Scale Normal Mixture GARCH: An Acccount for Black Swans

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Outline

History

- Since Bob Engle's Nobel Prize winning ARCH model, the vast amount of researches has been focused on how to improve the model.
- A paper by Tim Bollerslev documented a brief list of 110 generalisation or extension to the orginal ARCH/GARCH type model.
- In this paper we build on existing work on heavy tail GARCH model by relaxing the conditional error distribution from a Normal to a Scale normal mixture.

Motivation

- There has been much criticism on the GARCH frame work despite its popularity. One of them is the inability to capture the heavy tail nature of the financial time series.
- The aim of the work attmpts to address this shortfall by extending a single normal error distribution to a mixture of normal distribution with different variance.
- The generalisation not only improves the fit and the account for heavy tail, it also enable the understanding of the market structure.

The genral GARCH(p, q) model

GARCH(p, q)

$$x_t = \mu_t + \sigma_t \epsilon_t \qquad \epsilon_t \sim N(0,1)$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \alpha_j x_{t-j}^2$$

Mixture Distribution

In it's most simplistic form, the Scaled Normal Mixture(SNM) is just a mixture of Normal distribuions with different scale parameter (θ).

Scale Normal Mixture

$$f(y_i|x_i;\pi,\theta,\beta) = \sum_{i=1}^{m} \pi_j f(y_i|x_i;\theta_j,\beta)$$

Scale Normal Mixture GARCH (p, q)

A scaled normal mixture GARCH model is simply a GARCH model where the assumption of standard normal error distribution is replaced with a scale normal mixture.

Scale Normal Mixture GARCH

$$x_t = \mu_t + \sigma_t \epsilon_t \qquad \epsilon_t \sim SNM(0, G)$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \alpha_j x_{t-j}^2$$

Estimation

The estimation is done via the lsei package by Yong (2010) which estimates the semi-parametric model using maximum likelihood estimation. The following functions forms a gradient surface which the algorithm maximise to obtain the estimated parameters.

Objective functions

The NYSE

The data

Fit

Compasiron

Conclustion

Further Development