GARCH Modeling with R/Rmetrics

A Case Study presented at the Meielisalp Workshop on Computational Finance and Financial Engineering

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Outline

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Mean and variance equation

The mean equation of an univariate time series x_t can be described by the process

$$x_t = \mathrm{E}(x_t | \mathcal{F}_{t-1}) + \varepsilon_t \,, \tag{1}$$

where $\mathrm{E}(\,\cdot\,|\,\cdot\,)$ denotes the conditional expectation operator, $\mathcal{F}_{t-1}$ the information set at time $t-1$, and ε_t the innovations of the time series.

ARMA mean equation

The ARMA(m,n) process of autoregressive order *m* and moving average order *n* can be described as

$$x_t = \mu + \sum_{i=1}^{m} a_i x_{t-i} + \sum_{j=1}^{n} b_j \varepsilon_{t-j} + \varepsilon_t \,, \tag{2}$$

with mean μ, autoregressive coefficients a_i and moving average coefficients b_i.

GARCH variance equation

The mean equation does not take into account heteroskedastic effects typically observed in financial time series. Engle [1982] introduced the *Autoregressive Conditional Heteroskedastic* model, named ARCH, later generalised by Bollerslev [1986], named GARCH.

$$
\begin{aligned}
\varepsilon_t &= z_t \sigma_t \ , \\
z_t &\sim \mathcal{D}_\vartheta(0,1) \ , \\
\sigma_t^2 &= \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \ ,
\end{aligned}
\tag{3}
$$

APARCH variance equation

Ding [1993] introduced the APARCH(p,q) variance that can be expressed as

$$
\begin{aligned}
\varepsilon_t &= z_t \sigma_t \ , \\
z_t &\sim \mathcal{D}_\vartheta(0,1) \ , \\
\sigma_t^\delta &= \omega + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta \ ,
\end{aligned} \tag{4}
$$

where $\delta > 0$ and $-1 < \gamma_i < 1$.
This model adds the flexibility of a varying exponent with an asymmetry coefficient γ_i to take the leverage effect into account and the varying power δ to consider the Taylor effect.

Outline

An unique GARCH modelling approach

- garchSpec() - specifies a GARCH model.
- garchSim() - simulates an artificial GARCH time series.
- garchFit() - fits the parameters to the model using the maximum log-likelihood estimator.
- print, plot, summary, - are S3 methods for an object returned by the function garchFit().
- predict - is a generic function to forecast from an estimated model.

Outline

How to fit GARCH(1,1) model

In this example we estimate the parameters for a GARCH(1,1) model with normal innovations. The process can be decomposed in five steps :

- Parameter initialization
- Conditional distribution
- Log-likelihood function
- Parameter estimation
- Summary report

Parameter initialization

we initialize the set of model parameters θ, params, and the corresponding upper and lower bounds.

```
garchInit = function(series) {
        Mean = mean(series); Var = var(series); S = 1e-6
        params = c(mu = Mean, omega = 0.1*Var, alpha = 0.1, beta = 0.8)
        lowerBounds = c(mu = -10*abs(Mean), omega = S^2, alpha = S, beta = S)
        upperBounds = c(mu =  10*abs(Mean), omega = 100*Var, alpha = 1-S, beta = 1-S)
        cbind(params=params, lowerBounds=lowerBounds, upperBounds=upperBounds)
}
```

Conditional distribution

For the conditional distribution we use the Normal distribution dnorm().

```
garchDist = function(z, hh) {dnorm(x = z/hh)/hh }
```

Log-likelihood function

The quasi-maximum likelihood technique applied to a GARCH(1,1) process leads then to the following optimisation problem

$$\min \ \mathcal{L}_N(\theta) = \frac{1}{2}\sum_t \left[\ln 2\pi + \ln \sigma_t^2 + \frac{\varepsilon_t^2}{\sigma_t^2} \right]$$

subject to

$$x_t - \mu - \varepsilon_t = 0$$
$$\sigma_t^2 - \omega - \alpha\varepsilon_{t-1}^2 - \beta\sigma_{t-1}^2 = 0$$
$$-\omega \leq 0$$
$$-\alpha \leq 0$$
$$-\beta \leq 0$$
$$\alpha + \beta - 1 \leq 0$$

Log-likelihood function

In our example we use a fast and efficient filter representation for the variance equation.

```
garchLLH = function(parm, series) {
        mu = parm[1]; omega = parm[2]; alpha = parm[3]; beta = parm[4]
        z = (series-mu)
        Mean = mean(z^2)
        # Use Filter Representation:
        e = omega + alpha * c(Mean, z[-length(series)]^2)
        h = filter(e, beta, "r", init = Mean)
        hh = sqrt(abs(h))
        llh = -sum(log(garchDist(z, hh)))
        llh
}
```

Parameter estimation

We use the constrained solver nlminb() which is available in R and SPlus.

```
garch11Fit= function(series, params, lowerBounds, upperBounds) {
        fit = nlminb(start = params, objective = garchLLH,
                     lower = lowerBounds, upper = upperBounds,
                     control = list(trace=3),series=series)
        fit
}
```

Summary report

The results for the estimated parameters together with standard errors and t-values are summarized and printed. To compute standard errors and t-values we evaluate the Hessian matrix numerically.

```
Coefficient(s):
        Estimate  Std. Error  t value   Pr(>|t|)
[1,] -0.00619040  0.00846216 -0.73154 0.46444966
[2,]  0.01076140  0.00285270  3.77235 0.00016171 ***
[3,]  0.15313411  0.02652273  5.77369 7.7553e-09 ***
[4,]  0.80597365  0.03355252 24.02125 < 2.22e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Outline

Overview of garchFit()

We keep the same steps as described above, but with more options :

- A wide range of GARCH models
- Different conditional distribution
- Recursion initialization
- Different solver
- Diagnostic plots

Outline

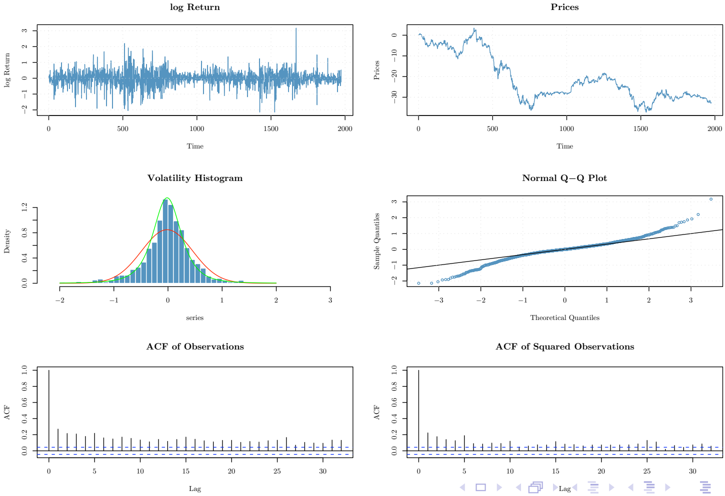
DEMGBP benchmark

Gabriele Fiorentini, Giorgio Calzolari, and Lorenzo Panattoni. 1996.

- Fiorentini et al. [1996] (FCP) took the daily percentage nominal returns for the German mark/British sterling exchange rate, henceforth (DEMGBP), as published by Bollerslev and Ghysels [1996].
- FCP calculated the set of parameters for the GARCH(1,1) process with the help of analytical derivatives and Hessian of the conditional log-likelihood.
- The FCP implementation constitutes today a well accepted benchmark.

DEMGBP benchmark

The series contains a total of 1975 daily observations sampled during the period from January 2, 1984, to December 31, 1991.



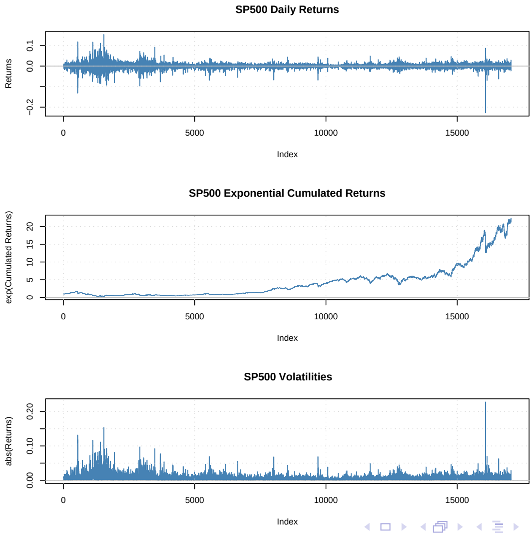
Results and comparison

	Rmetrics:			FCP Benchmark:			Ox/Garch:			Splus/Finmetrics:		
	Estimate	StdError	t Value	Estimate	StdError	t Value	Estimate	StdError	t Value	Estimate	StdError	t Value
μ	-0.0061904	0.0084621	-0.732	-0.0061904	0.0084621	-0.732	-0.006184	0.008462	-0.731	-0.006053	0.00847	-0.715
ω	0.010761	0.0028527	3.77	0.010761	0.0028527	3.77	0.010761	0.0028506	3.77	0.010896	0.0029103	3.74
α	0.15313	0.026523	5.77	0.15313	0.026523	5.77	0.15341	0.026569	5.77	0.15421	0.026830	5.75
β	0.80597	0.033553	24.0	0.80597	0.033553	24.0	0.80588	0.033542	24.0	0.80445	0.034037	23.6

Outline

APARCH : Ding et al.

The SP500 Index returns as discussed in the paper of Ding, Granger and Engle [1993], DGE.



APARCH : Ding et al.

	DGE Paper	**R Rmetrics**	rescaled	**Splus Finmetrics**	rescaled	**Ox G@RCH**	rescaled
μ	0.00021	0.02065	0.000207	0.02084	0.000208	0.02038	0.000204
a	0.145	0.1447		0.1447		0.1446	
ω	0.000014	0.009988	0.0000163	0.01003	0.0000159	0.009991	0.0000150
α	0.083	0.08380		0.08375		0.08377	
γ	0.373	0.3731		-0.3710		0.3765	
β	0.920	0.9194		0.9195		0.9199	
δ	1.43	1.435		1.429		1.416	

Thank you for your attention