Question 1

(a) $P(B|+j,+m)$ - variable elimination

- $P(+b|+j,+m)$
- $P(-b|+j,+m)$

① $P(+b|+j,+m) = \alpha P(+b,+j,+m) =$

$= \alpha P(+b) \sum_E P(E) \sum_A P(A|+b,E) P(+j|A) P(+m|A) =$

$= \alpha P(+b) \sum_E P(E) \left[P(+a|+b,E) P(+j|+a) P(+m|+a) + P(-a|+b,E) P(+j|-a) P(+m|-a) \right] =$

$= \alpha P(+b) \Big[P(+e) \left[P(+a|+b,+e) P(+j|+a) P(+m|+a) + P(-a|+b,+e) P(+j|-a) P(+m|-a) \right]$
$+ P(-e) \left[P(+a|+b,-e) P(+j|+a) P(+m|+a) + P(-a|+b,-e) P(+j|-a) P(+m|-a) \right] \Big]$

$= \alpha \cdot 0.001 \Big[0.002 \big[\overset{mult}{0.95 \cdot 0.9 \cdot 0.7} + \overset{mult}{0.05 \cdot 0.05 \cdot 0.01} \big] +$ → because one entry of $f_1(J,M,A|b,E)$
$+ 0.998 \big[\overset{mult}{0.94 \cdot 0.9 \cdot 0.7} + \overset{mult}{0.06 \cdot 0.05 \cdot 0.01} \big] \Big] =$

$= \alpha \cdot 0.001 \left[0.002 (0.5985 \overset{sum}{+} 0.000025) + 0.998 (0.5922 \overset{sum}{+} 0.00003) \right] =$

$= \alpha \cdot 0.001 \left[\overset{mult}{0.002 \cdot 0.598525} + \overset{mult}{0.998 \cdot 0.59223} \right] =$

$= \alpha \cdot 0.001 (0.00119705 \overset{sum}{+} 0.59104554) = \alpha \cdot \overset{mult}{0.001 \cdot 0.59224259} =$

$= \alpha \cdot 0.000592242$ ⇒ 7 mult and 3 sum

② $P(-b|+j,+m) = \alpha P(+b,+j,+m) =$

$= \alpha P(-b) \sum_E P(E) \sum_A P(A|-b,E) P(+j|A) P(+m|A) =$

$= \alpha P(-b) \sum_E P(E) \left[P(+a|-b,E) P(+j|+a) P(+m|+a) + P(-a|-b,E) P(+j|-a) P(+m|-a) \right] =$

$= \alpha P(-b) \Big[P(+e) \left[P(+a|-b,+e) P(+j|+a) P(+m|+a) + P(-a|-b,+e) P(+j|-a) P(+m|-a) \right]$
$+ P(-e) \left[P(+a|-b,-e) P(+j|+a) P(+m|+a) + P(-a|-b,-e) P(+j|-a) P(+m|-a) \right] \Big] =$

$= \alpha \cdot 0.999 \cdot \Big[0.002 \big[\overset{mult}{0.29 \cdot 0.9 \cdot 0.7} + \overset{mult}{0.71 \cdot 0.05 \cdot 0.01} \big] +$
$+ 0.998 \big[\overset{mult}{0.001 \cdot 0.9 \cdot 0.7} + \overset{mult}{0.999 \cdot 0.05 \cdot 0.01} \big] \Big] =$

$= \alpha \cdot 0.999 \left[0.002 (0.1827 \overset{sum}{+} 0.000355) + 0.998 (0.00063 \overset{sum}{+} 0.0004995) \right] =$

$= \alpha \cdot 0.999 \left[\overset{mult}{0.002 \cdot 0.183055} + \overset{mult}{0.998 \cdot 0.0011295} \right] =$

$= \alpha \cdot 0.999 (0.00036611 \overset{sum}{+} 0.001127241) = \alpha \cdot \overset{mult}{0.999 \cdot 0.001493351} =$

$= \alpha \cdot 0.001491857$ → 7 mult and 3 sum

③ $\alpha = \dfrac{1}{0.000592242 \overset{sum}{+} 0.001491857} \overset{div}{=} 479.8236552$

$P(+b|+j,+m) = 479.8236552 \overset{mult}{\cdot} 0.000592242 = 0.284171721$

$P(-b|+j,+m) = 479.8236552 \overset{mult}{\cdot} 0.001491857 = 0.715828278$

④ 7 mult + 7 mult + mult + mult = 16 multiplications

3 sum + 3 sum + sum = 7 summations

1 division

(b) $P(b \mid +j, +m)$ - enumeration

- $P(+b \mid +j, +m)$
- $P(-b \mid +j, +m)$

(1) $P(+b \mid +j, +m) = \alpha P(+b, +j, +m) =$

$= \alpha P(+b) \sum_E \sum_A P(E) P(A \mid +b, E) P(+j \mid A) P(+m \mid A) =$

$= \alpha P(+b) \sum_E \sum_A P(E) P(A, +m, +j \mid +b, E) =$

$= \alpha P(+b) \sum_E \sum_A P(+j, +m, A, E \mid +b) =$

$= \alpha P(+b) \sum_E [P(+j, +m, +a, E \mid +b) + P(+j, +m, -a, E \mid +b)] =$

$= \alpha P(+b) [P(+j, +m, +a, +e \mid +b) + P(+j, +m, +a, -e \mid +b) +$
$+ P(+j, +m, -a, +e \mid +b) + P(+j, +m, -a, -e \mid +b)] =$

$= \alpha P(+b) [P(+j \mid +a) P(+m \mid +a) P(+a \mid +b, +e) \cdot P(+e) +$
$+ P(+j \mid +a) P(+m \mid +a) P(+a \mid +b, -e) \cdot P(-e) +$
$+ P(+j \mid -a) P(+m \mid -a) P(-a \mid +b, +e) \cdot P(+e) +$
$+ P(+j \mid -a) P(+m \mid -a) P(-a \mid +b, -e) \cdot P(-e)] =$

$= \alpha \cdot 0.001 [\overbrace{0.9 \cdot 0.7 \cdot 0.95 \cdot 0.002}^{\text{mult mult}} + \overbrace{0.9 \cdot 0.7 \cdot 0.94 \cdot 0.998}^{\text{mult mult}} +$
$+ \overbrace{0.05 \cdot 0.01 \cdot 0.05 \cdot 0.002}^{\text{mult mult}} + \overbrace{0.05 \cdot 0.01 \cdot 0.06 \cdot 0.998}^{\text{mult mult}}] =$

$= \alpha \cdot 0.001 \cdot [0.001197 \overset{\text{sum}}{+} 0.5910156 \overset{\text{sum}}{+} 0.00000005 \overset{\text{sum}}{+} 0.00002994] =$ 9 mult 3 sum

$= \alpha \cdot 0.001 \overset{\text{mult}}{\cdot} 0.59224259 = \alpha \cdot 0.000592242$ ← same as in (a)

(2) $P(-b \mid +j, +m) = \alpha P(-b, +j, +m) = \alpha P(-b) \sum_E \sum_A P(E) P(A \mid -b, E) P(+j \mid A) P(+m \mid A) = \alpha P(-b) \sum_E \sum_A P(E) P(A, +m, +j \mid -b, E) =$

$= \alpha P(-b) \sum_E \sum_A P(A, +j, +m, E \mid -b) =$

$= \alpha P(-b) \sum_E [P(+a, +j, +m, E \mid -b) + P(-a, +j, +m, E \mid -b)] =$

$= \alpha P(-b) \sum_E [P(+a, +j, +m, +e \mid -b) + P(-a, +j, +m, +e \mid -b] +$

$= \alpha P(-b) [\cancel{P(+e)} [P(+a, +j, +m, -e \mid -b) + P(-a, +j, +m, -e \mid -b)] =$
$+ \cancel{P(-e)} [P(+a, +j, +m, \ldots$

$= \alpha P(-b) [P(+j \mid +a) P(+m \mid +a) P(+a \mid -b, +e) \cdot P(+e) +$
$+ P(+j \mid -a) P(+m \mid -a) P(-a \mid -b, +e) \cdot P(+e) +$
$+ P(+j \mid +a) P(+m \mid +a) P(+a \mid -b, -e) \cdot P(-e) +$
$+ P(+j \mid -a) P(+m \mid -a) P(-a \mid -b, -e) \cdot P(-e)] =$

$= \alpha \cdot 0.999 [\overbrace{0.9 \cdot 0.7 \cdot 0.29 \cdot 0.002}^{\text{mult mult}} + \overbrace{0.05 \cdot 0.01 \cdot 0.71 \cdot 0.002}^{\text{mult}} +$ 9 mult
$+ \overbrace{0.9 \cdot 0.7 \cdot 0.001 \cdot 0.998}^{\text{mult mult}} + \overbrace{0.05 \cdot 0.01 \cdot 0.999 \cdot 0.998}^{\text{mult mult}}] =$ 3 sum

$= \alpha \cdot 0.999 [0.0003654 \overset{\text{sum}}{+} 0.00000071 \overset{\text{sum}}{+} 0.00062874 \overset{\text{sum}}{+} 0.000498501] =$

$= \alpha \cdot 0.999 \overset{\text{mult}}{\cdot} 0.001493351 = \alpha \cdot 0.001491857$ ← same as in (a)

(3) From (a), $\alpha = 479.8236552$, $P(+b \mid +j, +m) = 0.284171721$
$P(-b \mid +j, +m) = 0.715828278$

(4) 9 mult + 9 mult + $\underbrace{\text{mult} + \text{mult}}_{\text{FROM (a) (4)}}$ =

= **20 multiplications**

3 sum + 3 sum + $\underbrace{\text{sum}}_{\text{FROM (a) (4)}}$ =

= **7 summations**

1 division FROM (a) (4)

Question 2

Eliminate D:

(a) $f_2(A, +c, E, F) = \sum_{d} P(E|d) P(F|d) f_1(A, +c, d)$

Factors:

$P(A)$ $P(+c)$ $P(G|+c, F)$ $f_2(A, +c, E, F)$

Eliminate G:

$f_3(+c, F) = \sum_{g} P(g|+c, F)$

Factors:

$P(A)$ $P(+c)$ $f_2(A, +c, E, F)$ $f_3(+c, F)$

Eliminate F:

$f_4(A, +c, E) = \sum_{f} f_2(A, +c, E, f) f_3(+c, f)$

Factors:

$P(A)$ $P(+c)$ $f_4(A, +c, E)$

(b) $P(A, E|+c) \propto P(A) P(+c) f_4(A, +c, E)$ with renormalization

(c) $f_2(A, +c, E, F)$ is the largest factor. A, E, F are not instantiated, thus the table would have $2^3 = 8$ rows.

(d)

Variable Eliminated	Factor Generated
B	$f_1(A, +c, D)$
F	$f_2(G, +c, D)$
G	$f_3(+c, D)$
D	$f_4(E, A, +c)$

Question 3

(a) I. $\boxed{5/8}$

II. $2/3$

Given samples:

I
+a +b −c −d
+a −b +c −d
−a +b +c −d
−a −b +c −d

+a −b ~~+c~~ −c +d
+a +b +c −d
−a +b −c +d
+a −b +c −d

II.
+a +b −c −d
+a −b +c −d
~~−a +b +c −d~~
~~−a −b +c −d~~

~~+a −b −c +d~~
+a +b +c −d
~~−a +b −c +d~~
~~−a −b +c −d~~

(b)

Sample	Weight
−a +b +c −d	$P(+b\mid -a)P(-d\mid +c) = 1/3 \cdot 5/6 = 5/18$
+a +b +c −d	$P(+b\mid +a)P(-d\mid +c) = 1/5 \cdot 5/6 = 5/30 = 1/6$
+a +b −c −d	$P(+b\mid +a)P(-d\mid -c) = 1/5 \cdot 1/8 = 1/40$
−a +b −c −d	$P(+b\mid -a)P(-d\mid -c) = 1/3 \cdot 1/8 = 1/24$

(c) $P(-a\mid +b,-d) = \dfrac{P(-a,+b,-d)}{P(+b,-d)} = \dfrac{5/18 + 1/24}{\frac{5}{18} + \frac{5}{30} + \frac{1}{40} + \frac{1}{24}} = \dfrac{\frac{23}{72}}{\frac{100+60+9+15}{360}} =$

$= \dfrac{\frac{23}{72}}{\frac{184}{360}} = \dfrac{23 \cdot 360}{72 \cdot 184} = \dfrac{115}{184} = \dfrac{5}{8}$

(d) $P(D\mid A)$ is better suited for likelihood weighting, because it is (the likelihood weighting) is determined by upstream evidence only.