Polynomial Functors in Lean4

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Chapter 1

Locally Cartesian Closed Categories

To be written

Chapter 2

Univaiate Polynomial Functors

In this section we develop some of the definitions and lemmas related to polynomial endofunctors that we will use in the rest of the notes.

Definition 2.0.1 (Polynomial endofunctor)**.** Let $\mathbb{C}$ be a locally Cartesian closed category (in our case, presheaves on the category of contexts). This means for each morphism $t : B \to A$ we have an adjoint triple

$$\begin{array}{ccc} & \mathbb{C}/B & \\ & \uparrow & \\ t_! \swarrow & \dashv \; t^* \; \dashv & \searrow t_* \\ & \downarrow & \\ & \mathbb{C}/A & \end{array}$$

where t^* is pullback, and $t_!$ is composition with t.

Let $t : B \to A$ be a morphism in $\mathbb{C}$. Then define $P_t : \mathbb{C} \to \mathbb{C}$ be the composition

$$P_t := A_! \circ t_* \circ B^*$$

$$\mathbb{C} \xrightarrow{B^*} \mathbb{C}/B \xrightarrow{t_*} \mathbb{C}/A \xrightarrow{A_!} \mathbb{C}$$

Chapter 3

Multivariate Polynomial Functors

To be written