Lecture 10: Heterogeneous Treatment Effects

The application of machine learning in causal inference

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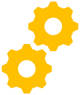
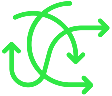
Outline

Outline

The power of machine learning



Predictive models automatically run adapted.



Dealing with seemingly complex problems.

Preventing overfitting by bias-efficiency tradeoff.

In recent years, an active area in causal inference is causal machine learning that applies ML methods to causal inference problems.

Some examples

- Using machine learning models to calculate propensity scores (Lee et al. 2010).
  - To alleviate the concerns over specification errors.
- Genetic matching (Diamond and Sekhon 2005).
  - To use genetic algorithm to automate the process of finding a good match.
- Selection of control variables for adjustment in RDD (Anastasopoulos 2019).
  - With an automatic LASSO procedure.

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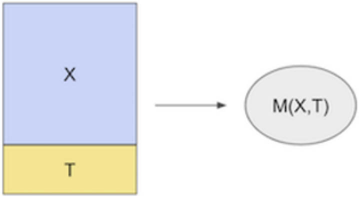
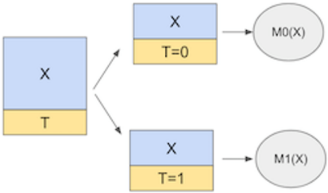
Meta learners for heterogeneous treatment effects

If we want to generalize the treatment effects, we need to know HTE function $\tau(x) = E\left(Y^1 - Y^0 \mid X_i = x\right)$

- S(ingle)-learner: fit a single ML model to $E(Y \mid D, X)$.
- T(two)-learner: fit two ML models to $E\left(Y^0 \mid D, X\right)$ and $E\left(Y^1 \mid D, X\right)$.



(a) S-learner



(b) T-learner

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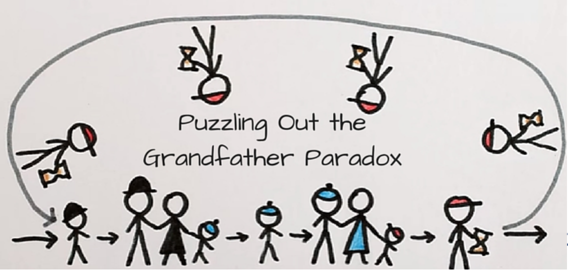
Outline

Back to the fundamental problem

Fact (The Fundamental Problem of Causal Inference)

For a unit, only one causal state can be realized, and the investigator can only observe the potential outcome from the realized causal state.

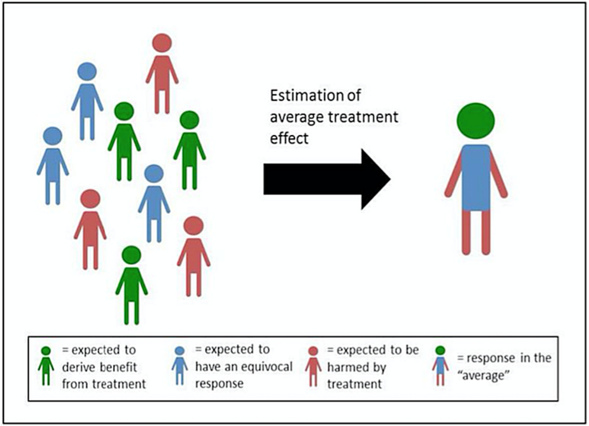
Implications: the individual treatment effects are inherently unknowable.



Back to the fundamental problem

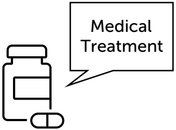
Solution 1: To assume ~~homogeneity~~ of units.
Solution 2: Potential outcome framework $\rightarrow$ ATE instead of ITE.

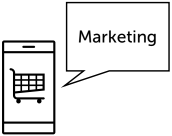
Problem: who's who?

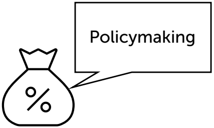


Two problems without HTEs

We identify ATE, but cannot predict how a particular person responds to the treatment...



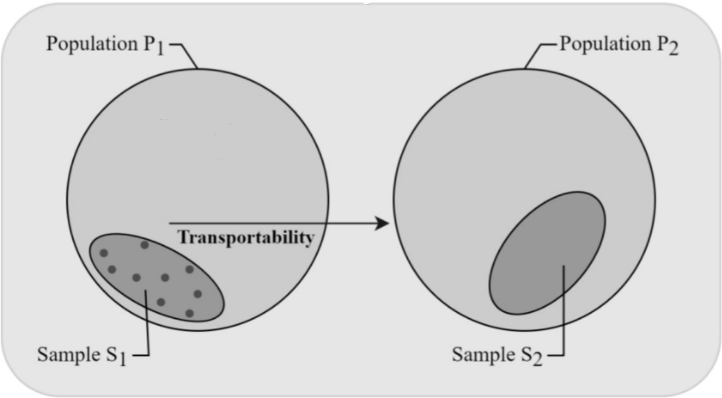




A treatment does not work for all, but may work for some.

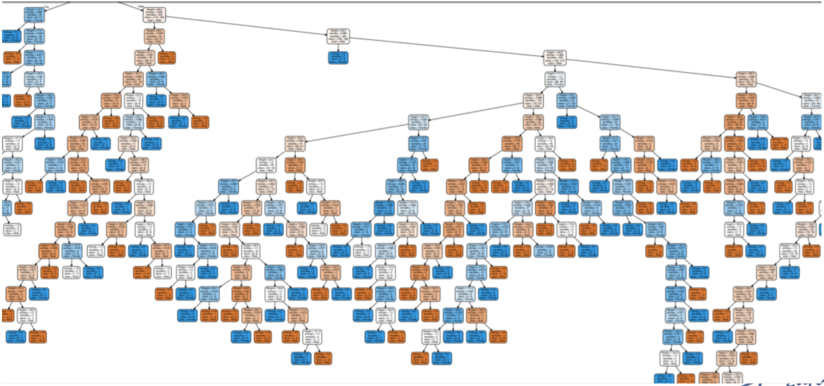
Predicting the treatment effects...

To predict the treatment effect for a new finite-population:



Background: data with rich features

We have accumulated and compiled data with rich sets of features...



Outline

The traditional approach

1. Specify a linear model:

$$Y = \alpha + \beta D + \varepsilon$$

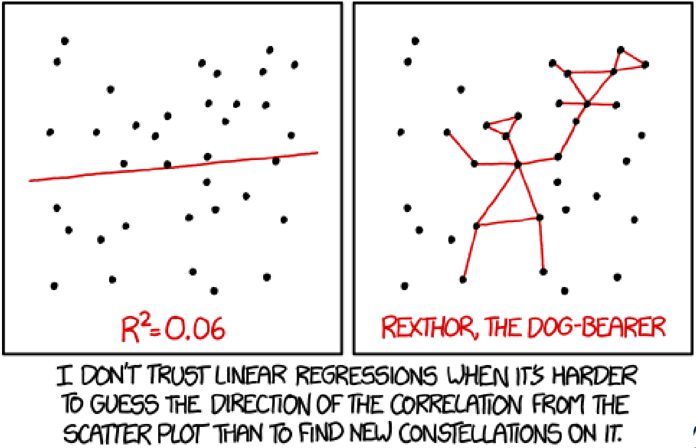
2. Adding interaction terms:

$$\begin{aligned} Y = \alpha + \beta D + & \underbrace{\lambda_1 DX_1 + \cdots + \lambda_K DX_K}_{\text{Two-way interactions}} \\ & + \underbrace{\lambda_{11} DX_1^2 + \cdots + \lambda_{KK} DX_K^2}_{\text{Three-way interactions}} \\ & + \cdots + \varepsilon \end{aligned}$$

3. Gather all λ's and mission accomplished.

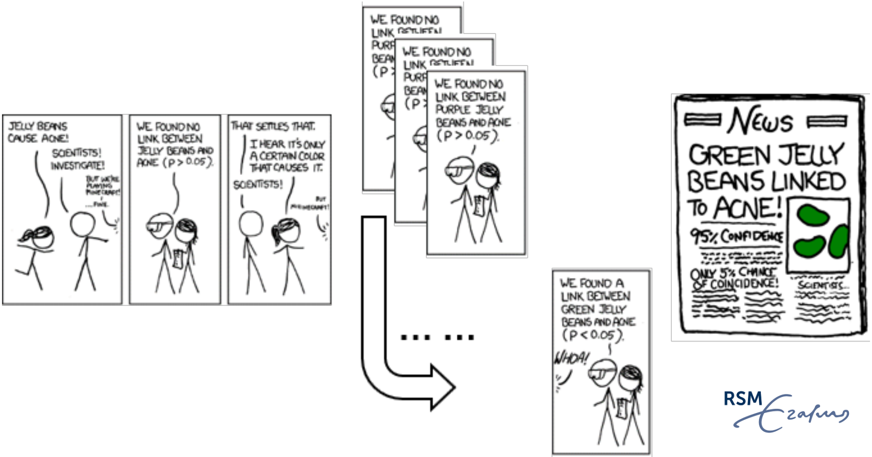
Problems of the traditional approach

It's parametric: linear, additive and separable.



Problems of the traditional approach

False discoveries from multiple testing, especially with **many features**...



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Naive applications of machine learning methods to HTEs[1]

S(ingle)-learner

1. estimate $\mu_d(x) = E(Y_i \mid D_i = d, X_i = x)$ using a single model.
2. compute $\hat{\tau}(x) = \hat{\mu}_1(x) - \hat{\mu}_1(x)$.

Example (S-learner)

LASSO with SVM to regularize over-fitting.

T(wo)-leaner

1. estimate $\mu_d(x) = E(Y_i \mid D_i = d, X_i)$ separately for $d = \{0, 1\}$.
2. compute $\hat{\tau}(x) = \hat{\mu}_1(x) - \hat{\mu}_0(x)$.

Example (T-learner)

Decision trees with regularization (tree depth).

[1] See Kunzel et al. (2019) for more details.

Naive applications of machine learning to HTEs

X-learner (X stands for "exchange")

1. estimate $\mu_d(x) = E(Y_i \mid D_i, X_i)$ separately for $D_i = 0$ or 1.
2. impute missing potential outcomes as an out-of-sample prediction (treatment $\hat{\mu}_1(x) \rightleftarrows$ control $\hat{\mu}_0(x)$).
3. impute the individual treatment effects $\tau_i(X_i)$ with observed outcomes Y_i^{obs} and imputed potential outcomes.
4. use the imputed ITEs $\hat{\tau}_i(X_i)$ as the response variable and use any supervised learning method with $\hat{\tau}_i(X_i) = f(X_i)$.

Naive applications of machine learning to HTEs

X-learner (X stands for "exchange")

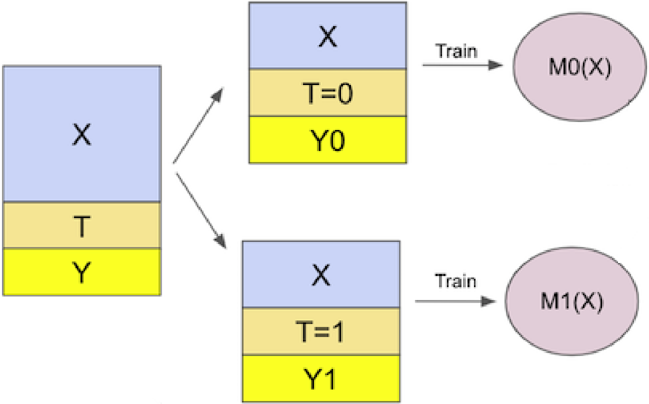
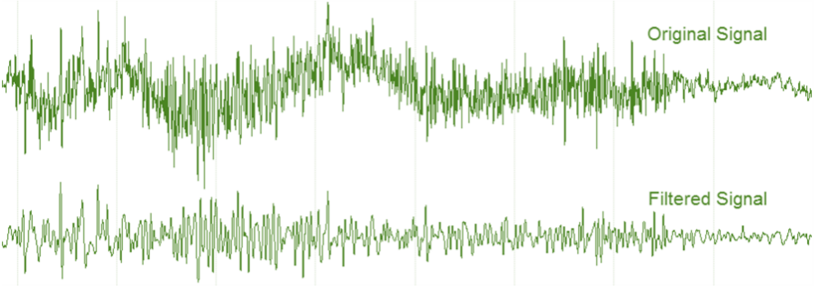


Figure: X-Learner

Problems with naive applications

HTEs require "good estimates" of **variance of ATE's**.
Naive applications: **no inference on the variance or second-moments**.

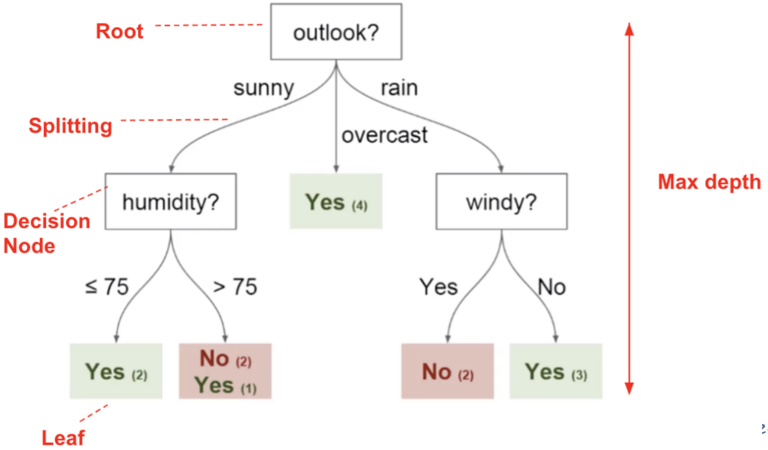


"Filtering the nuisance"

Outline

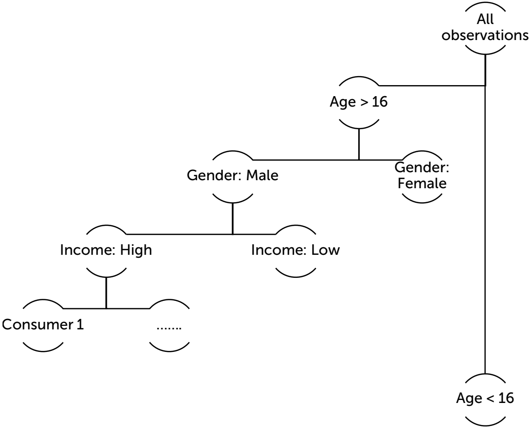
The ABCs of decision trees

Some terminologies with *a classifier of 365 days with weather conditions*:



Build a tree to predict purchases

1. Given some data (e.g. age, gender, and income), build a tree.
2. For a new case, check which leave the case is in.
3. Use $\bar{Y}$ of the leave as the predicted purchase.



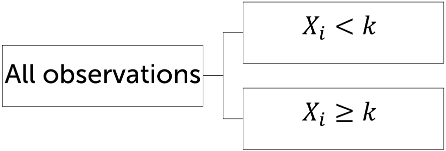
Tree building: how to make a partition

How to make a split?

- Choose a cutoff k to minimize a loss function
- Example, mean squared errors (MSE) with

$$\text{MSE} = \frac{1}{N}\sum_{i=1}^{N}\left(Y_i - \overline{Y}_{j:j\in l(X_i|\Pi)}\right)^2,$$

with Π a partition and $l(\cdot)$ a leaf



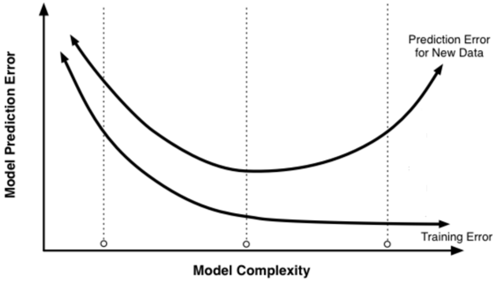
Choose k to minimize MSE

Tree building: when to stop splitting?

How many leaves to have (and/or the max depth)?

- With enough fine partitions $\mapsto$ 1 consumer in a leaf.
  - Perfect (in-sample) fit but uselessly high variance.

Regularization: to keep splits that improve MSE by at least c.



Bias-variance trade-off

Extending decision trees to causal trees

In a **decision tree**, the loss function is defined as MSE:

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^{N} \left(Y_i - \overline{Y}_{j:j \in l(X_i|\Pi)} \right)^2$$

Similarly, we can define a **causal tree** on treatment effects:

$$\text{MSE}_{\text{Causal}} = \frac{1}{N} \sum_{i=1}^{N} \left(\tau_i - \overline{\tau}_{j:j \in l(X_i|\Pi)} \right)^2$$

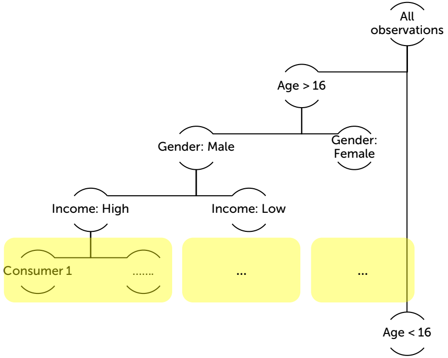
Fact (Challenge in defining causal $\text{MSE}_{\text{Causal}}$)

τ_i is unobserved, because of the fundamental problem of causal inference!

Extending decision trees to causal trees

Think about the idea of subclassification in Lecture 4.

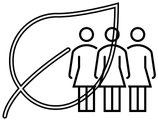
- For each sub-class, we can estimate the treatment effects!
- In a decision tree, **a sub-class = a leaf in a tree**.



Extending decision trees to causal trees

Each leaf is a subclass, and $\tau(X_i)$ is defined for a leaf $l(X_i \mid \Pi)$.
Summarize over all the leaves to obtain treatment effects.

Decision Trees
Y_i

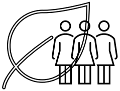


Causal Trees
τ_i

The ATE of a leaf: $\text{ATE}_l = \overline{Y}_l^1 - \overline{Y}_l^0$.

Extending decision trees to causal trees

Decision Trees
Y_i



Causal Trees
τ_i

Sample size requirement

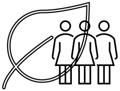
To estimate $I(X_i \mid \Pi)$ for each leaf, we need enough samples for each treatment condition within a leaf, i.e., restricting trees to have at least $2 \cdot k$ samples for any leaf.

The insight is from PO

We focus on ATE to “avoid” the fundamental problem.

Extending decision trees to causal trees

Decision Trees
Y_i



Causal Trees
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How to build causal trees?

Naive application of the traditional MSE criterion:

$$\text{MSE}_0 = \frac{1}{N_l} \sum_{l=1}^{N_l} \Bigg(\underbrace{\tau_l}_{\text{ATE of a leaf}} - \underbrace{\overline{\tau}_l}_{\text{Average of ATEs of all leaves}} \Bigg)^2$$

Question: is this MSE adequate for our purpose?

MSE_0: the cross-leaf variance.
Minimize MSE_0?

But we want to find heterogeneity.
MSE_0 is against our objective!

How to build causal trees?

Reverse the sign of MSE_0

$$\mathrm{MSE}_1 = -\frac{1}{N_l}\sum_{l=1}^{N_l}\left(\underbrace{\tau_l}_{\text{ATE of a leaf}} - \underbrace{\overline{\tau}_l}_{\text{Average of ATEs of all leaves}}\right)^2$$

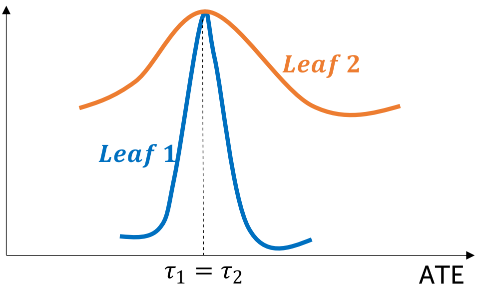
To minimize $\mathrm{MSE}_1 \Rightarrow$ to maximize the cross-leaf variance.
We intend to find heterogeneity of treatment effects $\tau(X_i)$.

Question: is MSE_1 adequate?

How to build causal trees?

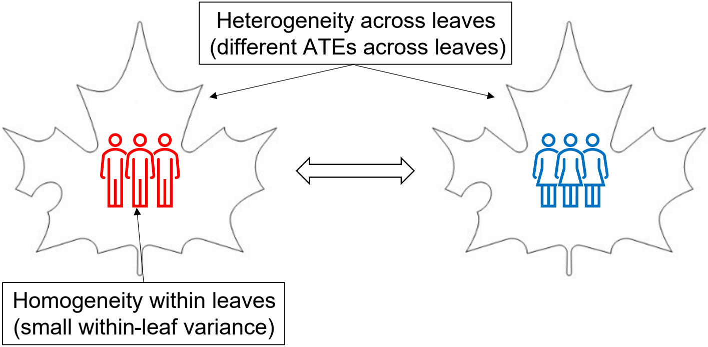
In a causal tree, suppose we have two leaves of the same ATE.

- By definition of MSE_1, these two leaves are equivalent.
- But, the variances of leaf-specific ATEs are different.



Question: which leaf is better for prediction?

How to build causal trees?



To balance cross-leaf vs. within-leaf variance:

$$\mathrm{MSE}_2 = -\frac{1}{N_l}\sum_{l=1}^{N_l}\underbrace{\left(\tau_l - \overline{\tau}_l\right)^2}_{\text{Cross-leaf Variance}} + \frac{1}{N_l}\sum_{l=1}^{N_l}\left(\underbrace{\frac{S_1^2(l)}{N_1(l)} + \frac{S_0^2(l)}{N_0(l)}}_{\text{Within-leaf Variance}}\right)$$

Building causal trees: some notes

- The MSE_2 introduced here is not exactly the one in Athey and Imbens (2016), but the intuition is the same.
- The authors first extended the standard MSE for decision trees and then generalized it for causal trees.
- Here, we work "backwards" to understand the intuitions of building causal trees.
- For more details, please check Athey and Imbens (2016).

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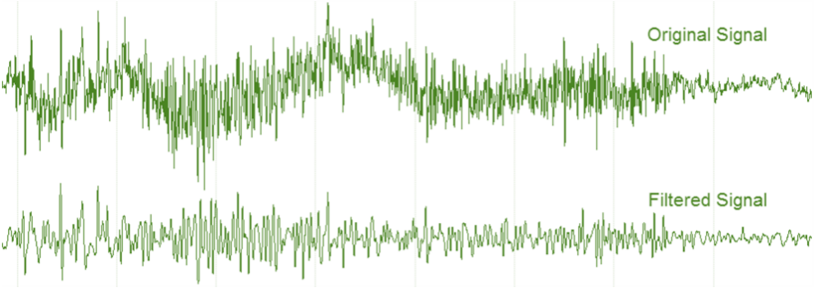
How to build causal trees?

- Now we have a criterion to produce partitions/splits...
- Still need **an inference procedure** for the variance of ATE.
  - So we know how heterogeneous the ATE is.



"Filtering the nuisance"

The inference procedure: resampling

Sub-sampling: to sample a fraction of all observations (without replacement) to create a subsample.

Sub-sampling

Given a data $X = \{X_1, \cdots, X_N\}$, the size of a subsample S, and a statistic $T(X)$:

- Sample $X^* = \{X_1^*, \cdots, X_S^*\}$ from X without replacement.
- Calculate the statistic $T(X^*)$.
- Repeat many time (at most $\binom{N}{S}$ times).

Why sub-sampling?

Why sub-sampling instead of bootstrapping?

- Bootstrapping may not work here.
- A deterministic operation creates **"holes" in the distribution of the statistics** $T(X^*)$.
- **Causal trees**: to use MSE to determine splits (a minimization).

Example (The failure of bootstrapping)

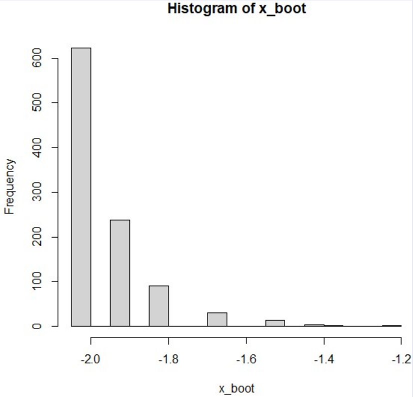
Suppose $X_1, \cdots, X_N \sim \text{Uniform}(0, 1)$, and a statistic: $T(X) = \min(X_1, \cdots, X_N)$. If you bootstrap, the test statistic would not converge to the true distribution.

Why sub-sampling?

Run the example in R

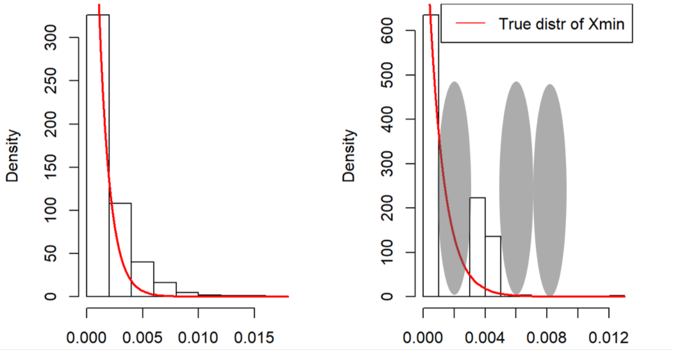
```
x <- rnorm(100)
x_boot <- rep(0,1000)

for (i in 1:1000) {
xs <-sample(x,100,replace = T)
x_boot[i] <- min(xs)
}
```

The distribution of X_{min}



Why sub-sampling?

Illustration: deterministic transformations create “holes” in the distribution.

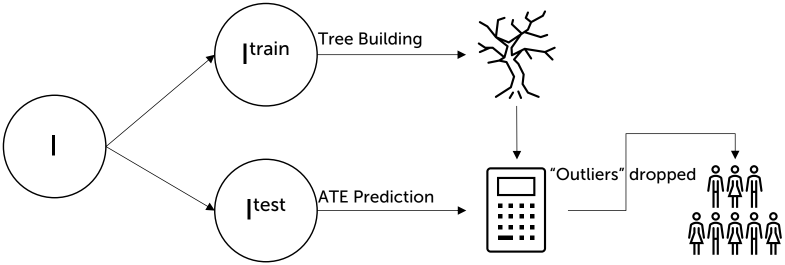


True Distribution Bootstrap Distribution

Building “honest trees”

In Wager and Athey (2018), a procedure to build “honest trees”.

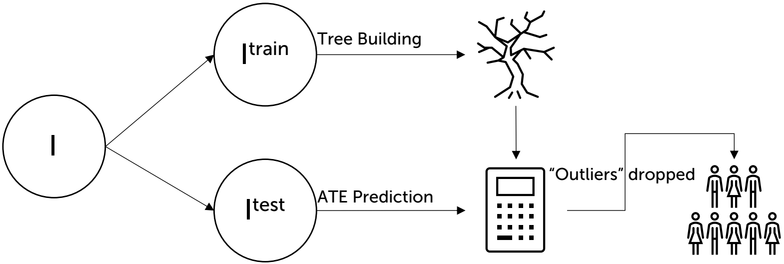
1. Split I sample (evenly) into a train set I^{train} and a test set I^{test}.
2. Build a tree with $I^{train}$and predict leaf-specific ATEs with I^{test}.



Building "honest trees"

The key insights into "honest trees"

- The splitting into $\{I^{\text{train}}, I^{\text{test}}\}$ is **cross-validation** from ML.
- The idea of **trimming** from CI: leaves in I^{train}that do not produce an ATE for I^{test} would be dropped.



The "dropping" reduces bias in HTEs.

Putting things together

Given data of a tuple $\{X_i, D_i, Y_i\}$ of N observations, the block size α, the minimum sample size per condition per leaf k, and the total number of repetition S, run the following:

A general procedure to causal forest

At a particular repetition s:

1. Draw a random subsample of size αN without replacement as I_s.
2. Split I_s to I_s^{Train} and I_s^{Test}.
3. Grow a tree T_s with I_s^{Train} using $\text{MSE}_{\text{Causal}}$ and restrict size of leaves $> k$.
4. Assign observations in I_s^{Test} with T_s and calculate $\tau_l^s\left(I_s^{\text{Test}}\right)$.
5. With full sample N, assign people with T_s, and calculate $\tau_i^s(N)$.
6. Repeat 1-5 S times and let $\tau_i = 1/S \sum_s \tau_i^s(N)$.

Outline

Doubly robust estimator

A modified weighting estimator which is robust as long as the propensity score or the regression model is correctly specified.

$$
\begin{aligned}
\tau_{\text{DR}} = & \frac{1}{N} \sum_{i=1}^{N} \left[\frac{D_i Y_i}{e(X_i)} - \frac{D_i - e(X_i)}{e(X_i)} \underbrace{m_1(X_i)}_{\text{Fit } Y_i^1 \sim X_i} \right] \\
& - \frac{1}{N} \sum_{i=1}^{N} \left[\frac{(1 - D_i) Y_i}{1 - e(X_i)} + \frac{D_i - e(X_i)}{1 - e(X_i)} \underbrace{m_0(X_i)}_{\text{Fit } Y_i^0 \sim X_i} \right]
\end{aligned}
$$

Three conditional distributions,

$$
\begin{aligned}
e(X_i) &= E(D_i \mid X_i) \\
m_1(X_i) &= E(Y_i \mid X_i, D_i = 1) \\
m_0(X_i) &= E(Y_i \mid X_i, D_i = 0)
\end{aligned}
$$

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Technical notes

Why it's doubly robust?

The first term in τ_{DR} is (a similar procedure for the second term):

$$
\begin{aligned}
E\left[\frac{D_i Y_i}{e(X_i)} - \frac{D_i - e(X_i)}{e(X_i)} m_1(X_i)\right] &= E\left[\frac{D_i Y_i^1}{e(X_i)} - \frac{D_i - e(X_i)}{e(X_i)} m_1(X_i)\right] \\
&= E\left[Y_i^1 + \frac{D_i - e(X_i)}{e(X_i)}\left(Y_i^1 - m_1(X_i)\right)\right] \\
&= E\left(Y_i^1\right) + E\left[\frac{D_i - e(X_i)}{e(X_i)}\left(Y_i^1 - m_1(X_i)\right)\right]
\end{aligned}
$$

The second term is zero, either one of the conditions hold,

$$
\begin{cases}
e(X_i) & = E(D_i \mid X_i) \\
m_1(X_i) & = E(Y_i \mid X_i, D_i = 1)
\end{cases}
$$

In other word, $e(X_i)$ or $m_1(X_i)$ consistently estimates the conditional expectations.

Tuning the trees

A new loss function to tune the parameters of trees:

$$\widetilde{\tau}(\cdot) = \operatorname{argmin}_{\tau} \left(\frac{1}{N} \sum_{i=1}^{N} \left[\underbrace{(Y_i - m^*(X_i)) - (D_i - e^*(X_i))\,\tau(X_i)}_{\text{Doubly Robust ATE Estimators}} \right]^2 + \underbrace{\Lambda_N(\tau(\cdot))}_{\text{Regularization}} \right)$$

For more details, see Wager and Athey (2018).

Generalized with moment conditions

See more details in Athey et al. (2019).
A package "grf" for R, and the online resource is here:
`https://grf-labs.github.io/grf/index.html`
Applications:

- generalized weighting estimator
- instrumental variables
- treatment heterogeneity
- ...

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