THEORY AND FORMULAE SHEETS

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Here's a concise theory and formula sheet for **Rectangular (Cartesian), Cylindrical, and Spherical coordinate systems**, including their interconversion formulas

**Theory**

1. **Rectangular (Cartesian) Coordinates**
   - Represented as **(x, y, z)**.
   - Defines a point in a 3D space using three perpendicular axes:
     - **x**: Horizontal axis.
     - **y**: Vertical axis.
     - **z**: Depth axis.
2. **Cylindrical Coordinates**
   - Represented as **(r, θ, z)**.
   - A mix of 2D polar coordinates **(r, θ)** and the height **z**:
     - **r**: Radial distance from the origin to the point in the xy-plane.
     - **θ**: Angle with the positive x-axis (measured in radians).
     - **z**: Same as the Cartesian z-coordinate.
3. **Spherical Coordinates**
   - Represented as **(ρ, θ, ϕ)**.
   - Fully defines a point in 3D space with:
     - **ρ**: Radial distance from the origin to the point.
     - **θ**: Same angular coordinate as in cylindrical coordinates (angle in xy-plane).
     - **ϕ**: Polar angle (angle between the radial vector and the positive z-axis).

**Interconversion Formulas**

**1. Rectangular to Cylindrical**

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) \quad (\theta \in [0, 2\pi))$$

$$z = z$$

**2. Cylindrical to Rectangular**

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$z = z$$

**3. Rectangular to Spherical**

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\phi = \cos^{-1}\left(\frac{z}{\rho}\right)$$

**4. Spherical to Rectangular**

$$x = \rho \sin\phi \cos\theta$$

$$y = \rho \sin\phi \sin\theta$$

$$z = \rho \cos\phi$$

**5. Cylindrical to Spherical**

$$\rho = \sqrt{r^2 + z^2}$$

$$\phi = \tan^{-1}\left(\frac{r}{z}\right) \quad (z \neq 0)$$

$$\theta = \theta$$

**6. Spherical to Cylindrical**

$$r = \rho \sin\phi$$

$$z = \rho \cos\phi$$

$$\theta = \theta$$

## ELECTRIC FIELD

**Theory of Electric Field**

1. **Definition**
    - The **electric field** (E) at a point in space represents the force experienced per unit positive charge at that point.
    - It is a **vector quantity**, having both magnitude and direction

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

- $\mathbf{F}$: Electric force (N)
- $q$: Test charge (Coulombs, C)

**Source**

- Electric fields are produced by charges, either **point charges**, continuous charge distributions, or changing magnetic fields.

## Electric Field Formulas

### 1. Electric Field Due to a Point Charge

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

- $q$: Source charge (C)
- $r$: Distance from the charge to the point of interest (m)
- $\hat{r}$: Unit vector pointing away (for $q > 0$) or toward (for $q < 0$) the charge.
- $\epsilon_0$: Permittivity of free space, $8.85 \times 10^{-12}\ \mathrm{F/m}$.

## 2. Electric Field Due to a Continuous Charge Distribution

- Linear Charge Distribution ($\lambda$)

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda\, dl}{r^2} \hat{r}$$

- Surface Charge Distribution ($\sigma$)

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma\, dA}{r^2} \hat{r}$$

- Volume Charge Distribution ($\rho$)

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\rho\, dV}{r^2} \hat{r}$$

### 3. Electric Field on an Infinite Plane (Uniform Surface Charge Density)

$$\mathbf{E} = \frac{\sigma}{2\epsilon_0}$$

- $\sigma$: Surface charge density (C/m$^2$).

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### 4. Electric Field Due to a Dipole

- At an axial point:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3}$$

- At an equatorial point:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$$

- $p = q \cdot d$: Dipole moment, $r$: Distance from the dipole center.

**5. Relationship Between Electric Field and Potential**

The electric field is the negative gradient of the electric potential ($V$):

$$\mathbf{E} = -\nabla V$$

- In 1D:

$$E_x = -\frac{dV}{dx}$$

**Key Properties**

1. Electric field lines:
   - Emanate from positive charges and terminate on negative charges.
   - Do not intersect.
   - Are denser where the field is stronger.
2. In a conductor in electrostatic equilibrium:
   - Electric field inside = 0.
   - Electric field at the surface is perpendicular.

**1. Gradient**

**Theory**

- The **gradient** of a scalar field f(x,y,z)f(x, y, z)f(x,y,z) is a vector field that points in the direction of the greatest rate of increase of fff and whose magnitude represents the rate of change of fff in that direction.

**Formula**

$$\nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

- $\nabla f$: Gradient of $f$.
- $\hat{i}, \hat{j}, \hat{k}$: Unit vectors along $x, y, z$ directions.

**2. Divergence**

**Theory**

- The **divergence** of a vector field F(x,y,z)\mathbf{F}(x, y, z)F(x,y,z) measures the net rate of "outflow" of the field from a given point. It is a scalar.

**Formula**

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

- $\nabla \cdot \mathbf{F}$: Divergence of the vector field $\mathbf{F}$.
- $F_x, F_y, F_z$: Components of $\mathbf{F}$.

### 3. Curl

**Theory**

- The **curl** of a vector field $\mathbf{F}(x, y, z)$ measures the rotation or "circulation" of the field at a point. It is a vector.

**Formula**

$$\nabla \times \mathbf{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

Expanding the determinant:

$$\nabla \times \mathbf{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{i} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{j} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{k}$$

## Relationships

1. **Gradient-Divergence Identity**
   - The divergence of a gradient of any scalar field $f$ is the Laplacian:

$$\nabla \cdot (\nabla f) = \nabla^2 f$$

2. **Divergence-Curl Identity**
   - The divergence of a curl of any vector field $\mathbf{F}$ is always zero:

$$\nabla \cdot (\nabla \times \mathbf{F}) = 0$$

3. **Curl-Gradient Identity**
   - The curl of a gradient of any scalar field $f$ is always zero:

$$\nabla \times (\nabla f) = 0$$

**Theory of Electric Potential**

1. **Definition**
   - The **electric potential** VVV at a point in space is the work done per unit positive charge to move the charge from a reference point (often infinity) to that point in an electric field.
   - It is a **scalar quantity**.

$$V = \frac{W}{q}$$

- $W$: Work done (Joules, $J$).
- $q$: Charge (Coulombs, $C$).

2. **Relation to Electric Field**
   - The electric field $\mathbf{E}$ is the negative gradient of the electric potential $V$:

$$\mathbf{E} = -\nabla V$$

   - In one dimension:

$$E_x = -\frac{dV}{dx}$$

3. **Potential Energy**
   - The **electric potential energy** ($U$) of a charge $q$ at a point with electric potential $V$:

$$U = qV$$

## Electric Potential Formulas

### 1. Electric Potential Due to a Point Charge

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

- $q$: Source charge (C).
- $r$: Distance from the charge to the point (m).
- $\epsilon_0$: Permittivity of free space ($8.85 \times 10^{-12}\,\mathrm{F/m}$).

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### 2. Electric Potential Due to Multiple Charges (Superposition Principle)

$$V = \sum_{i=1}^{n} \frac{1}{4\pi\epsilon_0}\frac{q_i}{r_i}$$

- Sum the potentials from all charges at a point.

**Key Notes**

1. **Potential is Relative**
   - Electric potential is often measured relative to infinity, where V=0V = 0V=0.
2. **Equipotential Surfaces**
   - Surfaces where the electric potential is constant.
   - No work is required to move a charge along an equipotential surface.
   - Equipotential surfaces are perpendicular to electric field lines.

3. **Sign of Potential**
   - Positive for positive source charges.
   - Negative for negative source charges.
4. **Potential Gradient**
   - The steeper the potential gradient, the stronger the electric field

## 1. Infinite Cylinder

### (a) Non-Conducting Cylinder (Uniform Charge Distribution)

- **Charge Distribution**: Uniform volume charge density ($\rho$) within radius $R$.
- Using **Gauss's Law**, $\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{enc}}}{\epsilon_0}$:

Electric Field ($E$)

1. **Inside the cylinder** ($r < R$):

$$E = \frac{\rho r}{2\epsilon_0}$$

   - $q_{\text{enc}} = \rho \cdot \pi r^2 \cdot L$.

2. **Outside the cylinder** ($r \geq R$):

$$E = \frac{\rho R^2}{2\epsilon_0 r}$$

   - $q_{\text{enc}} = \rho \cdot \pi R^2 \cdot L$.

Electric Potential ($V$)

1. **Outside the cylinder** ($r \geq R$):

$$V = \frac{\rho R^2}{2\epsilon_0} \ln\left(\frac{R}{r}\right) + V_0$$

   - $V_0$: Reference potential, often set to 0 at $r \to \infty$.

2. **Inside the cylinder** ($r < R$):

$$V = \frac{\rho}{4\epsilon_0}\left(R^2 - r^2\right) + V_{\text{surface}}$$

**(b) Conducting Cylinder**

- All charge resides on the **surface** (uniform surface charge density $\sigma$).

Electric Field ($E$)

1. **Inside the cylinder** ($r < R$):

$$E = 0$$

   - Electric field inside a conductor is zero in electrostatic equilibrium.

2. **Outside the cylinder** ($r \geq R$):

$$E = \frac{\sigma}{\epsilon_0 r}$$

   - Equivalent to the field of a line charge.

Electric Potential ($V$)

1. **Outside the cylinder** ($r \geq R$):

$$V = \frac{\sigma}{\epsilon_0} \ln\left(\frac{R}{r}\right) + V_0$$

2. **Inside the cylinder** ($r < R$):

$$V = \text{constant}$$

   - The potential is constant everywhere inside a conductor.

## 2. Sphere

### (a) Non-Conducting Sphere (Uniform Charge Distribution)

- **Charge Distribution**: Uniform volume charge density ($\rho$) within radius $R$.
- Using **Gauss's Law**:

Electric Field ($E$)

1. **Inside the sphere** ($r < R$):

$$E = \frac{\rho r}{3\epsilon_0}$$

   - $q_{\text{enc}} = \frac{4}{3}\pi r^3 \rho$.

2. **Outside the sphere** ($r \geq R$):

$$E = \frac{\rho R^3}{3\epsilon_0 r^2}$$

Electric Potential ($V$)

1. **Outside the sphere** ($r \geq R$):

$$V = \frac{\rho R^3}{3\epsilon_0 r} + V_0$$

2. **Inside the sphere** ($r < R$):

$$V = \frac{\rho}{6\epsilon_0}\left(3R^2 - r^2\right) + V_{\text{surface}}$$

**(b) Conducting Sphere**

- All charge resides on the **surface** (uniform surface charge density $\sigma$).

**Electric Field ($E$)**

1. **Inside the sphere** ($r < R$):

$$E = 0$$

2. **Outside the sphere** ($r \geq R$):

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

**Electric Potential ($V$)**

1. **Outside the sphere** ($r \geq R$):

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

2. **Inside the sphere** ($r < R$):

### 3. Electric Potential Due to a Continuous Charge Distribution

- **Linear Charge Distribution ($\lambda$)**

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda\, dl}{r}$$

- **Surface Charge Distribution ($\sigma$)**

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma\, dA}{r}$$

- **Volume Charge Distribution ($\rho$)**

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{\rho\, dV}{r}$$