

Určete definiční obor funkcí

1. $f(x) = \arcsin(2x + 3) + \operatorname{arctg}(3x - 9)$

$$-1 \leq 2x + 3 \leq 1 \quad / -3$$

$$-4 \leq 2x \leq -2 \quad / :2$$

$$-2 \leq x \leq -1$$

$$D_f = [-2; -1]$$

4. $f(x) = \sqrt{x^2 + 7x + 10} + 5 \cdot \arccos \frac{3-x}{9-x}$

$$x^2 + 7x + 10 \geq 0 \quad \wedge \quad -1 \leq \frac{3-x}{9-x} \leq 1$$

Viét: $x_1 + x_2 = -7$
 $x_1 x_2 = 10$

$$\downarrow$$

$$x_1 = -5, x_2 = -2$$

$$(x+5)(x+2) \geq 0$$



$$* x \in (-\infty; -5] \cup [-2; +\infty)$$

$$\textcircled{1} \frac{3-x}{9-x} \geq -1$$

$$\frac{3-x}{9-x} + 1 \geq 0$$

$$\frac{3-x+9-x}{9-x} \geq 0$$

$$\frac{12-2x}{9-x} \geq 0$$

$$\frac{2x-12}{x-9} \geq 0 \quad (:2)$$

$$\frac{x-6}{x-9} \geq 0$$



$$\textcircled{1} x \in (-\infty; 6] \cup (9; +\infty)$$

$$\textcircled{2} \frac{3-x}{9-x} \leq 1$$

$$\frac{3-x}{9-x} - 1 \leq 0$$

$$\frac{3-x-9+x}{9-x} \leq 0$$

$$\frac{-6 \cdot (-1)}{9-x \cdot (-1)} \leq 0$$

$$\frac{6}{x-9} \leq 0$$

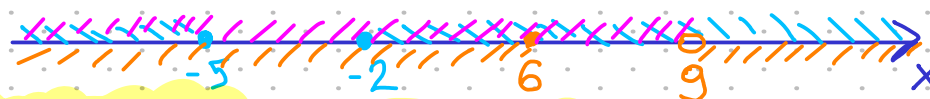
6 > 0 platí vždy, takže aby zlomek byl ≤ 0, číselník musí být < 0

$$x-9 < 0$$

$$x < 9$$

$$\textcircled{2} x \in (-\infty; 9)$$

Uděláme průnik $*$, $\textcircled{1}$ a $\textcircled{2}$:



$$D_f = (-\infty; -5] \cup [-2; 6]$$

Zderivujte následující funkce a výsledek co nejvíce zjednodušte

$$1. y' = (5x^2 - 3x + 8)' = (5x^2)' - (3x)' + (8)' = 5(x^2)' - 3(x)' + 0 = 5 \cdot 2x - 3 \cdot 1 = 10x - 3$$

$$2. y' = \left(\frac{1}{x} - \sqrt{x} + \frac{1}{3} \right)' = (x^{-1})' - (x^{\frac{1}{2}})' + \left(\frac{1}{3} \right)' = -1 \cdot x^{-2} - \frac{1}{2} x^{-\frac{1}{2}} + 0 = -\frac{1}{x^2} - \frac{1}{2\sqrt{x}}$$

$$4. y' = (\sqrt{x} \cdot (5x + x^2))' = (\sqrt{x})' (5x + x^2) + \sqrt{x} \cdot (5x + x^2)' = (x^{\frac{1}{2}})' \cdot (5x + x^2) + \sqrt{x} \cdot ((5x)' + (x^2)') = \frac{1}{2} x^{-\frac{1}{2}} (5x + x^2) + \sqrt{x} \cdot (5 + 2x) = \frac{5}{2} x^{\frac{1}{2}} + \frac{1}{2} x^{\frac{3}{2}} + 5\sqrt{x} + 2x\sqrt{x} = \frac{5}{2}\sqrt{x} + \frac{1}{2}x\sqrt{x} + 5\sqrt{x} + 2x\sqrt{x} = \frac{15}{2}\sqrt{x} + \frac{5}{2}x\sqrt{x} = \frac{5}{2}\sqrt{x}(3+x)$$

$$9. y' = (e^x \cdot \cos x \cdot (3x + 7))' = (e^x)' (\cos x \cdot (3x + 7)) + e^x \cdot (\cos x \cdot (3x + 7))' = e^x \cdot \cos x \cdot (3x + 7) + e^x \cdot ((\cos x)' \cdot (3x + 7) + \cos x \cdot (3x + 7)') = e^x \cos x (3x + 7) + e^x (-\sin x \cdot (3x + 7) + \cos x \cdot 3) = e^x (\cos x \cdot (3x + 7) - \sin x \cdot (3x + 7) + \cos x \cdot 3) = e^x ((\cos x - \sin x) \cdot (3x + 7) + 3 \cos x)$$

$$10. y' = \left(\frac{x}{3} + \frac{5}{x^2} \right)' = \left(\frac{1}{3} x \right)' + \left(5 \cdot \frac{1}{x^2} \right)' = \frac{1}{3} (x)' + 5 (x^{-2})' = \frac{1}{3} + 5 \cdot (-2) \cdot x^{-3} = \frac{1}{3} - \frac{10}{x^3}$$

$$12. y' = \left(\frac{x}{x^2 + 1} \right)' = \frac{x' (x^2 + 1) - x \cdot (x^2 + 1)'}{(x^2 + 1)^2} = \frac{1 \cdot (x^2 + 1) - x ((x^2)' + (1)')}{(x^2 + 1)^2} = \frac{x^2 + 1 - x (2x + 0)}{(x^2 + 1)^2} = \frac{x^2 + 1 - 2x^2}{(x^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)^2}$$

$$16. y' = \left(\frac{\ln x}{x^2} \right)' = \frac{(\ln x)' \cdot x^2 - \ln x \cdot (x^2)'}{(x^2)^2} = \frac{\frac{1}{x} \cdot x^2 - \ln x \cdot 2x}{x^4} = \frac{x - 2x \ln x}{x^4} = \frac{x(1 - 2 \ln x)}{x^4} = \frac{1 - 2 \ln x}{x^3}$$