

$$MSE = \text{bias}^2 + \text{var}$$

The MSE of an estimator, $\hat{\theta}$, $p(\theta)$ for θ
 $= MSE(\hat{\theta})$
 $= E(\hat{\theta} - \theta)^2 | \theta$

$$\text{bias}(\hat{\theta}) = E(\hat{\theta}) - \theta$$

$$E(\hat{\theta} - \theta)^2$$

$$\text{let } \mu = E\hat{\theta}$$

$$\begin{aligned} E(\hat{\theta} - \theta)^2 &= E((\hat{\theta} - \mu) + (\mu - \theta))^2 \\ &= E((\hat{\theta} - \mu)^2 + 2(\hat{\theta} - \mu)(\mu - \theta) + (\mu - \theta)^2) \end{aligned}$$

$$= E(\hat{\theta} - \mu)^2 + E(\mu - \theta)^2$$

$$= \underbrace{E(\hat{\theta} - \mu)^2}_{\text{var}(\hat{\theta})} + (\mu - \theta)^2$$

$$\text{var}(\hat{\theta}) + \text{bias}^2$$

$$\therefore MSE(\hat{\theta}) = \text{bias}(\hat{\theta})^2 + \text{var}(\hat{\theta})$$