"We believe in God the inventor of Mathematics and the greatest mathematician of all time"

THE ROUND TWO INTER-CLASS MATHEMATICS CONTEST

TIME;

[XX.IIIIIVIIIXIIVIIIIV…]^3

SECONDS

[002]

VII:XV P.M—IX:XLV P.M THURSDAY.8TH.JUNE.2023.

VENUE:THE LIBRARY.

INSTRUCTIONS TO PARTICIPANTS

1. Read all questions **CAREFULLY.**
2. A silent scientific calculator is **ALLOWED**
3. Mathematical tables are **NOT ALLOWED.**
4. Giving of exact answers is **OPTIONAL** i.e., $\mathbf{8\pi}$, $\sqrt{6}$, $\frac{\pi}{7}$ also well-approximated answers **ARE PERMITTED.**
5. **DO NOT WRITE ANY MARKS ON THIS TEST BOOKLET.**
6. Crying is allowed but silently.
7. For every even numbered question divide your answer by a half.
8. Screaming is allowed but outside.
9. **DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO.**
10. Participants are expected to answer the questions of **THEIR CLASS ONLY** and the open section.
11. You are expected to answer only **SEVEN QUESTIONS.**
12. Each question is worth **7 POINTS.**
13. This is a test of both **SOLUTIONS AND ANSWERS**. Write the **WORKING WITH THE ANSWERS** in the **PLAIN SHEET** provided. Use a visible **PENCIL OR PEN** and arrange your work **NEATLY.**

NOTE:

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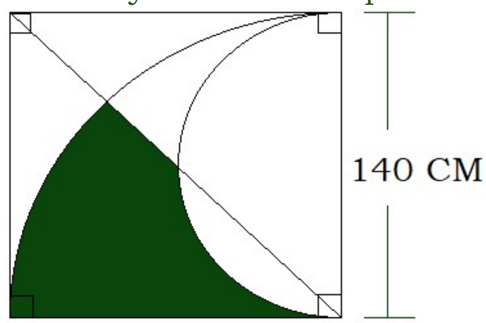
- The Executive Committee of the Alliance High School Mathematics Club (ECAHSMC) reserves the right to disqualify all scores of any participant if it determines that the required security procedures have not been followed.

FORM ONE TEST

SECTION I

1. Determine the area of the shaded region without application of Calculus but limiting yourself to Trigonometry and Geometry. **ABCD** is a square of side 140cm.



{**Hint**; Diagram not drawn to scale}

2. Let **A** and **B** be some real numbers such that;

$$\mathbf{221A - 34B = 323}$$

Determine the exact value of $\mathbf{143A - 22B}$.

SECTION II

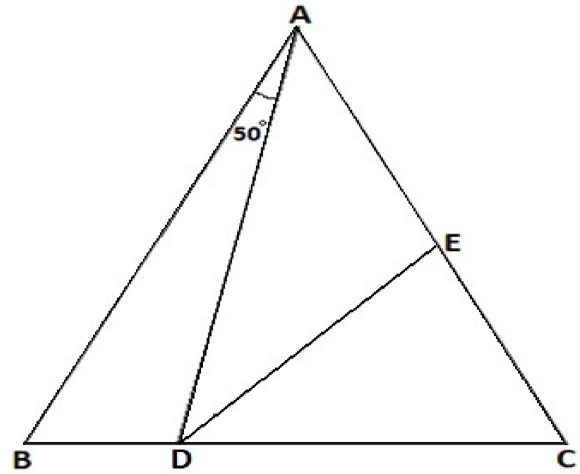
3. Given that **x** and **y** are some positive integers such that $\mathbf{735x = y^5}$. Determine the smallest possible value of $(\mathbf{x - y})(\mathbf{x + y + xy})$.

4. Determine the smallest number such that when it is divided by **7**, it leaves a remainder of **4**, when it is divided by **13** it leaves a remainder of **7**, when it is divided by **17** it leaves a remainder of **12** and when it is divided by **6** it leaves a remainder of **4**.

SECTION III

5. Given that some numbers **a**, **b** and **c** are real numbers such that: $|a - b| = 12$, $|b - c| = 6$, $|c - a| = 9$ and $abc = 160$.
Then find the value of: $\frac{a}{bc} + \frac{b}{ca} + \frac{c}{ab} - \frac{1}{a} - \frac{1}{b} - \frac{1}{c}$.

6. Consider the figure below where $\mathbf{AB = AC}$, $< \mathbf{BAD = 50°}$ and $\mathbf{AE = AD}$. Determine the exact size of $< \mathbf{CDE}$.



{**Hint**; Diagram not drawn to scale}

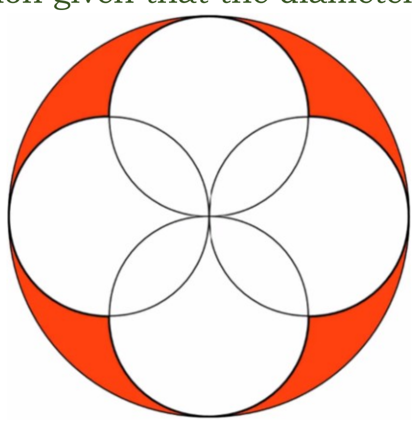
7. Determine all values of $\boldsymbol{\varepsilon}$ that completely satisfy the equation below.

$$\boldsymbol{\varepsilon^5 + \varepsilon^4 + \varepsilon^3 + \varepsilon^2 + \varepsilon^1 = -1}$$

FORM TWO TEST

SECTION I

1. In the figure below, four concentric circles are inside a larger circle. Each of the four circles is tangent to a point in the large circle. Each of the four circles intersects with two adjacent circles. All the four circles intersect at the center of the larger circle. Determine the exact area of the shaded region given that the diameter of the larger circle is **168cm**.



2. Determine the smallest possible value of $\mathbf{k}$ such that;

$$\mathbf{k + 3k + 5k + 7k + 9k + 11k + \cdots + 2023k}$$

is a perfect cube.

SECTION II

3. Given that **a** and **b** are some positive integers such that $\mathbf{a^2 - b^2 = 2023}$. Determine all ordered pairs $(\boldsymbol{a}, \boldsymbol{b})$.

4. Given that, $\sin\vartheta = 3\cos\vartheta$. Determine the exact value of $2023\{\sin^5\vartheta\cos^3\vartheta\}$. Give also the approximate if not the exact value of ϑ.

SECTION III

5. A right-angled triangle **ABC** has perimeter **2023cm** and area **26299cm²**. Find its length, width and hypotenuse.

6. Determine all the integer values of **p** for which $p^2 + 19p + 97$ is a perfect square.

7. Alix determined the sum of the first **n** consecutive integers and his answer was **2047423**. He later noticed that he had counted a digit **g** thrice, a digit **k** twice and a digit **d** twice. What are these three digits and what is their sum?

FORM THREE TEST

SECTION I

1. Three well-armed policemen; Matilu, Nzioka and Mukoya are manning a prestigious bank by the name, **'Khiser Commercial Bank'**. On his way in, a thief by the name Onsarigo has to pass through Matilu, Nzioka and Mukoya. The probability that Onsarigo is caught by Matilu, Nzioka and Mukoya is **0.6**, **0.4** and **0.75** respectively. On his way out, the probability that Onsarigo is caught by Matilu, Nzioka and Mukoya is **0.5**, **0.2** and **0.4** respectively. Represent the above situation on a tree diagram.
   a) Using the above information determine the probability that Onsarigo is caught.
   b) Determine the probability that Onsarigo is caught by Officer Matilu.
   c) Determine the probability that Onsarigo is not caught given that on his way out, Mukoya had gone to pick a call from his beautiful wife.

2. The speed of Geoffrey's boat in still and calm waters is $45km/h$. the boat moves from a point **K** to another point **X** upstream and back to **K** in **7 hours 45 minutes**. If the river current is flowing at a speed of $6km/h$, calculate the distance between **K** and **X**.

SECTION II

3. Using the concept logarithms evaluate the value of **E**. Given that;

$$\mathbf{E} = \left[\sqrt{\frac{(7.653 - 3.451)2.226^{\frac{1}{3}}}{\{(0.4572)^5 \times 4575\}^{2.5}}}\right]^{5.5}$$

(HINT; DO NOT USE A MATH TABLE)

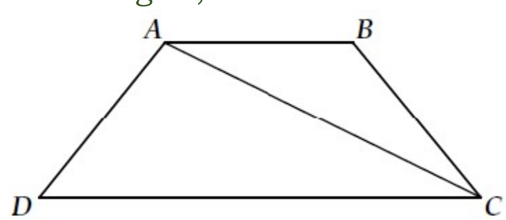
4. A unique sequence is formed by relating the corresponding values of an A.P and a G.P such that the first term of the unique sequence is obtained by adding the product of the corresponding terms of the A.P and G.P and the corresponding term of the A.P. The common ratio of the G.P is 6.
   a) Given that the first three terms of the unique sequence are 20,152 and 1199. Find
      i. The first term of the G.P, the first term of the A.P. and the common difference of the A.P.
      ii. Determine the **2023rd** term of the unique sequence.
      iii. Determine the sum of the first seven terms of the unique sequence.

SECTION III

5. Determine the value of **N**. Given that;

$$\mathbf{N} = \left[\mathbf{1} - \frac{\mathbf{67}}{\mathbf{2}}\right] \times \left[\mathbf{1} - \frac{\mathbf{67}}{\mathbf{3}}\right] \times \left[\mathbf{1} - \frac{\mathbf{67}}{\mathbf{4}}\right] \times \left[\mathbf{1} - \frac{\mathbf{67}}{\mathbf{5}}\right] \times \cdots \times \left[\mathbf{1} - \frac{\mathbf{67}}{\mathbf{2022}}\right] \times \left[\mathbf{1} - \frac{\mathbf{67}}{\mathbf{2023}}\right]$$

6. **ABCD** is an isosceles trapezium with **AB** parallel to **DC**, **AC = DC** and **AD = BC**. If the height of the trapezium is equal to **AB**, find the exact ratio of **AB** to **DC** and hence determine the ratio of the area of triangles; **ADC** and **ABC**.



7. Let $\boldsymbol{m} = \mathbf{100}$. Evaluate the infinite sum **K** given that;

$$\mathbf{K} = (\log_m 2)^0 (\log_m 5^{2^0}) + (\log_m 2)^1 (\log_m 5^{2^1}) + (\log_m 2)^2 (\log_m 5^{2^2}) + \cdots$$

FORM FOUR TEST

SECTION I

1. Given that $\mathbf{P^3 + Q^3 = 637}$ and $\mathbf{P + Q = 13}$. Determine the exact value of $(\mathbf{P - Q})^2$.

2. Gesimba has three harvesters; *Isuzu*, *Honda* and *Toyota*. It takes all the three harvesters **4 hours 30 minutes** to complete a job. On one occasion, all began working together. After **40 minutes**, the *Isuzu* broke down. After another **1 hour**, the *Honda* broke down when the job was half way. If it took **6 hours** to complete the job. Determine how long it would take each harvester working alone to complete the job.

SECTION II

3.
   a) Expand and simplify $(\mathbf{3}-\boldsymbol{k})^{\mathbf{13}}$ up to the $\mathbf{11}^{\boldsymbol{th}}$ term.
   b) Use the simplified expression in (a) above to estimate the value of $(\mathbf{2.97})^{\mathbf{13}}$ giving your answer correct to 9 significant figures.

4. Determine all positive integers **a**, **b** and **c** that completely satisfy the following equation.
   $\mathbf{a}^{\mathbf{b}^{\mathbf{c}}-\mathbf{2}} \times \mathbf{b}^{\mathbf{c}^{\mathbf{a}}-\mathbf{2}} \times \mathbf{c}^{\mathbf{a}^{\mathbf{b}}-\mathbf{2}} = \mathbf{83}$.

SECTION III

5.

a) Complete the table below for the function.

$\boldsymbol{y} = \mathbf{2}\boldsymbol{x}^{\mathbf{3}} + \mathbf{10}\boldsymbol{x}^{\mathbf{2}} + \mathbf{7}\boldsymbol{x} + \mathbf{5}.$ (Give y values correct to 2dp)

x	-4.50	-4.00	-3.50	-3.00	-2.50	-2.00	-1.50	-1.00	-0.50	0.00	0.50	1.50	2.00
y	-6.25			20.00					3.75			**44.75**	**75.00**

NOTE: Draw the table in your plain sheet.

b) Draw the graph of $\mathbf{y} = \mathbf{2x^3} + \mathbf{10x^2} + \mathbf{7x} + \mathbf{5}$ for $-\mathbf{4.5} \le \mathbf{x} \le \mathbf{0.5}$ on the graph paper provided. (Using 1cm to represent 0.5 on the x- axis and 1cm to represent 1.0 on the y axis)

c) Use your graph to solve the following simultaneous equations.

   i. $\boldsymbol{y} = \mathbf{2}\boldsymbol{x}^{\mathbf{3}} + \mathbf{10}\boldsymbol{x}^{\mathbf{2}} + \mathbf{7}\boldsymbol{x} + \mathbf{5}$
   ii. $\boldsymbol{y} = \mathbf{3}\boldsymbol{x} + \mathbf{15}$

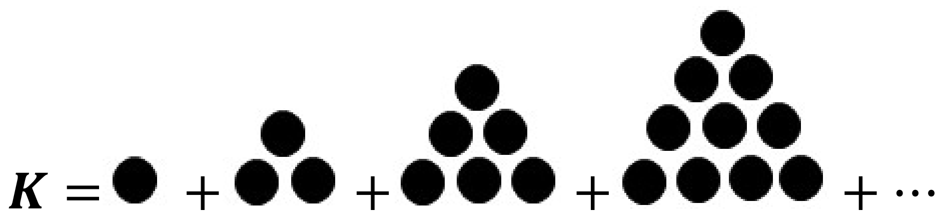
d) Determine a cubic equation in **x** whose roots are the **x** values you found in (c) above.

e) From your graph state the root of $\mathbf{2}\boldsymbol{x}^{\mathbf{3}} + \mathbf{10}\boldsymbol{x}^{\mathbf{2}} + \mathbf{7}\boldsymbol{x} + \mathbf{5} = \mathbf{0}$.

f) From the graph determine the area bounded by the curve and the line of the function $\boldsymbol{y} = \mathbf{3}\boldsymbol{x} + \mathbf{15}$.

6. Using a ruler and a compass **only.** Construct $\Delta\mathbf{KEM}$ such that $\mathbf{KE = 7.7cm}$, $< \mathbf{MKE} = \mathbf{83°}$ and $< \mathbf{MEK} = \mathbf{80°}$. (**Hint**: Do not use a protractor).
   a) Construct a locus of point **X** such that $< \mathbf{KEM} =< \mathbf{MXK}$, $< \mathbf{MKE} =< \mathbf{MXE}$ and $< \mathbf{EMK} =< \mathbf{EXK}$.
   b) Construct a locus of point **A** which is equidistant from the line segments; **KE**, **EM** and **KM**.
   c) Locate $\mathbf{Q_1}$ and $\mathbf{Q_2}$ on **X** such that $\mathbf{Q_1Q_2}$ intersects with the lines **MK** and **ME** dividing them in the ratio $\mathbf{1:3}$ and $\mathbf{4:4}$ respectively.
   d) Determine the ratio of the area of $\Delta\mathbf{KEM}$ and the area $\Delta\mathbf{Q_1Q_2A}$.
   e) Construct a perfect circle such that it touches **ME**, **KE** and **KM** produced and is also opposite to $< \mathbf{EKM}$.

7. Consider Khisa wishes to define a new sequence **K** made up of terms of consecutive triangular numbers. Such that $\mathbf{K = 1, 3, 6, 10, 15, \cdots}$. Define and derive a formular for the expression of the $\mathbf{n^{th}}$ term and the summation of infinite terms of the sequence. **(Hint)**



OPEN SECTION

1. Given that $\boldsymbol{\pi} = \mathbf{4}$.Provide proofs as to why this assumption may be true.

2. Given that $\mathbf{0 = 1}$. Provide proofs that satisfy this assumption.

3. If $\lim\limits_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{\delta y}{\delta x} = ax^n$,then $y = \frac{ax^{n+1}}{n+1} + c$ for all integral values of **n.** Prove or disapprove.

4. Given that $[0 \times \infty] = 0$. Prove;
   a) By the concept of ‘Limits’.
   b) By the concept of ‘Traditional Arithmetic’.
   c) By the concept of The ‘Wheel Algebra’.

Whether the above assumption is true or not.

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EXAMINER – MUKOYA KHISA {**Head of the Examination and Analysis Department**}

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Approval signature,

Sir Jonathan Mbithi,

Mathematics Club Patron.

Courtesy of The Alliance High School Mathematics Club in partnership with The Alliance High School Mathematics Department and The Alliance High School Mathematics Community (AHSMC).