Mixture distribution GARCH: An Account for Black Swans

Cheng-Jan Kao
Supervisor : Dr Yong Wang

Dept. of Statistics, University of Auckland

Outline

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2. Data Exploration
3. An R package
4. Implementation and Analysis
5. Conclusion
6. Acknowledgment

Background

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Motivation

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Data Exploration

- The data were collected every half an hour for each household.
- Each consumption series consists of 19008 observation or 396 days.
- Depicted below are four time series chosen to illustrate the variability in the household consumption usage pattern.

Data Exploration

Data Exploration

The Cramer Representation

In essence, The Cramer representation states that many time series can be arbitrarily approximated as a weighted sum of complex exponentials.
When applied to real-valued series, the result says that many stationary time series can be approximated by a sum of the form:

Equation

$$x_t \approx \sum_{\lambda} A_\lambda \cos(\lambda t + \phi_\lambda)$$

Dealing with Non-stationarity

However, as we have seen in the exploratory section that our data is clearly a non-stationary one. Complex Demodulation is a technique in which trades frequency resolution for time resolution and allows the amplitude and phase to vary with respect to time. Thus we arrive at the following representation for a time series:

Equation

$$x_t \approx \sum_{\lambda} A_{\lambda,t} \cos(\lambda t + \phi_{\lambda,t})$$

Complex Demodulation

- $y_t = x_t \times e^{-i\lambda_0 t}$
- $y'_t = \text{smooth}(y_t)$
- Then we can obtained the amplitude and phase from the y'_t.

Complex Demodulation

Package Description

To carry out the analysis, an R package was written and developed. The package container's two main functions:

`mdemodulation` for extracting the amplitude and the phase.
`mmodulation` for obtaining the filtered series y'_t.

Several other accessor functions are also available.

Implementation and Analysis

- Demodulate the time series at 1/48, 1/336 and their first two harmonics. They correspond to daily and weekly cycle.
- Demodulate at zero to obtain the moving mean.
- Below we show the amplitude and phase of the demodulated series.

Implementation and Analysis

Implementation and Analysis

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Conclusion

- Advantage: Its flexible, efficient and deals with non-stationarity.
- Also enabling to observe patterns that were difficult to observe in the original plot.
- limitation: need to obtain a sensible set of frequency. Can be difficult when the series contain too much noises.
- Statistics can be produced from the decomposition for market segmentation through cluster analysis.

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