Cvičení 2 - 26.9.2024

červené - spolu
modré - samostatně

(učebnice s. 99)

1. Vypočtěte limitu posloupnosti

(a) $\lim \dfrac{2n^5 + n^3 - 7}{3n^5 - n + 1}$ (b) $\lim \dfrac{n^2 + n + 3}{5n^3 + n}$ (c) $\lim \dfrac{n^2 + 3n - 1}{n - 4}$

(d) $\lim \dfrac{n^2 - 7}{n^2 + n + 4}$ (e) $\lim \dfrac{n^4}{n^2 - 2}$ (f) $\lim \dfrac{5}{n^2 + n - 1}$

(g) $\lim \dfrac{5^n}{2^n - 1}$ (h) $\lim \dfrac{3^n + 2^n}{1 - 3^n}$ (i) $\lim \dfrac{3 + n + 7^n}{\left(\frac{1}{2}\right)^n - 2}$

(j) $\lim (n^2 + \sqrt{n} - 1)$ (k) $\lim (5n^2 - n + 3)$ (l) $\lim (\sqrt{n} - \sqrt{n+1})$

(m) $\lim (n^3 + 7n - 3n^5)$ (n) $\lim (\sqrt{n^2 + 1} - 2n)$ (o) $\lim (2 \cdot 3^n - 4^n)$

(p) $\lim (9^n - (\frac{1}{9})^n + n)$ (q) $\lim \dfrac{2 - 2^n}{4^n - 3^n}$ (r) $\lim (\sqrt{n^2 + 2n + 2} - n)$

(s) $\lim (\sqrt{2n^2 + n} - n)$ (t) $\lim \dfrac{(-1)^n}{4n - 3}$ (u) $\lim \dfrac{1 - \left(\frac{1}{2}\right)^n}{\left(\frac{1}{3}\right)^n + \left(\frac{1}{4}\right)^n}$

(v) $\lim (9^n - 6^n + 10)$ (w) $\lim \dfrac{2 - 2n}{n + \sqrt{3}}$ (x) $\lim (\sqrt{2n + 1} - \sqrt{2n - 1})$

(y) $\lim (\sqrt{n^2 + n} - n)$ (z) $\lim \left(\dfrac{n + 2}{3n + 1} - 2^n\right)$

2. Určete reálný parametr a tak, aby platilo

(a) $\lim (an^2 - 5n + 1) = -\infty$ (b) $\lim (a + 5)^n = 0$

(c) $\lim (a^2 + 5a + 7)^n = \infty$ (d) $\lim (an^2 - 3n + n^2) = \infty$

(e) $\lim (a - 3)^n = 1$ (f) $\lim \left(\dfrac{n^2 + 1}{2n} - an\right) = 0$

(g) $\lim \left(\dfrac{an^2}{3 - 2n} + 2n + 1\right) = \infty$ (h) $\lim \left(6 + an - \dfrac{3n^2}{n - 2}\right) = 0$

(i) $\lim (a^n - (\frac{1}{9})^n + 3) = 3$ (j) $\lim \left(n + \dfrac{an^2}{n - 1}\right) = \infty$

(k) $\lim (\sqrt{n^2 + 1} - an) = -\infty$ (l) $\lim \left(\dfrac{n^2 + 4}{n} - an\right) = 0$

4. Vypočtěte limitu funkce

$(a)\ \lim\limits_{x\to 1} \dfrac{x^2+x+3}{5x^3+x}$ $\quad (b)\ \lim\limits_{x\to\infty} \dfrac{x^2+x+3}{2x^2+7}$ $\quad (c)\ \lim\limits_{x\to\infty} (x+\sin x)$

$(d)\ \lim\limits_{x\to -2_+} \dfrac{x-1}{x^2+5x+6}$ $\quad$ [redacted] $\lim\limits_{x\to -3_-} \dfrac{x-1}{x^2+5x+6}$ $\quad$ [redacted] $\lim\limits_{x\to\infty} \dfrac{x-1}{x^2+5x+6}$

[redacted] $\lim\limits_{x\to 0} \dfrac{x-1}{x^2+5x+6}$ $\quad (h)\ \lim\limits_{x\to 2} \dfrac{5}{x-2}$ $\quad (i)\ \lim\limits_{x\to -\infty} \dfrac{5}{x-2}$

$(j)\ \lim\limits_{x\to 0} \dfrac{5}{x-2}$ $\quad (k)\ \lim\limits_{x\to\infty} (5x^2-x+3)$ $\quad (l)\ \lim\limits_{x\to\infty} (\sqrt{x}-x)$

$(m)\ \lim\limits_{x\to 3_+} \dfrac{x^2}{9-x^2}$ $\quad (n)\ \lim\limits_{x\to 3_-} \dfrac{x^2}{9-x^2}$ $\quad (o)\ \lim\limits_{x\to -3} \dfrac{x^2}{9-x^2}$

$(p)\ \lim\limits_{x\to 1} \dfrac{2}{x^2-2x+1}$ $\quad (q)\ \lim\limits_{x\to\infty} e^{-x^2}$ $\quad (r)\ \lim\limits_{x\to\infty} \sqrt{\dfrac{7}{x-1}}$

$(s)\ \lim\limits_{x\to\infty} \dfrac{\cos x}{x}$ $\quad (t)\ \lim\limits_{x\to\infty} \operatorname{arctg} \dfrac{3}{x-1}$ $\quad (u)\ \lim\limits_{x\to 1_+} \operatorname{arctg} \dfrac{3}{x-1}$

$(v)\ \lim\limits_{x\to\infty} \sin(\pi - \operatorname{arctg} x)$ $\quad (w)\ \lim\limits_{x\to\infty} \ln \dfrac{x^2+1}{x^2+2x+3}$ $\quad (x)\ \lim\limits_{x\to\infty} \operatorname{arccotg} \dfrac{1}{x}$

$(y)\ \lim\limits_{x\to 0_+} \operatorname{arccotg} \dfrac{1}{x}$ $\quad (z)\ \lim\limits_{x\to\infty} \left(\dfrac{x+2}{3x+1} - 2^x\right)$

Výsledky

1. (a) $\frac{2}{3}$, (b) 0, (c) ∞, (d) 1, (e) ∞, (f) 0, (g) ∞, (h) -1, (i) $-\infty$, (j) ∞, (k) ∞, (l) 0, (m) $-\infty$, (n) $-\infty$, (o) $-\infty$, (p) ∞, (q) 0, (r) 1, (s) ∞, (t) 0, (u) ∞, (v) ∞, (w) -2, (x) 0, (y) $\frac{1}{2}$, (z) $-\infty$.

2. (a) $a \le 0$, (b) $a \in (-6,-4)$, (c) $a \in (-\infty,-3) \cup (-2,\infty)$, (d) $a > -1$, (e) $a = 4$, (f) $a = \frac{1}{2}$, (g) $a < 4$, (h) $a = 3$, (i) $a \in (-1,1)$, (j) $a > -1$, (k) $a > 1$, (l) $a = 1$.

4. (a) $\frac{5}{6}$, (b) $\frac{1}{2}$, (c) ∞, (d) $-\infty$, (e) $-\infty$, (f) 0, (g) $-\frac{1}{6}$, (h) neexistuje, (i) 0, (j) $-\frac{5}{2}$, (k) ∞, (l) $-\infty$, (m) $-\infty$, (n) ∞, (o) neexistuje, (p) ∞, (q) 0, (r) 0, (s) 0, (t) 0, (u) $\frac{\pi}{2}$, (v) 1, (w) 0, (x) $\frac{\pi}{2}$, (y) 0, (z) $-\infty$.