"We believe in God, The Inventor of Mathematics and The Greatest Mathematician of all time"

THE ALLIANCE HIGH SCHOOL MATH OLYMPIAD

Instructions to participants.

TIME: $[90]^2$ SECS

JUNIOR TEST

1. Read all problems **carefully.**
2. All calculators and mathematical tables are ALLOWED if they may help.
3. Geometrical sets and pens with green or colourless inks are allowed.
4. Only use a pencil on the answer sheet.
5. Give the exact answers only i.e., **8π,** $\sqrt{6}$ **,** $\frac{\pi}{7}$**.**
6. Do not write any marks on the question paper.
7. Crying is allowed but silently.
8. For every $\mathbf{k^{th}}$ numbered question divide your answer by **k** where $k = 2a$ and $a \geq 1$
9. Screaming is allowed but outside.
10. **Do not open the question booklet until you are told to do so**.
11. In **section I** each problem is worth **5 points**, in **section II** each problem is worth **8 points** and in **section III** each question is worth **10 points**.
12. This is a test of both **solutions and answers**. Write the **answers** in the answer sheet provided and the **solutions** in the plain sheet given and attach. Use a visible **pencil or pen** and arrange your work **neatly.**

MATHEMAGICS CLUB EXECUTIVE COMMITTEE

1. MUKOYA KHISA	**CHAIRMAN**	5. GEOFFREY GIKONYO	- **FORM 3 REP 3**
2. MATILU MUYEKU	**VICE CHAIRMAN**	6. CLIVE MAINA	- **FORM 3 REP 2**
3. IVAN MAYABI	**SECRETARY**	7. SAMWEL WAREGA	- **FORM 3 REP 1**
4. NIMROD NYABERI	**ORGANISING SECRETARY**		

- ✓ The Executive Committee reserves the right to nullify results of any participant who commits any form of malpractices.
- ✓ The test will be revised in the first Tuesday of the next term in the class 3B in the Admin block.

SECTION I [40 POINTS]

1. Given that there exist some distinct numbers $\propto_1 \propto_2, \ldots, \propto_{n-1} \propto_n$. Such that.

$$(\propto_1 + \propto_2 + \propto_3 + \ldots + \propto_{n-2} + \propto_{n-1} + \propto_n)^2 = \propto_1^3 + \propto_2^3 + \propto_2^3 + \ldots + \propto_{n-2}^3 + \propto_{n-1}^3 + \propto_n^3$$

Then find $(\propto_1 + \propto_2 + \propto_3 + \ldots + \propto_{n-2} + \propto_{n-1} + \propto_n)$ where $n = 2023$.

2. Determine the last four digits of **M**. Given that.

$$M = 1 + 13 + 26 + 39 + 52 + \ldots + 2912 + 2925 + 2938$$

3. According to the disturbed Indian Mathematician by the name Srinivasan Ramanujan, the number **e** is a special number since it is the smallest number expressible as a sum of two cubes in exactly two ways. What is the value of **e** and what are the two expressible summations?

4. Given that the product of two roots of the equation $4x^4 - 24x^3 + 31x^2 + 6x - 8 = 0$ is 2023^0. Determine all the roots of the equation.

5. Given the sum below, determine its exact value and derive a formula to determine the sum.

$$1.3^1 + 2.3^2 + 3.3^3 + \ldots + n - 2.3^{n-2} + n - 1.3^{n-1} + n.3^n$$

6. Given that a, b, c and d are some distinct integers such that.

$(k-a)(k-b)(k-c)(k-d) = 9$, Then prove that $4|(a+b+c+d)$.

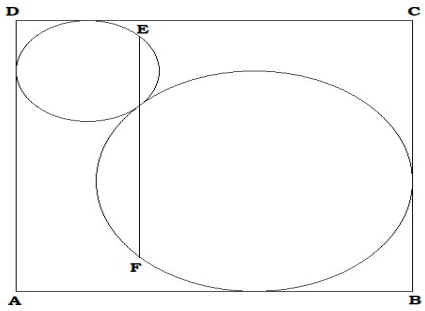
7. Determine the exact number of digits in K. Given that; $K = 2023^{2023}$.

8. Determine the number of trailing zeros in **2023**!

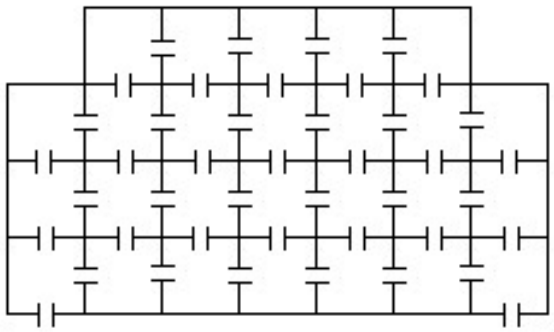
SECTION II [40 POINTS]

1. At Khisa's party, there are some 2023 people, who are either liars or truth tellers; liars always lie while truth tellers always tell the truth. After the party is over, you ask each person, "How many truth tellers did you shake hands with?" Each person gave a different answer, ranging from 0 to 2022.(***The answers were***; **1, 2, 3, 4…, 2020 , 2021 , 2022**). How many liars were at Khisa's party?

2. Determine the exact length of **DC**, given that **EF** is **6** and $AD = 8$.



3. Khisa visited all the **26** rooms without re-entering any one of them, then determine and show on the diagram below the route that Khisa took.



4. Given that.

$$x + y = 1 \text{ and } x^2 + y^2 = 2$$

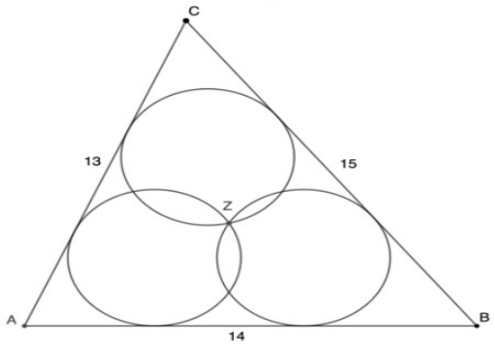
Then determine the value of $x^{11} + y^{11}$.

5. Determine the exact value of;

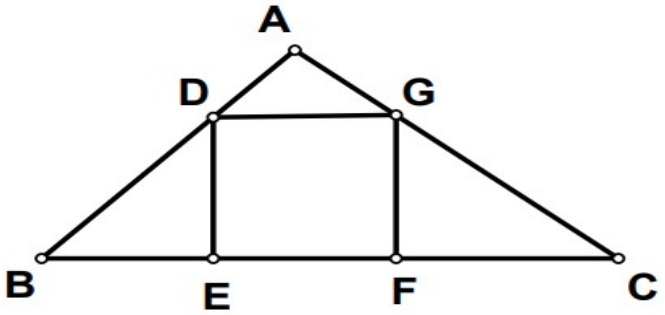
$$\sqrt{\underbrace{11111\ldots1111}_{\text{2024 Digits}} - \underbrace{22222\ldots222}_{\text{2012 Digits}}}$$

SECTION III [40 POINTS]

1. The figure below shows a **13-14-15** triangle of which three congruent circles each tangent to two sides of the triangle are concurrent at Z. If it is possible to determine their radius, then find it, express it in the form $\frac{p}{q}$, and determine **p + q** if it is not possible then prove it.

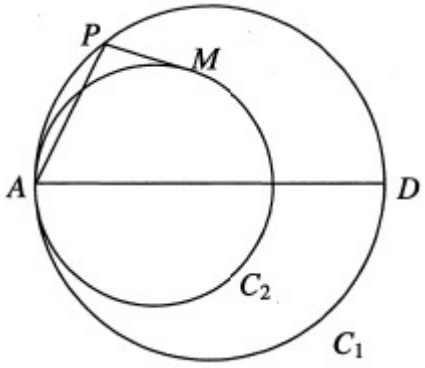


2. In the figure below $< \mathbf{BAC} = \mathbf{90°}$ and **DEFG** is a square. If the length of BC is $\frac{\mathbf{185}}{\sqrt{\mathbf{6}}}$ and the area of **ABC** is **1369**, then determine the exact area of the square **DEFG**.



3. Given that $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ and $\boldsymbol{d}$ are some positive integers with the sum **63**. Show by proof the maximum value of $\boldsymbol{ab} + \boldsymbol{bc} + \boldsymbol{dc}$.

4. Two circles $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$ of radii **10 cm** and **8 cm** respectively are tangent to each other internally at a point **A**. **AD** is the diameter of $\boldsymbol{C_1}$ and **P** and **M** are points on $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$ respectively such that **PM** is tangent to $\boldsymbol{C_2}$, as shown in the figure below. If $\boldsymbol{PM} = \sqrt{\mathbf{20}}\ \boldsymbol{cm}$ and $< \boldsymbol{PAD} = \boldsymbol{k}°$ find the value of **k**.



SETER – MUKOYA KHISA {Head of the Examination and Analysis Department}

THE END.