ARCH and GARCH Models

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Objectives

By the end of this meeting, participants should be able to:

- Identify when heteroscedasticity might be corrected with a log versus a model of conditional heteroscedasticity.
- Model conditional heteroscedasticity with ARCH and GARCH models.
- Explain how ARCH and GARCH resemble ARMA processes.
- Derive the log-likelihood function for an ARIMA or transfer function model. (Time permitting.)

Conditional Heteroscedasticity

- Normal Box-Jenkins setup:
    - $z_t = N_t$ with $\sigma_t^2 = \sigma^2$ for all t
- Expected variance is a constant.
- ARCH lets σ_t^2 vary over time and, in particular, as a function of a_{t-1}.
- Intuition: big errors at t-1 give you an expectation of atypically variable behavior at t.

ARCH (Autoregressive Conditional Heteroscedastic) Models

- A two equation setup:
  1. $z_t = N_t$ (a normal ARIMA specification)
  2. $\sigma_t^2 = \lambda a_{t-1}^2$
- Equation 2 lets variance be non-constant at the modest cost of one additional estimated parameter.

GARCH (Generalized Autoregressive Conditional Heteroscedastic)

- ARCH models $z_t = N_t$ and
  - $\sigma_t^2 = \lambda a_{t-1}^2$
- GARCH adds to the variance model:
  - $\sigma_t^2 = \lambda_1 a_{t-1}^2 + \lambda_2 \sigma_{t-1}^2$
- i.e., variance becomes a function of previous variance as well as previous disturbances.

Formal Model Specifictions

$$z_t = X\beta + \frac{1 - \theta_1 B - \cdots - \theta_Q B^Q}{1 - \phi_1 B - \cdots - \phi_P B^P}\epsilon_t \tag{1}$$

ARCH

$$\begin{aligned} \epsilon_t &= \nu_t \sqrt{h_t} \\ \sigma^2_\nu &= 1 \\ h_t &= \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \alpha_2 \epsilon^2_{t-2} + \cdots + \alpha_q \epsilon^2_{t-q} \end{aligned} \tag{2}$$

GARCH

$$h_t = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \cdots + \alpha_q \epsilon^2_{t-q} + \beta_1 h_{t-1} + \cdots + \beta_p h_{t-p} \tag{3}$$

Notation

- An ARCH process is defined with moving-average terms. We call this an ARCH(q) process.
- A GARCH process allows autoregressive and moving-average terms. We call it GARCH(p,q).
- Hence, an ARCH(q) could be referred to as a GARCH(0,q) in standard notation, which is consistent with ARMA notation.
- However, the R function we will use, "garchFit," actually asks for the MA terms first. Thus, to estimate an ARCH(q) process, the code would call for "garch(2,0)".
- Such is shareware.

Software

- R: "garchFit" (fGarch) is superior to "garch" (tseries), as it allows ARMA processes and is more robust. (Albeit "garch" uses more conventional notation.)
- Nothing in R allows exogenous covariates (yet).
- Hence, R is good at forecasting and diagnosis.

Stata

- "arch" command is like ARIMA, but adds extra arch and garch parameters.
- E.g., for an arch AR(1): arch z, AR(1) arch(1)
- Or for garch: arch z, AR(1) garch(1)
- We also could add a static covariate x: arch z x, AR(1) garch(1)
- Score one for the bad guys!
- To my knowledge, nothing allows for dynamic effect estimation with ARCH/GARCH processes.

For October 18 & 20

Prepare for the midterm.

For October 25

- Granger & Newbold. 1974. "Spurious Regressions in Econometrics." *Journal of Econometrics* 2:111-120.
- Hibbs. 1974. "Problems of Statistical Estimation and Causal Inference in Time-Series Regression Models." *Sociological Methodology* 4:252-307.