Určete definiční obor funkcí

1. $f(x) = \arcsin(2x+3) + \operatorname{arctg}(3x-9)$

$-1 \le 2x+3 \le 1 \quad / -3$

$-4 \le 2x \le -2 \quad / :2$

$-2 \le x \le -1$

$D_f = [-2;-1]$

4. $f(x) = \sqrt{x^2+7x+10} + 5 \cdot \arccos \dfrac{3-x}{9-x}$

$x^2 + 7x + 10 \ge 0 \quad \wedge \quad -1 \overset{1}{\le} \dfrac{3-x}{9-x} \overset{2}{\le} 1$

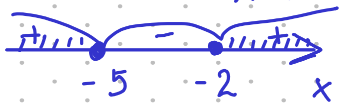
Viét: $x_1 + x_2 = -7$

$x_1 x_2 = 10$

$\downarrow$

$x_1 = -5, \ x_2 = -2$

$(x+5)(x+2) \ge 0$



(*) $\boxed{x \in (-\infty;-5] \cup [-2;+\infty)}$

(1) $\dfrac{3-x}{9-x} \ge -1$

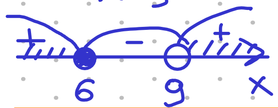
$\dfrac{3-x}{9-x} + 1 \ge 0$

$\dfrac{3-x+9-x}{9-x} \ge 0$

$\dfrac{12-2x}{9-x} \ge 0 \quad \cdot(-1)$

$\dfrac{2x-12}{x-9} \ge 0 \quad (:2)$

$\dfrac{x-6}{x-9} \ge 0$



(1) $\boxed{x \in (-\infty;6] \cup (9;+\infty)}$

(2) $\dfrac{3-x}{9-x} \le 1$

$\dfrac{3-x}{9-x} - 1 \le 0$

$\dfrac{3-x-9+x}{9-x} \le 0$

$\dfrac{-6}{9-x} \le 0 \quad \cdot(-1)$

$\dfrac{6}{x-9} \le 0$

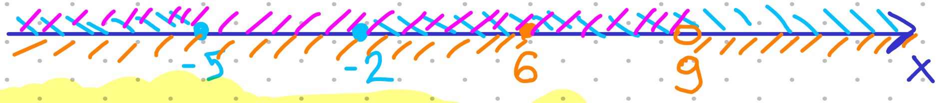
6>0 platí vždy,takže aby zlomek byl <=0, čitatel musí být <0

$x-9<0$

$x<9$

(2) $\boxed{x \in (-\infty;9)}$

Uděláme průnik (*), (1) a (2):



$D_f = (-\infty;-5] \cup [-2;6]$

Zderivujte následující funkce a výsledek co nejvíce zjednodušte

1. $y' = (5x^2 - 3x + 8)' = (5x^2)' - (3x)' + (8)' = 5(x^2)' - 3(x)' + 0 = 5\cdot 2x - 3\cdot 1 = 10x - 3$

2. $y' = \left(\frac{1}{x} - \sqrt{x} + \frac{1}{3}\right)' = (x^{-1})' - (x^{\frac{1}{2}})' + \left(\frac{1}{3}\right)' = -1\cdot x^{-2} - \frac{1}{2}x^{-\frac{1}{2}} + 0 = -\frac{1}{x^2} - \frac{1}{2\sqrt{x}}$

4. $y' = (\sqrt{x}\cdot(5x + x^2))' = (\sqrt{x})'(5x+x^2) + \sqrt{x}\cdot(5x+x^2)' = (x^{\frac{1}{2}})'\cdot(5x+x^2) + \sqrt{x}\cdot((5x)' + (x^2)') = \frac{1}{2}x^{-\frac{1}{2}}(5x+x^2) +$
$+ \sqrt{x}\cdot(5+2x) = \frac{5}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{\frac{3}{2}} + 5\sqrt{x} + 2x\sqrt{x} = \frac{5}{2}\sqrt{x} + \frac{1}{2}x\sqrt{x} + 5\sqrt{x} + 2x\sqrt{x} = \frac{15}{2}\sqrt{x} + \frac{5}{2}x\sqrt{x} = \frac{5}{2}\sqrt{x}(3+x)$

9. $y' = (\underbrace{e^x}_{f}\cdot \underbrace{\cos x\cdot(3x+7)}_{g})' = (e^x)'(\cos x\cdot(3x+7)) + e^x\cdot(\cos x\cdot(3x+7))' = e^x\cdot\cos x\cdot(3x+7) + e^x\cdot((\cos x)'\cdot(3x+7) +$
$+ \cos x\cdot(3x+7)') = e^x\cos x\,(3x+7) + e^x(-\sin x\cdot(3x+7) + \cos x\cdot 3) = e^x(\cos x\cdot(3x+7) - \sin x\,(3x+7) + \cos x\cdot 3) =$
$= e^x((\cos x - \sin x)\cdot(3x+7) + 3\cos x)$

10. $y' = \left(\frac{x}{3} + \frac{5}{x^2}\right)' = \left(\frac{1}{3}x\right)' + \left(5\cdot\frac{1}{x^2}\right)' = \frac{1}{3}(x)' + 5(x^{-2})' = \frac{1}{3} + 5\cdot(-2)\cdot x^{-3} = \frac{1}{3} - \frac{10}{x^3}$

12. $y' = \left(\frac{x}{x^2+1}\right)' = \frac{x'(x^2+1) - x\cdot(x^2+1)'}{(x^2+1)^2} = \frac{1\cdot(x^2+1) - x((x^2)' + (1)')}{(x^2+1)^2} = \frac{x^2+1-x(2x+0)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$

16. $y' = \left(\frac{\ln x}{x^2}\right)' = \frac{(\ln x)'\cdot x^2 + \ln x\cdot(x^2)'}{(x^2)^2} = \frac{\frac{1}{x}\cdot x^2 + \ln x\cdot 2x}{x^4} = \frac{x + 2x\ln x}{x^4} = \frac{\cancel{x}(1+2\ln x)}{x^{\cancel{4}3}} = \frac{1+2\ln x}{x^3}$