THE ALLIANCE HIGH SCHOOL

---MATH OLYMPIAD---

TIME: $[90]^2$ SECS

SENIOR TEST

Instructions

1. Read all questions **carefully**
2. All calculators are not allowed
3. Mathematical tables are not allowed
4. Geometrical sets and green pens allowed
5. Only use a pencil on the answer sheet
6. Give the exact answers only i.e., **8π, $3\sqrt{6}$**
7. Do not write any marks on the question paper
8. Crying is allowed but silently
9. Do not open the question paper until told to do so
10. In **section I** each problem is worth **5 points**, in **section II** each problem is worth **8 points** and in **section III** each question is worth **10 points**

Mathemagics Club Committee

1.	Collins Kerama	Chairman
2.	Vic Koech	Vice Chairman
3.	Dalton Omondi	Secretary
4.	Michael Mumo	Team Leader
5.	Victor Nthiwa	The Organising Secretary
6.	Stephen Adika	Treasurer
7.	Godwins Oloo	Form Three Representative

Note:

- ✓ Revision of the paper will be done in 3Q on Tuesday during the club meeting
- ✓ The Executive Committee reserves the right to nullify any results of any house that commits any form of malpractices.

SETER – MUKOYA KHISA

SECTION I {40 POINTS}

1. Determine the value of k.
$$\log_k k + \log_{k^2} k^2 + \log_{k^3} k^3 + \dots + \log_{k^{2021}} k^{2021} + \log_{k^{2022}} k^{2022} = 4044$$

2. Consider some positive integers **x, y** and **z** that satisfy the equation below. $x^2 + z^2 = y^2$**.** Determine ten possible values of **x, y** and **z.**

3. Given we have a regular polygon which is such that the interior angle is **2022** times the exterior angle, determine the number of sides of the polygon and name it.

4. Find the number of multiples of 13 from 1 to 1000000000 inclusive.

5. Estimate as accurate as possible the number of digits in$\mathbf{7^{2022}}$.

6. Determine the sum of number of trailing zeros in **2022!** and in $\mathbf{2016^3 \times 135^4 \times 15^6}$.

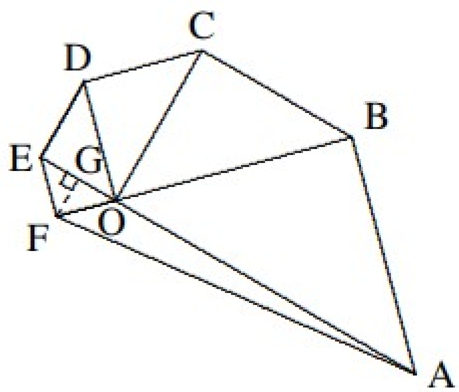
7. Given that **x** and **y** are two real numbers satisfying the following equations; $x + xy + y = 2 + 3\sqrt{2}$ *and* $x^2 + y^2 = 6$, determine the value of $x + y$.

8. Write 0.9999…. as a fraction in the form $\frac{a}{b}$ and determine the value of $a + b$.

SECTION II {40 POINTS}

1. Determine the exact value of the sum below.
$$\frac{1}{1 \times 2 \times 3} + \frac{1}{2 \times 3 \times 4} + \frac{1}{3 \times 4 \times 5} + \dots + \frac{1}{2020 \times 2021 \times 2022}.$$

2. The diagram shows a hexagon **ABCDEF** made up of five right-angled isosceles triangles **ABO, BCO, CDO, DEO, EFO,** and a triangle **AOF**, where **O** is the point of intersection of the lines BF and AE. Given that **OA = 8 cm,** find the area of **ΔAOF** in cm^2.



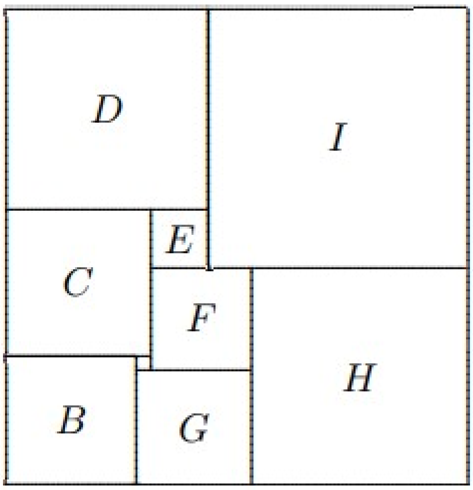
3. Given that a mother had some biscuits which she wished to share among some fellows who we don’t know how many they are. If she divides the biscuits in threes no biscuit remains, if she divides the biscuits in fives two biscuits remain, if she divides the biscuits in sevens three biscuits remain, if she divides the biscuits in thirteens nine biscuits remain. Then determine five possible number of biscuits she might be having.

4. Given that difference between the L.C.M and H.C.F of some two numbers is 132 and the difference between the numbers is 12. Determine the sum of the two numbers.

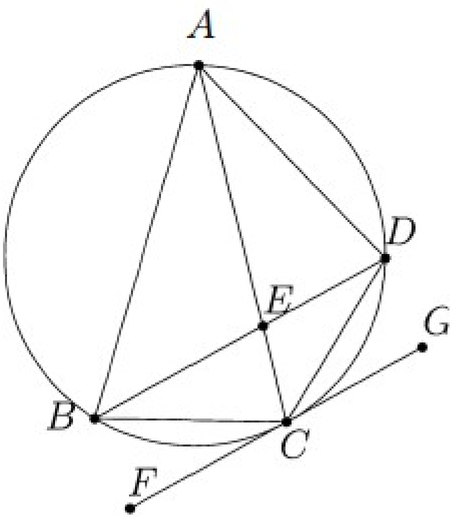
5. It is given that **a, b, c** and **d** are four positive prime numbers such that the product of these four prime numbers is equal to the sum of **55** consecutive positive integers. Find the smallest possible value of **a+ b + c +d**. (Remark: The four numbers **a, b, c, d** are not necessarily distinct.)

SECTION III {40 POINTS}

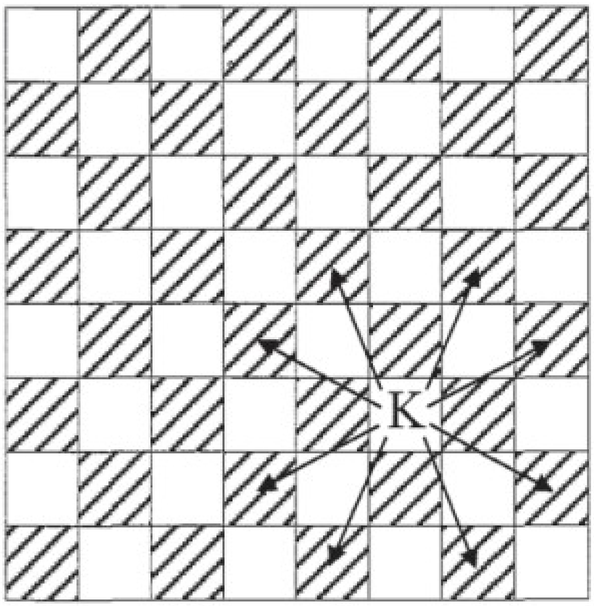
1. Nine squares are arranged to form a rectangle as shown. The smallest square has side of length 1. How big is the next smallest square? And how about the area of the rectangle?



2. In the diagram below, **ABCD** is a cyclic quadrilateral with **AB = AC**. The line FG is tangent to the circle at the point **C**, and is parallel to **BD**. If **AB = 6** and **BC = 4**, find the value of **3AE**.



3. As shown in the picture, the knight can move to any of the indicated squares of the **8×8** chess board in only one move. If the knight starts from position shown. Find the number of possible landing positions after **20** consecutive moves.



4. Initially, the number **10** is written on the board. In each subsequent move you can either erase the number **1** and replace it with a **10** or erase the number **10** and replace it with a 1 and a **25** or erase a **25** and replace it with two **10**. After sometime you notice that there are exactly **100** copies of **1** on the board. What is the least possible sum of all the numbers on the board at that moment?

THE END.