Cvičení 2 - 26.9.2024

červené - spolu

(učebnice s. 99)

modré - samostatně

Vypočtěte limitu posloupnosti

(a)
$$\lim \frac{2n^5 + n^3 - 7}{3n^5 - n + 1}$$

(b)
$$\lim \frac{n^2 + n + 3}{5n^3 + n}$$

(c)
$$\lim \frac{n^2 + 3n - 1}{n - 4}$$

(d)
$$\lim \frac{n^2 - 7}{n^2 + n + 4}$$

(e)
$$\lim \frac{n^4}{n^2-2}$$

$$(f) \lim \frac{5}{n^2+n-1}$$

$$(g) \lim \frac{5^n}{2^n - 1}$$

$$\underbrace{(h)}\lim \frac{3^n+2^n}{1-3^n}$$

(i)
$$\lim \frac{3+n+7^n}{(\frac{1}{2})^n-2}$$

(j)
$$\lim (n^2 + \sqrt{n} - 1)$$

$$(k) \lim (5n^2 - n + 3)$$

(l)
$$\lim (\sqrt{n} - \sqrt{n+1})$$

$$(m) \lim (n^3 + 7n - 3n^5)$$

(n)
$$\lim (\sqrt{n^2 + 1} - 2n)$$
 (o) $\lim (2 \cdot 3^n - 4^n)$

(o)
$$\lim (2 \cdot 3^n - 4^n)$$

$$(p) \lim \left(9^n - \left(\frac{1}{9}\right)^n + n\right)$$

(a)
$$\lim \frac{2-2^n}{4^n-3^n}$$

(r)
$$\lim (\sqrt{n^2 + 2n + 2} - n)$$

(s)
$$\lim \left(\sqrt{2n^2+n}-n\right)$$

(t)
$$\lim \frac{(-1)^n}{4n-3}$$

$$\frac{1 - \left(\frac{1}{2}\right)^n}{\left(\frac{1}{3}\right)^n + \left(\frac{1}{4}\right)^n}$$

$$(v)$$
 $\lim (9^n - 6^n + 10)$

$$(w) \lim \frac{2-2n}{n+\sqrt{3}}$$

(x)
$$\lim (\sqrt{2n+1} - \sqrt{2n-1})$$

$$(y) \lim \left(\sqrt{n^2 + n} - n\right)$$

$$(z) \lim \left(\frac{n+2}{3n+1} - 2^n\right)$$

2. Určete reálný parametr a tak, aby platilo

$$(a) \lim (an^2 - 5n + 1) = -\infty$$

(b)
$$\lim (a+5)^n = 0$$

$$(c) \lim (a^2 + 5a + 7)^n = \infty$$

$$(d) \lim (an^2 - 3n + n^2) = \infty$$

$$(e) \lim (a-3)^n = 1$$

$$(f) \lim \left(\frac{n^2+1}{2n}-an\right)=0$$

(g)
$$\lim \left(\frac{an^2}{3-2n}+2n+1\right)=\infty$$

$$(h) \lim \left(6 + an - \frac{3n^2}{n-2}\right) = 0$$

(i)
$$\lim (a^n - (\frac{1}{9})^n + 3) = 3$$

$$(j) \lim \left(n + \frac{an^2}{n-1} \right) = \infty$$

$$(k) \lim \left(\sqrt{n^2 + 1} - an\right) = -\infty$$

(l)
$$\lim \left(\frac{n^2+4}{n}-an\right)=0$$

4. Vypočtěte limitu funkce

(a)
$$\lim_{x \to 1} \frac{x^2 + x + 3}{5x^3 + x}$$

(b)
$$\lim_{x \to \infty} \frac{x^2 + x + 3}{2x^2 + 7}$$

(c)
$$\lim_{x \to \infty} (x + \sin x)$$

$$\lim_{x \to -2_+} \frac{x-1}{x^2 + 5x + 6}$$

(e)
$$\lim_{x \to -3_{-}} \frac{x-1}{x^2 + 5x + 6}$$

$$\lim_{x \to \infty} \frac{x-1}{x^2 + 5x + 6}$$

(g)
$$\lim_{x\to 0} \frac{x-1}{x^2+5x+6}$$

$$\lim_{x\to 2} \frac{5}{x-2}$$

$$\underbrace{(i)}_{x \to -\infty} \lim_{x \to -\infty} \frac{5}{x - 2}$$

$$(j) \lim_{x \to 0} \frac{5}{x - 2}$$

$$\lim_{x \to \infty} (5x^2 - x + 3)$$

(l)
$$\lim_{x\to\infty}(\sqrt{x}-x)$$

$$(m)$$
 $\lim_{x \to 3_+} \frac{x^2}{9 - x^2}$

(n)
$$\lim_{x \to 3_{-}} \frac{x^2}{9 - x^2}$$

$$\lim_{x \to -3} \frac{x^2}{9 - x^2}$$

(p)
$$\lim_{x \to 1} \frac{2}{x^2 - 2x + 1}$$

$$(q) \lim_{x \to \infty} e^{-x^2}$$

$$(r) \lim_{x \to \infty} \sqrt{\frac{7}{x - 1}}$$

(s)
$$\lim_{x \to \infty} \frac{\cos x}{x}$$

(t)
$$\lim_{x \to \infty} \arctan \frac{3}{x-1}$$

(u)
$$\lim_{x \to 1_+} \arctan \frac{3}{x-1}$$

$$(v) \lim_{x \to \infty} \sin(\pi - \arctan x)$$

(w)
$$\lim_{x \to \infty} \ln \frac{x^2 + 1}{x^2 + 2x + 3}$$

(x)
$$\lim_{x \to \infty} \operatorname{arccotg} \frac{1}{x}$$

$$(y) \lim_{x \to 0_+} \operatorname{arccotg} \frac{1}{x}$$

$$(z) \lim_{x \to \infty} \left(\frac{x+2}{3x+1} - 2^x \right)$$

Výsledky

1. (a) $\frac{2}{3}$, (b) 0, (c) ∞ , (d) 1, (e) ∞ , (f) 0, (g) ∞ , (h) -1, (i) $-\infty$, (j) ∞ ,

 $\text{(k)} \, \infty, \quad \text{(l)} \, \, 0, \quad \text{(m)} \, \, -\infty, \quad \text{(n)} \, \, -\infty, \quad \text{(o)} \, \, -\infty, \quad \text{(p)} \, \, \infty, \quad \text{(q)} \, \, 0, \quad \text{(r)} \, \, 1, \quad \text{(s)} \, \, \infty, \quad \text{(t)} \, \, 0,$

 $-(u) \infty$, $(v) \infty$, (w) -2, (x) 0, $(y) \frac{1}{2}$, $(z) -\infty$.

2. (a) $a \le 0$, (b) $a \in (-6, -4)$, (c) $a \in (-\infty, -3) \cup (-2, \infty)$, (d) a > -1,

(e) a = 4, (f) $a = \frac{1}{2}$, (g) a < 4, (h) a = 3, (i) $a \in (-1, 1)$,

(j) a > -1, (k) a > 1, (l) a = 1.

4. (a) $\frac{5}{6}$, (b) $\frac{1}{2}$, (c) ∞ , (d) $-\infty$, (e) $-\infty$, (f) 0, (g) $-\frac{1}{6}$, (h) neexistuje, (i) 0,

(j) $-\frac{5}{2}$, (k) ∞ , (l) $-\infty$, (m) $-\infty$, (n) ∞ , (o) neexistuje, (p) ∞ , (q) 0, (r) 0,

(s) 0, (t) 0, (u) $\frac{\pi}{2}$, (v) 1, (w) 0, (x) $\frac{\pi}{2}$, (y) 0, (z) $-\infty$.