Zderivujte následující funkce a výsledek co nejvíce zjednodušte

1. $y = \cos 5x$

$$y' = (\underbrace{\cos}_{f}\underbrace{5x}_{g(x)})' = \underbrace{-\sin(5x)}_{f'(g(x))} \cdot \underbrace{(5x)'}_{g'(x)} = -5\sin(5x)$$

5. $y = \sin^2 x$

$$y' = ((\underbrace{\sin x}_{g(x)})^{\overset{f}{2}})' = \underbrace{2\sin x}_{f'(g(x))}\underbrace{(\sin x)'}_{g'(x)} = 2\sin x\cos x$$

7. $y = \arcsin\sqrt{x}$

$$y' = (\underbrace{\arcsin}_{f}\underbrace{\sqrt{x}}_{g(x)})' = \underbrace{\frac{1}{\sqrt{1-(\sqrt{x})^2}}}_{f'(g(x))} \cdot \underbrace{(\sqrt{x})'}_{g'(x)} = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{(1-x)x}} = \frac{1}{2\sqrt{x-x^2}}$$

14. $y = \dfrac{3x+1}{\sqrt{x^2+4}}$

$$y' = \left(\frac{3x+1}{\sqrt{x^2+4}}\right)' = \frac{(3x+1)'\cdot\sqrt{x^2+4} - (3x+1)(\underset{f}{\sqrt{\underbrace{x^2+4}_{g(x)}}})'}{(\sqrt{x^2+4})^2} = \frac{3\sqrt{x^2+4} - (3x+1)\cdot\underbrace{\frac{1}{2\sqrt{x^2+4}}}_{f'(g(x))}\cdot\underbrace{(x^2+4)'}_{g'(x)}}{x^2+4} = \frac{3\sqrt{x^2+4} - (3x+1)\cdot\frac{1}{\cancel{2}\sqrt{x^2+4}}\cdot\cancel{2}x}{x^2+4} =$$

$$= \frac{3(x^2+4) - (3x+1)\cdot x}{(x^2+4)\sqrt{x^2+4}} = \frac{\cancel{3x^2}+12-\cancel{3x^2}-x}{(x^2+4)\sqrt{x^2+4}} = \frac{12-x}{(x^2+4)^{\frac{3}{2}}}$$

20. $y = \ln \dfrac{\cos x}{\sin^2 x}$

už jsme počítali
v příkladu 5

$$y' = \left(\ln\frac{\cos x}{\sin^2 x}\right)' = \underbrace{\frac{1}{\frac{\cos x}{\sin^2 x}}}_{f'(g(x))} \cdot \underbrace{\left(\frac{\cos x}{\sin^2 x}\right)'}_{g'(x)} = \frac{\sin^2 x}{\cos x}\cdot\frac{(\cos x)'\cdot \sin^2 x - \cos x\,(\sin^2 x)'}{(\sin^2 x)^2} =$$

$\sin^2 x + \cos^2 x = 1$

$$= \frac{\cancel{\sin^2 x}}{\cos x}\cdot\frac{-\sin^3 x - \cos x\cdot 2\sin x\cos x}{\sin^{\cancel{4}\,2} x} = \frac{-\sin^3 x - 2\sin x\cos^2 x}{\cos x\,\sin^2 x} = \frac{-\cancel{\sin x}\,(\sin^2 x + 2\cos^2 x)}{\cos x\,\sin^{\cancel{2}} x} = \frac{1+\cos^2 x}{\cos x\,\sin x}$$

Spočtěte hodnotu $f'(m)$, když

20. $f(x) = 3x^2 - 5x + 4;\ m = 4$

$$f'(x) = 6x - 5$$

$$f'(4) = 6\cdot 4 - 5 = 19$$

22. $f(x) = \sqrt[3]{x}\cdot(x-5);\ m = 8$

$$f'(x) = \left(x^{\frac{1}{3}}(x-5)\right)' = \left(x^{\frac{1}{3}}\right)'\cdot(x-5) + x^{\frac{1}{3}}\cdot(x-5)' = \frac{1}{3}x^{-\frac{2}{3}}(x-5) + x^{\frac{1}{3}}\cdot 1 = \frac{1}{3\sqrt[3]{x^2}}\cdot(x-5) + \sqrt[3]{x} = \frac{x-5}{3\sqrt[3]{x^2}} + \sqrt[3]{x}$$

$$f'(8) = \frac{8-5}{3\sqrt[3]{8^2}} + \sqrt[3]{8} = \frac{\cancel{3}}{\cancel{3}\sqrt[3]{64}} + 2 = \frac{1}{4} + 2 = 2{,}25$$