

$$f: y = \frac{x^2}{x-1}$$

- ① určit, kde je rostoucí /
klesající / extrémů
- ② určit konvexitu / konkavitu
inflexní body
- ③ graf

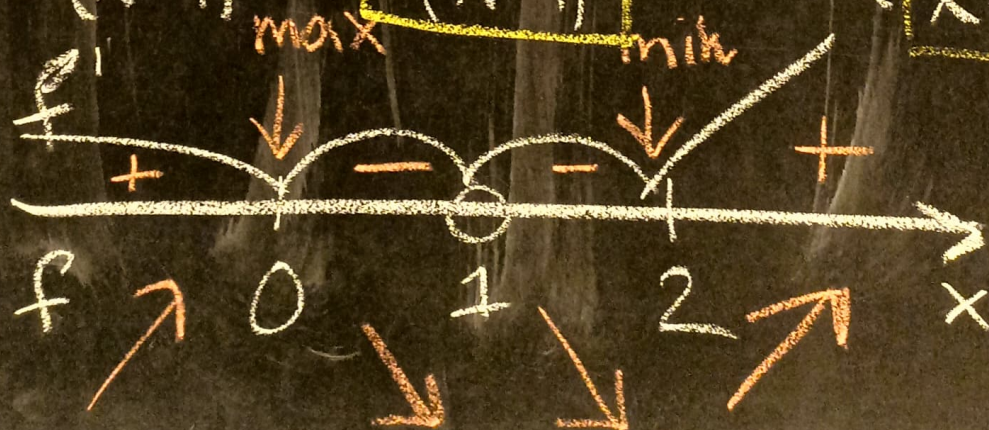
$$f: y = \frac{x^2}{x-1}$$

$$D(f) = (-\infty; 1) \cup (1; +\infty)$$

$$y' = \frac{(x^2)' \cdot (x-1) - x^2 \cdot (x-1)'}{(x-1)^2}$$

$$= \frac{2x \cdot (x-1) - x^2 \cdot 1}{(x-1)^2} = \frac{2x^2 - 2x - x^2}{(x-1)^2} =$$

$$= \frac{x^2 - 2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2} = 0 \quad \begin{cases} x=0 \\ x=2 \end{cases}$$



$$f'(x) = \frac{x^2 - 2x}{(x-1)^2}$$

$$f''(x) = \frac{(x^2 - 2x)' \cdot (x-1)^2 - (x^2 - 2x) \cdot ((x-1)^2)'}{((x-1)^2)^2}$$

$$= \frac{(2x-2)(x-1)^2 - (x^2-2x) \cdot 2(x-1)}{(x-1)^4} =$$

$$= \frac{2(x-1)^3 - 2x(x-2)(x-1)}{(x-1)^4} =$$

$$= \frac{2 \cancel{(x-1)} ((x-1)^2 - x(x-2))}{(x-1)^4} =$$

$$= \frac{2(\cancel{x^2} - \cancel{2x} + 1 - \cancel{x^2} + \cancel{2x})}{(x-1)^3} = \boxed{\frac{2}{(x-1)^3}}$$

$$3) \quad f'' = 0$$

$$(1-x) \frac{2}{(x-1)^3} = 0$$

f'' nemá řešení



$$= -x - (1 - \ln x) \cdot 2x = x(-1 - (1 - \ln x) \cdot 2)$$

$$= \frac{-1 - (1 - \ln x) \cdot 2}{x^3} = \frac{-1 - 2 + 2 \ln x}{x^3}$$