

HOMEWORK-3

1)  $f(x) = 5(x+47)^2$

$$f'(x) = \frac{df}{dx} = 5 \times 2 \times (x+47) \times 1 = 10(x+47)$$

$$f'(x) = 10x + 470$$

2)  $f(x) = 3x^3 + 15x^2$

$$f'(x) = 9x^2 + 30x$$

Solving for  $f'(x) = 0$  to get critical points

$$9x^2 + 30x = 0$$

$$x(9x + 30) = 0$$

$$3x(3x + 10) = 0$$

$$x(3x + 10) = \frac{0}{3} = 0$$

$x = 0$  or  $x = -\frac{10}{3}$  are the critical points

$$f''(x) = 18x + 30$$

$$f''(0) = 18(0) + 30 = 30 > 0$$

$x = 0$  is the local minimum

$$f''\left(-\frac{10}{3}\right) = 18 \times -\frac{10}{3} + 30 = -60 + 30 = -30$$

$$f''\left(-\frac{10}{3}\right) < 0$$

$x = -\frac{10}{3}$  is the local maximum

∴ Maximum value for  $f(x) =$

$$f\left(-\frac{10}{3}\right) = \text{scribbled out}$$

$$f\left(-\frac{10}{3}\right) = 3 \times \left(-\frac{10}{3}\right)^3 + 15 \times \left(-\frac{10}{3}\right)^2$$

$$= 3 \times \frac{-1000}{27} + 15 \times \frac{100}{9}$$

$$= \frac{-1000}{9} + \frac{1500}{9}$$

$$\text{Maximum value} = \frac{500}{9}$$

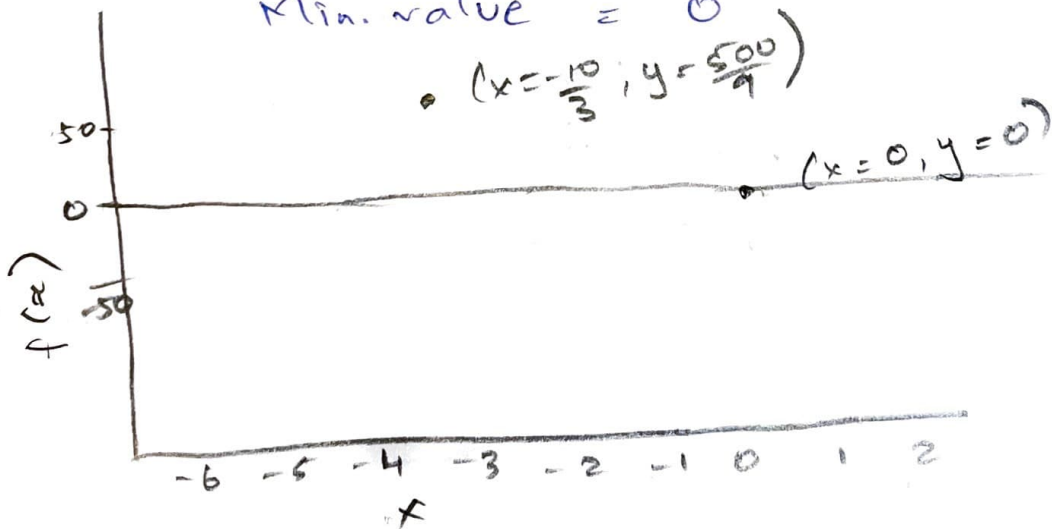
Minimum value for  $f(x) = f(0)$

$$= 3 \times 0^3 + 15 \times 0^2$$

$$\text{Min. value} = 0$$

$$\bullet \left(x = -\frac{10}{3}, y = \frac{500}{9}\right)$$

$$\bullet (x = 0, y = 0)$$



$$3) f(x, y) = 3x + 4y$$

$$\frac{\partial f}{\partial x} = 3 \quad (y \text{ is considered a constant})$$

$$\frac{\partial f}{\partial y} = 4 \quad (x \text{ is considered a constant})$$

$$4) f(x, y) = xy^3 + x^2y^2$$

$$\frac{\partial f}{\partial x} = y^3 + 2xy^2$$

$$\frac{\partial f}{\partial y} = 3xy^2 + 2x^2y$$

$$5) f(x, y) = x^3y + e^x$$

$$\frac{\partial f}{\partial x} = 3x^2y + e^x$$

$$\frac{\partial f}{\partial y} = x^3$$

$$6) f(x, y) = xe^{2x+3y}$$

$$\frac{\partial f}{\partial x} = e^{2x+3y} + xe^{2x+3y} \times 2$$

$$= e^{2x+3y} [1 + 2x]$$

$$\frac{\partial f}{\partial y} = x \times e^{2x+3y} \times 3 = 3xe^{2x+3y}$$

$$7) J(\omega_0, \omega_1) = \frac{1}{2m} \sum_{i=1}^m (\omega_0 + \omega_1 x_i - y_i)^2$$

$$\frac{\partial J}{\partial \omega_0} = \frac{1}{2m} \sum_{i=1}^m 2x (\omega_0 + \omega_1 x_i - y_i)$$

$$\frac{\partial J}{\partial \omega_0} = \frac{1}{m} \sum_{i=1}^m \omega_0 + \omega_1 x_i - y_i$$

$$\frac{\partial J}{\partial \omega_1} = \frac{1}{2m} \sum_{i=1}^m 2(\omega_0 + \omega_1 x_i - y_i) x_i$$

$$= \frac{1}{m} \sum_{i=1}^m (\omega_0 + \omega_1 x_i - y_i) x_i$$

$$8) f(x) = \frac{1}{1+e^{-x}}$$

$$\frac{df}{dx} = \frac{-1}{(1+e^{-x})^2} \times (-e^{-x})$$

$$\frac{df}{dx} = \frac{e^{-x}}{(1+e^{-x})^2}$$