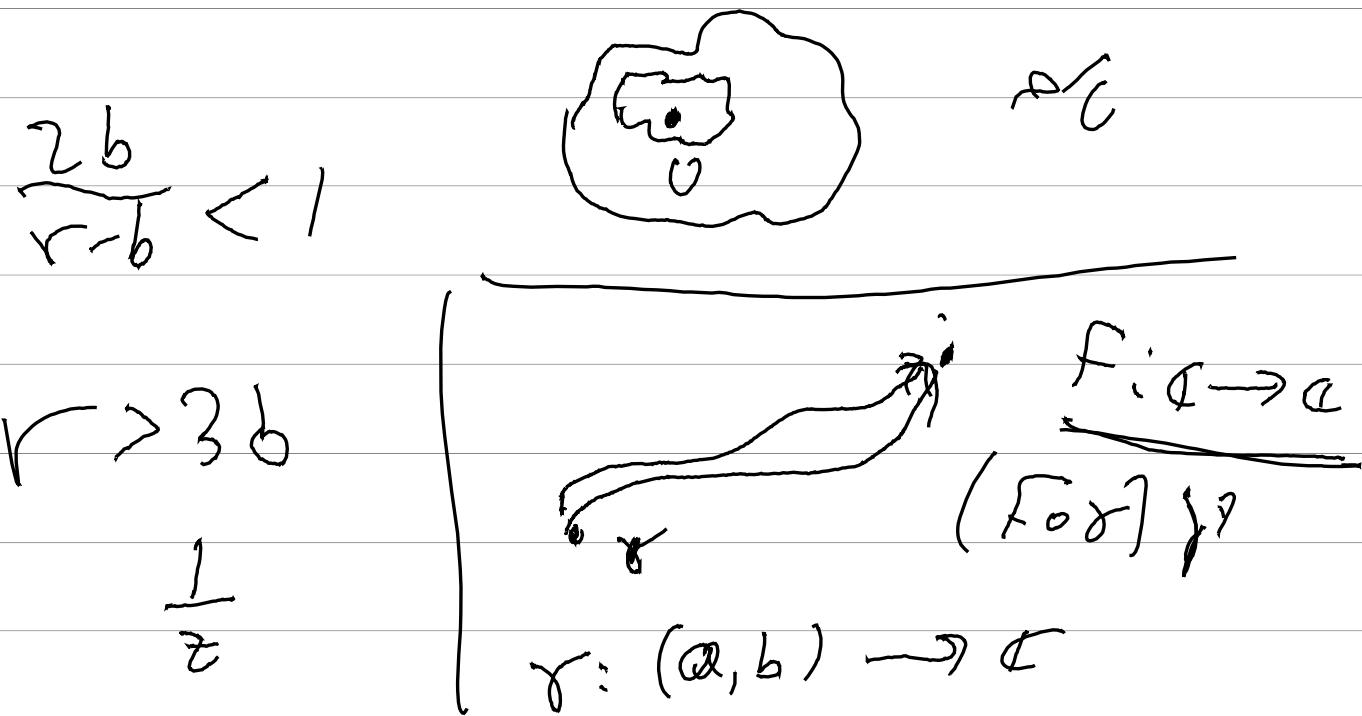
$$\underbrace{\|z-x\|}_{\geq r-b}$$

$$\|f^{(k)}(x)\| \leq \frac{k!}{2\pi} \frac{M_N}{(r-b)^{k+1}} \cdot 2\pi r \Rightarrow$$

$$\frac{M_n \cdot (2b)^n}{n!} \leq \frac{n! \cdot M_0}{(r-b)^{n+1}} \cdot r \cdot (2b)^n =$$

$n!$

$$\frac{M_0 \cdot r}{r-b} \cdot \left(\frac{2b}{r-b}\right)^n \rightarrow 0$$



רְאֵבָנָה | גִּבְעָה אֶלְגָּרָה  
 $([-1, 1] \setminus \{0\})$  || • 160 גִּבְעָה ה

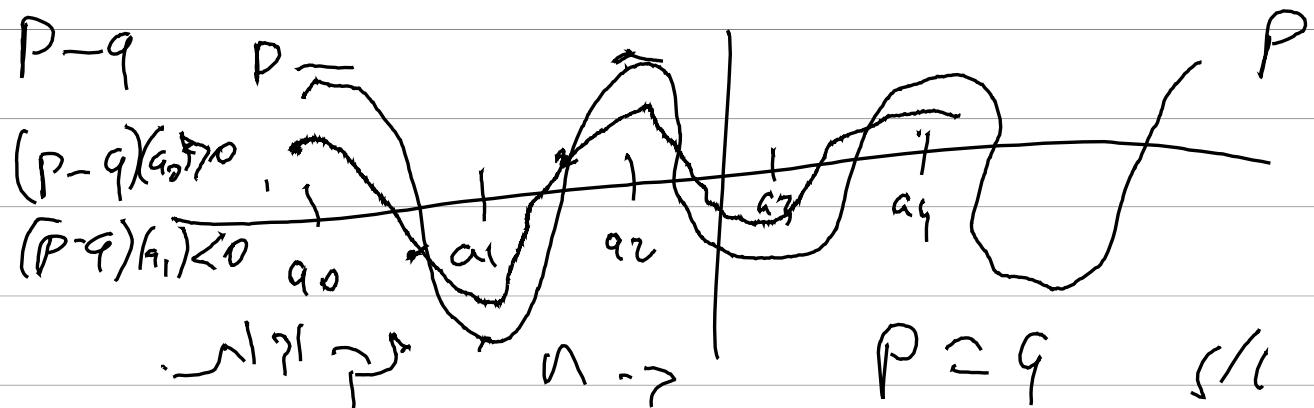
? גִּבְעָה ?

רְאֵבָנָה | גִּבְעָה אֶלְגָּרָה P אַלְכָּה

ולא  $\|P\|_\infty = |P(a_i)|$  -& > n  
 $P(a_i) = -P(a_{i+1})$   
 $P$  ס/י  $a_i$  מ/פ/ $P$  n+1

רְאֵבָנָה  $\|P\|_\infty$  וְאֶלְגָּרָה ח/א

$\|q\| < \|p\|$  וְאֶלְגָּרָה ח/א



replay 323 n/a surf

input  $\sim \sim \sim$

$$T_n(\operatorname{Re} z) = \underbrace{\operatorname{Re}(z^n)}$$

$$\Rightarrow T_n(x) = 0$$

$$\text{ne } z^n$$

$\Leftrightarrow$

$$x = \cos\left(\pi \frac{2k+1}{2n}\right)$$

$$z^n = \pm i$$

$$y = \cos\left(\pi \frac{k}{n}\right)$$

$$z^n = i \Leftrightarrow$$

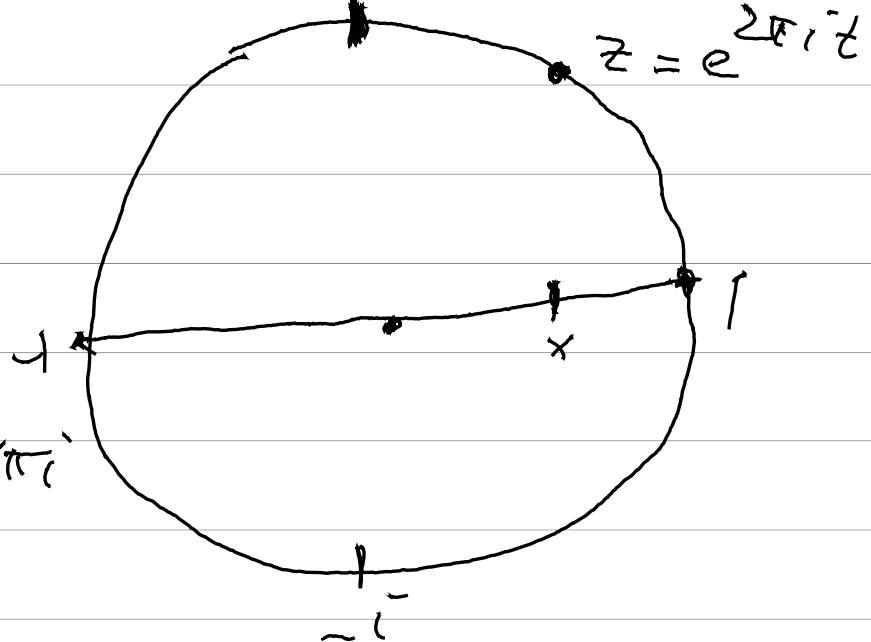
$$k=0, \dots, n$$

$$z = e^{2\pi i \frac{k}{n}}$$

$$e^{2\pi i \operatorname{int}} = i$$

$$2\pi i \operatorname{int} = \frac{\pi i}{2} + k\pi i$$

$$\Rightarrow \operatorname{int} = \frac{2k+1}{4n}$$



Ex  $\left| \frac{1}{2} \right| \rightarrow \text{gegen 0}$  für  $T_n$

$$2^{n-1} \cdot k \cdot T_n$$

Prim 's' da  $\frac{1}{T_n} = \frac{1}{2^{n-1}} \cdot T_n$  s/c

abgängc  $\left\| \frac{1}{T_n} \right\| = \frac{1}{2^{n-1}}$ ,  $n \rightarrow \infty$

$a_i = \cos\left(\frac{\pi i}{n}\right)$  ↗ 1/2/1/2

Gründen wir 's' n. d. o.  $0 \leq i \leq n$

positiv zu  $\frac{1}{T_n}$  &  $\frac{1}{T_n}$

ausklammre 's'  $\int_0^1 \frac{1}{T_n} \sim \int_0^1 \sin$   
→  $\int_0^1 \sin x dx$  - A  $\rightarrow \infty$

$$||f - \Pi_{\tilde{C}^{(n)}}(f)|| \leq \frac{|f^{(n+1)}(\xi)|}{(n+1)!} \cdot \|T_n\|_\infty =$$

$$\frac{|f^{(n+1)}(\xi)|}{(n+1)!} \cdot 2^{n-1}$$

—

$$(c_0, c_1, \dots) \rightarrow \text{S'0'0'IC 2230}$$

$$P_0, P_1, \dots \quad \deg(P_i) < i$$

$\delta_i$

$P_i$

$$P_{i+1}(x) = P_i(x) + a_{i+1,0}(x-c_0) \dots (x-c_c)$$

$$a_{i+1} (c_{i+1} - c_0) \dots (c_{i+1} - c_i) = f_{i+1} - p_i f_{i+1}$$

$$\underline{a_{i+1}} = \underline{(c_{i+1} - c_0) \dots (c_{i+1} - c_i)} = \frac{f_{i+1} - p_i(f_{i+1})}{f_{i+1} - p_i(f_{i+1})}$$

$$\underline{[c_0, \dots, c_{i+1}]} f$$

$$\underline{[c_0, \dots, c_{i+1}]} f \subseteq \underline{[c_0, \dots, c_i]} f \sim \underline{[c_1, \dots, c_{i+1}]} f$$

$c_{i+1} - c_0$

$$\tilde{C} = (c_0, \dots, \overset{\text{:=} c_i}{c_{i+1}}, \dots, c_{i+1})$$

$$\underline{P_{\tilde{C}}(x)} = P_{\tilde{C}_{i+1}} - \frac{(x - c_0)}{(c_{i+1} - c_0)} (P_{\tilde{C}_{i+1}} - P_{\tilde{C}_0}) =: q(x)$$

$\text{if } 0 < j < i+1 \text{ do } :[n' 33$

$$q(c_j) = f_j = P_{\tilde{C}}(c_j)$$

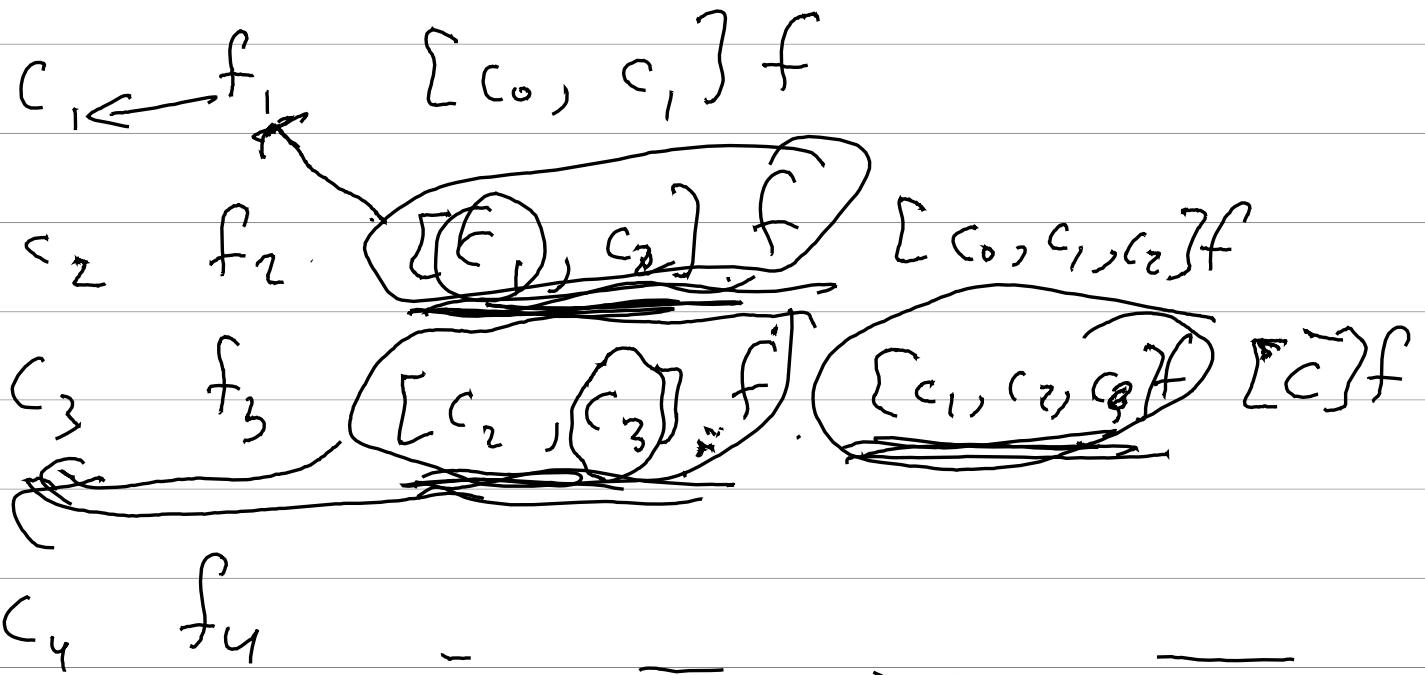
$$q(c_0) = P_{\tilde{C}_{i+1}}(c_0) = f_0 = P_{\tilde{C}}(c_0)$$

$$q(c_{i+1}) = P_{\tilde{C}_{i+1}}(c_{i+1}) - (P_{\tilde{C}_{i+1}}(c_{i+1})) -$$

$$P_{\tilde{C}_0}(c_{i+1}) = P_{\tilde{C}_0}(c_{i+1}) = f_{i+1} = P_C(f_{i+1})$$

$c$        $f$

$c_0$        $f_0$



$$\{c_1, c_2, c_3\}f = \frac{\{c_2, c_3\}f - \{c_1, c_2\}f}{c_3 - c_1}$$

$$\bar{c} = c_0, \dots, c_i$$

$$P_{\bar{c}}(x) = P_{\bar{c}_{\leq i}}(x) + [\bar{c}] f \cdot \prod_{j < i} \pi(x - c_j)$$

$$\|P_{\bar{c}}(x) - P_{\bar{c}_{\leq i}}(x)\| = \|[\bar{c}] f \cdot \prod_{j < i} \pi(x - c_j)\|$$

$$\hookrightarrow \underbrace{\left[ P_{\bar{c}}^{(i+1)}(\xi) \cdot \prod_{j < i} \pi(x - c_j) \right]}_{(i+1)}$$

$$\underline{\underline{f}} \left[ \begin{array}{c} c_0, c_1, \dots, c_n \\ f_0, f_1, \dots, f_n \end{array} \right]$$

$$P_{\bar{C}}(x) = [\bar{c}] f(x^n) + \dots =$$

$$[\bar{c}] f(x - c_0) \dots (x - c_n) + P_{\bar{c}_{n+1}}$$

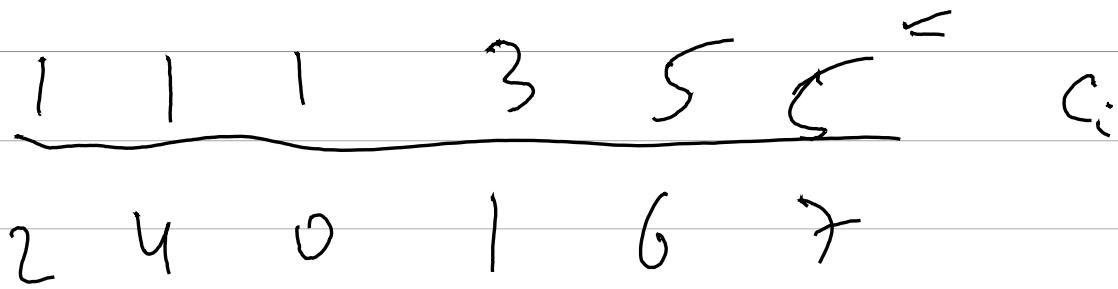
$$\hat{c}_i = \{c_0, \dots, c_n\} \setminus \{c_i\}$$

$$[\bar{c}] f = \frac{[\hat{c}_0] f - [\hat{c}_n] f}{c_n - c_0}$$

$c_0, \dots, c_h$

$f_0, \dots, f_n$

$$\bar{C} = 3 \cdot \{ \} + \{ \} + 2 \cdot \{ \}$$



$$P(1)=2, \quad P'(1)=4, \quad P''(1)=0$$

$$P(3)=1, \quad P(5)=6, \quad P'(5)=7$$

$$P_{\bar{C}}(x) = (\bar{C} \bar{f}) \cdot x^4 + \dots$$

$$\hat{c}_i = \bar{c} - \{c_i\}$$

$$\bar{C} = \sum n_i \{c_i\} \quad \deg(\bar{C}) = \sum n_i = n$$

$$\int_C c_i \neq c_j \text{ and } \underline{\text{is simple}}$$

$$\{\tilde{c}\} f = \frac{\sum \tilde{c}_i f - \sum \tilde{c}_j f}{c_j - c_i}$$

$$\tilde{c} = \sum_{k=1}^m a_k [c_k] \quad \underline{\text{closed}}$$

$$n = \deg(\tilde{c}) = \sum a_k$$

$$N(\tilde{c}) = n = \infty \quad \leftarrow \text{not possible}$$

$$n_k > 1 \quad \epsilon' \leq \epsilon \quad n-k > 0 \quad \text{and}$$

$$\tilde{d} = d_\varepsilon = \tilde{c} - \{c_n\} + \{c_n + \varepsilon\}$$

$$\int_C \tilde{d} = \int_C c_n + \varepsilon \quad \varepsilon > 0 \quad \text{and}$$

$$N(\tilde{d}) < N(\tilde{c})$$

$$[\tilde{d}]f = \frac{[\tilde{d}_i]f - [\tilde{d}_j]f}{\tilde{d}_j - \tilde{d}_i}$$

$$\text{Definim } \sum_{i=1}^n \tilde{d}_i f_i = \sum_{i=1}^n c_i f_i$$

$$\text{at } \tilde{c}_i = \frac{\tilde{d}_i f_i}{\sum_{j=1}^n \tilde{d}_j f_j}$$

$$= \lim_{\epsilon \rightarrow 0} \int_{\tilde{d}_i - \epsilon}^{\tilde{d}_i + \epsilon} f(x) dx$$

$$[(n-1)[\tilde{c}_0] + [c_0 + \epsilon]]f \rightarrow \underbrace{[n[c_0]]f}_{\epsilon \rightarrow 0}$$

$$[c_0, c_0 + \epsilon]f = \frac{f(c_0 + \epsilon) - f(c_0)}{\epsilon} \xrightarrow{\epsilon \rightarrow 0} f'(c_0)$$

$$P_n(c_0) = \sum_{k=0}^n \frac{f^{(k)}}{k!} (x - c_0)^k$$

$$c_0 = c_1 = c_2$$

$$c_5 = c_6$$

$$c_0 \quad f_0$$

$$c_1 \quad f_0 = [f]_{c_0, c_1} f = f_1$$

$$c_2 \quad f_0 \cdot f_1 \quad f_2$$

$$c_3 \quad f_3 \quad [c_2, c_3] f = \frac{f_3 - f_0}{c_3 - c_0} \cdot [c_1, c_2, c_3] f = \frac{[c_2, c_3] f - f_1}{c_3 - c_0}$$

$$c_4 \quad f_4$$

$$c_5 \quad f_5$$

$$c_6 \quad f_5 \quad f_6$$

$$c_7 \quad f_0 \quad [c_6, c_7] f = \frac{f_0 - f_5}{c_7 - c_6}$$

$c_6^{11}$

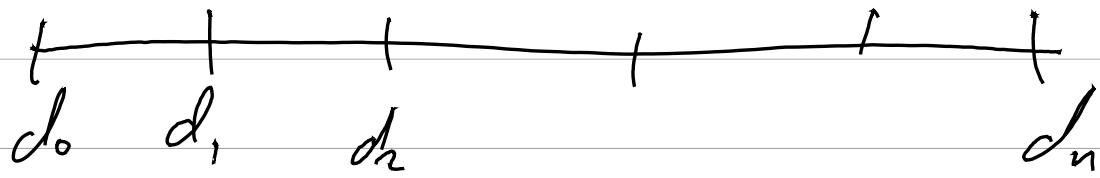
$$[c_5, c_3] f$$

$$\underbrace{p'j's \circ}_{a < b \in R}$$

$$1g/\delta n \quad \sim \gamma / p$$

$$a = d_0 < d_1 < \dots < d_n = b$$

↓

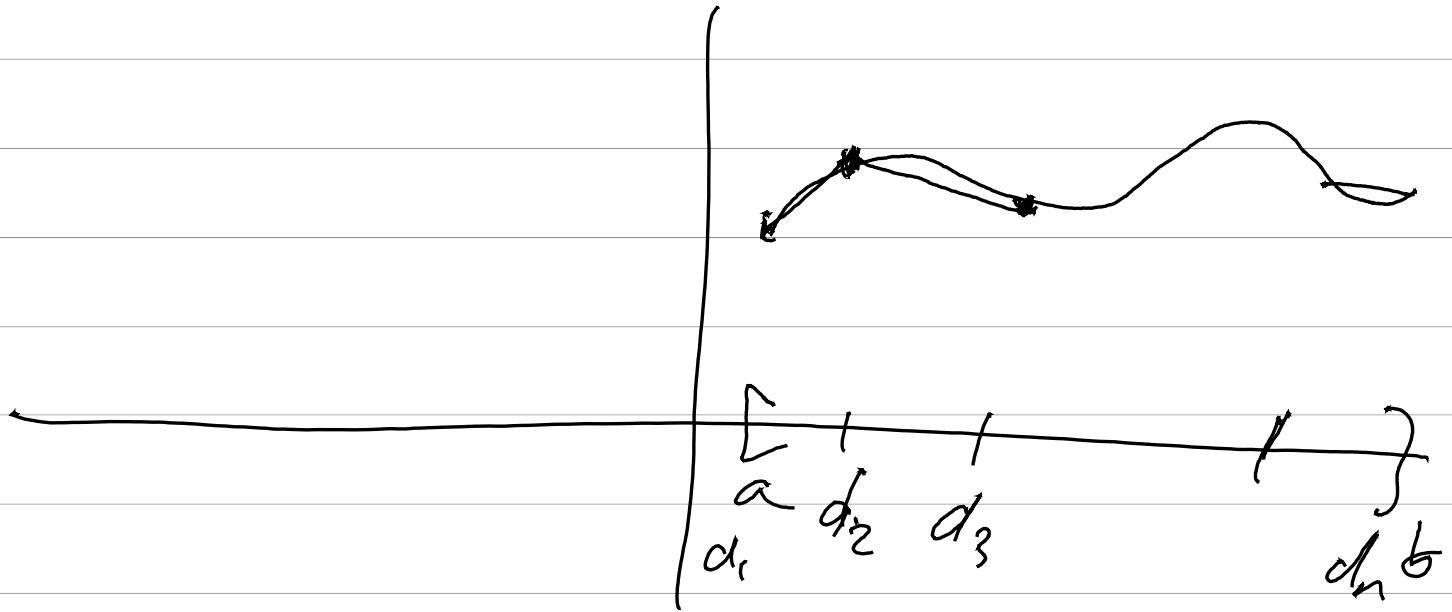


$$S_m^k(D) = \left\{ s \in C^k[a, b] \mid s|_{[d_i, d_i]} \right\}$$

$m \geq 1, 2, \dots, k$

$$k < m$$

$$S_1^{\circ}(\delta) = \int_{\sqrt{d}}^{\sqrt{d+\delta}} \sin^{-1} f(x) dx$$



スレーブの間接積分法

$f$  の  $\int_a^b f(x) dx$  を  $\sum_{i=1}^n f(\xi_i) \Delta x$  で近似する

$$|f(x) - S(x)| \leq \frac{M}{2} \cdot |(x-d_i)(x-d_{i+1})| \leq$$

$$\frac{M}{8} \cdot [(d_{i+1} - d_i)^2]$$

$$M = \max_{x \in [d_i, d_{i+1}]} f''(x)$$

(a)  $\rightarrow$   $\lambda$ )  $\wedge$   $\forall \lambda$   $\exists K$   $\exists c$   $\forall x$

$$\frac{M}{8} \cdot |\Delta|^2$$

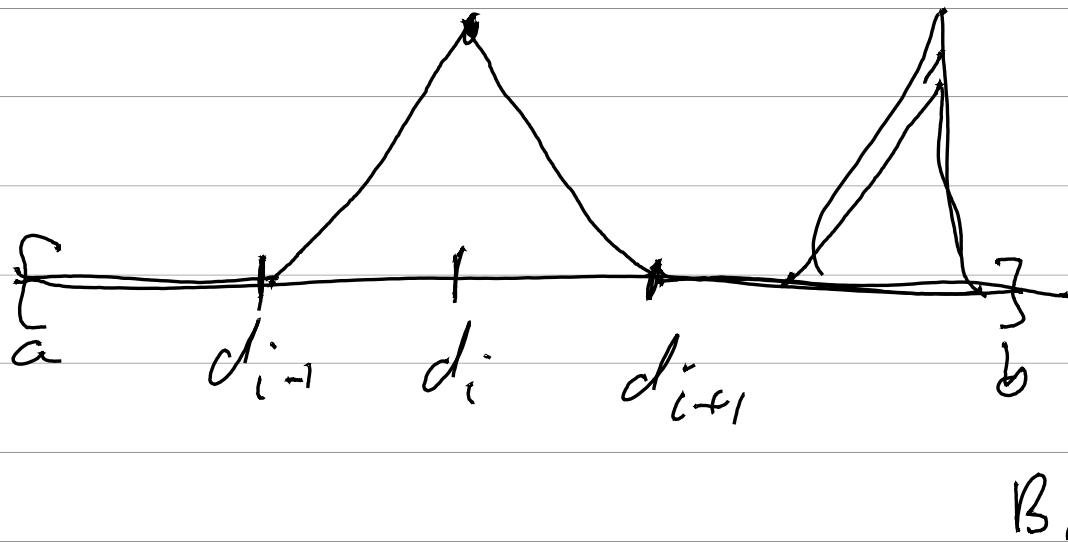
$|\Delta| = \text{max } \sqrt{\tau} \text{ for } \tau \in C$

$$\|f - \varsigma_1\| \leq \underbrace{\|f - \varsigma_1^0\|}_{\text{error}}$$

$V \rightarrow \partial \tau \text{ in } \Lambda \text{ near } \varsigma_1^0(\delta)$

17267 0.025  $\rightarrow$   $\tau$

$R^{d_1, \dots, d_n}$   $\mu$



$$\langle B_i, B_j \rangle \neq 0$$

~~$|i-j| \leq 1$~~   $\forall i, j$

~~such that~~

$$\hat{s} = \sum c_i B_i$$

~~such that~~  $f \in \mathcal{F}_{\gamma_1}$

~~such that~~  $f \in \mathcal{F}_{\gamma_1}$

$$T \bar{c} = \bar{d}$$

$$T = \left( \langle B_i, B_j \rangle \right)_{i,j} \in \mathbb{R}^{C \times C}$$

$$d_i := \langle f, B_i \rangle$$

$$T = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

•  $\sum_i d_i^2 / C$  "average"

$$\|f - \hat{g}\|_\infty \leq 4 \cdot \overbrace{d(f, S_1^\circ(\delta))}^{> 3\gamma/\delta}$$

$\therefore 3 \rightarrow \text{approx} \approx 1520$

and '3' for  $f$  to  $1520$

we approx.  $s_3'(d)$   $\approx$

- $c$ )  $\approx$   $c$ '  $\approx$  '3' for  $f$

$s(d_i) = f(d_i)$  i.e.

$s|_{[d_{i-1}, d_i]}$   $\geq$  approx 1520

$s'(d_i) = m_i$

$m_i = \underline{\underline{f'(d_i)}} \cdot l_c$

$\rightarrow$   $m_i$  part of  $\approx 1520$

$s \in s_3^2(D)$

1) def and lin

. (def)  $f$  for  $\boxed{f: \mathbb{R} \rightarrow \mathbb{R}}$

for map is  $f$

. (def) or if  $[a, b]$

numbers 113w in range

$f(x) = 0$  if else

then return if else return  $x$

return  $\left( \begin{array}{c} x \\ \vdots \\ x_n \end{array} \right)$

$F(t, x, x', \dots, x^{(n)}) = 0$  .?

$x(1) = x_1$ ,  $x(0) = x_0$

$f(a) \cdot f(b) \leq 0$   $\checkmark$  C is so,

Now we can see  $\exists c' \in ]a, b[$  such that

$f(a) \cdot f(c') < 0$  i.e.  $a' \in f^{-1}(0)$

$f(a) \cdot f(b) \leq 0$   $\checkmark$  "  $c = \frac{a+b}{2}$

$a' < c$ ,  $f(a) \cdot f(c) \leq 0$   $\checkmark$

$\{a, c\} \cup \{c, b\}$   $\Rightarrow$   $f$

$\exists c' \in ]a, b[$

$f(c) \geq 0$   
 $\leq 0$

$X_n - a \in \mathbb{Q}$   $\checkmark$   $c$  is rational  
 $c \in \mathbb{R}$   $\checkmark$   $c$  is real

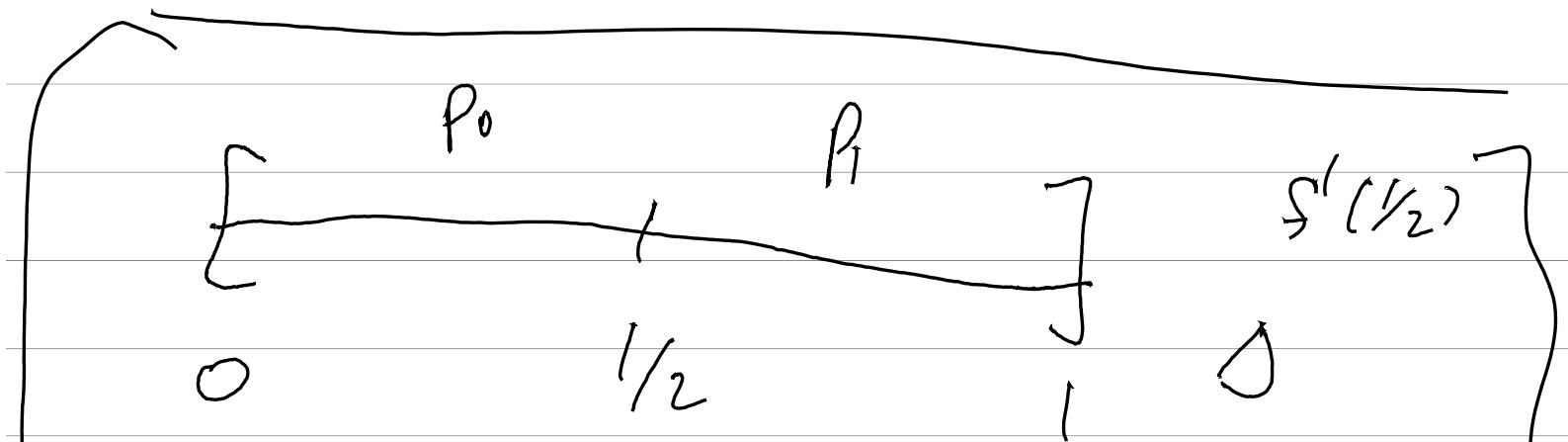
$$|x_n - a| \leq \frac{(b-a)}{2^n} = \varepsilon_n$$

→  $\rho \approx 2$

$$\frac{\varepsilon_{n+1}}{\varepsilon_n} \underset{n \rightarrow \infty}{\sim} \frac{1}{2}$$

$\sqrt{c}$   $P > 30\%$  value

$$\cdot (P > 1 \text{ value}) \quad \overbrace{\varepsilon_{n+1} / \varepsilon_n}^{\rightarrow c > 0} \rightarrow c > 0$$



$$f \quad S \in S_3'( \Delta ) \quad P_0'(1/2) = P_1'(1/2)$$

$$P_0(0) = f(0), \quad P_0(1/2) = f(1/2), \quad P_1(1/2) = f(1/2), \quad P_1(1) = f(1)$$

$$\cdot f(x) = 0 \quad [a, b]$$

$$x_i \rightarrow 0 \quad f(c) = 0$$

$$\left| \frac{x_{i+1}}{x_i} \right| < \varepsilon_i \quad \frac{\varepsilon_{i+1}}{\varepsilon_i} \rightarrow c < 1$$

$$r^l, c, \delta \sim \mathcal{O}(1)$$

$$p > k$$

$$\frac{\varepsilon_{i+1}}{\varepsilon_i} \rightarrow c > 0$$

• Prenav navra

1' 3' 1' 2' 3' 0' : 1' 1'  
 $f_0, \dots, f_n$       1' 3'

For  $n \in \mathbb{N}$  define  $\varphi_{\{a,b\}}$  by  
 $\{a,b\} \subset \text{range } \varphi_{\{a,b\}}^n$

$f_{n+1} = -f_n$ ,  $f_{-1} = 0$       /not

(1')  $f_n$  a ND range ref)

$r \in \{a,b\}$  So  $0 \leq i \leq n$   $\exists$

$f_{i+1}(r), f_{i-1}(r) \in S \cap f_i(r) = 0$   $\rightsquigarrow$

or  $\exists c \in \Gamma(x)$   $\begin{cases} \text{no } \\ \text{if } \end{cases}$

$f_0(x), \dots, f_n(x)$  1' 3' 0' e u m g d)

1' (a)-oth b.  $f_n$  se m g d)  $\begin{cases} \text{no } \\ \text{if } \end{cases}$

ר $\in$   $\{a, b\}$   $\Rightarrow$   $f(r) \in \underbrace{\{a, b\}}$

ר $\in$   $[c, d]$   $\Rightarrow$   $f(r) \in [a, b]$

$[a, b] \rightarrow$  ר $\in$   $[c, d]$

ר $\in$   $[c, d]$   $\Rightarrow$   $f(r) \in [a, b]$

ר $\in$   $[c, d]$   $\Rightarrow$   $f(r) \in [a, b]$

$\left[ \begin{array}{c} f \\ x \end{array} \right] \in \left[ \begin{array}{c} x \\ x \end{array} \right] \in \left[ \begin{array}{c} x \\ x \end{array} \right] \in \left[ \begin{array}{c} x \\ x \end{array} \right]$

ר $\in$   $[a, b]$   $\Rightarrow$   $f(r) \in [a, b]$

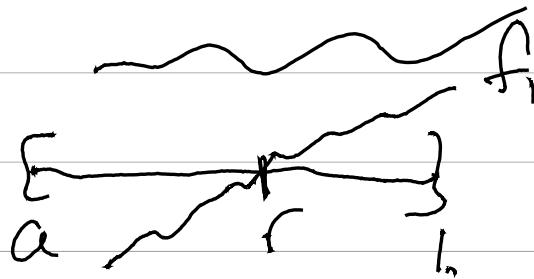
$r \in [a, b] \Rightarrow f(r) \in [f(a), f(b)]$

$n \in \mathbb{N}$   $\exists r \in \mathbb{R}^+$  :  $f_{n+1}(x) < f_n(x)$

$\checkmark n=0$

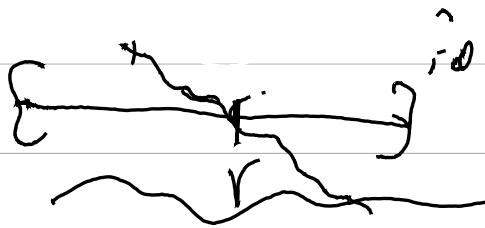
$\exists r \in \mathbb{R}^+$   $\forall x \in \mathbb{R} : f_0(x) < r$   $\underline{n=1}$

$\exists r \in \mathbb{R}^+$   $\forall x \in \mathbb{R} : f_1(x) < r$



$$\underline{f_0(a) = 0}$$

$$f_1(b) = r$$



$$f_1(r) = 0 \Rightarrow f_0(r) \cdot f_1'(r) < 0$$

$$f_0' = -f_1'(r)$$

$$f_0'(r) < 0, f_1'(r) < 0$$

$\epsilon' \rightarrow n - 5$  ו $L_0$  מינימום  $f'$

: מינימום  $f'$

לפ  $x^k$  ר'ו  $f'_k(x)$  מינימום  $f_k(r) \neq 0$  . א

ר'ו  $f_1, \dots, \underline{f_{n+1}}$  ניגו,

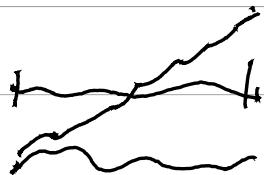
ר'ו  $f_1, \dots, f_n$ ,  $f_{n+1}$  מינימום  $f'$

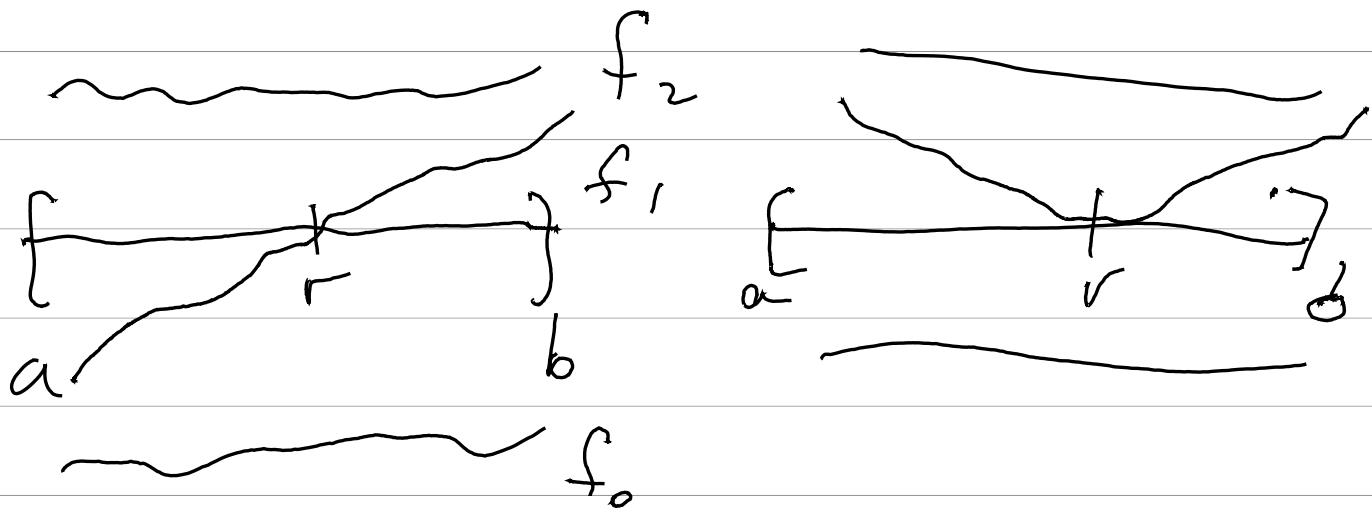
ר'ו  $f_1, \dots, f_n$ ,  $f_{n+1} = 0$

$f_0, f_1 \rightarrow$  מינימום  $f_1(r) \approx 0$  . ס

$f_0(r) \cdot f_1(r) < 0$  ניגו ס

$f_2(r) \neq 0$  ס





$\omega \cup \{f_i\}_{i=0}^n$   $\sim$   $\wedge' \cup J \cup \{\underbrace{f_i}_{\text{crisis}}\}_{i=0}^n$

$\sim$   $\wedge' G \cup \omega \cup \omega' \cup \omega$

$$f_{n-1} = p^l, f_n = p \quad \gamma' \beta \cup$$

~~$$f_{k+1} = q_k \cdot f_k - f_{k-1}$$~~

$$n \leq k-1 \Rightarrow \deg(f_{kn}) < \deg(f_k) \quad \text{reduces}$$

$$\vdash \forall n \forall j \forall i \forall l \forall m \forall p \forall q \forall r \forall s \forall t \forall u \forall v \forall w \forall x \forall y \forall z \forall k \forall l$$

רנינ גודל פולינום ב/c

בנין גודל פולינום ב/c  
 $f_{n+1} \cdot f_{n-1} = -\underbrace{(f')^2}_{\geq 0} \leq 0$

המונומיאים  $\{f_i\}$  נקראים מונומיאים של  $f$ .  
 $\deg(f_i) = i$

המונומיאים  $\{f_i\}$  נקראים מונומיאים של  $f$ .

$$f_{i+1} = (t-a_i) \cdot f_i - b_i f_{i-1}$$

$$\cdot b_i > 0 \rightarrow$$

$$\text{לפיכך } f_i(b) > 0, b > 0 \text{ ו } c \\ \sigma(b) \leq 0 \text{ ס'ו}$$

$\tau(a) \cap g(a) \subset \omega_1$

numeral  $i$  is  $f_i$

$\rho' / \text{JC}$

$\tau_2' / c$   $\infty \delta \delta$  and  $/ c$   $\rho \delta$

$\rightarrow f_{k+1} \in \omega_1$   $(3n)$

$\tau(a) \subseteq \omega_1$

$$x_{k+1} = \underbrace{x_k + x_{k+1}}_{\infty}, \in \omega_1$$

$\tau(x_{k+1}) \in \omega_1$

$\tau(x_{k+1}) \in \omega_1$

$\tau(x_{k+1}) \in \omega_1$

For  $f(x) = 0$  if  $\exists \gamma \in \text{sm}$

$\Rightarrow \exists \beta > \gamma$ ,  $f$ ,  $[a, b]$   $\forall f \geq 0$

$$f(a) \cdot f(b) < 0$$

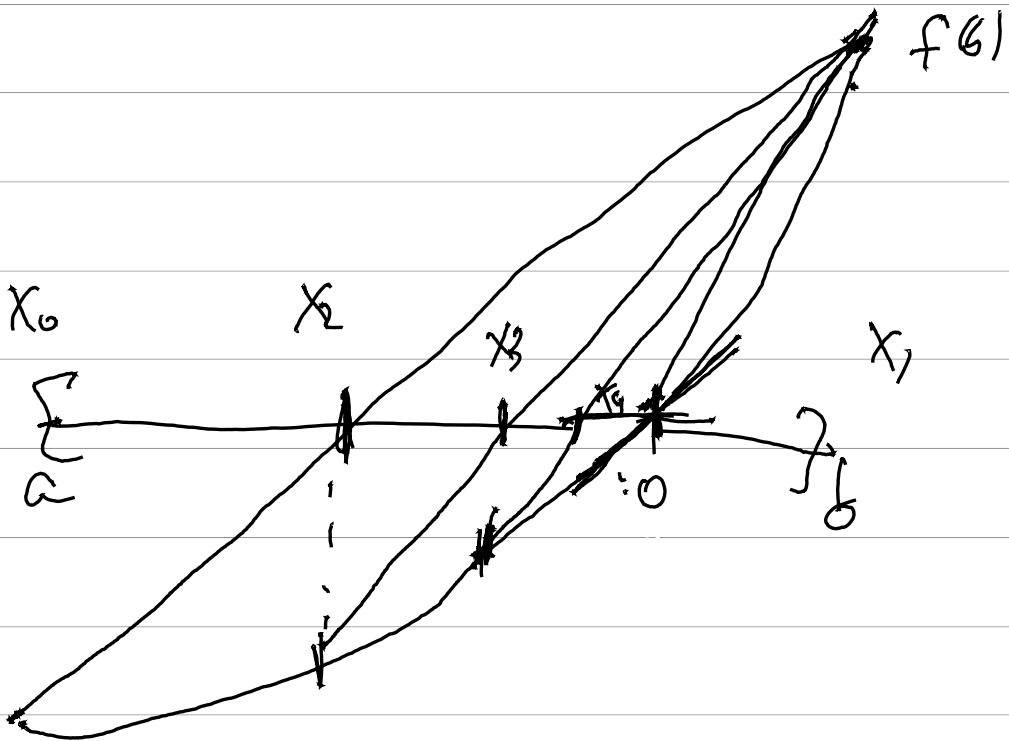


$$\frac{f(b)}{b-a}(x-a) + \frac{f(a)}{a-b}(x-b) = 0$$

$\Downarrow$

$$f(b)(x-a) - f(a)(x-b) = 0$$

$$x = \frac{f(b)a - f(a)b}{f(b) - f(a)}$$



$$x_{n+1} = \frac{f(x_n) \cdot x_{n-1} - f(x_{n-1}) \cdot x_n}{f(x_n) - f(x_{n-1})}$$

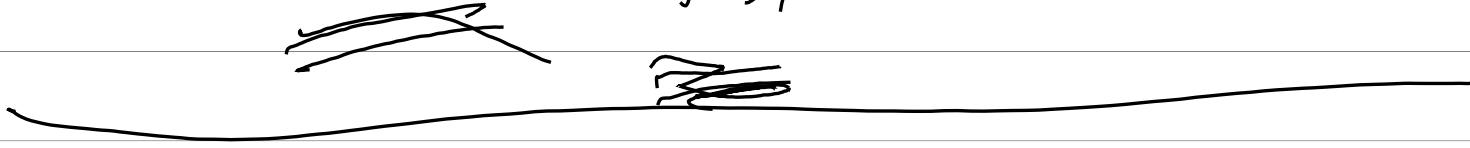
$$\frac{(f(x_n) - f(x_{n-1})) \cdot x_{n-1} + f(x_n) \cdot (x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

$$x_n' = f(x_n) \cdot \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$$

$$\frac{x_{n+1}}{x_n} = 1 - \frac{f(x_n)}{x_n} \cdot \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} =$$

$$1 - \frac{f(x_n)}{x_n} \cdot \frac{x_n - b}{f(x_n) - f(b)} \rightarrow :$$

$$1 - f'(0) \cdot \frac{b}{f(b)} =: c$$



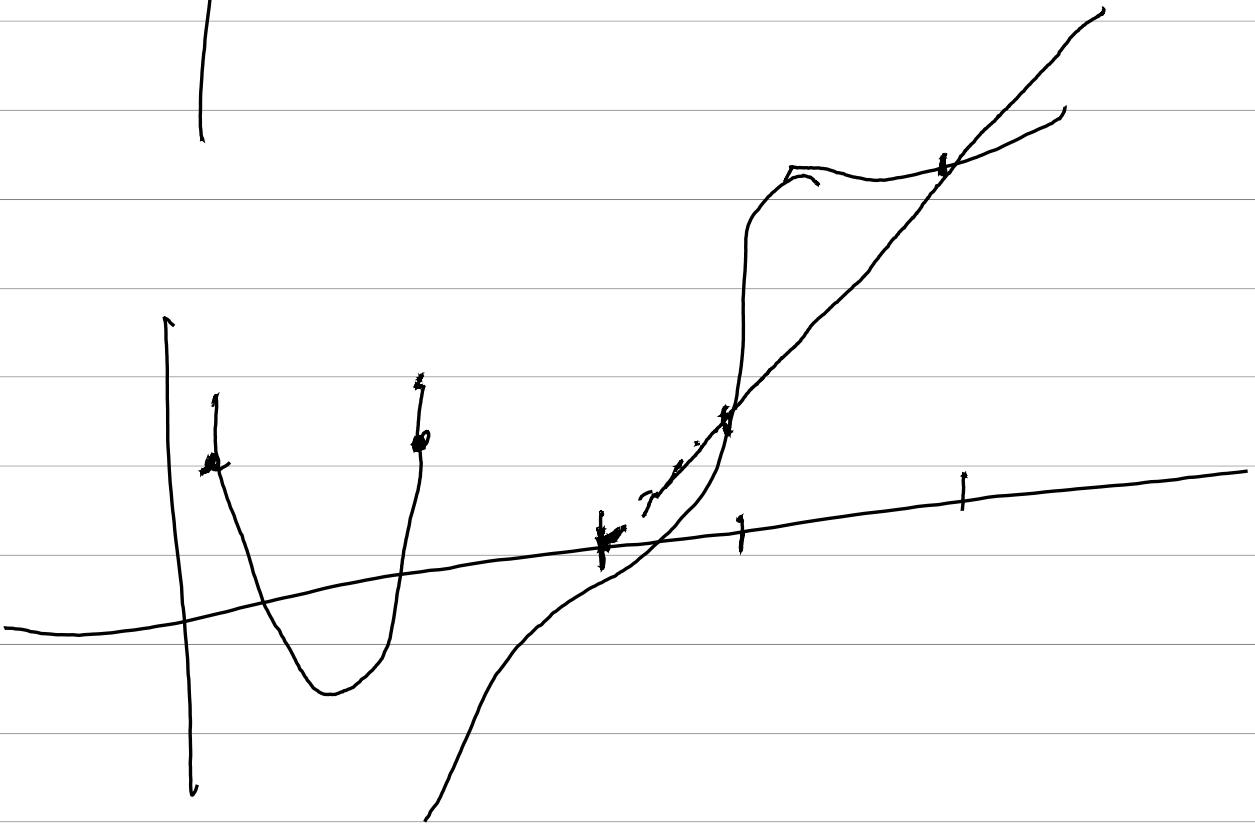
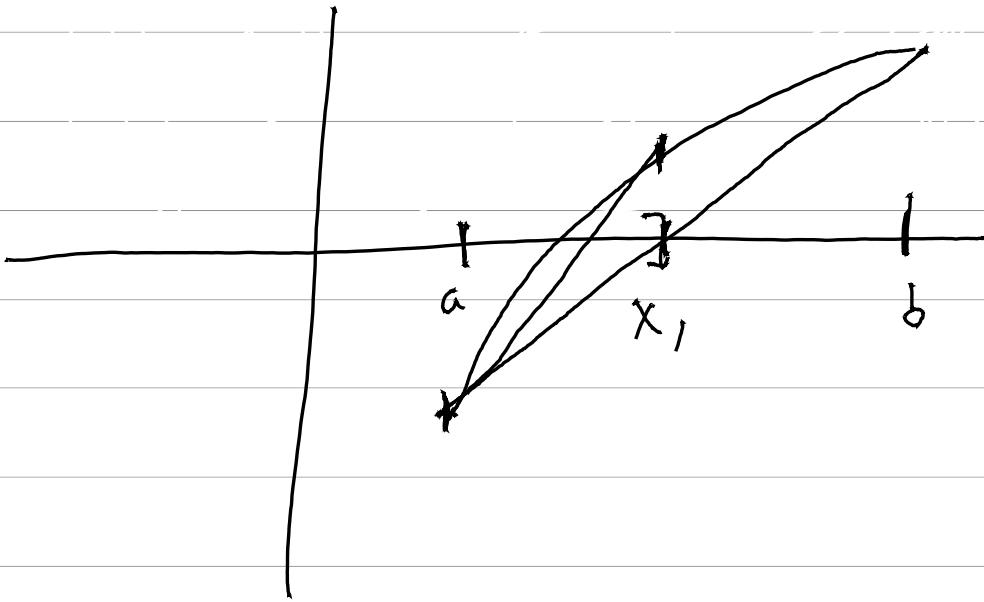
?  $\subset \mathbb{N}$ ,  $n+1 - ? \supset \mathbb{N}$

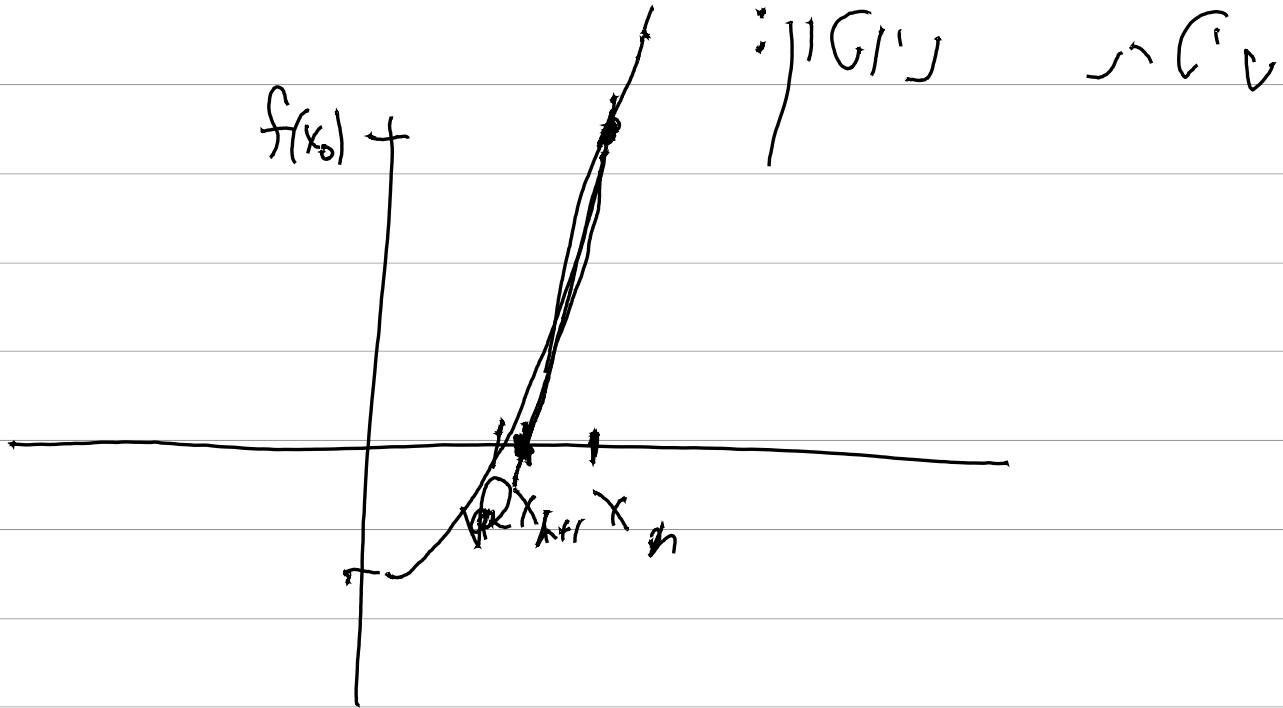
$$d = \frac{f(x_n) \cdot x_{n-1} - f(x_{n-1}) \cdot x_n}{f(x_n) - f(x_{n-1})}$$

$$x_{n+1} = d \quad \text{if } f(d) \cdot f(x_n) < 0 \text{ or } \\ x_{n+1} = x_n, \quad x_n = d \quad \text{if } f(d) \cdot f(x_n) \geq 0 \text{ or }$$

Secant ->  $\cup C_1$ .1

$| \cup C_1$   $\cup C_2$ .2





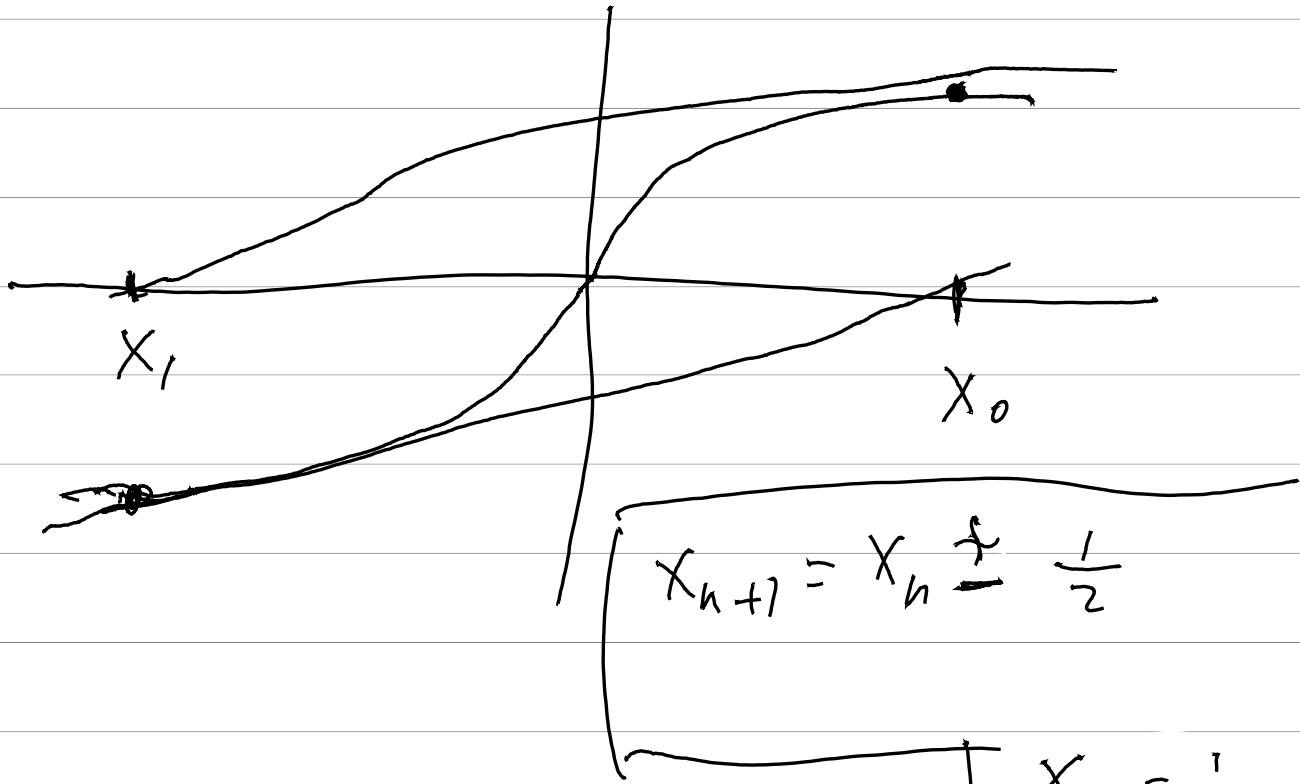
$$\left[ \frac{x_{n+1} - x_n}{0 - f(x_n)} = \frac{1}{f'(x_n)} \right] \Rightarrow$$

$$x_{n+1} = x_n \circ \underbrace{\begin{pmatrix} f(x_n) \\ f'(x_n) \end{pmatrix}}_{\text{f(x_n)}}$$

$\sqrt{a}$  src  $\overbrace{c \circ f^{-1}}^{\text{?}}$  lceil d / 3

$$x^2 - a = 0 \quad (\Rightarrow a > 0)$$

$$x_{n+1} = x_n - \frac{x_n^2 - a}{2x_n} = \frac{1}{2} \left( x_n + \frac{a}{x_n} \right)$$



$$f(x) = \begin{cases} \sqrt{x} & x \geq 0 \\ -\sqrt{-x} & x \leq 0 \end{cases}$$

$$x_0 = \frac{1}{4}$$

$$x_1 = -\frac{1}{4}$$

$$x_2 = \frac{1}{16} = \infty$$

$$f'(x) = \begin{cases} \frac{1}{2\sqrt{x}} & x \geq 0 \\ -\frac{1}{2\sqrt{-x}} & x \leq 0 \end{cases}$$

$$\varphi(x) = x - \frac{f(x)}{f'(x)}$$

$$x_{n+1} = \varphi(x_n)$$

Se f ist  
auf  $\mathbb{R}$  stetig und  
durchl.

$$\varphi(x) = x - \frac{f(x)}{f'(x)}$$

in  $\mathbb{R}$  mit  
einem  
Grenz

$$\text{in } M \text{ mit } (M, d) \text{ mit}$$

$\varphi: M \rightarrow M$  ist  
stetig

$$d(\varphi(x), \varphi(y)) \leq c \cdot d(x, y)$$

$c > 0$  mit  $x, y \in M$

$X_{n+1} = \varphi(X_n)$  ,  $\forall n \geq 0$ ,  $\exists \lim_{n \rightarrow \infty}$

$\Rightarrow \text{Lip } X_0$

,  $i \leq j \Rightarrow \int_{x_i}^{x_j} \dot{x}(t) dt$

$d(x_i, x_j) \leq c^i \cdot \underbrace{d(x_0, x_{j-i})}_{\dots}$

$d(x_0, x_k) \leq d(x_0, x_1) + \dots + d(x_{k-1}, x_k) \leq$

$d(x_0, x_k) / (1 + c + c^2 + \dots + c^{k-1}) = \frac{1-c}{1-c} d(x_0, x_k)$

$\Rightarrow d(x_i, x_j) \leq \frac{c^i}{1-c} d(x_0, x_j) \rightarrow 0$

$c' \subset [c] \cap \cup_{i=0}^{\infty} (X_i)_{i \geq 0} \subset$

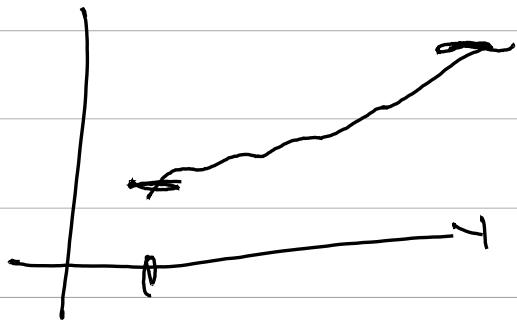
$(r^2, r^3, r^4) \subset \Delta \cup \{r\}$

$\varphi(\alpha) = \varphi(\lim_{i \rightarrow \infty} x_i) = \lim_{i \rightarrow \infty} \varphi(x_i) = \lim_{i \rightarrow \infty} x_i = \alpha$

$\text{Def}(\nu)$   
 $\nu \in \mathbb{N}^{\mathbb{N}} \quad M \subseteq \mathbb{R}$  - e  $\lambda' \cup$

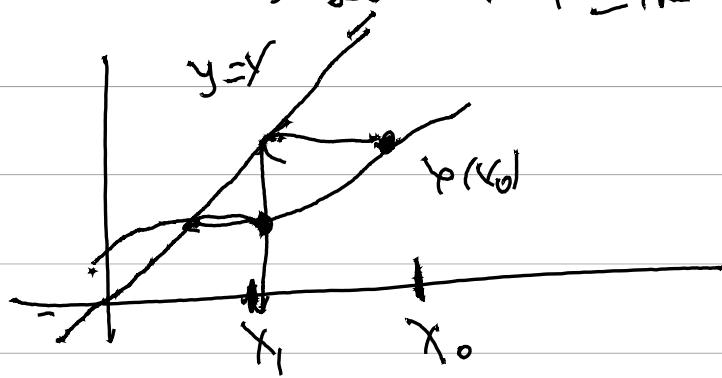
-1, No 3  $\Rightarrow$   $\lambda' \subseteq \varnothing$

all  $\nu$   $\exists c \cdot x \in M \Rightarrow \forall \epsilon < c$



Def  $\nu \in \mathbb{N}^{\mathbb{N}} \rightarrow \mathbb{R}_0 \rightarrow \mathbb{R}$

$\psi: M \rightarrow \mathbb{R}$   
such that  $M \subseteq \mathbb{R}^n$   $\rightarrow \mathbb{R}$



$\alpha^p \approx \alpha^{p-1} \alpha$   $\wedge$   $\varphi$  is VC

$$\varphi'(\alpha) = \varphi''(\alpha) = \dots = \varphi^{(p-1)}(\alpha) = 0 \quad -1$$

$(\varphi \in C^p(M)) \cdot \underbrace{\varphi^{(p)}(\alpha) \neq 0}_{\text{---}}$  -1

: p 1.  $\alpha$   $\approx$   $\alpha^{p-1} \alpha$   $\approx$   $\alpha^p$

$$\frac{x_{n+1} - \alpha}{(x_n - \alpha)^p} \rightarrow \frac{\varphi^{(p)}(\alpha)}{p!}$$

$$\varphi(x) = \alpha + \varphi'(\alpha)(x - \alpha) + \frac{\varphi^{(p-1)}(\alpha)}{(p-1)!}(x - \alpha)^{p-1} + \frac{\varphi^{(p)}(\alpha)}{p!}(x - \alpha)^p$$

$$\alpha + \frac{\varphi^{(p)}(u)}{p!}(x - \alpha)^p \Rightarrow u \in \{\alpha, x\}$$

$$x_{n+1} - \alpha = \frac{\varphi^{(p)}(u)}{p!} \cdot (x_n - \alpha)^p$$

לעסן גראונד

$$\varphi(x) = x - \frac{f(x)}{f'(x)}$$

$$\varphi'(x) = 1 - \frac{f'^2 - f \cdot f''}{f'^2} = \frac{f \cdot f''}{|f'|^2}$$

$$\varphi'(x) = 0$$

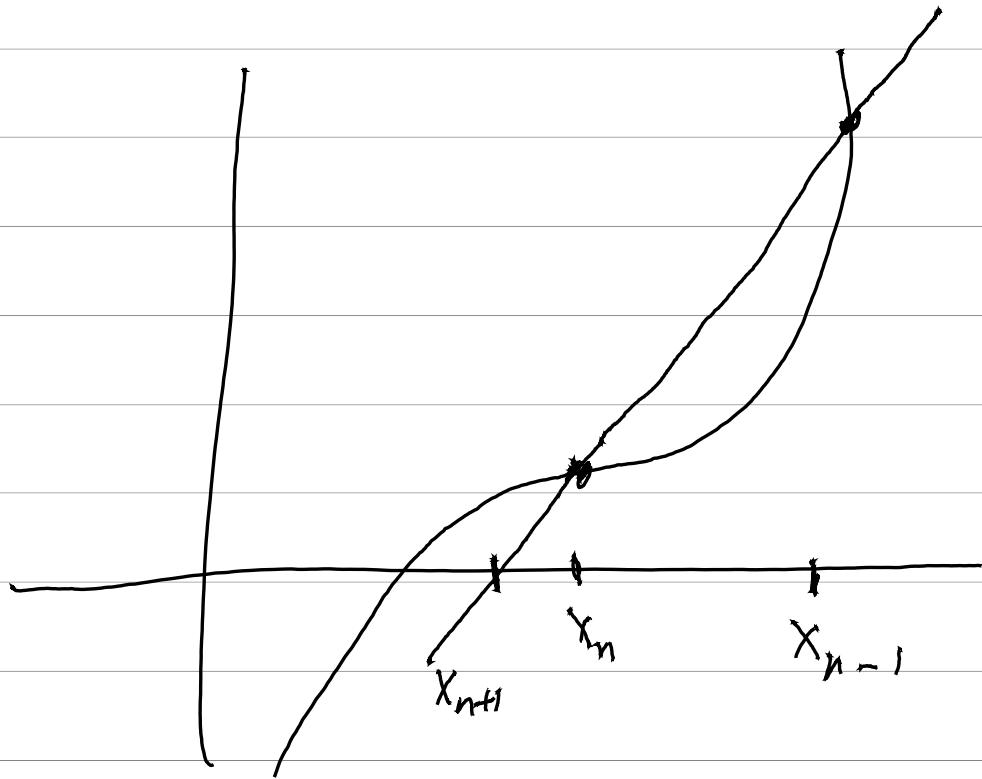
נתקה (ונרמזו) גראונד

ב' גראונד

$$u \text{ sod } \varphi'(u) \leq 0 \Leftrightarrow f'(u) \neq 0$$

ולא נרמזו גראונד

Secant  $\rightarrow$   $C_V$



$$\frac{x_{n+1} - x_n}{-f(x_n)} = \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$$

$$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \cdot f(x_n)$$

$\alpha$  (red) en  $\ell'$  e  $\wedge \vee$

$$x_{n+1} - \alpha = x_n - \alpha + \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n) =$$

$$(x_n - \alpha) \left( 1 - \frac{f(x_n)}{(x_n - \alpha)[x_n, x_{n-1}] f} \right) =$$

$$(x_n - \alpha) \left( 1 - \frac{[x_n, \alpha] f}{(x_n, x_{n-1}) f} \right) =$$

$$(x_n - \alpha) \left( \frac{[x_n, x_{n-1}] f - [x_n, \alpha] f}{[x_n, x_{n-1}] f} \right) =$$

$$(x_n - \alpha)(x_{n-1} - \alpha) \cdot \left( \frac{[x_n, x_{n-1}, \alpha] f}{[x_n, x_{n-1}] f} \right)$$

$$(x_n - \alpha)(x_{n-1} - \alpha) \cdot M$$

$$m = \max_{t, s_Q} \left| \frac{f''(s)}{2f'(t)} \right| \rightarrow R_f, \rightarrow$$

∴  $f'(x_0)$

290 | { } 290 ↗

$$U_\alpha = \{x \mid |x - a| < \varepsilon\}$$

$$\text{If } \varepsilon \cdot n < 1 - \delta \quad \text{and} \quad x_{n-1}, x_n \in K$$

$\overbrace{\hspace{10em}}$

$$|x_{n+1} - x| \leq \varepsilon^2 n \leq \varepsilon \cdot 1 = \varepsilon$$

$$\therefore x_n \rightarrow d \Rightarrow n' \in \mathbb{N} \text{ s.t. } |x_{n'} - d| < \epsilon$$

$$|X_{n+1} - \alpha| \leq |X_n - \alpha| \cdot |X_{n-1} - \alpha| \cdot M \leq$$

$$\varepsilon \cdot |X_{n-1} - \alpha| \leq \varepsilon^2 (|X_n - \alpha|)^2$$

$$\underbrace{(\varepsilon \cdot M)}_1^n \cdot |X_1 - \alpha|$$

$$E_n = \underbrace{|X_n - \alpha|}_1 \cdot M$$

$$\frac{|X_{n+1} - \alpha|}{(|X_n - \alpha|)^P} = \frac{E_{n+1}/M}{\frac{E_n^P}{M}} \leq \phi$$

$$E_{n+1} \leq E_n \cdot E_{n-1}$$

$$\underbrace{P^2 = P + 1}_{||}$$

$$P = \frac{1 + \sqrt{5}}{2}$$

$$E_n \leq E \cdot M^n$$

$$E = \max(E_0, E^{1/P})$$

$$E_{n+1} \leq E_n \cdot E_{n-1} \leq E^P \cdot E^{P^{n-1}} = E^{P^{n-1}(P+1)} = E^{P^{n+1}}$$

$$[x_0, x_1]f \doteq f'(u)$$

$$u \in \{x_0, x_1\}$$

$$[x_0, \dots, x_k] = \frac{f^{(k)}(u)}{k!}$$

$$u \in [a, b]$$

$$\therefore r^{\prime} N \cup S \rightarrow \int \parallel C's \quad \text{sum}$$

$$x_{n+1} = x_n - \underbrace{\frac{f(x_n)}{f'(x_n)}}$$

$$x^n + a \begin{cases} b_n = c_n = 1 \\ b_k = t b_{k+1} + c_k \\ c_k = t c_{k+1} + b_k \end{cases}$$

$$(x-t)(x^{n-1} + b_{n-1}x^{n-2} + \dots + b_1) + b_0 = P(x)$$

$$b_n = 1 \quad b_k = t b_{k+1} + c_k$$