Cvičení 4 - 4.10.2024

červené - spolu
modré - samostatně

(učebnice s. 123)

Příklady na mechanické derivování

1. Vypočtěte derivaci funkce dané předpisem

(a) $f(x) = x^2 + 3x - 7$

(b) $f(x) = \dfrac{6+4x}{9-4x^2}$

(c) $f(x) = 4x^7 - \dfrac{x}{3} + \sqrt{2}$

(d) $f(x) = x^7 + 5^x - \ln 2$

(e) $f(x) = 12 + x^2 \cdot 2^x$

(f) $f(x) = \dfrac{3}{x} - \dfrac{1}{x^2} - \dfrac{2}{x^3}$

(g) $f(x) = x^2\sqrt{x} + 12$

(h) $f(x) = e^x(x^3 - x^2 + 6)$

(i) $f(x) = \dfrac{1}{\cos x}$

(j) $f(x) = \dfrac{1 + x\cdot\sin x}{1 - x\cdot\sin x}$

(k) $f(x) = x \cdot e^x \sin x$

(l) $f(x) = \dfrac{x^2-1}{x^2+1}$

2. Vypočtěte derivaci složené funkce dané předpisem

(a) $f(x) = \ln\sqrt{x}$

(b) $f(x) = (x^2 + x - 1)^7$

(c) $f(x) = e^{\operatorname{tg} x}$

(d) $f(x) = \arcsin(5x - 3)$

(e) $f(x) = \operatorname{arctg}^5 x$

(f) $f(x) = \cos 5x$

(g) $f(x) = \sqrt{1 - x^2}$.

(h) $f(x) = x \cdot \ln\dfrac{x-1}{x+1}$

(i) $f(x) = 5^{\sqrt{x}}$

(j) $f(x) = \ln(x + \sqrt{x^2+1})$

(k) $f(x) = \cos(x-1)^2$

(l) $f(x) = \sqrt{5 + \sin x}$

(m) $f(x) = \sqrt[3]{\cos x^2 - 1}$

(n) $f(x) = \sin^3 \frac{x}{4}$

3. Vypočtěte druhou derivaci funkce dané předpisem

(a) $f(x) = x^2 + 3x - 7$

(b) $f(x) = \dfrac{6+4x}{9-4x^2}$

(c) $f(x) = x \cdot \operatorname{tg} x + \ln \cos x$

(d) $f(x) = \ln(x + \sqrt{x^2+1})$

(e) $f(x) = \frac{1}{2}x - \frac{1}{2}\sin x \cos x$

(f) $f(x) = \operatorname{tg}^4 x - 2\operatorname{tg}^2 x - 4 \ln \cos x$

(g) $f(x) = e^x(x^3 - 3x^2 + 6x - 6)$

(h) $f(x) = \operatorname{arctg} \dfrac{x-1}{x+1}$

(i) $f(x) = \arccos x$

(j) $f(x) = \operatorname{arctg} \frac{1}{x}$

(k) $f(x) = (1-2x)^4$

(l) $f(x) = (\sqrt{x} - 1)\, e^{\sqrt{x}}$

9. Najděte Taylorův polynom T_3 funkce f v bodě a (není v .pdf učebnici)

(a) $f(x) = \operatorname{tg} x, \quad a = 0$

(b) $f(x) = \ln x, \quad a = 1$

(c) $f(x) = \operatorname{arccotg} x, \quad a = 0$

(d) $f(x) = \operatorname{tg} 2x, \quad a = 0$

(e) $f(x) = \sin 2x, \quad a = 0$

(f) $f(x) = \sqrt[3]{1+3x}, \quad a = 0$

Výsledky

1. (a) $f'(x) = 2x + 3$

(b) $f'(x) = \dfrac{4}{(2x-3)^2}$

(c) $f'(x) = 28x^6 - \frac{1}{3}$

(d) $f'(x) = 7x^6 + 5^x \ln 5.$

(e) $f'(x) = x\,2^x(2 + x\ln 2)$

(f) $f'(x) = -\dfrac{3}{x^2} + \dfrac{2}{x^3} + \dfrac{6}{x^4}$

(g) $f'(x) = \frac{5}{2}x\sqrt{x}$

(h) $f'(x) = e^x(x^3 + 2x^2 - 2x + 6)$

(i) $f'(x) = \dfrac{\sin x}{\cos^2 x}$

(j) $f'(x) = \dfrac{2(\sin x + x \cdot \cos x)}{(1 - x \cdot \sin x)^2}$

(k) $f'(x) = e^x(\sin x + x \sin x + x \cos x)$

(l) $f'(x) = \dfrac{4x}{(x^2+1)^2}$

2. (a) $f'(x) = \dfrac{1}{2x}$

(b) $f'(x) = 7(x^2 + x - 1)^6(2x + 1).$

(c) $f'(x) = e^{\operatorname{tg} x} \dfrac{1}{\cos^2 x}$

(d) $f'(x) = \dfrac{5}{\sqrt{1 - (5x-3)^2}}$

(e) $f'(x) = 5\operatorname{arctg}^4 x \cdot \dfrac{1}{1+x^2}$

(f) $f'(x) = -5 \sin 5x$

(g) $f'(x) = -\dfrac{x}{\sqrt{1-x^2}}$

(h) $f'(x) = \ln \dfrac{x-1}{x+1} + \dfrac{2x}{x^2-1}$

(i) $f'(x) = 5^{\sqrt{x}} \cdot \ln 5 \cdot \dfrac{1}{2\sqrt{x}}$

(j) $f'(x) = \dfrac{1}{\sqrt{x^2+1}}$

(k) $f'(x) = 2(1-x)\sin(x-1)^2$

(l) $f'(x) = \dfrac{\cos x}{2\sqrt{5 + \sin x}}$

(m) $f'(x) = \dfrac{-2x \sin x^2}{3\sqrt[3]{(\cos x^2 - 1)^2}}$

(n) $f'(x) = \frac{3}{4} \sin^2 \frac{x}{4} \cos \frac{x}{4}$

3. (*a*) $f''(x) = 2$

(*b*) $f''(x) = -\dfrac{16}{(2x-3)^3}$

(*c*) $f''(x) = \dfrac{2x \cdot \operatorname{tg} x + 1}{\cos^2 x}$

(*d*) $f''(x) = -\dfrac{x}{\sqrt{(x^2+1)^3}}$

(*e*) $f''(x) = \sin 2x$

(*f*) $f''(x) = 20 \operatorname{tg}^4 x \cdot \dfrac{1}{\cos^2 x}$

(*g*) $f''(x) = x^2(x+3)\, e^x$

(*h*) $f''(x) = -\dfrac{2x}{(1+x^2)^2}$

(*i*) $f''(x) = -\dfrac{x}{\sqrt{(1-x^2)^3}}$

(*j*) $f''(x) = \dfrac{2x}{(1+x^2)^2}$

(*k*) $f''(x) = 48\,(2x-1)^2$

(*l*) $f''(x) = \frac{1}{4\sqrt{x}}\, e^{\sqrt{x}}$

9. (a) $T_3(x) = x + \frac{1}{3}x^3$,

(b) $T_3(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$,

(c) $T_3(x) = \frac{\pi}{2} - x + \frac{1}{3}x^3$,

(d) $T_3(x) = 2x + \frac{8}{3}x^2$,

(e) $T_3(x) = 2x - \frac{4}{3}x^3$,

(f) $T_3(x) = 1 + x - x^2 + \frac{5}{3}x^3$.