Mechanisms for a No-Regret Agent: Beyond the Common Prior

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Introduction

Motivation

Policy success or failure often influenced by...

- Environmental details (e.g. consumer demand, labor supply)
- Individuals' beliefs about environment (e.g. inflation expectations)

Inherently dynamic

- Economic conditions evolve in unpredictable ways
- Individuals learn and adapt over time

Ideal policies would adapt to environment over time

- E.g. minimum wage adapted to current, local market conditions

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High Level Questions

Can we develop dynamic policies that adapt to their environment over time?

Without making assumptions on how the environment evolves?

- I.e. prior-free or adversarial

With permissive assumptions on agent behavior?

- Ex-ante optimal: Bayesian agents *want to* satisfy assumptions
- Ex-post feasible: non-Bayesian algorithm guaranteed to satisfy assumptions

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Framework

Repeated interaction between policymaker and single agent

Hidden state of nature observed after each interaction

Minimum viable case?

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Contribution

Standard behavioral assumptions are insufficient, allow for odd behaviors

Refine existing assumptions to counterfactual calibration

- Agent fully and consistently exploits any private information

Propose calibrated policy that adapts over time using historical data

Conditions under policymaker's regret is bounded relative to best static policy

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Related Literature

Robust dynamic mechanisms, e.g. Chassang 2013, Penta 2015, Madarász and Prat 2016, Mirrokni, Paes Leme, Tang, and Zuo 2020, Carroll 2020, ...

Data-driven auction design, e.g. Blum and Hartline 2005, Elkind 2007, Balcan, Blum, Hartline, and Mansour 2008, Cole and Roughgarden 2014, ...

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Model

Stage Game

Policymaker sets policy $p \in \mathcal{P}$, sends message $m \in \mathcal{M}$

Agent chooses response $r \in \mathcal{R}$

Hidden state of nature $s \in \mathcal{S}$

Payoffs $u^P(p, r, s)$ and $u^A(p, r, s)$

Assumption: $\mathcal{P}, \mathcal{M}, \mathcal{R}, \mathcal{S}$ finite

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Repeated Game

Stage game repeated T times

Period t − 1 → **Period t** → **Period t + 1**

⋮

1. Policymaker sets policy p_t and message m_t
2. Agent chooses response r_t
3. State s_t is observed

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Policymaker's mechanism : $\text{history} \to p_t, m_t$

Agent's strategy : $\text{history}, p_t, m_t \to r_t$

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Policymaker's Regret

$r_{1:T}$ = agent's responses to actual policies $p_{1:T}$

$r^p_{1:T}$ = agent's responses to fixed policy p

Definition: policymaker's regret compares actual policies with best fixed policy, i.e.

$$\underbrace{\max_p \frac{1}{T}\sum_{t=1}^{T} u^P(p, r^p_t, s_t)}_{\text{utility under best fixed policy } p} - \underbrace{\frac{1}{T}\sum_{t=1}^{T} u^P(p_t, r_t, s_t)}_{\text{utility under actual policies } p_{1:T}}$$

Note: standard no-regret guarantees do not apply since $r^p_t \neq r_t$.

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Price Regulation

Running Example

Firm produces indivisible good at cost for price to maximize profit.

- Response r : $\text{cost} \to \text{price}$
- State s = firm's cost, buyer's value

Policymaker regulates price to maximize welfare.

- Policy $p = (\text{price floor}, \text{price cap})$

Outcome is $\text{sale} = \mathbf{1}(\text{value} \geq \text{price}) \cdot \mathbf{1}(\text{price floor} \leq \text{price} \leq \text{price cap})$

- Profit $= \text{sale} \cdot (\text{price} - \text{cost})$
- Welfare $= \text{sale} \cdot (\text{value} - \text{cost})$

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Repeated Price Regulation

Running Example

Sequence of buyers t with value_t

State $s_t = (\text{value}_t, \text{cost}_t)$ observed after period t

- Replace value_t with sale_t if needed

$\text{price}_t, \text{price floor}_t, \text{price cap}_t$ depend on observed history

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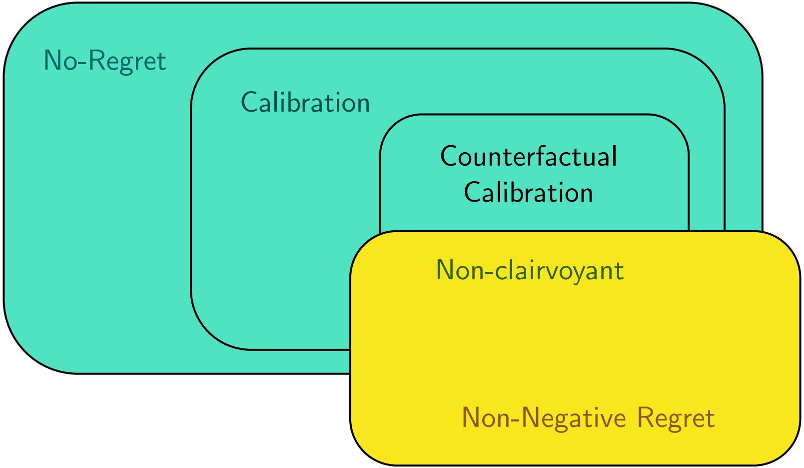
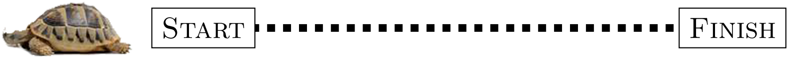
Agent's Behavior

Preview of Definitions



Dealing with Information

Tortoise travels 1km in 1h : uninformed agent satisfies no-regret.



Hare travels 1km in 1h : informed agent satisfies no-regret.

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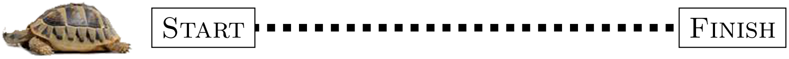

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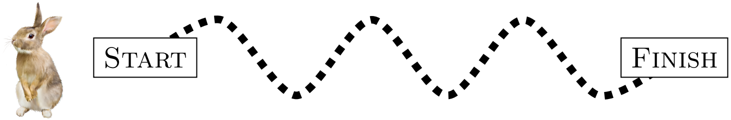

Dealing with Information

Tortoise travels 1km in 1h : uninformed agent satisfies no-regret.

START ········ FINISH

Hare travels 1km in 2min : informed agent satisfies no-regret conditioned on her information.

START ········ FINISH

Agent's Conditional Regret

Definition: agent's regret conditioned on information I_t is

$$\max_{h:\mathcal{I}\to\mathcal{R}} \quad \underbrace{\frac{1}{T}\sum_{t=1}^{T} u^A(p_t, h(I_t), s_t)}_{\text{utility under modified responses } h(I_t)} \quad - \quad \underbrace{\frac{1}{T}\sum_{t=1}^{T} u^A(p_t, r_t, s_t)}_{\text{utility under actual responses } r_{1:T}}$$

where modification rule h maps information I_t to response $h(I_t)$.

No-Regret

Definition: regret = regret conditioned on public information

$$I_t = (p_t, m_t)$$

Definition: no-regret = regret $\to 0$ as $T \to \infty$

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Behavior Reveals Information

Game of **rock-paper-scissors** between agent and nature.

$s_t =$	R	P	S	R	P	S	R	P	S	R	P	S

An uninformed strategy; no correlation between response and state

$r_t =$	S	R	R	P	P	P	S	P	R	R	P	R
$u_t^A =$	-1	-1	1	1	0	-1	-1	0	1	0	0	1

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Behavior Reveals Information

Game of **rock-paper-scissors** between agent and nature.

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An informed strategy; perfect correlation between response and state

$r_t =$ R P S R P S R P S R P S

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Calibration

Definition: internal regret = regret conditioned on information revealed on-path, i.e.

$$I_t = (p_t, m_t, r_t)$$

Definition: calibration = internal regret $\rightarrow 0$ as $T \rightarrow \infty$

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No-Regret to Calibration

Example: no-regret allows strange behavior that calibration rules out

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Tortoise strategy: uninformed, optimal, no-regret

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$s_t =$ R R R R R R P P P P P P

Lazy hare strategy: informed, suboptimal, no-regret

$r_t =$ R R R R R R S S S S S S

$u_t^A =$ 0 0 0 0 0 0 1 1 1 1 1 1

Impossibility Result

Proposition

There exists a strategy for the agent where no mechanism can guarantee non-trivial bound on policymaker's regret across all $s_{1:T}$ where agent's strategy is calibrated.

Takeaway: calibration is not enough for low-regret policy design

- Even if we know the agent's strategy in advance

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Takeaway: calibration is not enough for low-regret policy design

- Even if we know the agent's strategy in advance

Counterfactual Calibration

Definition: counterfactual internal regret = regret conditioned on information revealed on- and off-path, i.e.

$$I_t = \left(p_t, m_t, r_t, r_t^{p_1}, \ldots, r_t^{p_n}\right)$$

Definition: counterfactual calibration = CIR $\rightarrow 0$ as $T \rightarrow \infty$

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Calibration to Counterfactual Calibration

Example: calibration allows strange behavior that counterfactual calibration rules out

$s_t =$	R	R	R	R	R	R	P	P	P	P	P	P

If policymaker follows mechanism, play tortoise strategy

$r_t =$	P	P	P	P	P	P	P	P	P	P	P	P
$u_t^A =$	1	1	1	1	1	1	0	0	0	0	0	0

If policymaker follows fixed policy, play active hare strategy

$r_t =$	P	P	P	P	P	P	S	S	S	S	S	S
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Calibration to Counterfactual Calibration

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Non-clairvoyance

Definition: non-clairvoyance = counterfactual calibration & non-negative regret

Intuition:

1. Counterfactual calibration $\implies$ agent fully exploits her private info
2. Non-negative regret $\implies$ agent doesn't outperform best use of public info
3. Therefore, her private info must not be useful

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Calibrated Policy

Preview of Mechanism

In each period t...

1. Form probabilistic forecast of state s_t
2. Assume agent shares forecast
3. Choose ϵ-robust policy based on forecast

Informal result: works well when agent is non-clairvoyant

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ϵ-Robustness

Stage Game

π_t^s = state distribution

Definition: mixed response π^r is ϵ-best reply to policy p if

$$\underbrace{\max_{r'} \mathrm{E}_{\pi^s}\left[u^A(p, r', s)\right]}_{\text{optimal utility}} - \underbrace{\mathrm{E}_{\pi^s,\pi^r}\left[u^A(p, r, s)\right]}_{\text{actual utility}} \leq \epsilon$$

Definition: policymaker's worst-case utility given ϵ-best reply is

$$\mathrm{WC}_\epsilon(p, \pi^s,) = \min_{\pi^r} \mathrm{E}_{\pi^s,\pi^r}\left[u^P(p, r, s)\right] \quad \text{s.t. } \pi^r \text{ is } \epsilon\text{-best reply}$$

ϵ-Robustness *Stage Game*

π_t^s = state distribution

Definition: mixed response π^r is ϵ-best reply to policy p if

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Calibrated Policy

ϵ = robustness parameter

$\tilde{\pi}_t^s$ = calibrated forecast with grid width δ

Policy $\mathbf{p_t}$:= ϵ-robust policy assuming forecast is correct, i.e.

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1. Agent is non-clairvoyant
   counterfactual calibration + non-negative regret

2. Information useless to agent under any policy $\Rightarrow$ not harmful to policymaker
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Regret Bound

Theorem

Policymaker's regret from calibrated policy is less than

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$\tilde{\pi}_t^s$ = *forecast with grid width* δ

CIR_T = *counterfactual internal regret*

$\mathcal{S}$ = *state space*
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Tradeoff: $\epsilon\uparrow \implies$ sensitivity to miscalibration $\downarrow$ & cost of robustness $\uparrow$

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Robustness Lemma

Lemma

For any distribution π^s, *policy* p, *and constants* $\epsilon' > \epsilon > 0$,

$$\mathrm{WC}_{\epsilon'}(p, \pi^s) \geq \mathrm{WC}_{\epsilon}(p, \pi^s) - O\left(\frac{\epsilon' - \epsilon}{\epsilon}\right)$$

Calibrated Price Regulation

Running Example

Typical tradeoff:

- price cap_t too large $\implies$ price_t too large, fewer sales
- price cap_t too small $\implies$ risk of $\text{cost}_t > \text{price cap}_t$, firm shutdown

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Conclusion

Ways to Deal with Private Information

1. Assume it doesn't exist
2. Assume it exists but is well-understood
3. Optimize against worst-case private information (extension)
4. Adapt to private information over time (work-in-progress)

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Thank you!