DERSE GELMEDEN ÖNCE...

Sayısal Fizik Kitabının pdf link aşağıdadır.
[Sayisal Fizik Bekir Karaoğlu 2nciBaski (https://www.seckin.com.tr/kitap/966813697)](https://www.seckin.com.tr/kitap/966813697)
Bu dosyadaki 2. üniteyi Gauss İntegrali kısmına kadar okuyup anlamaya çalışın...

In [1]:

```python
import IPython.display as ipd
vid = ipd.YouTubeVideo('hr1xp3tVgUQ', start=17)
link = ipd.Markdown(r'## [Programlama öğrenmenin yaşı yok!](https://youtu.be/hr1xp3tVgUQ?t=16)')
display(link)
display(vid)
```

Out[1]:

IyMgW1Byb2dyYW1sYW1hIMO2xJ9yZW5tZW5pbiB5YcWfxLEgeW9rIV0oaHR0cHM6Ly95b3

Out[1]:

SAYISAL TÜREV ve İNTEGRAL

SAYISAL TÜREV

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ **veya** $f'(x) = \lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0}$

In [2]:

```
import sympy as sm
from sympy.parsing.sympy_parser import parse_expr
import IPython.display as ipd
f, x, x_0, h, = sm.symbols('f, x, x_0, h')
f = sm.Function('f')

ft = sm.series(f(x), x, x_0, 4)
ft_tex0 = sm.latex(ft)
ft_texp = sm.latex(ft.subs({x:x_0+h, x_0:x}))
ft_texm = sm.latex(ft.subs({x:x_0-h, x_0:x}))

ft_tex0 = ft_tex0.replace(r'\rightarrow', r'\rightarrow ') # rightarrow'ün sağına bir b
oşluk ekle
ipd.display(ipd.Markdown("### $f(x) = %s$"%ft_tex0))
ipd.display(ipd.Markdown("### $f(x+h) = %s$"%ft_texp))
ipd.display(ipd.Markdown("### $f(x-h) = %s$"%ft_texm))
```

Out[2]:

$$f(x) = f(x_0) + (x - x_0) \frac{d}{d\xi_1} f(\xi_1) \Big|_{\xi_1=x_0} + \frac{(x-x_0)^2 \frac{d^2}{d\xi_1^2} f(\xi_1) \Big|_{\xi_1=x_0}}{2} + \\ + O\left((x - x_0)^4; x \rightarrow x_0\right)$$

Out[2]:

$$f(x+h) = f(x) + h \frac{d}{d\xi_1} f(\xi_1) \Big|_{\xi_1=x} + \frac{h^2 \frac{d^2}{d\xi_1^2} f(\xi_1) \Big|_{\xi_1=x}}{2} + \frac{h^3 \frac{d^3}{d\xi_1^3} j}{}$$

Out[2]:

$$f(x-h) = f(x) - h \frac{d}{d\xi_1} f(\xi_1) \Big|_{\xi_1=x} + \frac{h^2 \frac{d^2}{d\xi_1^2} f(\xi_1) \Big|_{\xi_1=x}}{2} - \frac{h^3 \frac{d^3}{d\xi_1^3} j}{}$$

In [3]:

```
import sympy as sm
import IPython.display as ipd
f, x, x0, h = sm.symbols('f, x, x0, h')
f = sm.Function('f')

def ft(f, x, h=0, n=3):
    x0 = sm.symbols('x0')
    theSeries = sm.series(f(x), x, x0, n).doit()
    if h == 0:
        ss = theSeries
    else:
        ss = theSeries.subs({x:x0+h, x0:x})
    ssl = sm.latex(ss).replace(r'\rightarrow', r'\rightarrow ')
    return ss, ssl

fxph, fxpl = ft(f, x, h)
fxmh, fxml = ft(f, x, -h)
fx_left = f(x+h) - f(x-h)
fx_right = fxph - fxmh

solwO = sm.expand(sm.solve(fx_left-fx_right, sm.Derivative(f(x), x))[0])
sol = solwO.removeO()
OO = solwO - sol
solO = sm.simplify(sol) + OO

ipd.display(ipd.Markdown("$%s = %s$"%(f(x+h), fxpl)))
ipd.display(ipd.Markdown("$%s = %s$"%(f(x-h), fxml)))
ipd.display(ipd.Markdown("$%s = %s$"%(fx_left, sm.latex(fx_right))))
ipd.display(ipd.Markdown("$%s = %s$"%(sm.latex(sm.Derivative(f(x), x)), sm.latex(solO
))))
```

Out[3]:

$$f(h+x) = f(x) + h\frac{d}{dx}f(x) + \frac{h^2 \frac{d^2}{dx^2} f(x)}{2} + O\left(h^3\right)$$

Out[3]:

$$f(-h+x) = f(x) - h\frac{d}{dx}f(x) + \frac{h^2 \frac{d^2}{dx^2} f(x)}{2} + O\left(h^3\right)$$

Out[3]:

$$-f(-h+x) + f(h+x) = 2h\frac{d}{dx}f(x) + O\left(h^3\right)$$

Out[3]:

$$\frac{d}{dx}f(x) = \frac{-f(-h+x)+f(h+x)}{2h} + O\left(h^2\right)$$

$\xi_1 \to x$

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{6}f'''(x) + \mathcal{O}\left(h^4\right)$$

$$hf'(x) = f(x+h) - f(x) - \frac{h^2}{2}f''(x) - \frac{h^3}{6}f'''(x) + \mathcal{O}\left(h^4\right)$$

$$f'(x) = \frac{1}{h}\left[f(x+h) - f(x) - \frac{h^2}{2}f''(x) - \frac{h^3}{6}f'''(x) + \mathcal{O}\left(h^4\right)\right]$$

$$f'(x) = \frac{f(x+h)-f(x)}{h} - \frac{h}{2}f''(x) - \frac{h^2}{6}f'''(x) + \frac{1}{h}\mathcal{O}\left(h^4\right)$$

$$f'(x) = \frac{f(x+h)-f(x)}{h} - h\left[\frac{1}{2}f''(x) - \frac{h}{6}f'''(x) + \frac{1}{h^2}\mathcal{O}\left(h^4\right)\right]$$

$$f'(x) = \frac{f(x+h)-f(x)}{h} + \mathcal{O}(h)$$

$$f'(x) \approx \frac{f(x+h)-f(x)}{h}$$

In [4]:

```
# Yukarıdaki yöntemi kullanarak verilen herhangi bir f(x) fonksiyonun
# x = x_0 noktasında (x_0 herhangi bir reel sayı) sayısal türevini alan
# programı yazınız.

# ileri türev
x = 1.
h = 0.1

def f(x):
    return x**2 + x**3

fts = (f(x+h) - f(x))/h
ftt = 2*x + 3*x**2

print ftt, fts, fts-ftt
```

```
5.0 5.41 0.41
```

In [5]:

```
# simetrik türev
x = 1.
h = 0.1

def f(x):
    return x**2 + x**3

fts = (f(x+h) - f(x-h))/(2.*h)
ftt = 2*x + 3*x**2

print ftt, fts, fts-ftt
```

```
5.0 5.01 0.01
```

$$f'(x) = \frac{f(x-2h) - 8f(x-h) + 8f(x+h) - f(x+2h)}{12h}$$

In [6]:

```
import sympy as sm
import IPython.display as ipd
f, x, x0, h = sm.symbols('f, x, x0, h')
f = sm.Function('f')

def ft(f, x, h=0, n=3):
    x0 = sm.symbols('x0')
    theSeries = sm.series(f(x), x, x0, n).doit()
    if h == 0:
        ss = theSeries
    else:
        ss = theSeries.subs({x:x0+h, x0:x})
    ssl = sm.latex(ss).replace(r'\rightarrow', r'\rightarrow ')
    return ss, ssl

n=5
fxph, fxpl = ft(f, x, h, n)
fxmh, fxml = ft(f, x, -h, n)
fxp2h, fxp2l = ft(f, x, 2*h, n)
fxm2h, fxm2l = ft(f, x, -2*h, n)


fx_left = f(x-2*h) - 8*f(x-h) + 8*f(x+h) - f(x+2-h)
fx_right = fxm2h - 8*fxmh + 8*fxph - fxp2h

solwO = sm.expand(sm.solve(fx_left-fx_right, sm.Derivative(f(x), x))[0])
sol = solwO.removeO()
OO = solwO - sol
solO = sm.simplify(sol) + OO

ipd.display(ipd.Markdown("$%s = %s$"%(f(x+h), fxpl)))
ipd.display(ipd.Markdown("$%s = %s$"%(f(x+2*h), fxp2l)))
ipd.display(ipd.Markdown("$%s = %s$"%(f(x-h), fxml)))
ipd.display(ipd.Markdown("$%s = %s$"%(f(x-2*h), fxm2l)))
ipd.display(ipd.Markdown("$%s = %s$"%(sm.latex(fx_left), sm.latex(fx_right))))
ipd.display(ipd.Markdown("$%s = %s$"%(sm.latex(sm.Derivative(f(x), x)), sm.latex(solO
))))
```

Out[6]:

$$f(h+x) = f(x) + h\frac{d}{dx} f(x) + \frac{h^2 \frac{d^2}{dx^2} f(x)}{2} + \frac{h^3 \frac{d^3}{dx^3} f(x)}{6} + \frac{h^4 \frac{d^4}{dx^4} f(x)}{24} + O\left(h^5\right)$$

Out[6]:

$$f(2*h+x) = f(x) + 2h\frac{d}{dx} f(x) + 2h^2 \frac{d^2}{dx^2} f(x) + \frac{4h^3 \frac{d^3}{dx^3} f(x)}{3} + \frac{2h^4 \frac{d^4}{dx^4} f(x)}{3} + O\left(h^5\right)$$

Out[6]:

$$f(-h+x) = f(x) - h\frac{d}{dx} f(x) + \frac{h^2 \frac{d^2}{dx^2} f(x)}{2} - \frac{h^3 \frac{d^3}{dx^3} f(x)}{6} + \frac{h^4 \frac{d^4}{dx^4} f(x)}{24} + O\left(h^5\right)$$

Out[6]:

$$f(-2*h+x) = f(x) - 2h\frac{d}{dx} f(x) + 2h^2 \frac{d^2}{dx^2} f(x) - \frac{4h^3 \frac{d^3}{dx^3} f(x)}{3} + \frac{2h^4 \frac{d^4}{dx^4} f(x)}{3} + O\left(h^5\right)$$

Out[6]:

$$f(-2h+x) - 8f(-h+x) + 8f(h+x) - f(-h+x+2) = -(8f(x) - 8h\frac{d}{dx} f(x)$$
$$+ 8f(x) + 4h\frac{d}{dx} f(x) + 4h^2 \frac{d^2}{dx^2} f(x) - \frac{4h^3 \frac{d^3}{dx^3} f(x)}{3} + \frac{h^4 \frac{d^4}{dx^4} f(x)}{3} + O\left(h^5\right)$$

Out[6]:

$$\frac{d}{dx} f(x) = \frac{f(-2h+x) - 8f(-h+x) + 8f(h+x) - f(-h+x+2)}{12h} + O\left(h^4\right)$$

In [7]:

```
x = 1.
h = 0.1

def f(x):
    return x**2 + x**3

fts = (f(x-2*h) - 8*f(x-h) + 8*f(x+h) - f(x+2*h))/(12.*h)
ftt = 2*x + 3*x**2

print ftt, fts, fts-ftt
```

5.0 5.0 1.7763568394e-15

In [8]:

```
# simetrik türev
x = 1.
h = 0.1

def f(x):
    return x**2 + x**3

fts1 = (f(x+h) - f(x-h))/(2.*h)
fts2 = (f(x+h) -  2*f(x) + f(x-h))/h**2.

ftt1 = 2*x + 3*x**2
ftt2 = 2 + 6*x

print ftt1, fts1, fts1-ftt1
print ftt2, fts2, fts2-ftt2
```

```
5.0 5.01 0.01
8.0 8.0 4.97379915032e-14
```

Sayısal Türev Kavramının Bir Fizik Problemine Uygulanması

Bir boyutlu hareket eden bir sistemin konumunun ve hızının zaman bağlılığı Newton'un 2nci ($\vec{F} = m\vec{a}$) yasası kullanılarak aşağıdaki gibi hesaplanabilir.

$F = m\frac{\Delta v}{\Delta t} \implies \frac{\Delta v}{\Delta t} = \frac{F}{m} \implies v(t+\Delta t) = v(t) + \frac{F}{m}\Delta t$

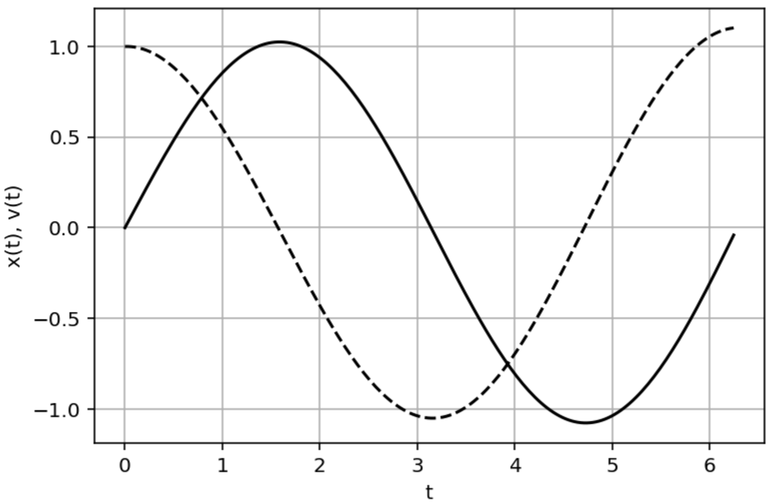
$v = \frac{\Delta x}{\Delta t} \implies \Delta x = v\Delta t \implies x(t+\Delta t) = x(t) + v(t)\Delta t$

$F = -kx$ kuvveti uygulanan yay için örnek:

In [9]:

```
# Kitapta pylab ve numpy kütüphaneleri kullanılarak
# F = -kx kuvvetli yay sistemi için çözüm.

from pylab import *
from numpy import *

def f(x,v,t):
    return -x

n=200
h=6.28/float(n)
t=zeros(n,float)
x=zeros(n,float)
v=zeros(n,float)
v[0]=1.0

for i in range(1,n):
    t[i]=h*i
    x[i]=x[i-1]+v[i-1]*h
    v[i]=v[i-1]+f(x[i-1],v[i-1],t[i-1])*h

plot(t,x,"k-")
plot(t,v,"k--")
xlabel("t")
ylabel("x(t), v(t)")
grid()
show()
```

Out[9]:

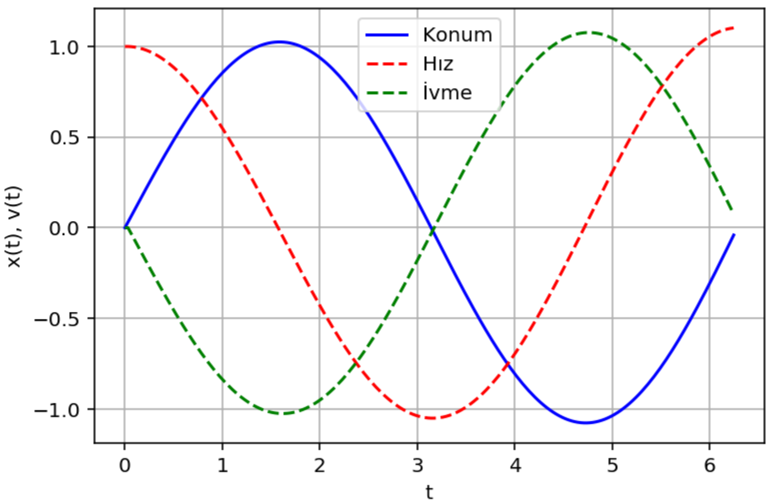


$F = -kx$ **kuvveti uygulanan yay için örnek: (Python programı farklı)**

In [10]:

```
# Hesaplama için standart Python, çizim için matplotlib.pyplot
# kullanılarak F = -kx kuvvetli yay sistemi için çözüm.

import matplotlib.pyplot as plt
%matplotlib inline

def f(x,v,t):
    return -x # harmonik salınıcı
n=200
h=6.28/float(n)
t = [0]
x = [0]
v = [1.0]
a = [0.]

for i in range(1,n):
    t = t + [h*i]
    x = x + [ x[i-1] + v[i-1]*h]
    v += [v[i-1] + f(x[i-1],v[i-1],t[i-1])*h]
    a += [(v[i]-v[i-1])/h]

plt.plot(t,x,"b-", label="Konum")
plt.plot(t,v,"r--", label=u"Hız")
plt.plot(t,a,"g--", label=u"İvme")
plt.xlabel("t")
plt.ylabel("x(t), v(t)")
plt.grid()
plt.legend()
plt.show()
```

Out[10]:



Kinematik fonksiyonu

Yukarıdaki hesaplamaları ve çizimi yapan kısım Python fonksiyonu olarak yeniden yazıldı. Böylece sadece kuvvet fonksiyonu ve başlangıç değerleri tanımlanarak sistemler incelenebilir.

In [11]:

```
import matplotlib.pyplot as plt
%matplotlib inline

def kinematik(n = 100, t0=0., tn=10.0, x0=0., v0=1., a0=-10.):
    t, x, v, a = [t0], [x0], [v0], [a0]
    dt = abs(tn-t0)/float(n)
    for i in range(1, n):
        t += [dt*i]
        x += [ x[i-1] + v[i-1]*dt]
        v += [v[i-1] + f(x[i-1],v[i-1],t[i-1])*dt]
        a += [(v[i]-v[i-1])/dt]

    plt.plot(t,x,"b-", label="Konum")
    plt.plot(t,v,"r--", label=u"Hız")
    plt.plot(t,a,"g--", label=u"İvme")
    plt.xlabel("t")
    plt.ylabel("x(t), v(t)")
    plt.grid()
    plt.legend()
    plt.show()
```

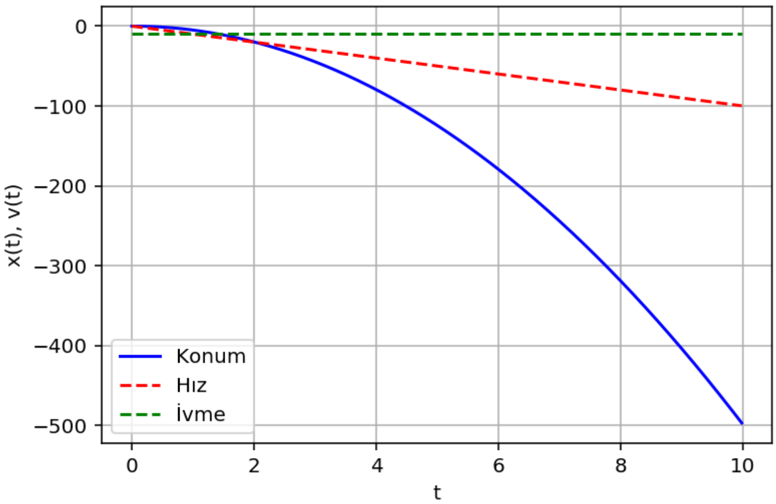
Serbest düşen cisim:

$$\Sigma F = -mg$$

In [12]:

```
def f(x,v,t):
    return -10

#kinematik(n = 100, t0=0., tn=10.0, x0=0., v0=1., a0=-10.)
kinematik(n=500, v0=0.)
```

Out[12]:



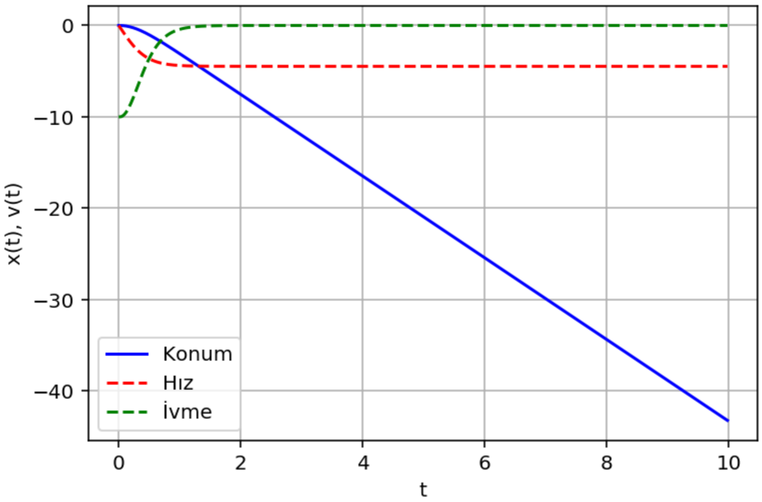
Direnç kuvveti varken:

$\Sigma F = -mg + cv^2$

In [13]:

```
def f(x,v,t):
    return -10+.5*v**2

#kinematik(n = 100, t0=0., tn=10.0, x0=0., v0=1., a0=-10.)
kinematik(n=500, v0=0.)
```

Out[13]:



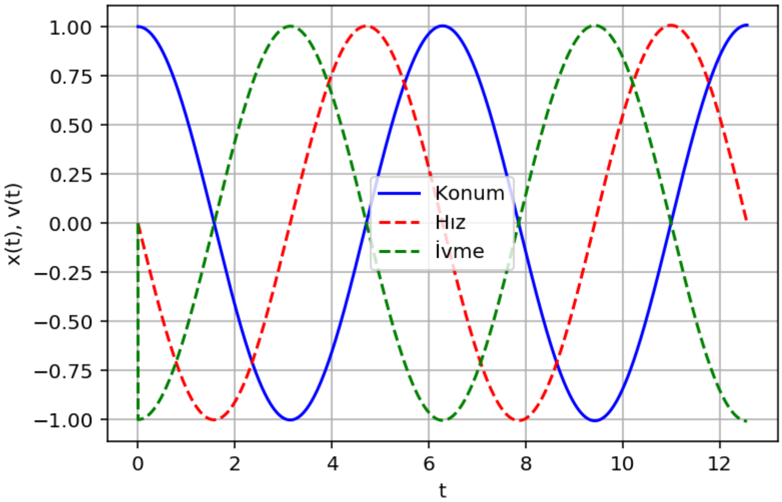
Harmonik salıncı

$\Sigma F = -kx$ **durumu:**

In [14]:

```
def f(x,v,t):
    return -x

pi = 3.14159
kinematik(n = 10000, tn=4*pi, x0=1., v0=0., a0=0.)
```

Out[14]:



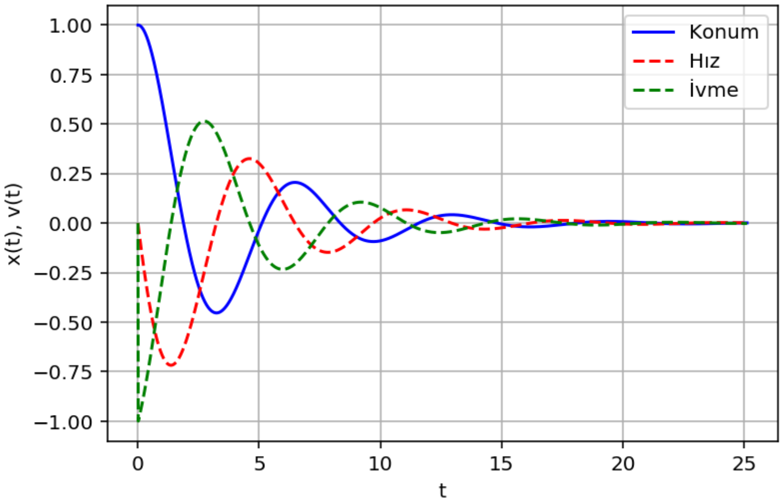
Sönümlü yayın davranışı:

$\Sigma F = -kx - bv$ **durumu:**

In [15]:

```
def f(x,v,t):
    return -x - .5*v # sönümlü harmonik salınıcı

pi = 3.14159
kinematik(n = 2000, tn=8*pi, x0=1., v0=0., a0=0.)
```

Out[15]:

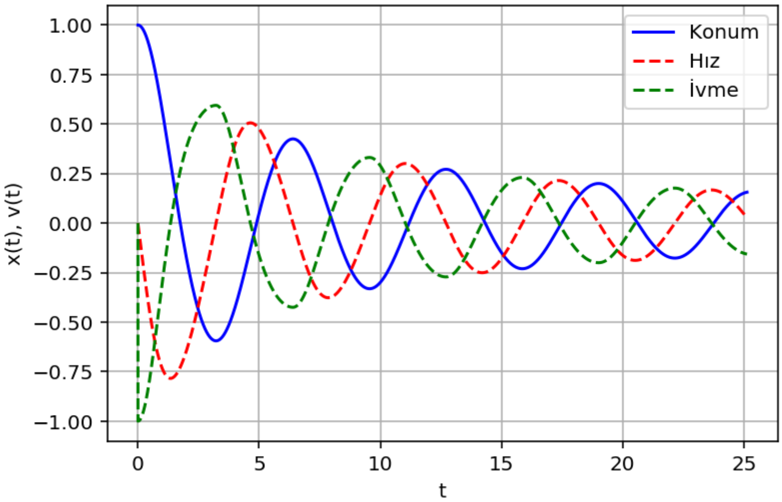


Direnç kuvveti $\Sigma F = -kx - cv^2$ durumu:

In [16]:

```
def f(x,v,t):
    return -x - .5*v**3/abs(v) # sönümlü harmonik salınıcı

pi = 3.14159
kinematik(n = 20000, tn=8*pi, x0=1., v0=1e-17, a0=0.)
```

Out[16]:

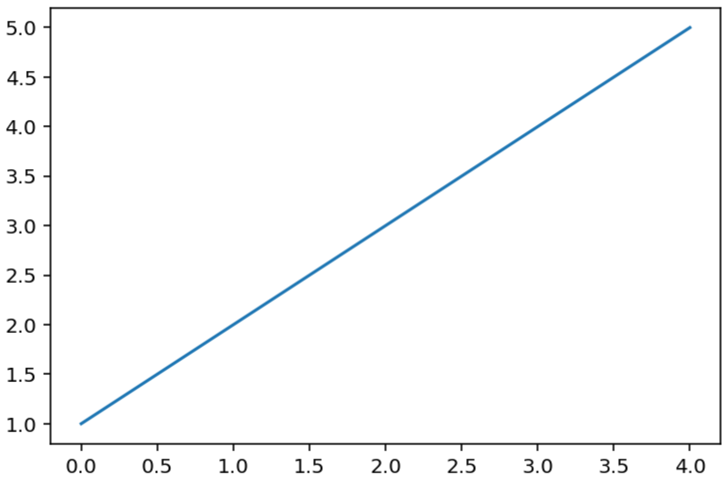


y=f(x) şeklinde bir fonksiyonun çizimi

In [17]:

```
import matplotlib.pyplot as plt
%matplotlib inline

plt.plot([1,2,3,4,5])
plt.show()
```

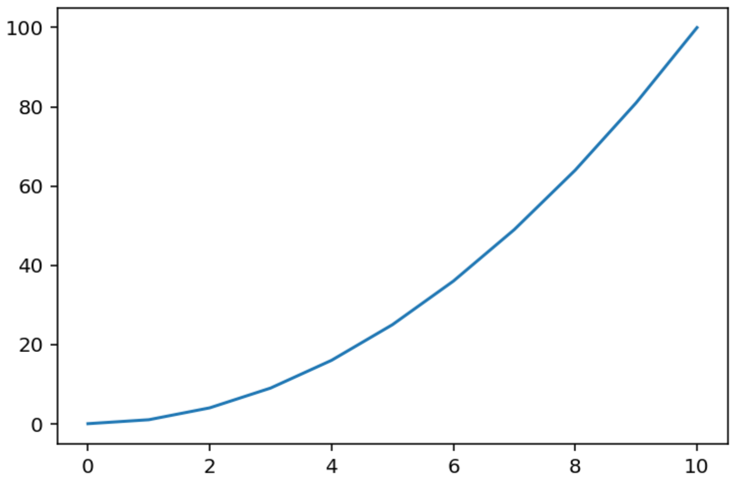
Out[17]:



In [18]:

```
import matplotlib.pyplot as plt
%matplotlib inline

def f(x):
    return x**2

xi = 0.
xs = 10.
nn = 10.
dx = abs(xs-xi)/nn

xl = []
yl = []

x = xi
while x<=xs:
    xl = xl + [x]
    yl = yl + [f(x)]
    x = x + dx

plt.plot(xl, yl)
plt.show()
```

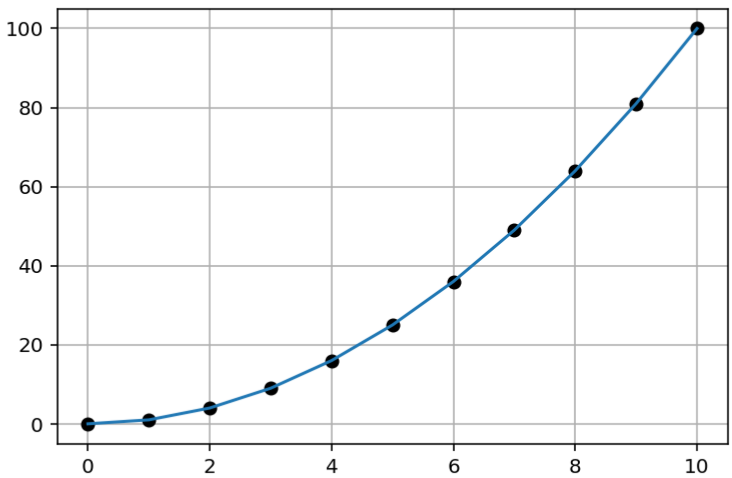
Out[18]:



In [19]:

```
import matplotlib.pyplot as plt
%matplotlib inline

def f(x):
    return x**2

xi = 0.
xs = 10.
nn = 10.
dx = abs(xs-xi)/nn

xl = []
yl = []

x = xi
while x<=xs:
    xl = xl + [x]
    yl = yl + [f(x)]
    x = x + dx

# renkler
# {'b', 'g', 'r', 'c', 'm', 'y', 'k', 'w'}
plt.plot(xl, yl, "ko")
plt.plot(xl, yl)
plt.grid()
plt.show()
```

Out[19]:



SAYISAL İNTEGRAL

f(x) = x fonksiyonunun sayısal integrali

Sol kenar kullanılarak alınan integral

In [20]:

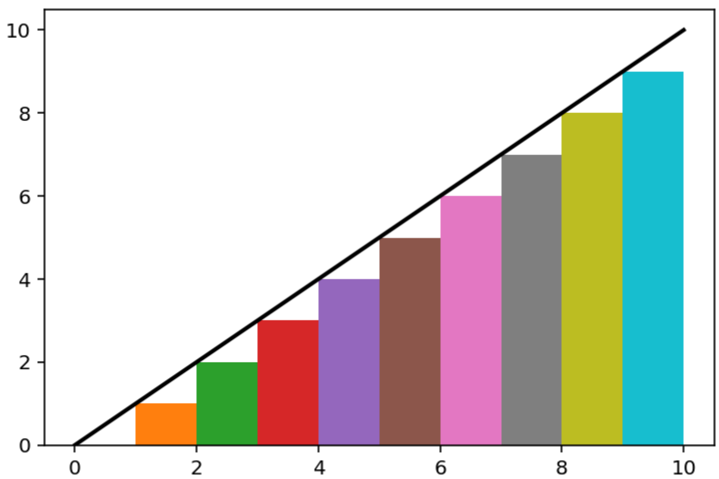
```
def f(x):
    return x

xi = 0.
xs = 10.
nn = 10.
dx = abs(xs-xi)/nn

i = 0.
toplam = 0.
print "%5s %5s"%("x", "f(x)")

while i<nn:
    x = xi + i*dx
    i = i + 1
    toplam = toplam + f(x)
    print "%5s %5s"%(x, f(x))

toplam = toplam*dx
print "f(x)'in %s ile %s aralığındaki integrali = %s"%(xi, xs, toplam)
```

```
    x  f(x)
  0.0   0.0
  1.0   1.0
  2.0   2.0
  3.0   3.0
  4.0   4.0
  5.0   5.0
  6.0   6.0
  7.0   7.0
  8.0   8.0
  9.0   9.0
f(x)'in 0.0 ile 10.0 aralığındaki integrali = 45.0
```

In [21]:

```
import matplotlib.pyplot as plt
%matplotlib inline


xl = [xi+i*dx for i in range(int(nn+1))]
fl = [f(x) for x in xl]

for i, x in enumerate(xl[:-1]):
    plt.bar(x+dx/2., fl[i], dx)

plt.plot(xl, fl, "k-", lw="2")
plt.show()
```

Out[21]:



f(x) = x fonksiyonunun sayısal integrali

Sağ kenar kullanılarak alınan integral

In [22]:

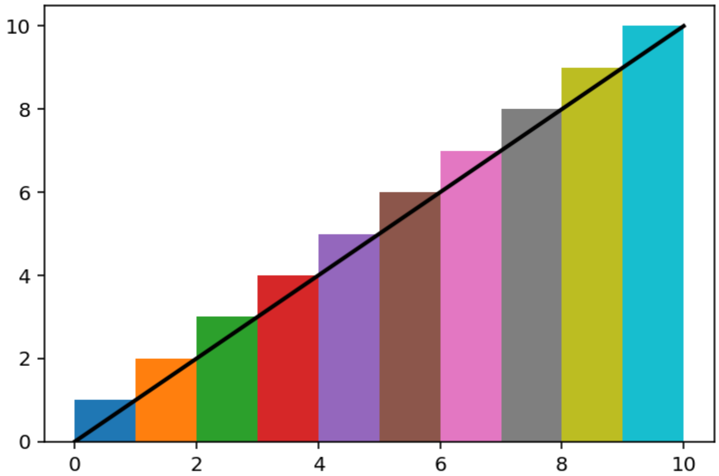
```python
def f(x):
    return x

xi = 0.
xs = 10.
nn = 10.
dx = abs(xs-xi)/nn

i = 1.
toplam = 0.
print "%5s %5s"%("x", "f(x)")
while i<=nn:
    x = xi + i*dx
    i = i + 1
    toplam = toplam + f(x)
    print "%5s %5s"%(x, f(x))

toplam = toplam*dx
print "f(x)'in %s ile %s aralığındaki integrali = %s"%(xi, xs, toplam)
```

```
    x  f(x)
  1.0   1.0
  2.0   2.0
  3.0   3.0
  4.0   4.0
  5.0   5.0
  6.0   6.0
  7.0   7.0
  8.0   8.0
  9.0   9.0
 10.0  10.0
f(x)'in 0.0 ile 10.0 aralığındaki integrali = 55.0
```

In [23]:

```
import matplotlib.pyplot as plt
%matplotlib inline

xl = [xi+i*dx for i in range(int(nn+1))]
fl = [f(x) for x in xl]

for i, x in enumerate(xl[1:]):
    plt.bar(x-dx/2., fl[i+1], dx)

plt.plot(xl, fl, "k-", lw="2")
plt.show()
```

Out[23]:



f(x) = exp(x) fonksiyonunun integrali

Sol kenar kullanılarak alınan integral

In [24]:

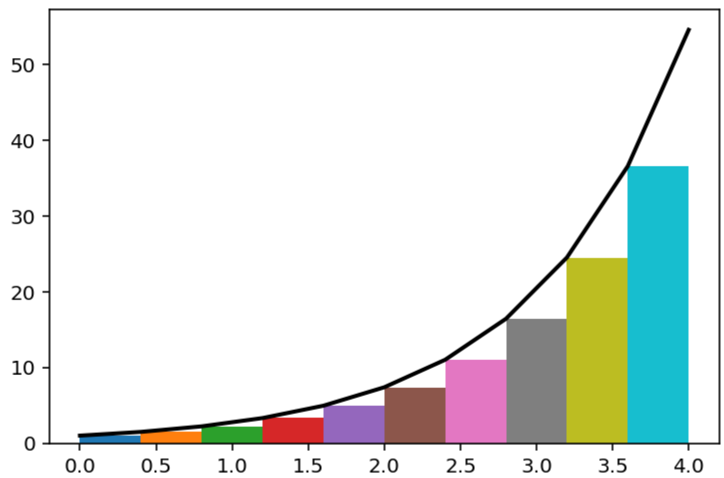
```
from math import *

def f(x):
    return exp(x)

xi = 0.
xs = 4.
nn = 10.
dx = abs(xs-xi)/nn

i = 0.
toplam = 0.
print "%5s %5s"%("x", "f(x)")
while i<nn:
    x = xi + i*dx
    i = i + 1
    toplam = toplam + f(x)
    #print "%5s %5s"%(x, f(x))

toplam = toplam*dx
print "f(x)'in %s ile %s aralığındaki integrali = %s"%(xi, xs, toplam)
print "f(x)'in tam integrali = %s"%(exp(4)-1)
```

```
    x  f(x)
f(x)'in 0.0 ile 4.0 aralığındaki integrali = 43.5912635459
f(x)'in tam integrali = 53.5981500331
```

In [25]:

```
import matplotlib.pyplot as plt
%matplotlib inline

xl = [xi+i*dx for i in range(int(nn+1))]
fl = [f(x) for x in xl]

for i, x in enumerate(xl[:-1]):
    plt.bar(x+dx/2., fl[i], dx)

plt.plot(xl, fl, "k-", lw="2")
plt.show()
```

Out[25]:



f(x) = exp(x) fonksiyonunun integrali

Sağ kenar kullanılarak alınan integral

In [26]:

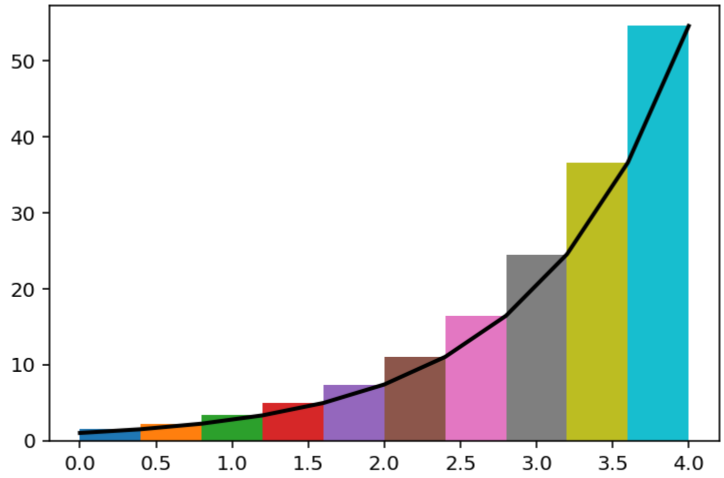
```
from math import *

def f(x):
    return exp(x)

xi = 0.
xs = 4.
nn = 10.
dx = abs(xs-xi)/nn

i = 1.
toplam = 0.
print "%5s %5s"%("x", "f(x)")
while i<=nn:
    x = xi + i*dx
    i = i + 1
    toplam = toplam + f(x)
    print "%5s %5s"%(x, f(x))

toplam = toplam*dx
print "f(x)'in %s ile %s aralığındaki integrali = %s"%(xi, xs, toplam)
```

```
    x  f(x)
  0.4 1.49182469764
  0.8 2.22554092849
  1.2 3.32011692274
  1.6 4.9530324244
  2.0 7.38905609893
  2.4 11.0231763806
  2.8 16.4446467711
  3.2 24.5325301971
  3.6 36.5982344437
  4.0 54.5981500331
f(x)'in 0.0 ile 4.0 aralığındaki integrali = 65.0305235591
```

In [27]:

```
import matplotlib.pyplot as plt
%matplotlib inline

xl = [xi+i*dx for i in range(int(nn+1))]
fl = [f(x) for x in xl]

for i, x in enumerate(xl[1:]):
    plt.bar(x-dx/2., fl[i+1], dx)

plt.plot(xl, fl, "k-", lw="2")
plt.show()
```

Out[27]:



f(x) = sin(x) fonksiyonunun integrali

Sol kenar kullanılarak alınan integral

In [28]:

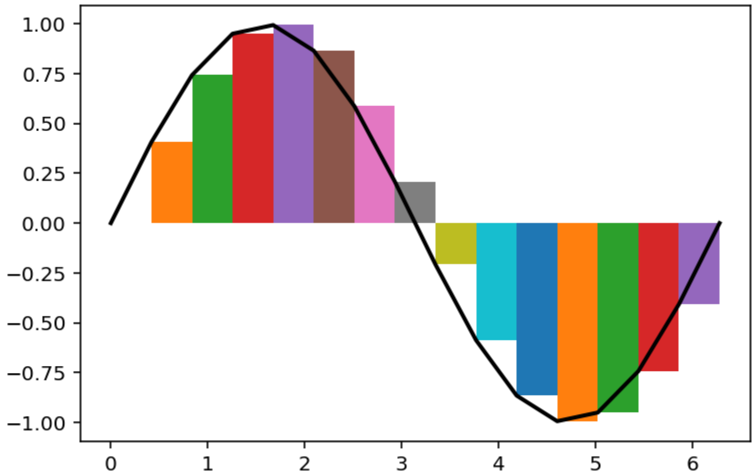
```
from math import *

def f(x):
    return sin(x)

xi = 0.
xs = 2*pi
nn = 15.
dx = abs(xs-xi)/nn

i = 0.
toplam = 0.
print "%5s %5s"%("x", "f(x)")
while i<nn:
    x = xi + i*dx
    i = i + 1
    toplam = toplam + f(x)
    print "%5s %5s"%(x, f(x))

toplam = toplam*dx
print "f(x)'in %s ile %s aralığındaki integrali = %s"%(xi, xs, toplam)
```

```
    x  f(x)
  0.0   0.0
0.418879020479 0.406736643076
0.837758040957 0.743144825477
1.25663706144 0.951056516295
1.67551608191 0.994521895368
2.09439510239 0.866025403784
2.51327412287 0.587785252292
2.93215314335 0.207911690818
3.35103216383 -0.207911690818
3.76991118431 -0.587785252292
4.18879020479 -0.866025403784
4.60766922527 -0.994521895368
5.02654824574 -0.951056516295
5.44542726622 -0.743144825477
5.8643062867 -0.406736643076
f(x)'in 0.0 ile 6.28318530718 aralığındaki integrali = 1.86019653227e-16
```

In [29]:

```
import matplotlib.pyplot as plt
%matplotlib inline

xl = [xi+i*dx for i in range(int(nn+1))]
fl = [f(x) for x in xl]

for i, x in enumerate(xl[:-1]):
    plt.bar(x+dx/2., fl[i], dx)

plt.plot(xl, fl, "k-", lw="2")
plt.show()
```

Out[29]:



f(x) = sin(x) fonksiyonunun integrali

Sağ kenar kullanılarak alınan integral

In [30]:

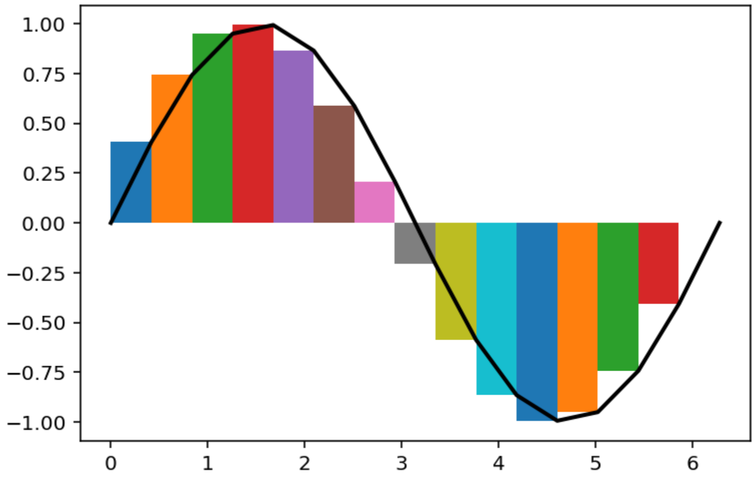
```
from math import *

def f(x):
    return sin(x)

xi = 0.
xs = 2*pi
nn = 15.
dx = abs(xs-xi)/nn

i = 1.
toplam = 0.
print "%13s %26s"%("x", "f(x)")
while i<=nn:
    x = xi + i*dx
    i = i + 1
    toplam = toplam + f(x)
    print "%+25.20f %+25.20f"%(x, f(x))

toplam = toplam*dx
print "f(x)'in %s ile %s aralığındaki integrali = %s"%(xi, xs, toplam)
```

```
            x                       f(x)
  +0.41887902047863906363   +0.40673664307580015276
  +0.83775804095727812726   +0.74314482547739413310
  +1.25663706143591724640   +0.95105651629515353118
  +1.67551608191455625452   +0.99452189536827340088
  +2.09439510239319526264   +0.86602540378443870761
  +2.51327412287183449280   +0.58778525229247324813
  +2.93215314335047327887   +0.20791169081775973115
  +3.35103216382911250903   -0.20791169081775906502
  +3.76991118430775173920   -0.58778525229247302608
  +4.18879020478639052527   -0.86602540378443837454
  +4.60766922526502931134   -0.99452189536827328986
  +5.02654824574366898560   -0.95105651629515364220
  +5.44542726622230777167   -0.74314482547739457718
  +5.86430628670094655774   -0.40673664307580092991
  +6.28318530717958623200   -0.00000000000000024493
f(x)'in 0.0 ile 6.28318530718 aralığındaki integrali = 8.34238828953e-17
```

In [31]:

```
import matplotlib.pyplot as plt
%matplotlib inline

xl = [xi+i*dx for i in range(int(nn+1))]
fl = [f(x) for x in xl]

for i, x in enumerate(xl[1:]):
    plt.bar(x-dx/2., fl[i+1], dx)

plt.plot(xl, fl, "k-", lw="2")
plt.show()
```

Out[31]:



TRAPEZ Yöntemi

In [32]:

```
from sympy import *
from IPython.display import Markdown, display
init_printing()

X, i, dx = symbols('X, i, dx')
x = IndexedBase('x')
f = IndexedBase('f')

def b(X):
    return (X-x[i+1])/(x[i]-x[i+1])

def c(X):
    return (X-x[i])/(x[i+1]-x[i])

P = lambda X: b(X)*f[i] + c(X)*f[i+1]

integ1 = simplify(integrate(P(X),(X, x[i], x[i+1])))
integ2 = simplify(integ1.subs(x[i+1], x[i]+dx))

S = lambda j: integ2.subs(i,j)
```

In [33]:

```
display(Markdown("$\Large P(X) = %s$"%latex(P(X))))

display(Markdown("$s_i \eqsim \int\limits^{x_{i+1}}_{x_{i}} P(x) dx = %s$"%latex(integ
1)))

display(Markdown("$\Large s_i \eqsim \int\limits^{x_{i}+dx}_{x_{i}} P(x) dx = %s$"%late
x(integ2)))

display(Markdown("$S \eqsim \large %s$"%latex(simplify(sum([S(j) for j in range(1,16,
1)])))))
```

Out[33]:

$$P(X) = \frac{(X - x_{i+1}) f_i}{-x_{i+1} + x_i} + \frac{(X - x_i) f_{i+1}}{x_{i+1} - x_i}$$

Out[33]:

$$s_i \eqsim \int\limits^{x_{i+1}}_{x_i} P(x) dx = \frac{f_{i+1} x_{i+1}}{2} - \frac{f_{i+1} x_i}{2} + \frac{f_i x_{i+1}}{2} - \frac{f_i x_i}{2}$$

Out[33]:

$$s_i \eqsim \int\limits^{x_i+dx}_{x_i} P(x) dx = \frac{dx (f_{i+1} + f_i)}{2}$$

Out[33]:

$$S \eqsim \frac{dx (2f_{10} + 2f_{11} + 2f_{12} + 2f_{13} + 2f_{14} + 2f_{15} + f_{16} + f_1 + 2f_2 + 2f_3 + 2f_4 + 2f_5 + 2f_6 + 2f_7 + 2f_8 + 2f_9)}{2}$$

In [34]:

```
def trapez_integral(xi=0., xs=10., nn=10.):
    dx = abs(xs-xi)/nn
    i = 1.
    toplam = 0.
    print "%5s %5s"%("x", "f(x)")
    while i<nn:
        x = xi + i*dx
        i = i + 1
        toplam = toplam + f(x)
        #print "%5s %5s"%(x, f(x))

    toplam = (toplam + (f(xi)+f(xs))/2.)*dx
    return toplam

def yamuk(x1,x2,y1,y2):
    return plt.fill([x1, x1, x2, x2], [0, y1, y2, 0])

def trapez_grafik(xi=0., xs=10., nn=10.):
    dx = abs(xs-xi)/nn

    # yamuklar için gerekli bilgiler
    xl = [xi+i*dx for i in range(int(nn+1))]
    fl = [f(x) for x in xl]

    # asıl fonksiyon için gerekli bilgiler
    xlf = [xi+i*dx/10. for i in range(int(10*nn+1))]
    flf = [f(x) for x in xlf]

    # trapez için gerekli yamukları çiziyor
    for i, x in enumerate(xl[:-1]):
        yamuk(xl[i], xl[i+1], fl[i], fl[i+1])

    # asıl fonksiyonu çiziyor
    plt.plot(xlf, flf, "k-", lw="2")
    plt.show()
```

In [35]:

```
import matplotlib.pyplot as plt
%matplotlib inline

def f(x):
    return x

xi, xs, nn = 0., 10., 10.

toplam = trapez_integral(xi, xs, nn)
print "f(x)'in %s ile %s aralığındaki integrali = %s"%(xi, xs, toplam)
trapez_grafik(xi, xs, nn)
```

```
    x  f(x)
f(x)'in 0.0 ile 10.0 aralığındaki integrali = 50.0
```

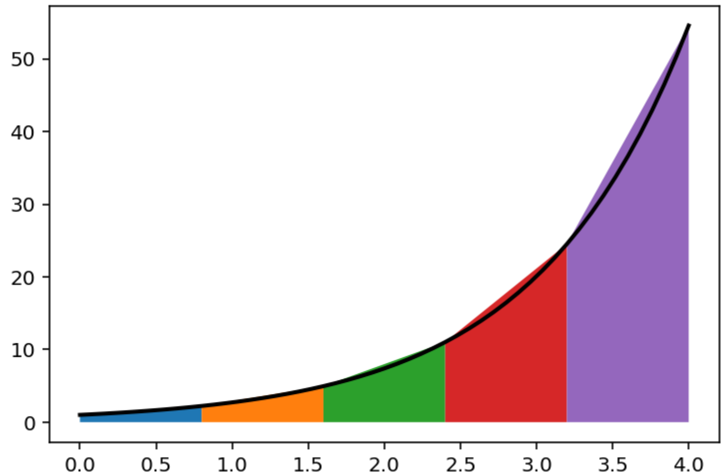
Out[35]:

In [36]:

```
from math import *
import matplotlib.pyplot as plt
%matplotlib inline

def f(x):
    return exp(x)

xi, xs, nn = 0., 4., 5.

toplam = trapez_integral(xi, xs, nn)
print "f(x)'in %s ile %s aralığındaki integrali = %s"%(xi, xs, toplam)
trapez_grafik(xi, xs, nn)
```

```
    x  f(x)
f(x)'in 0.0 ile 4.0 aralığındaki integrali = 56.4266839578
```

Out[36]:

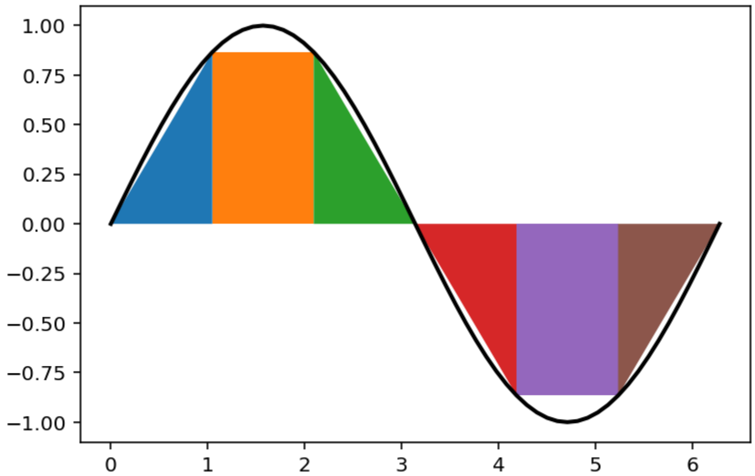


In [37]:

```
from math import *
import matplotlib.pyplot as plt
%matplotlib inline

def f(x):
    return sin(x)

xi, xs, nn = 0., 2*pi, 6.

toplam = trapez_integral(xi, xs, nn)
print "f(x)'in %s ile %s aralığındaki integrali = %s"%(xi, xs, toplam)
trapez_grafik(xi, xs, nn)
```

```
    x  f(x)
f(x)'in 0.0 ile 6.28318530718 aralığındaki integrali = -1.28244712915e-16
```

Out[37]:



SIMPSON Yöntemi

3 noktalı Simpson

Aşağıda verilen kod 3 nokta içeren Simpson integral yönteminin elde edilişini gösterir.

In [38]:

```
from sympy import *
from IPython.display import Markdown, display
init_printing()

X, i, dx = symbols('X, i, dx')
x = IndexedBase('x')
f = IndexedBase('f')

def a(X):
    return (X-x[i])*(X-x[i+1])/((x[i-1]-x[i])*(x[i-1]-x[i+1]))

def b(X):
    return (X-x[i-1])*(X-x[i+1])/((x[i]-x[i-1])*(x[i]-x[i+1]))

def c(X):
    return (X-x[i-1])*(X-x[i])/((x[i+1]-x[i-1])*(x[i+1]-x[i]))

P = lambda X: a(X)*f[i-1] + b(X)*f[i] + c(X)*f[i+1]

integ1 = simplify(integrate(P(X),(X, x[i-1], x[i+1])))
integ2 = simplify(integ1.subs(x[i-1], x[i]-dx).subs(x[i+1], x[i]+dx))

S = lambda j: integ2.subs(i,j)
```

In [39]:

```
display(Markdown("$\Large P(X) = %s$"%latex(P(X))))

display(Markdown("$s_i \eqsim \int\limits^{x_{i+1}}_{x_{i-1}} P(x) dx = %s$"%latex(inte
g1)))

display(Markdown("$\Large s_i \eqsim \int\limits^{x_{i}+dx}_{x_{i}-dx} P(x) dx = %s$"%l
atex(integ2)))

display(Markdown("$S \eqsim \large %s$"%latex(simplify(sum([S(j) for j in range(1,16,
2)])))))
```

Out[39]:

$$P(X) = \frac{(X-x_{i+1})(X-x_{i-1})f_i}{(-x_{i+1}+x_i)(-x_{i-1}+x_i)} + \frac{(X-x_{i+1})(X-x_i)f_{i-1}}{(-x_{i+1}+x_{i-1})(x_{i-1}-x_i)} + \frac{(X-x_{i-1}}{(x_{i+1}-x_i}$$

Out[39]:

$$s_i \eqsim \int\limits_{x_{i-1}}^{x_{i+1}} P(x)dx$$

$$= \frac{2f_{i+1}x_{i+1}^2x_{i-1}-2f_{i+1}x_{i+1}^2x_i-f_{i+1}x_{i+1}x_{i-1}^2-2f_{i+1}x_{i+1}x_{i-1}x_i+3f_{i+1}x_{i+1}x_i^2-f_{i+1}x_{i-1}^3+4f_{i+1}x_{i-1}^2x_i-3f_{i+1}x_{i-1}x_i^2+ \quad +2f_{i-1}x_{i-1}^2x_i-3f_{i-1}x_{i-1}x_i^2-f_ix_{i+1}^3+3f_ix_{i+1}^2x_{i-1}-3f_ix_{i+1}x_{i-1}^2+f_ix_{i-1}^3}{6(x_{i+1}x_{i-1}-x_{i+1}x_i-x_{i-1}x_i+x}$$

Out[39]:

$$s_i \eqsim \int\limits_{x_i-dx}^{x_i+dx} P(x)dx = \frac{dx(f_{i+1}+f_{i-1}+4f_i)}{3}$$

Out[39]:

$$S \eqsim \frac{dx(f_0+2f_{10}+4f_{11}+2f_{12}+4f_{13}+2f_{14}+4f_{15}+f_{16}+4f_1+2f_2+4f_3+2f_4+4f_5+2f_6+4f_7+2f_8+}{3}$$

4 noktalı Simpson

Aşağıda verilen kod 4 nokta içeren Simpson integral yönteminin elde edilişini gösterir.

In [40]:

```
from sympy import *
from IPython.display import Markdown, display
init_printing()

X, i, dx = symbols('X, i, dx')
x = IndexedBase('x')
f = IndexedBase('f')

def a(X):
    return (X-x[i])*(X-x[i+1])*(X-x[i+2])/((x[i-1]-x[i])*(x[i-1]-x[i+1])*(x[i-1]-x[i+2
]))

def b(X):
    return (X-x[i-1])*(X-x[i+1])*(X-x[i+2])/((x[i]-x[i-1])*(x[i]-x[i+1])*(x[i]-x[i+2]))

def c(X):
    return (X-x[i-1])*(X-x[i])*(X-x[i+2])/((x[i+1]-x[i-1])*(x[i+1]-x[i])*(x[i+1]-x[i+2
]))

def d(X):
    return (X-x[i-1])*(X-x[i])*(X-x[i+1])/((x[i+2]-x[i-1])*(x[i+2]-x[i])*(x[i+2]-x[i+1
]))

P = lambda X: a(X)*f[i-1] + b(X)*f[i] + c(X)*f[i+1] + d(X)*f[i+2]

integ1 = simplify(integrate(P(X),(X, x[i-1], x[i+2])))
integ2 = simplify(integ1.subs(x[i-1], x[i]-dx).subs(x[i+1], x[i]+dx).subs(x[i+2], x[i]+
2*dx))

S = lambda j: integ2.subs(i,j)
```

In [41]:

```
display(Markdown("$\Large P(X) = %s$"%latex(P(X))))

display(Markdown("$s_i \eqsim \int\limits^{x_{i+2}}_{x_{i-1}} P(x) dx = %s$"%latex(inte
g1)))

display(Markdown("$\Large s_i \eqsim \int\limits^{x_{i}+2dx}_{x_{i}-dx} P(x) dx = %s$"%
latex(integ2)))

display(Markdown("$S \eqsim \large %s$"%latex(simplify(sum([S(j) for j in range(1,16,
3)])))))
```

Out[41]:

$$P(X) = \frac{(X-x_{i+1})(X-x_{i+2})(X-x_{i-1})f_i}{(-x_{i+1}+x_i)(-x_{i+2}+x_i)(-x_{i-1}+x_i)} + \frac{(X-x_{i+1})(X-x_{i+2})(X-}{(-x_{i+1}+x_{i-1})(-x_{i+2}+x_{i-}}$$
$$+ \frac{(X-x_{i+2})(X-x_{i-1})(X-x_i)f_{i+1}}{(x_{i+1}-x_{i+2})(x_{i+1}-x_{i-1})(x_{i+1}-x_i)}$$

Out[41]:

$$s_i \eqsim \int\limits_{x_{i-1}}^{x_{i+2}} P(x)dx =$$
$$-\frac{\begin{gathered} f_{i+1}x_{i+2}^5x_{i-1}-f_{i+1}x_{i+2}^5x_i-2f_{i+1}x_{i+2}^4x_{i-1}^2-f_{i+1}x_{i+2}^4x_{i-1}x_i+3f_{i+1}x_{i+2}^4x_i^2+8f_{i+1}x_{i+2}^3x_{i-1}^2x_i-6f_{i+1}x_{i+2}^3x_{i-1}x_i^2-2f_{i+1} \\ +f_{i+1}x_{i+2}x_{i-1}^4x_i+6f_{i+1}x_{i+2}x_{i-1}^3x_i^2-6f_{i+1}x_{i+2}x_{i-1}^2x_i^3+f_{i+1}x_{i-1}^5x_i-3f_{i+1}x_{i-1}^4x_i^2+2f_{i+1}x_{i-1}^3x_i^3-4f_{i+2}x_{i+1}^3x_{i+2}^2x_{i-1}+ \\ +2f_{i+2}x_{i+1}^3x_{i-1}^3-8f_{i+2}x_{i+1}^3x_{i-1}^2x_i+6f_{i+2}x_{i+1}^3x_{i-1}x_i^2+3f_{i+2}x_{i+1}^2x_{i+2}^3x_{i-1}-3f_{i+2}x_{i+1}^2x_{i+2}^3x_i+3f_{i+2}x_{i+1}^2x_{i+2}^2x_{i-1}^2-3f_{i} \\ -3f_{i+2}x_{i+1}^2x_{i-1}^4+9f_{i+2}x_{i+1}^2x_{i-1}^3x_i-6f_{i+2}x_{i+1}^2x_{i-1}x_i^3-3f_{i+2}x_{i+1}x_{i+2}^3x_{i-1}^2+3f_{i+2}x_{i+1}x_{i+2}^3x_i^2+f_{i+2}x_{i+1}x_{i+2}^2x_{i-1}^3+3f_{i+} \\ -4f_{i+2}x_{i+1}x_{i+2}x_{i-1}x_i^3+f_{i+2}x_{i+1}x_{i-1}^5-9f_{i+2}x_{i+1}x_{i-1}^3x_i^2+8f_{i+2}x_{i+1}x_{i-1}^2x_i^3+3f_{i+2}x_{i+2}^3x_{i-1}^2x_i-3f_{i+2}x_{i+2}^3x_{i-1}x_i^2-f_{i+2} \\ -2f_{i+2}x_{i+2}x_{i-1}^2x_i^3-f_{i+2}x_{i-1}^5x_i+3f_{i+2}x_{i-1}^4x_i^2-2f_{i+2}x_{i-1}^3x_i^3-2f_{i-1}x_{i+1}^3x_{i+2}^3-2f_{i-1}x_{i+1}^3x_{i+2}^2x_{i-1}+8f_{i-1}x_{i+1}^3x_{i+2}^2x_i+4 \\ +6f_{i-1}x_{i+1}^3x_{i-1}x_i^2+3f_{i-1}x_{i+1}^2x_{i+2}^4+3f_{i-1}x_{i+1}^2x_{i+2}^3x_{i-1}-9f_{i-1}x_{i+1}^2x_{i+2}^3x_i-3f_{i-1}x_{i+1}^2x_{i+2}^2x_{i-1}^2+3f_{i-1}x_{i+1}^2x_{i+2}^2x_{i-1}x_i- \\ -6f_{i-1}x_{i+1}^2x_{i-1}x_i^3-f_{i-1}x_{i+1}x_{i+2}^5-f_{i-1}x_{i+1}x_{i+2}^4x_{i-1}-f_{i-1}x_{i+1}x_{i+2}^3x_{i-1}^2+9f_{i-1}x_{i+1}x_{i+2}^3x_i^2+3f_{i-1}x_{i+1}x_{i+2}^2x_{i-1}^3-3f_{i-} \\ -3f_{i-1}x_{i+1}x_{i-1}^3x_i^2+4f_{i-1}x_{i+1}x_{i-1}^2x_i^3+f_{i-1}x_{i+2}^5x_i+f_{i-1}x_{i+2}^4x_{i-1}x_i-3f_{i-1}x_{i+2}^4x_i^2+f_{i-1}x_{i+2}^3x_{i-1}^2x_i-3f_{i-1}x_{i+2}^3x_{i-1}x_i^2- \\ +3f_{i-1}x_{i+2}x_{i-1}^3x_i^2-4f_{i-1}x_{i+2}x_{i-1}^2x_i^3+2f_ix_{i+1}^3x_{i+2}^3-6f_ix_{i+1}^3x_{i+2}^2x_{i-1}+6f_ix_{i+1}^3x_{i+2}x_{i-1}^2-2f_ix_{i+1}^3x_{i-1}^3-3f_ix_{i+1}^2x_{i+2}^4+ \\ -8f_ix_{i+1}x_{i+2}^3x_{i-1}^2+8f_ix_{i+1}x_{i+2}^2x_{i-1}^3-f_ix_{i+1}x_{i+2}x_{i-1}^4-f_ix_{i+1}x_{i-1}^5-f_ix_{i+2}^5x_{i-1}+2f_ix_{i+2}^4x_{i-1}^2-2f_ix_{i+2}^2x_{i-1}^4+f_ix_{i+2}x \end{gathered}}{\begin{gathered} 12x_{i+1}^3x_{i+2}x_{i-1}-12x_{i+1}^3x_{i+2}x_i-12x_{i+1}^3x_{i-1}x_i+12x_{i+1}^3x_i^2-12x_{i+1}^2x_{i+2}^2x_{i-1}+12x_{i+1}^2x_{i+2}^2x_i-12x_{i+1}^2x_{i+2}x_{i-1}^2+ \\ -12x_{i+1}x_{i+2}x_{i-1}x_i^2+12x_{i+1}x_{i+2}x_i^3-12x_{i+1}x_{i-1}^2x_i^2+12x_{i+1}x_{i-1}x_i^3-12x_{i+2}^2x_{i-1}^2x_i+12x_{i+2}^2x_{i-1}x_i^2+12x_{i+2}x_{i-} \end{gathered}}$$

Out[41]:

$$s_i \eqsim \int\limits_{x_i-dx}^{x_i+2dx} P(x)dx = \frac{3dx(3f_{i+1}+f_{i+2}+f_{i-1}+3f_i)}{8}$$

Out[41]:

$$S \eqsim \frac{3dx(f_0+3f_{10}+3f_{11}+2f_{12}+3f_{13}+3f_{14}+f_{15}+3f_1+3f_2+2f_3+3f_4+3f_5+2f_6+3f_7+3f_8+2f_9)}{8}$$

In [42]:

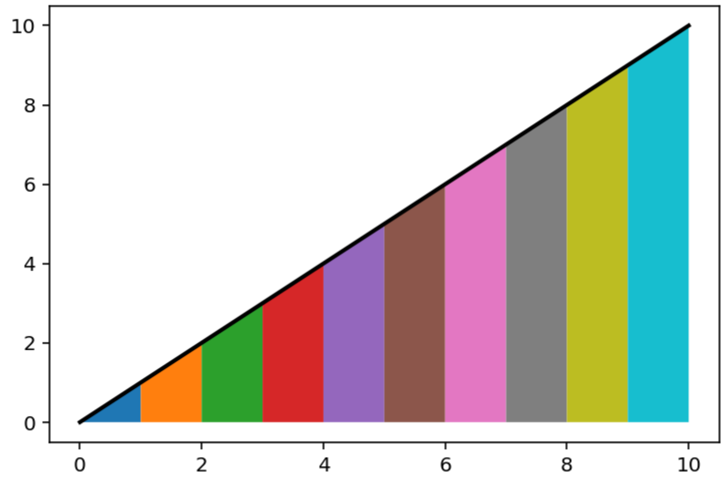
```
def simpson_integral(xi=0., xs=10., nn=10.):
    if nn%2==1:
        nn += 1
        print "nn çift sayıya çevirildi. yeni değeri %s'dir."%nn

    dx = abs(xs-xi)/nn
    # tek indisliler
    i = 1.
    toplam_tek = 0.
    #print "%5s %5s"%("x", "f(x)")
    while i<=nn-1:
        x = xi + i*dx
        i = i + 2
        toplam_tek +=  f(x)
        #print "%5s %5s"%(x, f(x))

    # çift indisliler
    i = 2.
    toplam_cift = 0.
    #print "%5s %5s"%("x", "f(x)")
    while i<=nn-2:
        x = xi + i*dx
        i = i + 2
        toplam_cift += f(x)
        #print "%5s %5s"%(x, f(x))

    toplam = (4*toplam_tek + 2*toplam_cift + f(xi) + f(xs))*dx/3.
    return toplam

# f(x)'i temsil eden polinom fonksiyon
def phi(x, xi, dx):
    phi1 = (x-xi)*(x-xi-dx)*f(xi-dx)/2.
    phi2 = -(x-xi+dx)*(x-xi-dx)*f(xi)
    phi3 = (x-xi+dx)*(x-xi)*f(xi+dx)/2.
    phiT = (phi1 + phi2 + phi3)/dx**2.
    return phiT

def simpson_grafik(xi=0., xs=10., nn=10.):
    if nn%2==1:
        nn += 1
        print "nn çift sayıya çevirildi. yeni değeri %s'dir."%nn
    dx = abs(xs-xi)/nn
    # polinom için gerekli x ve f değerleri
    xl = [xi + i*dx for i in range(int(nn))]
    fl = [f(x) for x in xl]
    # polinom çizimi
    for sx in xl[1::2]:
        xlphi = [sx-dx+i*dx/10. for i in range(21)]
        flphi = [phi(x, sx, dx/10.) for x in xlphi]
        plt.fill_between(xlphi[:11], flphi[:11])
        plt.fill_between(xlphi[10:21], flphi[10:21])

    # fonksiyon için gerekli x ve f değerleri
    xlf = [xi + i*dx/5. for i in range(int(5*nn+1))]
    flf = [f(x) for x in xlf]
    # fonksiyon çizimi
    plt.plot(xlf, flf, "k-", lw="2")

    plt.show()
```

In [43]:

```
from math import *
import matplotlib.pyplot as plt
%matplotlib inline

def f(x):
    return x

xi, xs, nn = 0., 10., 10.
toplam = simpson_integral(xi, xs, nn)
print "f(x)'in %s ile %s aralığındaki integrali = %s"%(xi, xs, toplam)
simpson_grafik(xi, xs, nn)
```

```
f(x)'in 0.0 ile 10.0 aralığındaki integrali = 50.0
```

Out[43]:

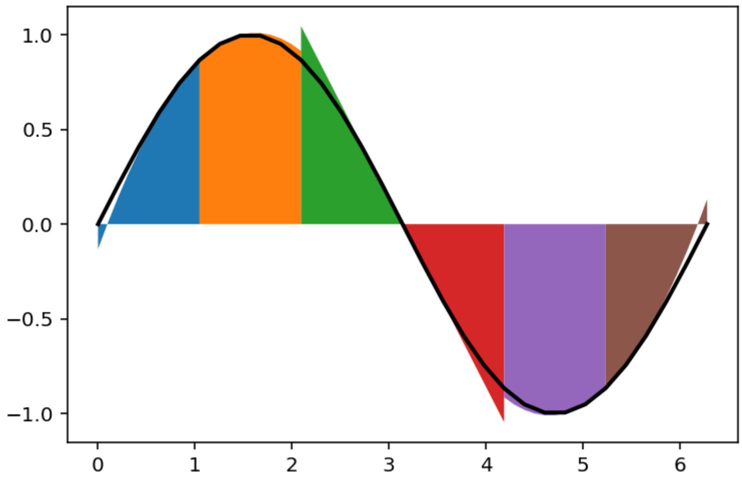


In [44]:

```
from math import *
import matplotlib.pyplot as plt
%matplotlib inline

def f(x):
    return sin(x)

xi, xs, nn = 0., 2*pi, 6.
toplam = simpson_integral(xi, xs, nn)
print "f(x)'in %s ile %s aralığındaki integrali = %s"%(xi, xs, toplam)
simpson_grafik(xi, xs, nn)
```

```
f(x)'in 0.0 ile 6.28318530718 aralığındaki integrali = -3.1802104181e-16
```

Out[44]:



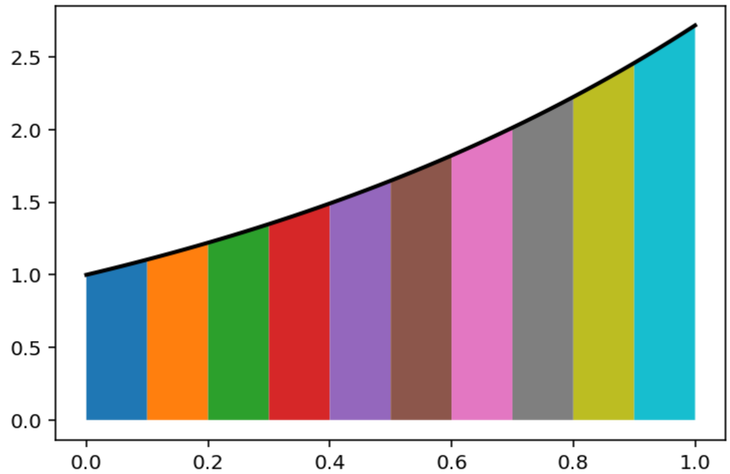
TRAPEZ ve SİMPSON Karşılaştırması

In [45]:

```
from math import *
import matplotlib.pyplot as plt
%matplotlib inline

def f(x):
    return exp(x)

xi, xs, nn = 0., 1., 10.
toplam = simpson_integral(xi, xs, nn)
print "f(x)'in %s ile %s aralığındaki integrali = %s"%(xi, xs, toplam)
simpson_grafik(xi, xs, nn)

int_simpson = toplam
```

```
f(x)'in 0.0 ile 1.0 aralığındaki integrali = 1.71828278192
```

Out[45]:

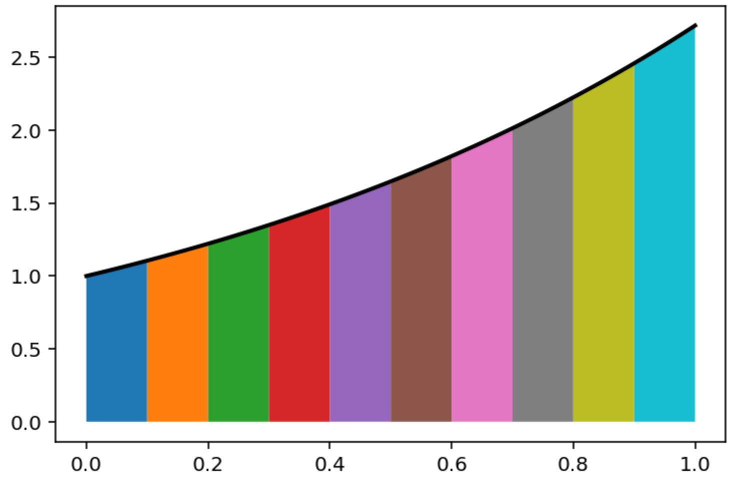


In [46]:

```
from math import *
import matplotlib.pyplot as plt
%matplotlib inline

def f(x):
    return exp(x)

xi, xs, nn = 0., 1., 10.
toplam = trapez_integral(xi, xs, nn)
print "f(x)'in %s ile %s aralığındaki integrali = %s"%(xi, xs, toplam)
trapez_grafik(xi, xs, nn)

int_trapez = toplam
```

```
    x  f(x)
f(x)'in 0.0 ile 1.0 aralığındaki integrali = 1.71971349139
```

Out[46]:



In [47]:

```
int_tam = exp(1.) - 1.
print "trapez        =", int_trapez
print "simpson       =", int_simpson
print "tam           =", int_tam
print "trapez - tam  =", int_trapez - int_tam
print "simpson - tam =", int_simpson - int_tam
```

```
trapez        = 1.71971349139
simpson       = 1.71828278192
tam           = 1.71828182846
trapez - tam  = 0.00143166293027
simpson - tam = 9.53465778775e-07
```

TEKİL İNTEGRALLER

$\int_0^{\infty} \frac{e^{-x}}{(1+e^{x})} dx \approx \int_0^{20} \frac{e^{-x}}{(1+e^{x})} dx$

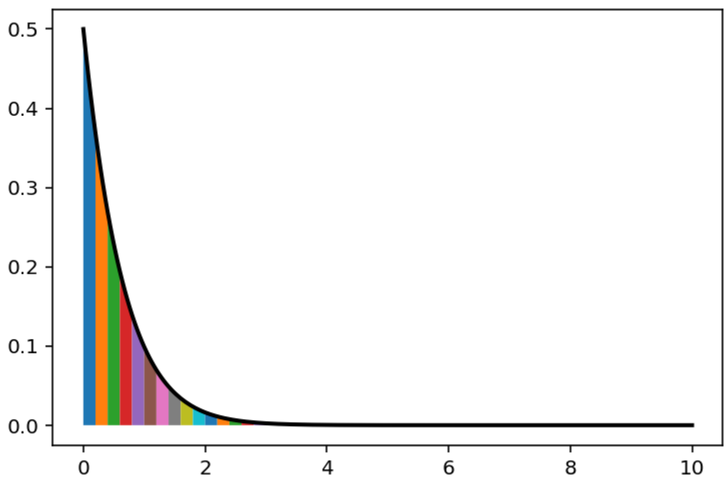
İntegrand (integralin içi) çok hızlı sıfıra yaklaşıtığı için sonsuz yerine uygun bir üst sınır alabiliriz.

In [48]:

```
from math import *

def f(x):
    return exp(-x)/(1+exp(x))

print simpson_integral(xi=0., xs=10., nn=50.)
simpson_grafik(xi=0., xs=10., nn=50.)
```

0.306862787035

Out[48]:



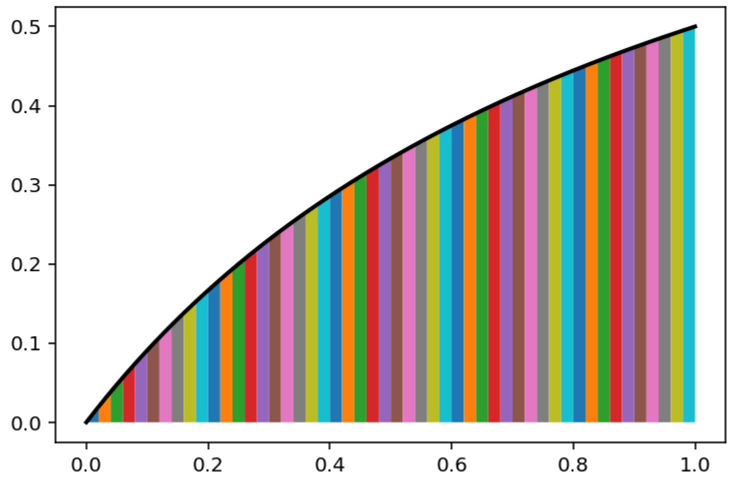
$u = e^{-x}$ değişken dönüşümü yapılırsa $\int_0^{1} \frac{u}{(u+1)} dx$

In [49]:

```
from math import *

def f(u):
    return u/(u+1.)

print simpson_integral(xi=0., xs=1., nn=50.)
simpson_grafik(xi=0., xs=1., nn=50.)
```

```
0.306852814445
```

Out[49]:



In [50]:

```
import numpy as np

ul = np.linspace(0,1, 10)

print "f(u) fonksiyonunun davranışı"
for u in ul:
    print "%.5f %5f"%(u, f(u))
```

```
f(u) fonksiyonunun davranışı
0.00000 0.000000
0.11111 0.100000
0.22222 0.181818
0.33333 0.250000
0.44444 0.307692
0.55556 0.357143
0.66667 0.400000
0.77778 0.437500
0.88889 0.470588
1.00000 0.500000
```

$$\int_0^1 \frac{\cos(x)}{\sqrt{x}} dx \approx \int_\varepsilon^1 \frac{\cos(x)}{\sqrt{x}} dx; \varepsilon \to 0$$

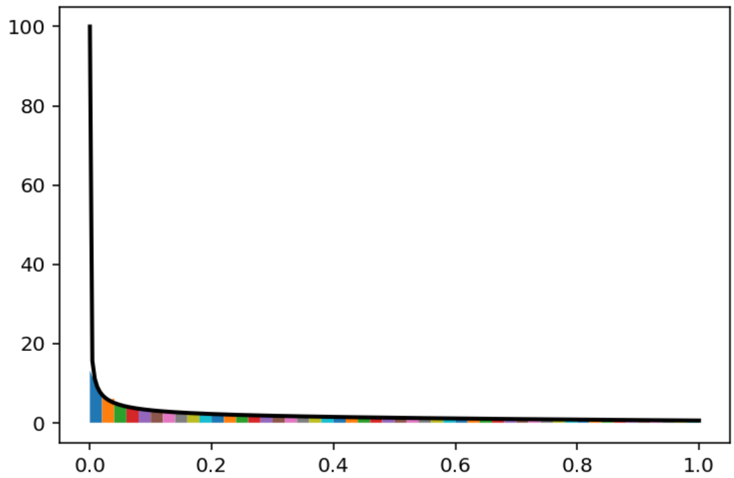
ε'u sıfıra yakın bir sayı seçerek tekillikten kurtulmaya çalışabiliriz.
Fakat bu durumda sayısal hesap güvenilmez hale gelebilir. Sadece ε'u küçültmek yeterli olmayabilir. Bu Δx'i de (integral alma adım miktarı) küçültmek gerekebilir.

In [51]:

```
from math import *

def f(x):
    return cos(x)/sqrt(x)

print simpson_integral(xi=0.0001, xs=1., nn=50.)
simpson_grafik(xi=0.0001, xs=1., nn=50.)
```

```
2.29663518785
```

Out[51]:



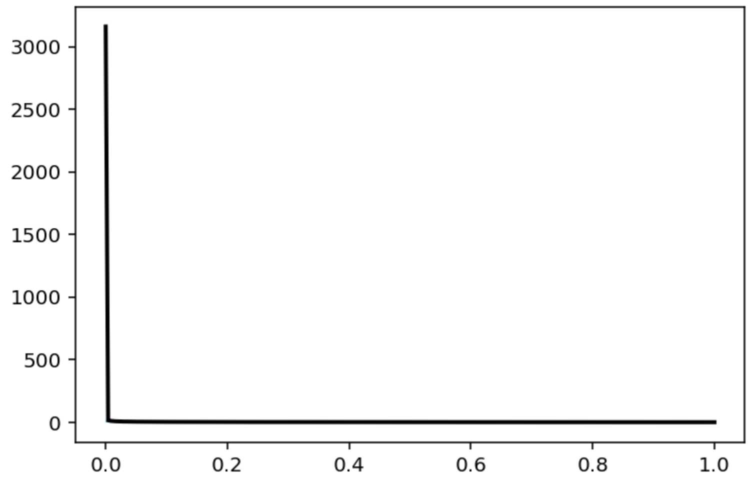
Hem ε hem de Δx çok küçük hale getirilirse...

In [52]:

```
from math import *

def f(x):
    return cos(x)/sqrt(x)

print simpson_integral(xi=0.0000001, xs=1., nn=500000.)
simpson_grafik(xi=0.0000001, xs=1., nn=50.)
```

```
1.80927726651
```

Out[52]:



$x = u^2$ dönüşümü ile $\int_0^1 \frac{\cos(x)}{\sqrt{x}} dx = \int_0^1 2\cos(u^2) du$ olur.

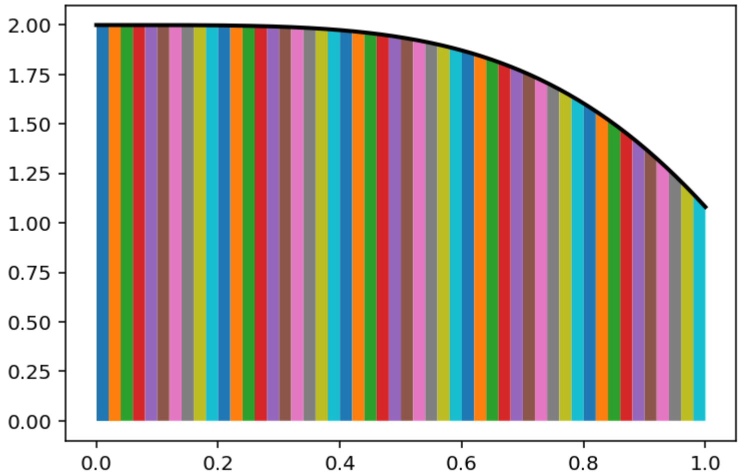
Bu şekilde tekillik ortadan kalkmış olur.

In [53]:

```
from math import *

def f(u):
    return 2*cos(u**2)

print simpson_integral(xi=0., xs=1., nn=50.)
simpson_grafik(xi=0., xs=1., nn=50.)
```

1.80904847623

Out[53]:



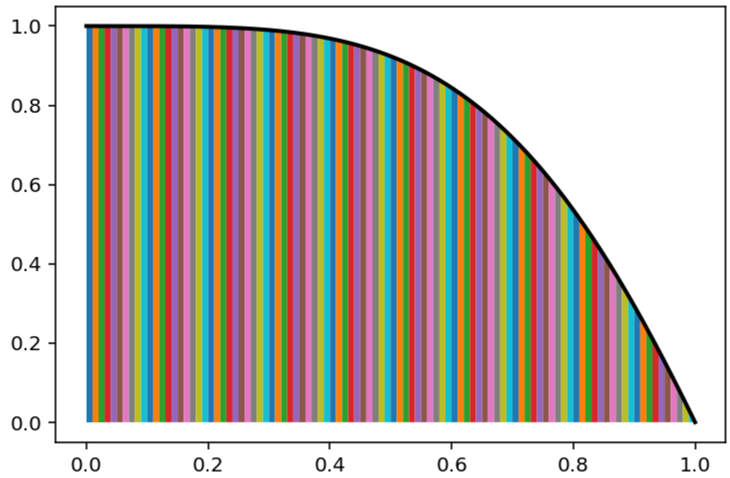
Fresnel İntegralleri

In [54]:

```
from math import *
import matplotlib.pyplot as plt
%matplotlib inline

def f(d):
    return cos(pi*d**2./2.)

xi, xs, nn = 0., 1., 100.
Cd = simpson_integral(xi, xs, nn)
print "f(x)'in %s ile %s aralığındaki integrali = %s"%(xi, xs, Cd)
simpson_grafik(xi, xs, nn)
```

```
f(x)'in 0.0 ile 1.0 aralığındaki integrali = 0.779893402099
```

Out[54]:

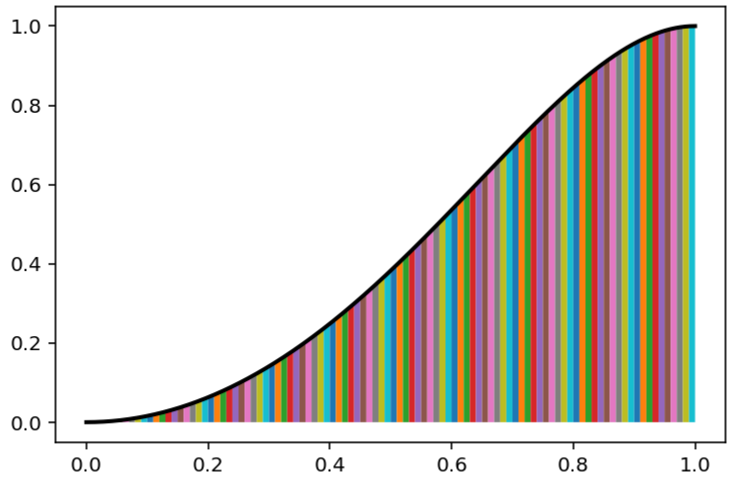


In [55]:

```
from math import *
import matplotlib.pyplot as plt
%matplotlib inline

def f(d):
    return sin(pi*d**2./2.)

xi, xs, nn = 0., 1., 100.
Sd = simpson_integral(xi, xs, nn)
print "f(x)'in %s ile %s aralığındaki integrali = %s"%(xi, xs, Sd)
simpson_grafik(xi, xs, nn)
```

f(x)'in 0.0 ile 1.0 aralığındaki integrali = 0.438259145745

Out[55]:



In [56]:

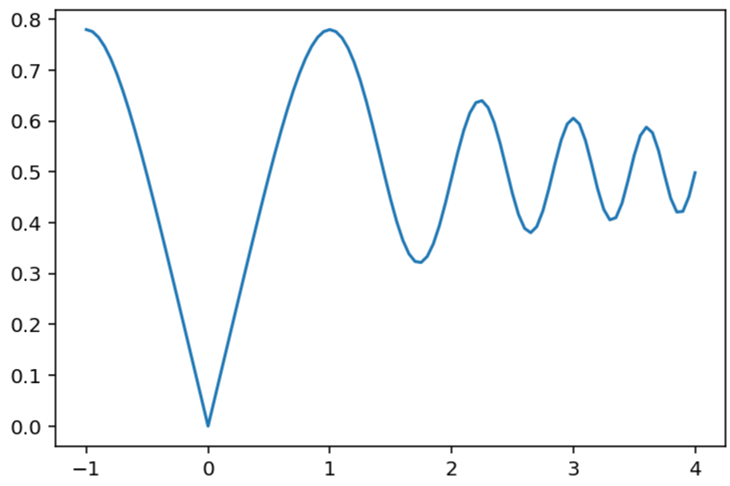
```
print ((Cd+.5)**2. + (Sd+.5)**2.)/2.
```

1.25922867266

In [57]:

```
from math import *
import matplotlib.pyplot as plt
%matplotlib inline

def f(d):
    return cos(pi*d**2./2.)

xi, nn = 0., 100.
di = -1.
ds = 4.
dd = abs(ds-di)/nn

d = []
Cd = []
for i in range(int(nn)+1):
    d += [di + i*dd]
    Cd += [simpson_integral(xi, d[i], nn)]

plt.plot(d, Cd)
plt.show()
```

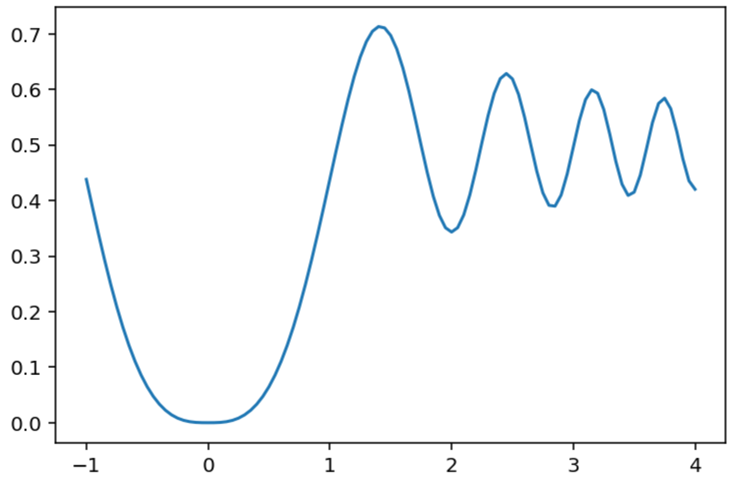
Out[57]:



In [58]:

```
from math import *
import matplotlib.pyplot as plt
%matplotlib inline

def f(d):
    return sin(pi*d**2./2.)

xi, nn = 0., 100.
di = -1.
ds = 4.
dd = abs(ds-di)/nn

d = []
Sd = []
for i in range(int(nn)+1):
    d += [di + i*dd]
    Sd += [simpson_integral(xi, d[i], nn)]

plt.plot(d, Sd)
plt.show()
```

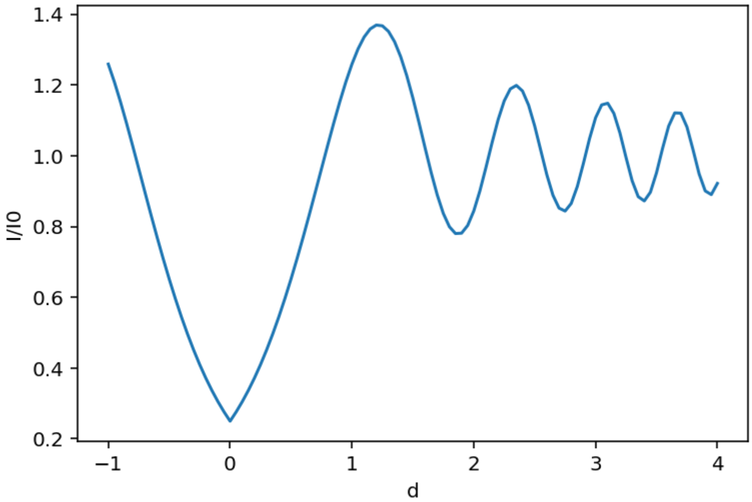
Out[58]:



In [59]:

```
Id = []
for i in range(int(nn)+1):
    Id += [((Cd[i] + 0.5)**2. + (Sd[i] + 0.5)**2.)/2.]

plt.plot(d, Id)
plt.xlabel('d')
plt.ylabel('I/I0')
plt.show()
```

Out[59]:



In [0]:

```

```