

Statistical Working Paper on Imputation and Validation Methodology for the FAOSTAT Production Domain

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Abstract

This paper proposes a new imputation method for the FAOSTAT production domain. The proposal provides resolve to many of the shortcomings of the current approach, and offers a flexible framework to incorporate further information to improve performance. We first examine the factors that drive changes in production by commodity, after which a brief account of the current approach and its shortcomings. Then, a description of the new methodology is provided, with a visual decomposition of the model and accompanying explanation.

Keywords: Imputation, Linear Mixed Model, Agricultural Production, EM.

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1. Introduction

Missing values are commonplace in the agricultural production domain, stemming from non-response in surveys or a lack of capacity by the reporting entity to provide measurement. Yet a consistent and non-sparse production domain is of critical importance to Food Balance Sheets (FBS), thus accurate and reliable imputation is essential and a necessary requisite for continuing work. This paper addresses several shortcomings of the current work and a new methodology is proposed in order to resolve these issues and to increase the accuracy of imputation.

The relationship between the variables in the production domain can be expressed as:

$$P_t = A_t \times Y_t \quad (1)$$

Where P , A and Y represent production, area harvested and yield of crops, respectively, indexed by time t . In the case of livestock, A represents number of slaughtered animal while Y represents the carcass weight per animal. The yield is, however, unobserved and can only be calculated when both production and area are available. For certain commodities, harvested area may not exist or sometimes it may be represented under a different context.

The primary objective of imputation is to incorporate all available and reliable information in order to provide best estimates of food supply in FBS.

2. Background and Review of the Current Methodology

There have been two classes of methodology proposed in the past in order to account for missing values in the production domain. The first type utilizes historical information and implements methods such as linear interpolation and trend regression; while the second class aims to capture the variation of relevant commodity and/or spatial characteristics through the application of aggregated growth rates. The imputation is carried out independently on both area and production, with the yield calculated implicitly as an identity.

Nevertheless, both approaches only utilize one dimension of information and improvements can be obtained if information usage can be married. Furthermore, these methods lack the ability to incorporate external information such as vegetation indices, precipitation or temperature that may provide valuable information and enhance the accuracy of imputation.

Simulation results of the prior attempts indicate that linear interpolation is a stable and accurate method but it lacks the capability to utilize cross-sectional information. Furthermore, it does not provide a solution for extrapolation where connection points are not available. As a result, the aggregation method was then implemented as it was found to provide a high coverage rate for imputation with seemingly satisfactory performance.

In short, the aggregation imputation method computes the commodity/regional aggregated growth of both area and production, the growth rate is then applied to the last observed value of the respective series. The formula of the aggregated growth can be expressed as:

$$r_{s,t} = \sum_{c \in S} X_{c,t} / \sum_{c \in S} X_{c,t-1} \quad (2)$$

Where S denotes the relevant set of products and countries within the relevant commodity group and regional classification after omitting the item to be imputed. For example, to compute the *country cereal aggregated growth* with the aim to impute wheat production, we sum up all the production of commodities listed in the cereal group in the same country excluding wheat. On the other hand, to impute by *regional item aggregated growth*, wheat production data within the regional profile except the country of interest are aggregated.

Imputation can then be computed as:

$$\hat{X}_{c,t} = X_{c,t-1} \times r_{s,t} \quad (3)$$

There are, however, several shortcomings of this methodology. The Achilles heel lies in the fact that area and production are imputed independently, cases of diverging area harvested and production have been observed that result in inconsistency between trends as well as exploding yields. The source of this undesirable characteristic is nested in the computation of the aggregated growth rate. Owing to missing values, the basket computed may not be comparable over time and consequently results in spurious growth or contraction. Furthermore, the basket to compute the changes in production and area may be considerably different.

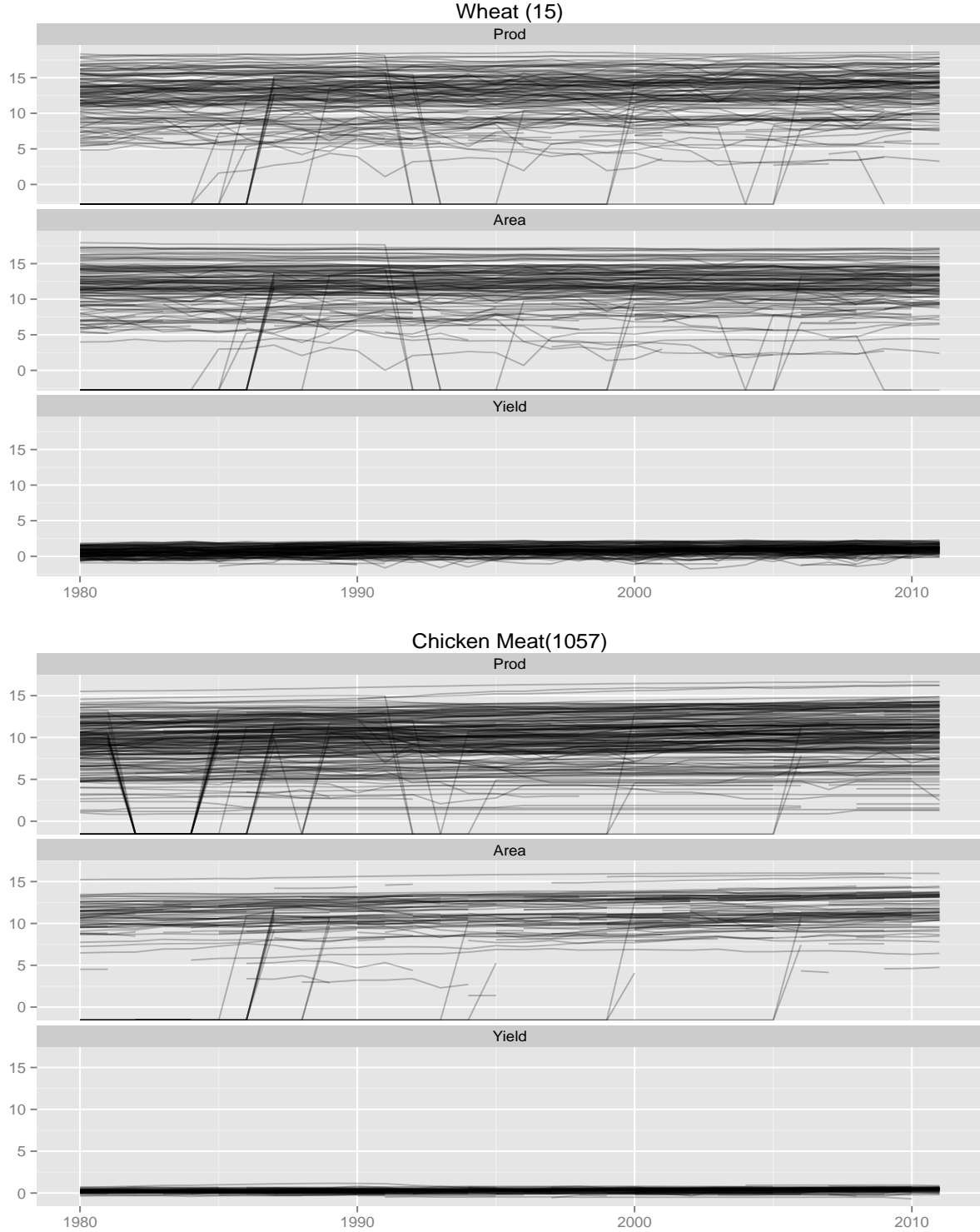
Finally, the methodology does not provide insight into the underlying driving factors of production that are required to better understand the phenomenon and hence for interpretation.

3. Exploratory Data Analysis

Before any modelling or statistical analysis, a grasp of the data is essential. This section is devoted to some basic exploratory analysis of the data in order to understand the nature of the series and their drivers. First, let us explore the relationship between the identity in equation

1. To make the relationship clearer we have log-transformed the data so the relationship becomes an additive one rather than multiplicative.

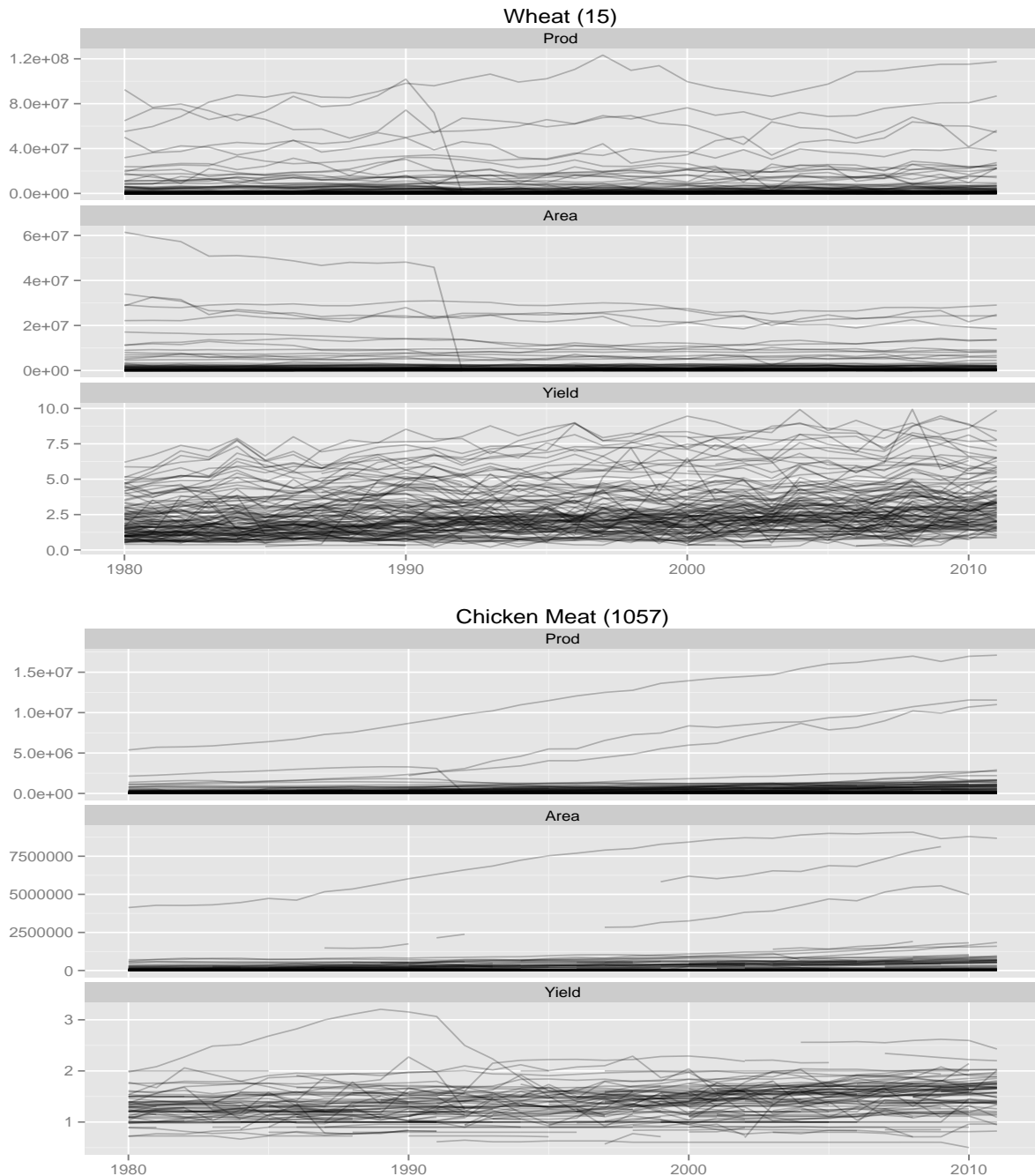
$$\log(P_t) = \log(A_t) + \log(Y_t) \quad (4)$$



In the above plots, the log of production, area and yield of a specific commodity is depicted within each panel for comparison. Each line represents a country and the production is the sum of area and yield. The first noteworthy feature we observe from the relationship between the series is that the level of production is mainly determined by the level of harvested area,

furthermore shocks are typically reflected by a significant change in the area rather than the yield. It is commonly believed that harvested area is stable and predictable over time while vulnerable to shocks stemming from the natural world. Secondly, the range of the variability for yield is very small in comparison to area, consistent with our intuition that there are physical constraints on the potential yield of a crop within a given size of area. The results are very similar even between different commodities.

After exploring the relationship between the identity, let us delve deeper into the constituents of production: area and yield. Depicted below are area and yields for the same set of commodities but on an original scale.



We can first observe that area is in general much more stable and smoother when compared to yield. The yield fluctuates from year-to-year with correlation to a certain extent, which is more prominently observed in wheat. This implies that there maybe underlying factors such as climatic shocks, which may impact the yield in different countries simultaneously.

However, this characteristic is not observed in the livestock and there are no reason to suspect that the carcass weight per animal vary significantly from year-to-year and correlated amongst countries.

The figures strongly suggests that both the trend and level of production is largely determined by area harvested, but the year-to-year fluctuation is driven by the yield, which may be associated with changing climate conditions. The exploratory data analysis reveals valuable insights into the nature of the time series, and underpins the proposed model decomposition of variability and in attributing the fluctuation to area and yield.

4. Proposed Methodology

In order to avoid identification problems and to capture the correlation of yield between countries, we propose to impute the yield and area in contrast to production and area. The added advantage of this approach, with well designed validation, almost guarantees that the series will not diverge as is the case with the current approach.

4.1. Imputation for Harvested area

From the exploratory analysis we can observe that the series is extremely stable, and it is impossible to predict the shocks given the current information set. The methodology proposed here is what we called naive imputation with linear interpolation and last observation carry forward or backward. This method has proven to work extremely well especially when support points are added after imputing the yield.

After imputing the yield and computing area and production where available, we then impute the area with linear interpolation and carry forward the last observation when both production and area are not available.

Following prior research and current investigation, we believe linear interpolation is suitable because much of the harvested area data exhibit extremely stable trends while linear interpolation yields a satisfactory result. Despite the stability, shocks are sometimes observed in the area series. However, without a further understanding of the nature and the source of the shocks, blindly applying the model will introduce vulnerability rather than an anticipated improvement of imputation performance. At the current stage, we have chosen to carry forward and backward the latest available data where linear interpolation is not applicable. The major advantage of this approach is that if production ceases to exist and both production and area are zero, we will not impute a positive value. Nevertheless, we are continuing to explore the data and investigate superior methods which may be applied to the imputation of area.

$$\hat{A}_t = A_{t_a} + (t - a) \times \frac{A_{t_b} - A_{t_a}}{t_b - t_a} \quad (5)$$

Then for values which we can not impute with linear interpolation, we impute with the latest value.

$$\hat{A}_t = A_{t_{nn}} \quad (6)$$

4.2. Imputation for Yield

The proposed model for imputing the yield is a linear mixed model, the usage of this model enables all information available both historical and cross-sectional to be incorporated. In addition, proposed indicators such as the vegetation index, CO₂ concentration and other drivers can be tested and incorporated if proven to improve predictive power.

The general form of the model can be expressed as:

$$\begin{aligned} \mathbf{y}_i &= \mathbf{X}_i\boldsymbol{\beta} + \mathbf{Z}_i\mathbf{b}_i + \epsilon_i \\ \mathbf{b}_i &\sim \mathbf{N}_q(\mathbf{0}, \boldsymbol{\Psi}) \\ \epsilon_i &\sim \mathbf{N}_{ni}(\mathbf{0}, \sigma^2\boldsymbol{\Lambda}_i) \end{aligned} \quad (7)$$

Where the fixed component $\mathbf{X}_i\boldsymbol{\beta}$ models the regional level and trend, while the random component of $\mathbf{Z}_i\mathbf{b}_i$ captures the country specific variation around the regional level. More specifically, the proposed model for FAOSTAT production has the following expression:

$$Y_{i,t} = \underbrace{\beta_{0j} + \beta_{1j}t + \beta_{2,i}\bar{Y}_{j,t}}_{\text{Fixed effect}} + \underbrace{b_{0,i} + b_{1,i}t}_{\text{Random effect}} + \epsilon_{i,t} \quad (8)$$

Where Y denotes yield with \bar{Y} being the grouped averaged yield, i for country, j represents the designated regional grouping and t denotes time. Where the grouped averaged yield is computed as:

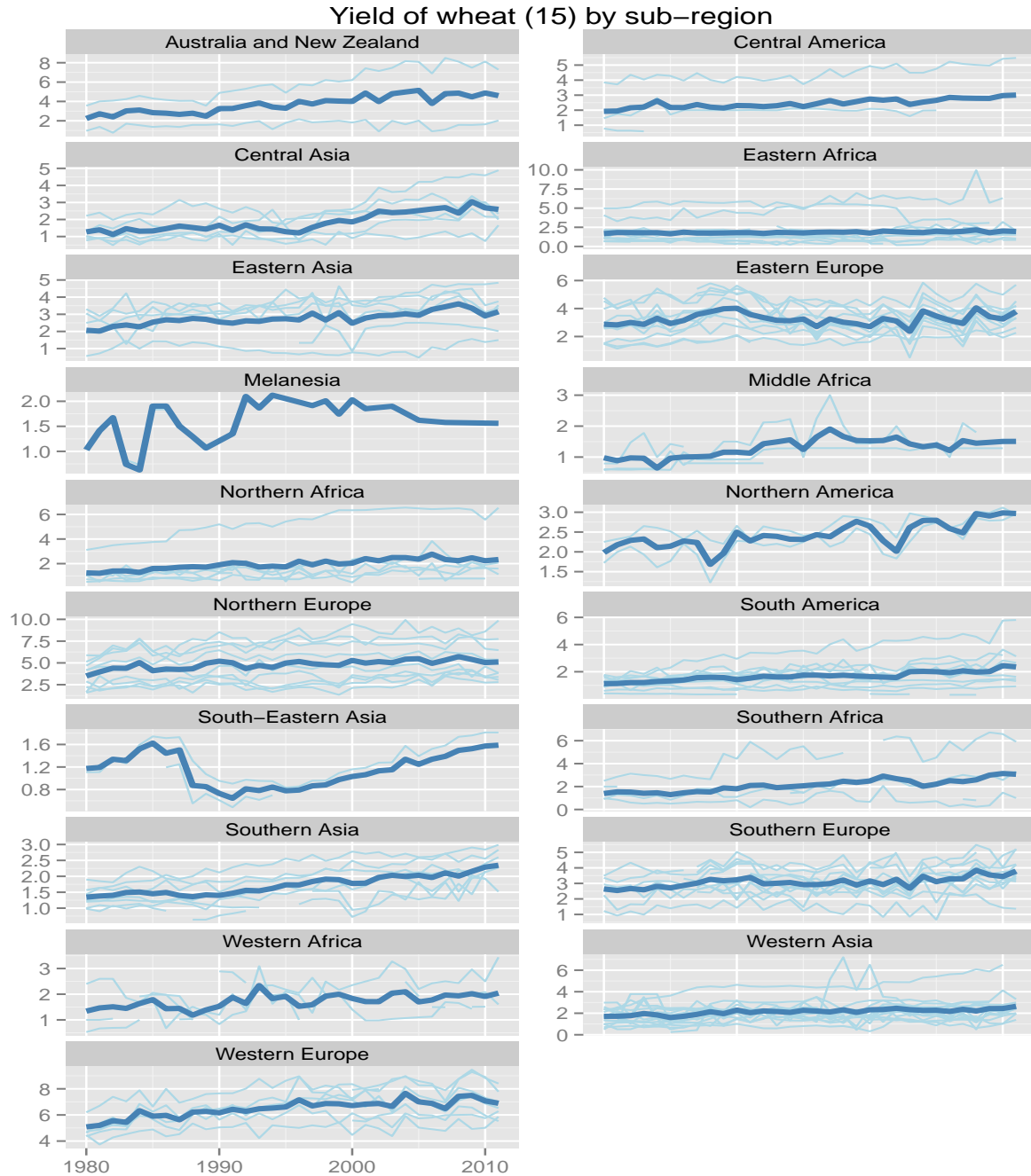
$$\bar{Y}_{j,t} = \frac{1}{N_i} \sum_{i \in j} \hat{Y}_{i,t} \quad (9)$$

However, as the grouped average yield is only partially observed given the missing values, the average yield is estimated through the EM-algorithm.

In essence, the imputation of the yield is based on the country specific level and historical regional trend while accounting for correlation between country and regional fluctuations. In contrast to the previous methodology, where the full effect of the change is applied, the proposed methodology measures the size of relationship between the individual time series and the regional variability to estimate the random effect for the country. Since both historical and cross-sectional information are utilized, imputed values display stable characteristics while reflecting changes in climatic conditions.

Nevertheless, we can observe that under the livestock circumstances the group average effect is not required. In this case, the algorithm will select the model with the lowest AIC and discard the group average regressor if it does not add value to the model.

To better understand the methodology, shown below is the sub-regional decomposition with both the regional level (black line) and the grouped yield (dark blue line) superimposed. The model assigns a sub-regional level and trend depicted as the black line to each series and models the correlation with the dark blue grouped yield series.



5. Conclusion and Future improvements

The aim of the paper has been to overhaul the current imputation methodology, with a more consistent, yet better performing approach.

The proposed model demonstrates the ability to resolve issues such as diverging area and production series and biased growth as a result of missing values. Furthermore, the proposal takes into account the attribute of incorporating relevant information as well as establishing a flexible framework to accommodate additional information.

This is, however, work in progress, as the technical teams are continuing to collaborate in order to seek a deeper understanding of the data in order to improve on the model. Application of the state-space model might be a candidate to succeed in this regard, as it allows for production, area and yield to be imputed simultaneously.

6. Acknowledgement

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Annex 1: Geographic and classification

The geographic classification follows the UNSD M49 classification at <http://unstats.un.org/unsd/methods/m49/m49regin.htm>. The definition is also available in the FAOregionProfile of the R package FAOSTAT.

Annex 2: Pseudo Codes

Algorithm 1: EM-Algorithm for Imputation

Initialization;

$$\hat{Y}_{i,t} \leftarrow f(Y_{i,t});$$

$$\mathcal{L}_{old} = -\infty;$$

$$\mathcal{E} = 1e-6;$$

$$n.iter = 1000;$$

(1) Estimate model without grouped average effect;

$$\hat{Y}_{i,t} \leftarrow \hat{\beta}_{0j} + \hat{\beta}_{1j}t + \hat{b}_{0i} + \hat{b}_{1i}t;$$

(2) Estimate model with grouped average effect;

begin

for $i=1$ to $n.iter$ **do**

 E-step: Compute the expected group average yield;

$$\bar{Y}_{j,t} \leftarrow 1/N \sum_{i \in j} \hat{Y}_i;$$

 M-step: Estimate the Linear Mix Model in 8;

if $\mathcal{L}_{new} - \mathcal{L}_{old} \geq \mathcal{E}$ **then**

$$\hat{Y}_{i,t} \leftarrow \hat{\beta}_{0j} + \hat{\beta}_{1j}t + \hat{\beta}_{2i}\bar{Y}_{j,t} + \hat{b}_{0i} + \hat{b}_{1i}t;$$

$$\mathcal{L}_{old} \leftarrow \mathcal{L}_{new};$$

end

else

 | break

end

end

end

(3) Select model with the smallest AIC

Algorithm 2: Imputation Procedure**Data:** Production (element code = 51) and Harvested area (element code = 31) data**Result:** ImputationMissing values are denoted \emptyset ;

Initialization;

begin **if** $A_t = 0 \wedge P_t \neq 0$ **then** $A_t \leftarrow \emptyset$; **end** **if** $P_t = 0 \wedge A_t \neq 0$ **then** $P_t \leftarrow \emptyset$; **end****end**

Start imputation;

begin **forall the commodities do**

(1) Compute the implied yield;

 $Y_{i,t} \leftarrow P_{i,t} / A_{i,t}$;

(2) Impute the missing yield with the imputation algorithm 1;

forall the imputed yield $\hat{Y}_{i,t}$ do **if** $A_t = \emptyset \wedge P_t \neq \emptyset$ **then** $\hat{A}_{i,t} \leftarrow P_{i,t} / \hat{Y}_{i,t}$; **end** **if** $P_t = \emptyset \wedge A_t \neq \emptyset$ **then** $\hat{P}_{i,t} \leftarrow A_{i,t} \times \hat{Y}_{i,t}$; **end** **end** (4) Impute area ($A_{i,t}$) with equation 5 then 6; **forall the imputed area $\hat{A}_{i,t}$ do** **if** $\hat{Y}_{i,t} \neq \emptyset$ **then** $\hat{P}_{i,t} \leftarrow \hat{A}_{i,t} \times \hat{Y}_{i,t}$; **end** **end** **end****end****Annex 3: Supplementary Resources**

The data, code implementation and documentation can all be found and downloaded from <https://github.com/mkao006/Imputation>. This paper is generated on July 16, 2013 and is subject to changes and updates.

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