5 Vertical Alignment

5.1 Objectives

1. Understand basic philosophies in establishing a vertical alignment
2. Apply criteria for selection of grades
3. Design a vertical Curve

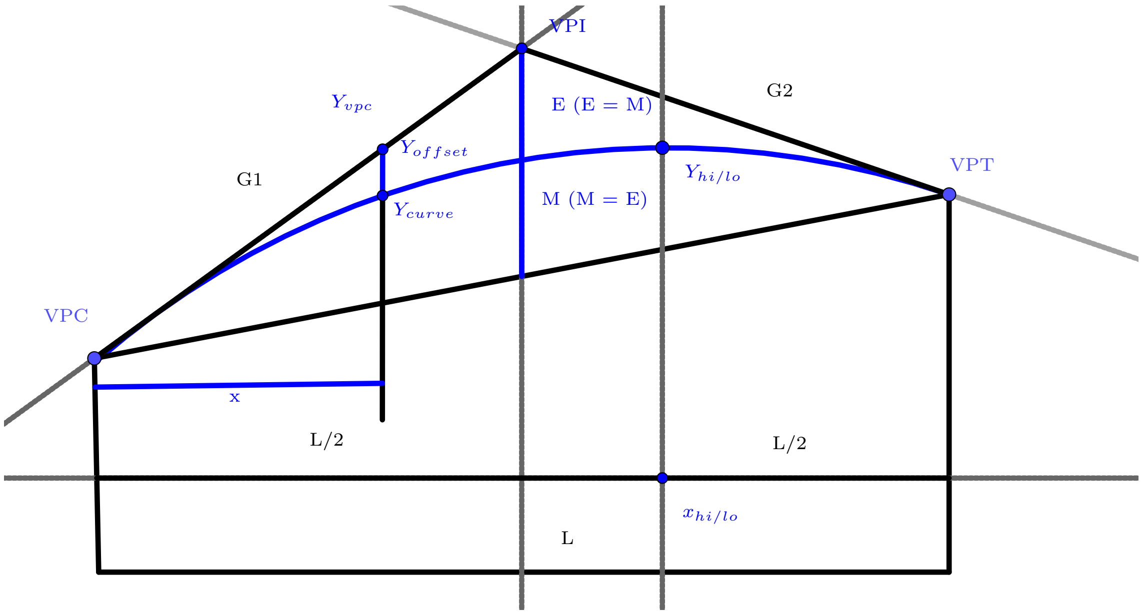
5.2 philosophies

1. conform to the existing terrain (within constraints of max grade and min lengths of vertical curves)
2. to minimize impacts (balance earthwork)
3. coordinate horizontal and vertical alignments (HC and VC)
   (a) avoid steep (near the max) grades and sharp (near min radius) horizontal curve
   (b) avoid placing the start of HC in the middle of VC; VC either at HC tangents or at HC vurves
   (c) avoid placing the start of HC at the bottom of a steep VC

5.3 maximum grades

1. Steepness and length heavily impacts heavy vehicles.
2. max grade design criteria is related with: design speed, the functional classification, and terrain
3. max grades: by AASHTO
   (a) freeway: 3-6%, +3.00% 70mph; max +4.00% for upgrade, max -5.00% for downgrade
   (b) arterials: +3.00% 60mph; up to +8.00% 40mph at mountainous
   (c) collectors: +4.00% 70mph; up to +14.00% 20 mph, mountainous
   (d) locals: up to +17.00% in mountainous terrain
4. min grades:
   (a) urban design(curb and gutter): an appropriate min grade is 0.5%, but grade of .30% ...
   (b) rural deisgn(shoulder and ditches): ... cross-slope is adaquate ...

5.4 Vertical Curve



If G_1 and G_2 are in slope, e.g. +0.02, x and L must in feet

$A = G_2 - G_1$

A: total change in grade, if negative, the curve is below the tangent

G_n: grade, like -0.08

$Y_{offset} = x^2 \dfrac{A}{2L} = x^2 \dfrac{G_2 - G_1}{2L}$

Y_{offset} : vertical offset from a tangent to a parabola, maybe negative!!!

$Y_{tan} = Y_{vpc} + G_1 x$

Y_{tan} : tangent elevation

$Y_{curve} = Y_{vpc} + G_1 x + \dfrac{A}{2L} x^2$

Y_{curve} : curve elevation

$Y_{tan} = Y_{offset} + Y_{curve}$

$x_{hi/lo} = L \dfrac{G_1}{G_1 - G_2}$

$x_{hi/lo}$: the highest/lowest point

5.5 stopping/passing sight distance on crest curves

$A = G_2 - G_1$

***SSD/PSD* minimum length of crest curve:**

$$L = \frac{|A|S^2}{200(\sqrt{h_1} + \sqrt{h_2})^2} \text{ when } S \leq L$$

$$L = 2S - \frac{200(\sqrt{h_1} + \sqrt{h_2})^2}{|A|} \text{ when } S \geq L$$

stopping: $h_1 = 3.5ft$, $h_2 = 2.0ft$

passing: $h_1 = 3.5ft$, $h_2 = 3.5ft$

***SSD* minimum length of crest curve:**

$$L_{SSD} = \frac{|A|S^2}{2158} \text{ when } S \leq L$$

$$L_{SSD} = 2S - \frac{2158}{|A|} \text{ when } S \geq L$$

A: in unit %, e.g. 3, the grade change of VC

L: the minimum length of the vertical curve ??? the arc length or the horizontal segment length???

S: stop sight distance, related with speed, reaction time, and coefficient of friction

h_1 : 3.5 feet, the driver eye height

h_2 : **2.0 feet, the object height**

5.6 sag sight distance

TODO diagram???

minimum length of sag curve

$$L = \frac{|A|S^2}{200(h + S \cdot tan\beta)} \text{ when } S \leq L$$

$$L = 2S - \frac{200(h + S \cdot tan\beta)}{|A|} \text{ when } S \geq L$$

$$L = \frac{|A|S^2}{400 + 3.5S} \text{ when } S \leq L$$

$$L = 2S - \frac{400 + 3.5S}{|A|} \text{ when } S \geq L$$

A: in unit %, e.g. 3, the grade change of VC

L: the horizontal length of sag curve

S: sag sight distance

h: 2 feet, the headlamp height

β : 1 degree, the headlamp beam angle

5.7 Vertical curve design - AASHTO elevation table - K^a factor

Another approach to determining curve length!

$L = K \cdot A$

L: in feet, curve length, minimum length for a given design speed

A: in unit %, change in grade, $A = G_2 - G_1$

K: rate of vertical curvature, K = required ft of curve length per 1% net change in grade

K: refter to K tables for a given design speed

K table for crest VC

K table for sag VC

Design Speed (mph)	**Stopping Sight Distance (ft)**	**Rate of Vertical Curvature, K^a**	
		Calculated	**Design**
15	80	3.0	3
20	115	6.1	7
25	155	11.1	12
30	200	18.5	19
35	250	29.0	29
40	305	43.1	44
45	360	60.1	61
50	425	83.7	84
55	495	113.5	114
60	570	150.6	151
65	645	192.8	193
70	730	246.9	247
75	820	311.6	312
80	910	383.7	384

Table 1: K Table for Crest Vertical Curve - U.S. Customary

Design Speed (mph)	**Stopping Sight Distance (ft)**	**Rate of Vertical Curvature, K^a**	
		Calculated	**Design**
15	80	9.4	10
20	115	16.5	17
25	155	25.5	26
30	200	36.4	37
35	250	49.0	49
40	305	63.4	64
45	360	78.1	79
50	425	95.7	96
55	495	114.9	115
60	570	135.7	136
65	645	156.5	157
70	730	180.3	181
75	820	205.6	206
80	910	231.0	231

Table 2: K Table for Sag Vertical Curve - U.S. Customary

5.8 K table for Crest VC, for a given design speed

5.9 K table for Sag VC, for a given design speed

5.10 elevation(height) in highway design

elevations are typically computed at the PVC, PVT, each 100-ft station, and the high or low point
procedure:

1. calc A - select vertical tangents
2. determine station and elevation of PVI
3. calculate the minimum length of the curve
   (a) - typicallly using SSD and K factor
   (b) - A vertical curve length can be selected first, and then checked to verify that K (based on L/A) $\geq K_{min}$
4. - Determine stations and elevations of PVC (BVC) and PVT (EVC)

$$
\begin{aligned}
Sta_{pvc} &= Sta_{pvi} - L/2 \\
Sta_{pvt} &= Sta_{pvi} + L/2 \\
Elev_{pvc} &= Elev_{pvi} - G_1 L/2 \\
Elev_{pvt} &= Elev_{pvi} + G_2 L/2
\end{aligned}
$$

G – grade in decimal, be sure to use correct sign on grade term

L – in ft, curve length

5. Compute elevations on initial tangent at full stations, typically to the nearest 0.01 ft

$Elev_x$ on initial tangent $= Elev_{pvc} + G_1\{(x)/100\}$

G – grade in percent; be sure to use correct sign on grade term

6. Compute the vertical distance y, **offset from tangent**, between initial tangent and curve

$$y = \frac{Ax^2}{200L}$$

y – in ft, offset from tangent

x – in ft, distance from PVC

7. Compute the elevation on the curve at each full station, or any station of interest!

   $Elev_x =$ Elev on initial tangent $+ offset$

   offset – positive for a sag curve, negative for a crest curve

8. Determine station and elevation of high or low point

$$Sta_{high/low} = Sta_{pvc} - \frac{G_1 \cdot L}{A}$$
$$Elev_{high/low} = Elev_{pvc} - \frac{G_1^2 \cdot L}{2A} \textbf{???}$$

5.11 Terms

gradient: slope rate grade: e.g. +4.00% a upward slope; -3.00% a downward slope

5.12 Rules

5.13 Formulas

5.14 Reference

5.15 In-class exam 1 - sag curve

A sag curve connects a downward 5% grade with an upward 2% grade. The length of the curve is 800 feet. The elevation of the beginning of the curve is 648.52 ft. The station of VPI is 31+00. Find the station and elevation of a low point of the curve.

$$
\begin{aligned}
G_1 &= -0.05 \\
G_2 &= +0.02 \\
L &= 800 \\
Y_{vpc} &= 648.52 \\
Sta_{vpi} &= 31+00 \\
x_{vpi} &= 3100
\end{aligned}
$$

Calculation

(1) station of low:

$$
\begin{aligned}
x_{low} &= L\frac{G_1}{G_1 - G_2} \\
&= 800 * \frac{-0.05}{-0.05 - 0.02} \\
&= 800 * 5/7 \\
&= 571.42 ft \\
Sta_x &= Sta_{vpi} - L/2 + x_{low} \\
&= 3100 - 800/2 + 571.42 \\
&= 3271.42 = 32 + 71.42
\end{aligned}
$$

(2) Tangent Elevation of low point:

$$
\begin{aligned}
Y_{tanlow} &= Y_{vpc} + G_1 x_{low} \\
&= 648.52 - 0.05 * 571.42 \\
&= 619.95 ft
\end{aligned}
$$

(2) curve elevation of low point:

$$
\begin{aligned}
Y_{low} &= Y_{vpc} + G_1 x_{low} + \frac{A}{2L} x_{low}^2 \\
&= 648.52 - 0.05 * 571.42 + 0.07/2/800 * 571.42^2 \\
&= 660.23 ft
\end{aligned}
$$

5.16 In-class exam 2 - low point example

A 600-ft equal-tangent sag vertical curve has the PVC at station 170+00 and elevation 1000 ft. The initial grade is -3.5% and the final grade is +0.5%. Determine the stationing and elevation of the PVI, the PVT, and the lowest point on the curve.

$$
\begin{aligned}
Given &: \\
L &= 600ft \\
Sta_{pvc} &= 170+00 \\
Ele_{pvc} &= 1000ft \\
G_1 &= -0.035 \\
G_2 &= +0.005
\end{aligned}
$$

(1) station and elevation of PVI:

TODO

(2) station and elevation of PVT:

TODO

(3) station and elevation of the lowest point:

$$
\begin{aligned}
x_{low} &= L\frac{G_1}{G_1 - G_2} \\
&= 600 * \frac{-0.035}{-0.035 - 0.005} \\
&= 525.00ft \\
Sta_{low} &= Sta_{pvc} + x_{low} \\
&= 17000 + 525.00 \\
&= 175 + 25.00 \\
Y_{tan} &= Y_{pvc} + G_1 x_{low} \\
&= 1000 - 0.035 * 525.00 \\
&= 981.63ft Y_{curve} & = Y_{tan} + \frac{A}{2L}x_{low}^2 \\
&= 981.63 + \frac{0.005 - -0.035}{2 * 600} 525.00^2 \\
&= 990.82ft
\end{aligned}
$$

5.17 In-class exam 3 - length example

A crest curve joins an upgrade of 4% with a downgrade of 3% on a multilane highway. The speed limit is 60 mph. What should be the minimum length of the vertical curve, making the most conservative assumptions.

5.18 Homework 5.1

A 500-meter equal-tangent sag vertical curve has the PVC at station 100+00 with an elevation of 1000 m. The initial grade is -4% and the final grade is +2%. Determine the stationing and elevation of the PVI, the PVT, and the lowest point on the curve.

5.19 Homework 5.2

A current roadway has a design speed of 100 km/hr, a coefficient of friction of 0.1, and carries drivers with perception-reaction times of 2.5 seconds. The drivers use cars that allows their eyes to be 1 meter above the road. Because of ample roadkill in the area, the road has been designed for carcasses that are 0.5 meters in height. All curves along that road have been designed accordingly.

The local government, seeing the potential of tourism in the area and the boost to the local economy, wants to increase the speed limit to 110 km/hr to attract summer drivers. Residents along the route claim that this is a horrible idea, as a particular curve called "Dead Man's Hill" would earn its name because of sight distance problems. "Dead Man's Hill" is a crest curve that is roughly 600 meters in length. It starts with a grade of +1.0% and ends with (-1.0)%. There has never been an accident on "Dead Man's Hill" as of yet, but residents truly believe one will come about in the near future.

A local politician who knows little to nothing about engineering (but thinks he does) states that the 600-meter length is a long distance and more than sufficient to handle the transition of eager big-city drivers. Still, the residents push back, saying that 600 meters is not nearly the distance required for the speed. The politician begins a lengthy campaign to "Bring Tourism to Town", saying that the residents are trying to stop "progress". As an engineer, determine if these residents are indeed making a valid point or if they are simply trying to stop progress?