

Simplicity and uniqueness of trace of group C^* -algebras

Bachelor thesis defense

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However, answering this question be quite hard, especially when G is non-discrete!

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