Cvičení 10 - 5.12.2024

červené - spolu
modré - samostatně

Vzorec $\dfrac{f'}{f}$

5. Vypočtěte integrály

(a) $\displaystyle\int \frac{3}{x-11}\,dx$ (b) $\displaystyle\int \frac{x}{x^2+7}\,dx$

Substituční metoda

3. Substituční metodou vypočtěte integrály

(a) $\displaystyle\int (x-2)^7\,dx$ (b) $\displaystyle\int \sqrt{3x+1}\,dx$ (c) $\displaystyle\int \frac{1}{\sqrt{7-x}}\,dx$

(d) $\displaystyle\int \frac{1}{(2x-3)^5}\,dx$ (e) $\displaystyle\int e^{3x-1}\,dx$ (f) $\displaystyle\int 2^{3-5x}\,dx$

(g) $\displaystyle\int \sin\frac{x}{2}\,dx$ (h) $\displaystyle\int \operatorname{tg} x\,dx$ (i) $\displaystyle\int \sin(4x-1)\,dx$

(j) $\displaystyle\int \operatorname{cotg} x\,dx$ (k) $\displaystyle\int \frac{\operatorname{arctg}^3 x}{1+x^2}\,dx$ (l) $\displaystyle\int \frac{1}{\sin^2 2x}\,dx$

(m) $\displaystyle\int x\,e^{x^2+1}\,dx$ (n) $\displaystyle\int x\cdot\cos x^2\,dx$ (o) $\displaystyle\int x^3\sqrt{1+x^4}\,dx$

(p) $\displaystyle\int e^x\cos e^x\,dx$ (q) $\displaystyle\int \frac{\sin x}{\cos^2 x}\,dx$ (r) $\displaystyle\int \cos^4 x\cdot\sin x\,dx$

Určitý integrál

6. Vypočtěte určité integrály

$(a)\ \int_1^e \frac{x+2}{2x}\,dx$ $(b)\ \int_2^6 \sqrt{x-2}\,dx$ $(c)\ \int_1^2 (x^2-2x+3)\,dx$

$(d)\ \int_{-1}^1 x^4 \sin x\,dx$ $(e)\ \int_0^{\frac{\pi}{2}} x \sin x\,dx$ $(f)\ \int_0^1 (x-2)^3\,dx$

$(g)\ \int_e^{e^2} \frac{1}{x\ln^3 x}\,dx$ $(h)\ \int_0^{\frac{\pi}{2}} \sin^4 x\,\cos x\,dx$ $(i)\ \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx$

$(j)\ \int_0^1 \frac{x}{x^2+3x+2}\,dx$ $(k)\ \int_0^{\frac{\pi}{4}} \operatorname{tg} x\,dx$ $(l)\ \int_0^1 x\,e^x\,dx$

Výsledky

5. $(a)\quad 3\ln|x-11|+c$ $(b)\quad \frac{1}{2}\ln(x^2+7)+c$

3. $(a)\quad \frac{(x-2)^8}{8}+c$ $(b)\quad \frac{2}{9}\sqrt{(3x+1)^3}+c$

$(c)\quad -2\sqrt{7-x}+c$ $(d)\quad -\frac{1}{8(2x-3)^4}+c$

$(e)\quad \frac{1}{3}e^{3x-1}+c$ $(f)\quad -\frac{2^{3-5x}}{5\ln 2}+c$

$(g)\quad -2\cos\frac{x}{2}+c$ $(h)\quad -\ln|\cos x|+c$

$(i)\quad -\frac{1}{4}\cos(4x-1)+c$ $(j)\quad \ln|\sin x|+c$

$(k)\quad \frac{1}{4}\operatorname{arctg}^4 x+c$ $(l)\quad -\frac{1}{2}\operatorname{cotg} 2x+c$

$(m)\quad \frac{1}{2}e^{x^2+1}+c$ $(n)\quad \frac{1}{2}\sin x^2+c$

$(o)\quad \frac{1}{8\sqrt{1+x^4}}+c$ $(p)\quad \sin e^x+c$

$(q)\quad \frac{1}{\cos x}+c$ $(r)\quad -\frac{1}{5}\cos^5 x+c$

6. (a) $\frac{1}{2}(e+1)$, (b) $\frac{16}{3}$, (c) $\frac{7}{3}$, (d) 0 (lichá funkce), (e) 1, (f) $-\frac{15}{4}$, (g) $\frac{3}{8}$
(h) $\frac{1}{5}$, (i) 2, (j) $\ln\frac{9}{8}$, (k) $-\ln\frac{\sqrt{2}}{2}$, (l) 1.