Postup na hledání tečny a normály v bodě T

Input: funkce $f(x)$, bod $T[a, ?]$

1) Vypočítat druhou souřadnici bodu T: $b = f(a)$

2) Vypočítat $f'(x)$

3) Vypočítat $f'(a)$: v derivaci $f'(x)$ dosadit a za x

4) Rovnice tečny: $y = f'(a)(x-a)+b$

5) Rovnice normály: $y = -1/f'(a)(x-a)+b$

Napište rovnici tečny a rovnici normály ke grafu funkce $f(x)$ v bodě T:

1. $f(x) = \dfrac{x}{1-\cos x}$, $\quad T = [\pi, ?]$

① $x_T = \pi$

$$f(\pi) = \frac{\pi}{1-\cos\pi} = \frac{\pi}{1-(-1)} = \frac{\pi}{2} \rightarrow T = \left[\pi; \frac{\pi}{2}\right]$$

② $$f'(x) = \left(\frac{x}{1-\cos x}\right)' = \frac{x'(1-\cos x) - x(1-\cos x)'}{(1-\cos x)^2} = \frac{1-\cos x - x\cdot\sin x}{(1-\cos x)^2}$$

③ $$f'(\pi) = \frac{1-\cos\pi - \pi\cdot\sin\pi}{(1-\cos\pi)^2} = \frac{1+1-0}{(1+1)^2} = \frac{2}{4} = \frac{1}{2}$$

④ **tečna:** $y = f'(\pi)(x-\pi) + f(\pi)$

$$y = \frac{1}{2}(x-\pi) + \frac{\pi}{2}$$

$$y = \frac{1}{2}x - \frac{\pi}{2} + \frac{\pi}{2}$$

$$y = \frac{1}{2}x$$

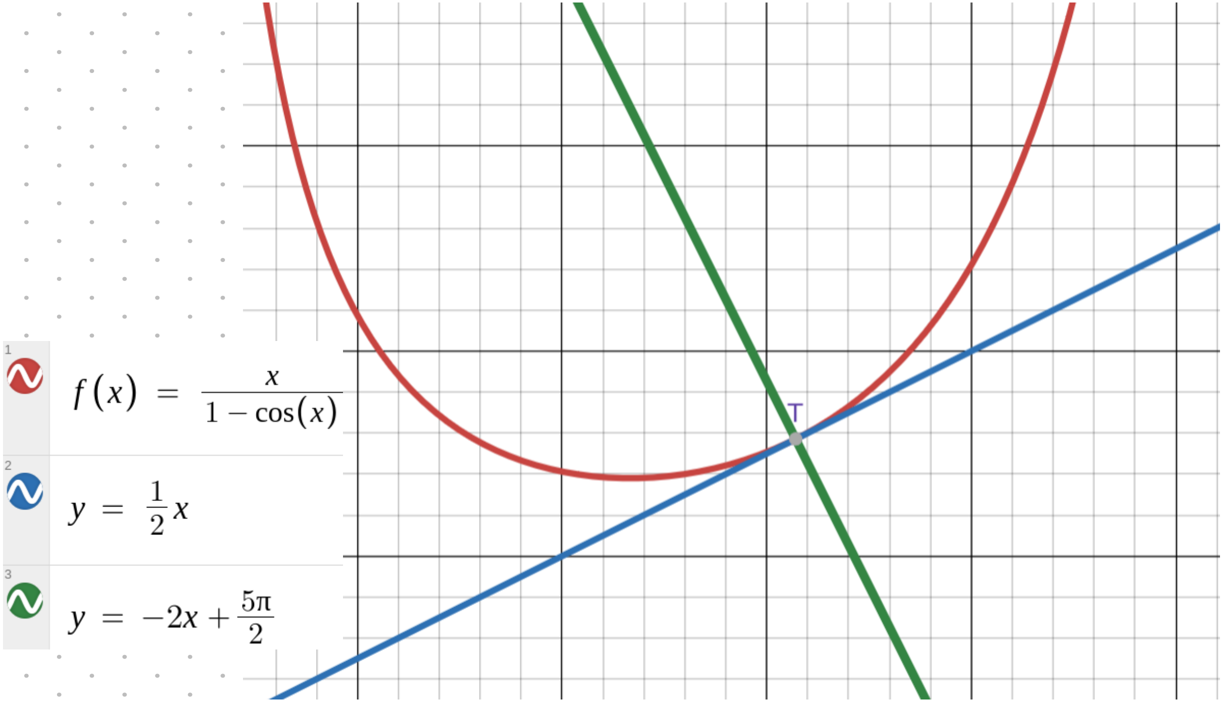
normála: $y = -\dfrac{1}{f'(\pi)}(x-\pi) + f(\pi)$

$$y = -2(x-\pi) + \frac{\pi}{2}$$

$$y = -2x + 2\pi + \frac{\pi}{2}$$

$$y = -2x + \frac{5\pi}{2}$$

https://www.desmos.com/calculator/yoyq0jzqjq



Napište rovnici tečny a rovnici normály ke grafu funkce $f(x)$ v bodě T:

3. $f(x) = \sqrt{x^2 - 3x - 1}, \quad T = [5, ?]$

① $x_T = 5$

$y_T = f(5) = \sqrt{5^2 - 3 \cdot 5 - 1} = \sqrt{25 - 15 - 1} = \sqrt{9} = 3 \rightarrow T\,[5;3]$

② $f'(x) = \left(\sqrt{x^2 - 3x - 1}\right)' = \frac{1}{2\sqrt{x^2 - 3x - 1}} \cdot (x^2 - 3x - 1)' = \frac{2x - 3}{2\sqrt{x^2 - 3x - 1}}$

složená funkce

③ $f'(5) = \frac{2 \cdot 5 - 3}{2\sqrt{5^2 - 3 \cdot 5 - 1}} = \frac{7}{2 \cdot 3} = \frac{7}{6}$

④ **tečna:**

$y = f'(5)(x - 5) + f(5)$

$y = \frac{7}{6}(x - 5) + 3$

$y = \frac{7}{6}x - \frac{35}{6} + 3$

$y = \frac{7}{6}x - \frac{17}{6}$

normála:

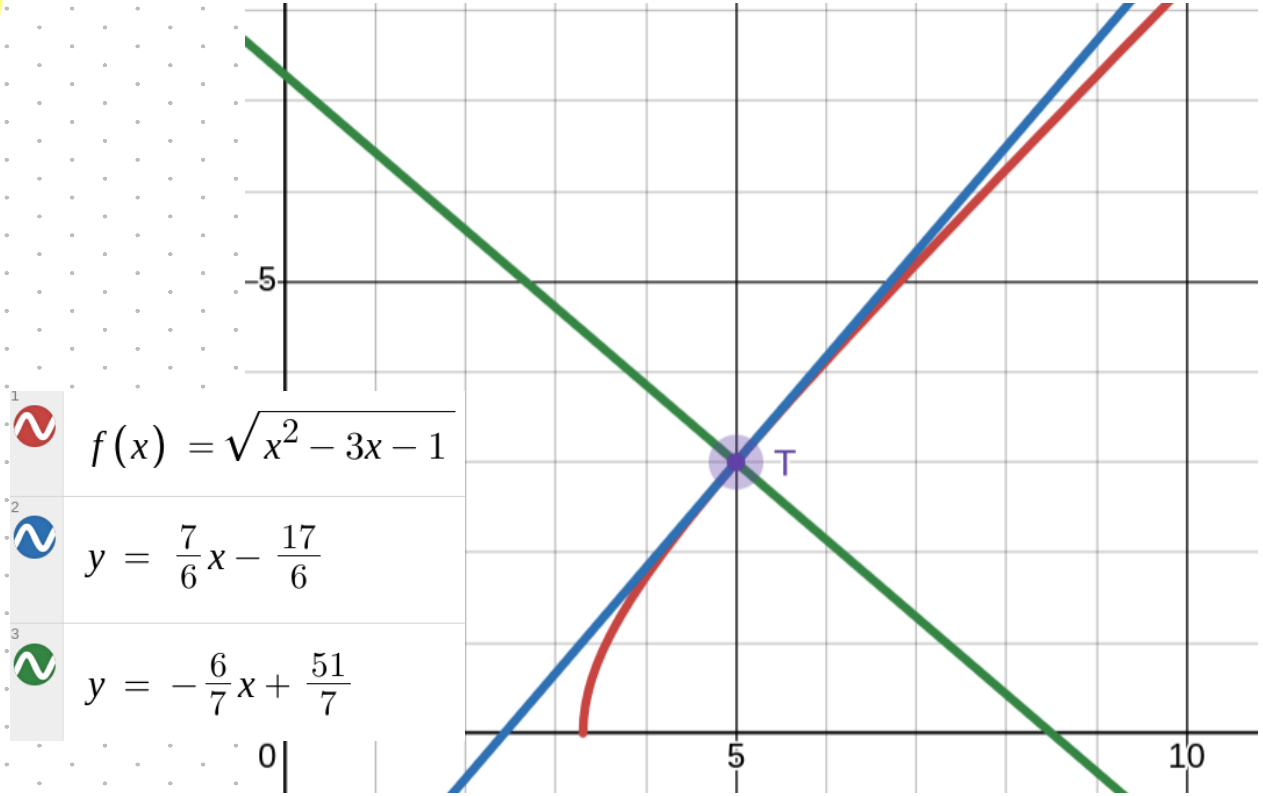
$y = -\frac{1}{f'(5)}(x - 5) + f(5)$

$y = -\frac{6}{7}(x - 5) + 3$

$y = -\frac{6}{7}x + \frac{30}{7} + 3$

$y = -\frac{6}{7}x + \frac{51}{7}$

https://www.desmos.com/calculator/btfcqzpjn2



Nalezněte rovnice tečen ke grafu $y = f(x)$, rovnoběžných s přímkou p, je-li dáno:

10. $f(x) = \operatorname{arctg}(2x+1)$, $\quad p: x - y + 3 = 0$;

$$f'(x) = (\operatorname{arctg}(2x+1))' = \frac{1}{1+(2x+1)^2} \cdot (2x+1)' = \frac{2}{1+4x^2+4x+1} = \frac{2}{2(1+2x+2x^2)} = \frac{1}{1+2x+2x^2}$$

tečny mají rovnici: $\quad y = f'(x_T)(x - x_T) + y_T$

p má rovnici: $\quad p: x - y + 3 = 0 \rightarrow y = x + 3$

aby byly rovnoběžné, musí mít stejnou směrnici: $\quad f'(x_T) = 1$

$$\frac{1}{1+2x_T+2x_T^2} = 1 \quad / \cdot (1+2x_T+2x_T^2)$$

$$\cancel{1} + 2x_T + 2x_T^2 = \cancel{1} \quad :2$$

$$x_T + x_T^2 = 0$$

$$x_T(1 + x_T) = 0$$

$$x_T = 0 \quad \text{nebo} \quad 1 + x_T = 0$$

$$x_T = -1$$

1. $x_T = 0$

$y_T = f(x_T) = \operatorname{arctg}(2\cdot 0+1) = \operatorname{arctg} 1 = \frac{\pi}{4} \rightarrow T = [0; \frac{\pi}{4}]$

Víme: $f'(x_T) = 1$

tečna: $y = f'(x_T)(x - x_T) + y_T$

$y = 1(x-0) + \frac{\pi}{4}$

$y = x + \frac{\pi}{4}$

2. $x_S = -1$

$y_S = f(x_S) = \operatorname{arctg}(2\cdot(-1)+1) = \operatorname{arctg}(-1) = -\frac{\pi}{4} \rightarrow S = [-1; -\frac{\pi}{4}]$

Víme: $f'(x_S) = 1$

tečna: $y = f'(x_S)(x - x_S) + y_S$

$y = 1\cdot(x+1) - \frac{\pi}{4}$

$y = x + 1 - \frac{\pi}{4}$

$f(x) = \arctan(2x+1)$

https://www.desmos.com/calculator/euihamtbob

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$x - y + 3 = 0$

3

$y = x + \frac{\pi}{4}$

4

$T = \left(0, \frac{\pi}{4}\right)$

Label: T

5

$y = x + 1 - \frac{\pi}{4}$

6

$S = \left(-1, -\frac{\pi}{4}\right)$

Label: S