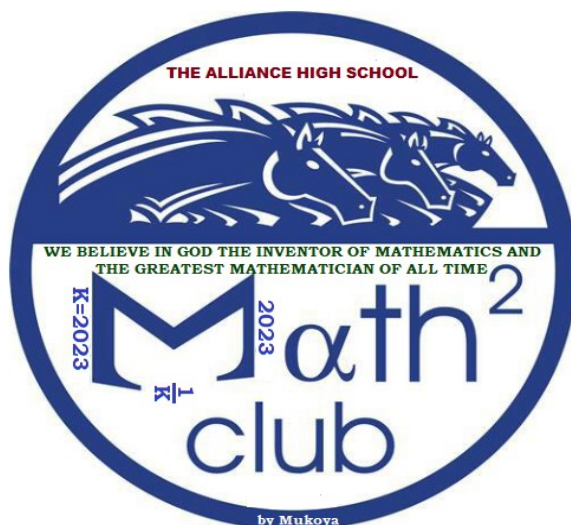


**“We believe in God the inventor of Mathematics and the greatest mathematician of all time”**



# THE ALLIANCE HIGH SCHOOL **SENIOR** CONTEST

## **Instructions to participants.**

## **FORM THREE TEST**

1. Read all questions **carefully**.
2. Calculators, mathematical tables and geometrical instruments are **ALLOWED** if they may help.
3. Use a pencil or a visible pen on the answer sheet.
4. Give the exact answers only i.e.,  $\sqrt{8\pi}$ ,  $\sqrt[3]{\pi}$ ,
5. Do your calculations in the spaces at the middle of the test booklet and only give the answers on the answer sheet.
6. This test booklet consists of **8** printed pages. Check to ascertain that it's true as indicated.
7. Write all your details i.e., name, stream and Adm. No in the spaces provided in the **answer sheet**.
8. **DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO.**
9. Each question is worth **1 POINT**.
10. The test takes exactly **60 minutes**. After the time has lapsed stop writing.

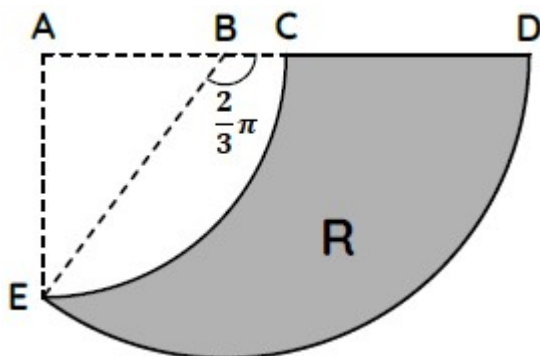
## **MATHEMAGICS CLUB EXECUTIVE COMMITTEE**

- |                   |                             |   |
|-------------------|-----------------------------|---|
| 1. MUKOYA KHISA   | <b>CHAIRMAN</b>             | 6. GEOFFREY GIKONYO - <b>FORM 3 REP 1</b> |
| 2. MATILU MUYEKU  | <b>VICE CHAIRMAN</b>        | 7. ALLAN NZIOKA - <b>FORM 3 REP 3</b>     |
| 3. IVAN MAYABI    | <b>SECRETARY</b>            | 8. SAMWEL WAREGA - <b>FORM 3 REP 2</b>    |
| 4. NIMROD NYABERI | <b>ORGANISING SECRETARY</b> |   |
| 5. ANDREW CHELIMO | <b>TREASURER</b>            | <b>EXAMINER – KHISA</b>                   |

- The Executive Committee of The Alliance High School Mathematics Club (ECAHSMC) reserves the right to disqualify all scores of any participant if it determines that the required security procedures have not been followed.

## QUESTIONS

- 1.** The figure below shows a part of Khisa’s blade viewed in two dimensions and constructed as follows.



- **EBD** is a circular sector with centre at **B** and radius **12** units, subtending an angle of  $\frac{2}{3}\pi$  radians at **B**.
- **EAC** is a quarter circle with centre at **A** and radius **r** units, so that **ABCD** is a straight line and **CAE** is a right angle.

The shaded region **R** is bounded by the arcs **ED** and **EC**, and the straight-line **CD**. Calculate the exact area of **R**. (Leave your answer in terms of  $\pi$  and surds.)

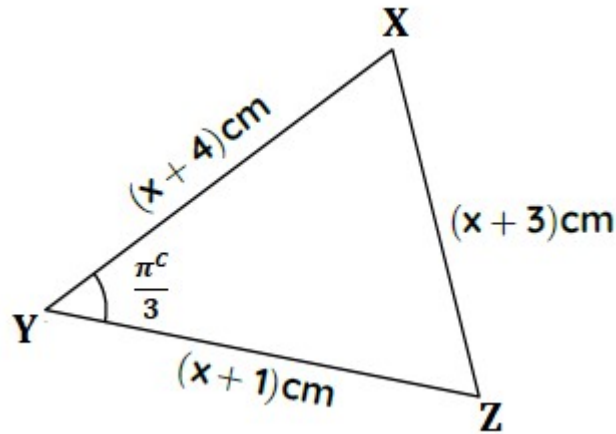
- 2.** Nyaberi needs to make **Ksh.45, 000** per week after tax in order to pay all his bills. The income tax rate is **25%**. What is the smallest pre-tax weekly salary Nyaberi can earn and still be able to pay his bills after he pays his income tax?
- 3.** The **1<sup>st</sup>**, **3<sup>rd</sup>** and **11<sup>th</sup>** term of an arithmetic progression are the first three terms of a geometric progression. It is further given that the sum of the first **13** terms of the arithmetic progression is **260**. Find, in any order, the common ratio of the geometric progression and the first term and common difference of the arithmetic progression.
- 4.** Given that  $a \neq 0, b \neq 0, y \neq 0$  and
- $$2 + \log_a b + 3 \log_a y = 2 \log_a (a^2 y).$$

Express **y** in terms of **a** and **b**, in a form not involving logarithms.

- 5.** It is given that if **k** is a constant then,
- $$(1 + K\sqrt{3})^4 = 892 - 336\sqrt{3}$$

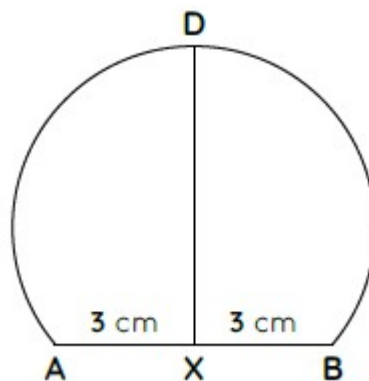
Determine the value of **k**.

- 6.** The figure below shows a triangle **XYZ** whose side lengths are given in terms of **x**.



Given that the angle **XYZ** is  $\frac{\pi}{3}$  radians, determine the exact area of the triangle.

- 7.** Tap **A** takes **3** hours to fill a tank when empty, Tap **B** takes **4** hours to fill the same tank when empty. Tap **C** takes **6** hours to empty the same tank when full. Tap **A** is opened, one hour later Tap **B** and Tap **C** are opened simultaneously. Calculate the total time it takes to fill the tank.
- 8.** Relative to a fixed origin **O**, the point **A** has coordinates  $(\mathbf{k}, 3, 5)$ , where **k** is a scalar constant. The points **B** and **C** are such so that  $\mathbf{BA} = 3\mathbf{i} - 2\mathbf{j}$  and  $\mathbf{BC} = 2\mathbf{i} + \mathbf{c}\mathbf{j} - 4\mathbf{k}$ , where **c** is a scalar constant. If the coordinates of **C** are  $(1, 4\mathbf{k}, 1)$ , determine the exact distance **BC**.
- 9.** In the figure below, not drawn to scale  $\mathbf{AX} = \mathbf{XB} = 3$  cm. Given that the circle has a radius of **4.5 cm**.



Calculate to 2 decimal places, the length **XD**.

- 10.** The cost (**C**) of hiring a venue for a delegates' conference is partly fixed and partly varies inversely to the number **N** of delegates. When **200** delegates attend the cost is Ksh.**8400** per delegate while for **150** delegates the cost is Ksh.**9600** per delegate. Calculate the fixed cost.

**11.** In a histogram the heights, **h** cm, of primary school pupils are plotted on the **x** axis. In this histogram the class  $120 \leq h < 130$  has a frequency of **72** and is represented by a rectangle of base 4.2cm and height 9cm. In the same histogram a different class is represented by a rectangle of base 2.1cm and height 8cm. Determine the frequency of this class.

**12.** Given that  $\frac{\sqrt{98}-\sqrt{8}}{1+\sqrt{2}} = x + y\sqrt{2}$  where **x** and **y** are integers, find **x + y**.

**13.** Rationalize the denominators of the following expressions and simplify if possible.

$$\frac{a^2}{b + \sqrt{a^2 + b^2}}$$

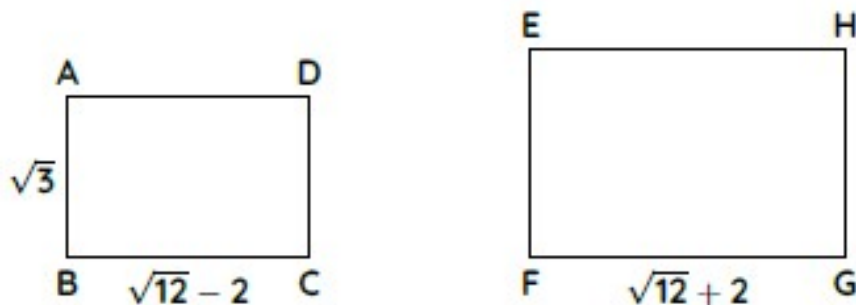
**14.** Warega lights a candle every **6** minutes. Each candle lasts **54** minutes. After **132** minutes, how many candles will be lit?

**15.** In the binomial expansion of  $(1 + ax)^k$ , where **a** and **k** are non-zero constants, the coefficient of **x** is **8** and the coefficient of **x<sup>2</sup>** is **30**. Determine the value of **a** and the value of **k**.

**16.** Two factories, **A** and **B**, undertook to fulfil an order in **12** days. After two days, factory **A** was closed down for repairs, while factory **B** continued with the fulfilment of the order. Knowing that **B** has an efficiency of  $66\frac{3}{2}\%$  that of **A**, determine in how many days the order will be completed.

**17.** The  $2 \times 2$  matrices **X** and **Y** are given by  $X = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$  and  $Y = \begin{pmatrix} 9 & 12 \\ 4 & 5 \end{pmatrix}$ . Find the  $2 \times 2$  matrix **K** that satisfy the equation **KX = Y**.

**18.** The two rectangles shown in the figure below are similar.



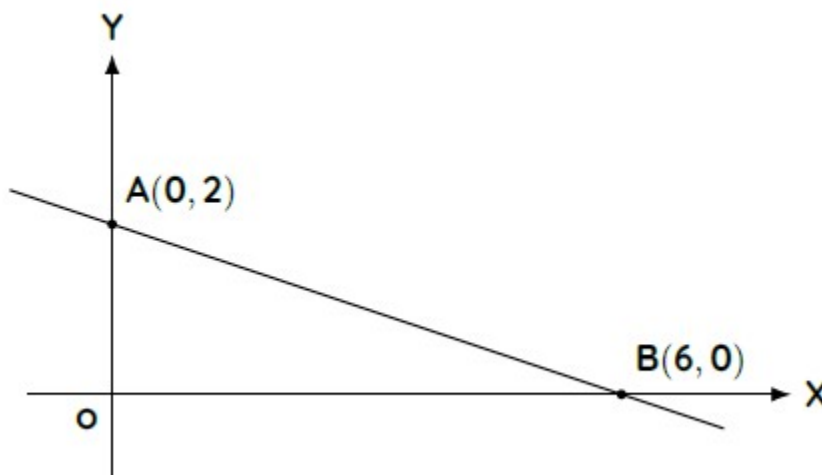
It is further given that in suitable units

$$AB = \sqrt{3}, BC = \sqrt{12} - 2 \text{ and } FG = \sqrt{12} + 2$$

Find the exact length of **EF**.

**19.** A box contains black and white balls only, in the ratio **3:2**. Two balls are randomly selected from the box without replacement. If the probability of selecting different colour balls is  $\frac{1}{2}$ , apply algebra to determine the total number of balls in the box.

**20.** The variables **x** and **y** are thought to obey a law of the form  $y = ax^n$  where **a** and **n** are non-zero constants. Let  $X = \log_{10} x$  and  $Y = \log_{10} y$ . The figure below shows the graph of **Y** against **X**.



Determine the value of **a** and the value of **n**.

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- 21.** Find the values of **x** and **y** that make the objective function  **$2x + 4y$**  maximum subject to the conditions shown below and find the value of the objective function at these values of **x** and **y**.

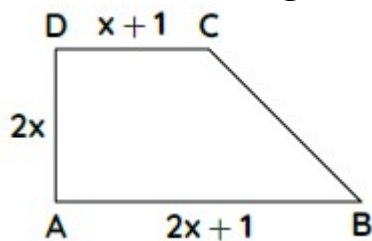
$$\begin{aligned}x &\geq 0 \\y &\geq 0 \\y &\leq x + 1 \\4y + x &\leq 10 \\y - x &\geq -3\end{aligned}$$

(Solve by calculation only)

- 22.** Solve the following logarithmic equation.

$$3 + 8 \log_{\frac{1}{k}} \left[ \sqrt{8 + 4\sqrt{3}} - \sqrt{8 - 4\sqrt{3}} \right] = 0 \quad \text{where } k > 0 \text{ and } k \neq 1$$

- 23.** A right-angled trapezium ABCD is shown in the figure below.



The trapezium has parallel sides **AB** and **CD** of lengths  **$(2x + 1)$  cm** and  **$(x + 1)$  cm**. The height of the trapezium **AD** is  **$2x$  cm**. Given that the area of the trapezium is  **$16\text{cm}^2$** , determine the **exact** length of **BC**.

- 24.** A number of the form **213gh**, where **g** and **h** are digits, has a remainder less than 10 when divided by 100. The sum of all the digits in the above number is equal to 13. Find the digit **g**.

**—THE END —**

# **ANSWER SHEET**

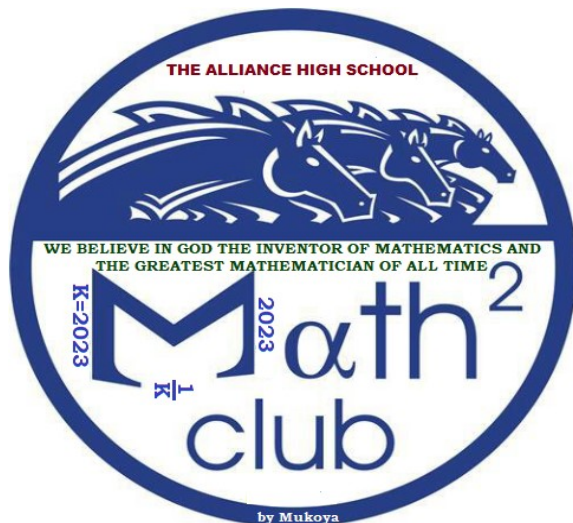
<b>NAME OF PARTICIPANT</b>	
<b>STREAM</b>	
<b>ADM.NO</b>	
<b>HOUSE</b>	
<b>MATH TEACHER</b>	
<b>PARTICIPANT'S SIGNATURE</b>	

**“We believe in God the inventor of Mathematics and the greatest mathematician of all time”**

<b>QUESTION</b>	<b>PARTICIPANT’S ANSWER</b>	<b>SCORE IN POINTS</b>	<b>EXAMINER’S MARK</b>
1.		1 Point	
2.		1 Point	
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24.		1 Point	



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# THE ALLIANCE HIGH SCHOOL **SENIOR** CONTEST

## **Instructions to participants.**

## **FORM FOUR TEST**

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15. Do your calculations in the spaces at the middle of the test booklet and only give the answers on the answer sheet.
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18. **DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO.**
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20. The test takes exactly **60 minutes**. After the time has lapsed kindly stop writing.

## **MATHEMAGICS CLUB EXECUTIVE COMMITTEE**

6. MUKOYA KHISA	<b>CHAIRMAN</b>	6. GEOFFREY GIKONYO - <b>FORM 3 REP 1</b>
7. MATILU MUYEKU	<b>VICE CHAIRMAN</b>	7. ALLAN NZIOKA - <b>FORM 3 REP 3</b>
8. IVAN MAYABI	<b>SECRETARY</b>	8. SAMWEL WAREGA - <b>FORM 3 REP 2</b>
9. NIMROD NYABERI	<b>ORGANISING SECRETARY</b>	
10. ANDREW CHELIMO	<b>TREASURER</b>	<b>EXAMINER – KHISA</b>

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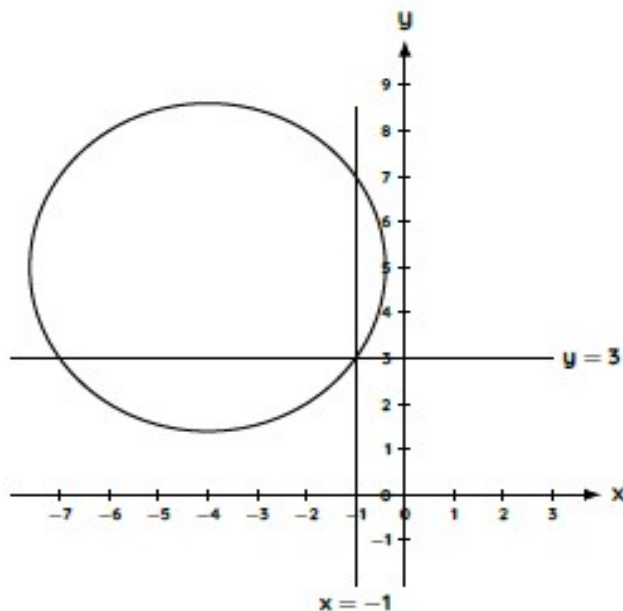
## QUESTIONS

**1.** A basket contains white and black balls only, in the ratio **2 : 3**. Two balls are randomly selected from the basket without replacement. If the probability of selecting different colour balls is  $\frac{1}{2}$ , use algebra to determine the total number of balls in the basket.

**2.** Determine the value of **k** in the following equation.

$$k^{7k^{11}} = 4^{63k^{11}}$$

**3.** The circle shown below cuts the line  $y = -1$  at **(-1, 3)** and **(-1, 7)**. It also cuts the line  $y = 3$  at **(-7, 3)** and **(-1, 3)**.

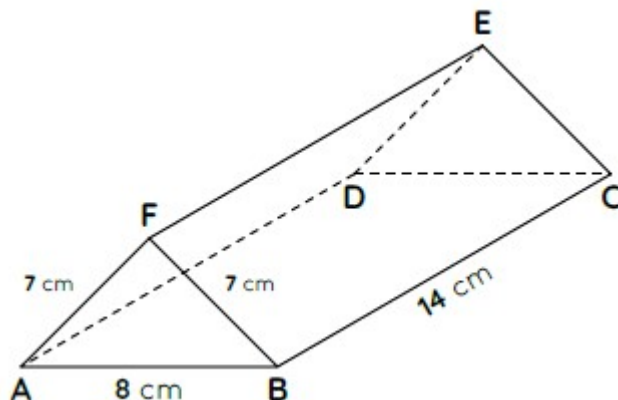


- (i) Find the radius of the circle, leaving your answer in surd form.
- (ii) Determine the equation of the circle in the form  $x^2 + y^2 + ax + by + c = 0$ .

**4.** Relative to a fixed origin **O**, the point **A** has coordinates **(k, 3, 5)**, where **k** is a scalar constant. The points **B** and **C** are such so that **BA = 3i - 2j** and **BC = 2i + cj - 4k**, where **c** is a scalar constant. If the coordinates of **C** are **(1, 4k, 1)**, determine the exact distance **BC**.

**5.** Two points **A** and **B** both lie **60°** south of the equator on **175°E** and **135°W** respectively. A plane left **A** on Tuesday **1200** hours for **B** using the shorter route along the latitude **60°S** at an average speed of **300** knots. Determine the local time and day of arrival in **B**.

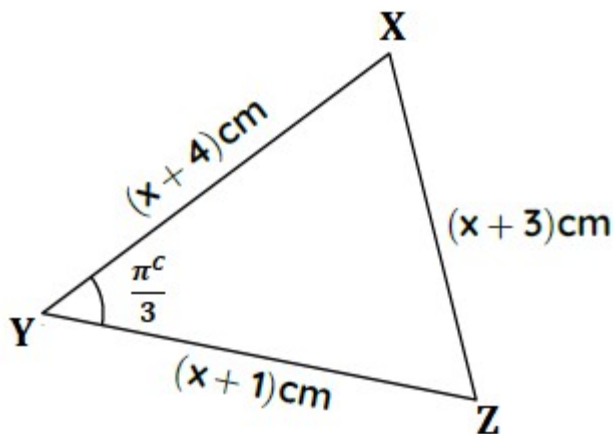
- 6.** The figure below represents a triangular prism. The faces **ABCD**, **ADEF** and **CBFE** are rectangles. **AB** = **8** cm, **BC** = **14** cm, **BF** = **7** cm and **AF** = **7** cm.



Calculate the angles between faces **BCEF** and **ABCD** correct to 1 decimal place.

- 7.** The area of a hall's floor equals **960,000** cm<sup>2</sup>. If the floor is to be covered with rhombus tiles with each of its diagonal measuring **48** cm and **25** cm, find the number of tiles required.

- 8.** The figure below shows a triangle **XYZ** whose side lengths are given in terms of **x**.

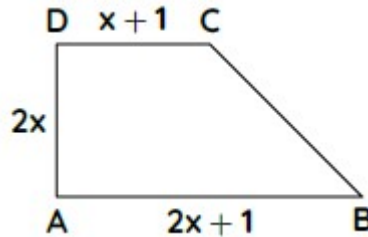


Given that the angle **XYZ** is  $\frac{\pi}{3}$  radians, determine the exact area of the triangle.

- 9.** In the binomial expansion of  $(1 + ax)^k$ , where **a** and **k** are non-zero constants, the coefficient of **x** is **8** and the coefficient of **x**<sup>2</sup> is **30**. Determine the value of **a** and the value of **k**.

- 10.** Find the image of the straight line, **L**<sub>1</sub> with equation **2x + 3y = 10**, under the transformation represented by the **2 × 2** matrix  $\begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ .

**11.** A right-angled trapezium **ABCD** is shown in the figure below.



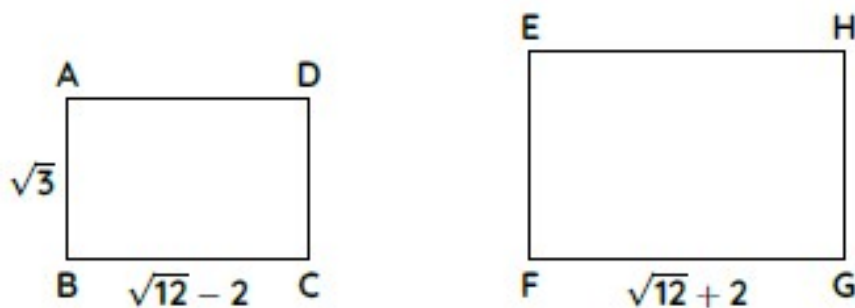
The trapezium has parallel sides **AB** and **CD** of lengths  $(2x + 1)$  cm and  $(x + 1)$  cm. The height of the trapezium **AD** is  $2x$  cm. Given that the area of the trapezium is  $16\text{cm}^2$ , determine the **exact** length of **BC**.

**12.** Two factories, **G** and **S**, undertook to fulfil an order in **12** days. After two days, factory **G** was closed down for repairs, while factory **S** continued with the fulfilment of the order. Knowing that **S** has an efficiency of  $66\frac{3}{2}\%$  that of **G**, determine in how many days the order will be completed.

**13.** A motorist travelling at a steady speed of **120 km/h** covers a section of a highway in **10** minutes. To minimize accidents a speed limit is imposed. Travelling at the maximum speed allowed, the motorist takes **5** minutes longer to cover the same section. Calculate the speed limit imposed.

**14.** The cost (**C**) of hiring a venue for a delegates' conference is partly fixed and partly varies inversely to the number **N** of delegates. When **180** delegates attend the cost is Ksh.**4500** per delegate while for **120** delegates the cost is Ksh.**6400** per delegate. Calculate the fixed cost.

**15.** The two rectangles shown in the figure below are similar.



It is further given that in suitable units

$$AB = \sqrt{3}, BC = \sqrt{12} - 2 \text{ and } FG = \sqrt{12} + 2$$

Find the exact length of **EF**.

**16.** The triangle  $T_1$  is mapped by the  $2 \times 2$  matrix  $B = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$  onto the triangle  $T_2$ , whose vertices have coordinates  $A_2(-7, -1)$ ,  $B_2(5, 5)$  and  $C_2(7, 16)$ . Determine the area of  $T_2$ .

**17.** The mean and standard deviation of the marks scored by a group of **10** students was found to be **47.0** and **11.0** respectively. An eleventh student had a score of **58** marks. Calculate the mean and standard deviation of the **11** students.

**18.** Given that  $e$  is measured in degrees, solve the following trigonometric equation

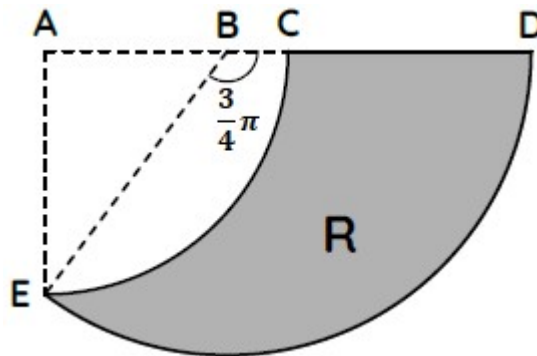
$$\frac{4}{\tan^2 3e} + 2 = \frac{7}{\sin 3e} \quad \text{for } 90 \leq e \leq 180$$

**19.** Find the values of  $x$  and  $y$  that make the objective function  $2x + 4y$  maximum subject to the conditions shown below and find the value of the objective function at these values of  $x$  and  $y$ .

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ y &\leq x + 1 \\ 4y + x &\leq 10 \\ y - x &\geq -3 \end{aligned}$$

(Solve by calculation only)

**20.** The figure below shows a part of Khisa’s blade viewed in two dimensions and constructed as follows.



- **EBD** is a circular sector with centre at **B** and radius **20** units, subtending an angle of  $\frac{3}{4}\pi$  radians at **B**.
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The shaded region **R** is bounded by the arcs **ED** and **EC**, and the straight-line **CD**. Calculate the exact area of **R**. (Leave your answer in terms of  $\pi$  and surds.)

**21.** Given that  $a \neq 0, b \neq 0, y \neq 0$  and

$$2 + \log_a b + 3 \log_a y = 2 \log_a (a^2 y).$$

Express  $y$  in terms of  $a$  and  $b$ , in a form not involving logarithms.

**22.** Solve the following logarithmic equation.

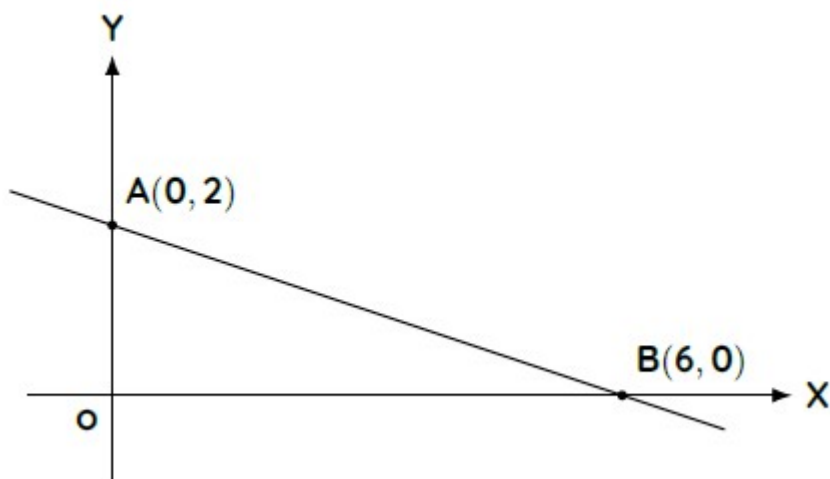
$$3 + 8 \log_{\frac{1}{k}} \left[ \sqrt{8 + 4\sqrt{3}} - \sqrt{8 - 4\sqrt{3}} \right] = 0 \quad \text{where } k > 0 \text{ and } k \neq 1$$

**23.** It is given that if  $k$  is a constant then,

$$(1 + K\sqrt{3})^4 = 892 - 336\sqrt{3}$$

Determine the value of  $k$ .

**24.** The variables  $x$  and  $y$  are thought to obey a law of the form  $y = hx^t$  where  $h$  and  $t$  are non-zero constants. Let  $X = \log_{10} x$  and  $Y = \log_{10} y$ . The figure below shows the graph of  $Y$  against  $X$ .



Determine the value of  $h$  and the value of  $t$ .

**—THE END —**

# **ANSWER SHEET**

<b>NAME OF PARTICIPANT</b>	
<b>STREAM</b>	
<b>ADM.NO</b>	
<b>HOUSE</b>	
<b>MATH TEACHER</b>	
<b>PARTICIPANT'S SIGNATURE</b>	

**“We believe in God the inventor of Mathematics and the greatest mathematician of all time”**

<b>QUESTION</b>	<b>PARTICIPANT’S ANSWER</b>	<b>SCORE IN POINTS</b>	<b>EXAMINER’S MARK</b>
1.		1 Point	
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