THE ALLIANCE HIGH SCHOOL MATH OLYMPIAD

SENIOR TEST

TIME: $[90]^2$ SECS

Instructions to participants.

1. Read all problems **carefully.**
2. All calculators and mathematical tables are allowed if they may help.
3. Geometrical sets and pens with green or colourless inks are allowed.
4. Only use a pencil on the answer sheet.
5. Give the exact answers only i.e., $\mathbf{8\pi}$, $\sqrt{6}$,$\frac{\pi}{7}$.
6. Do not write any marks on the question paper.
7. Crying is allowed but silently.
8. For every $\mathbf{k^{th}}$ numbered question divide your answer by **k** where $k = 2a$ and $a \geq 1$
9. Screaming is allowed but outside.
10. **Do not open the question booklet until you are told to do so**.
11. In **section I** each problem is worth **5 points**, in **section II** each problem is worth **8 points** and in **section III** each question is worth **10 points**.
12. This is a test of both **solutions and answers**. Write the **answers** in the answer sheet provided and the **solutions** in the plain sheet given and attach. Use a visible **pencil or pen** and arrange your work **neatly.**

MATHEMAGICS CLUB EXECUTIVE COMMITTEE

1. MUKOYA KHISA	**CHAIRMAN**	5. GEOFFREY GIKONYO	**- FORM 3 REP 3**
2. MATILU MUYEKU	**VICE CHAIRMAN**	6. CLIVE MAINA	**- FORM 3 REP 2**
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4. NIMROD NYABERI	**ORGANISING SECRETARY**		

- ✓ The Executive Committee reserves the right to nullify results of any participant who commits any form of malpractices.
- ✓ The test will be revised in the first Tuesday of the next term in the class 3B in the Admin block.

SECTION I [40 POINTS]

1. Consider we have a unique number $\boldsymbol{m}$ such that:

$$\frac{1}{\infty}+\frac{1}{2^1}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\dots+\frac{1}{2^{\infty-2}}+\frac{1}{2^{\infty-1}}+\frac{1}{2^\infty}=m$$

Given that **m** could be written in the form e^k *where* $e \geq 2023$.Then determine the exact value of k, and express it in the form $\frac{a}{b}$ and hence also find the value of $a+b$.

2. Given that $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{a+b+c}$ then prove that $\frac{1}{a^{2023}}+\frac{1}{b^{2023}}+\frac{1}{c^{2023}}=\frac{1}{a^{2023}+b^{2023}+c^{2023}}$.

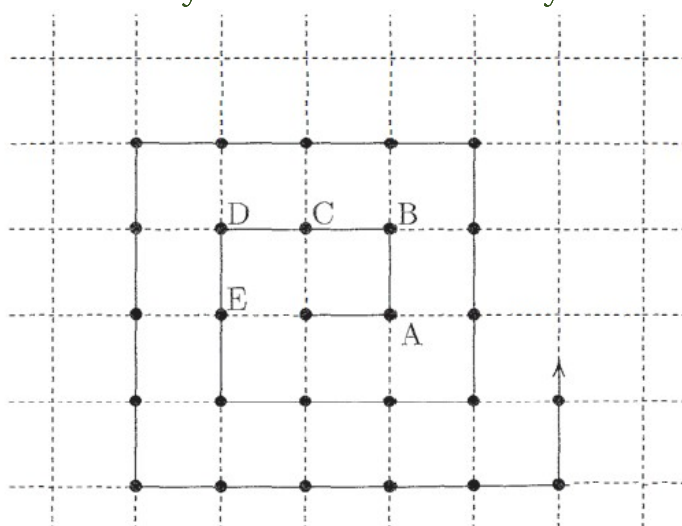
3. Determine the exact value of **K** given that:

$$K=1+4+9+\dots+2021^2+2022^2+2023^2.$$

4. Calculate the exact value of the following sum.

$$\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\frac{5}{32}+\dots+\frac{2022}{2^{2022}}+\frac{2023}{2^{2023}}$$

5. You walk a sprawling maze on the Cartesian plane as follows; *(0,0)* and the first five stops are at ***A (1,0), B (1,1) C (0,1), D (-1,1)*** and ***E (-1,0)***. Your ninth stop is at ***(1, -1)***. What are the coordinates of the point which you would arrive at on your 2023^{rd} stop.



6. Determine the product of.

$\mathbf{101 \times 10001 \times 100000001 \times \dots \times 10000\dots00000001}$ where the last term has exactly $2^{2023}-1$ zeros between the ones. Also determine the number of ones in the product.

7. Evaluate the following sum;

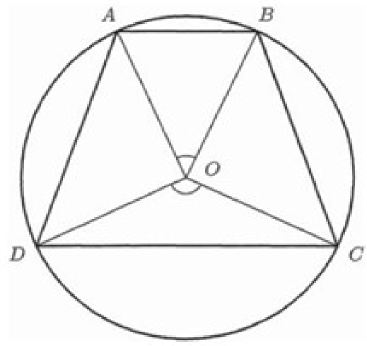
$$\frac{1}{1}+\frac{1}{2}+\frac{2}{2}+\frac{1}{3}+\frac{2}{3}+\frac{3}{3}+\frac{1}{4}+\frac{2}{4}+\frac{3}{4}+\frac{4}{4}+\frac{1}{5}+\dots$$

Up to the $\mathbf{2023^{rd}}$ term.

8. Determine the sum of all primes which can be written both as a sum of two primes and as a difference of two primes.

SECTION II [40 POINTS]

1. ABCD is a trapezium inscribed in a circle centred at O. Given that $\boldsymbol{AB//CD}$, and $<\boldsymbol{COD}=\boldsymbol{3}<\boldsymbol{AOB}$, and $\frac{\boldsymbol{AB}}{\boldsymbol{CD}}=\frac{\boldsymbol{2}}{\boldsymbol{5}}$.



Find the ratio $\frac{\boldsymbol{Area\ of\ \Delta BOC}}{\boldsymbol{Area\ of\ \Delta AOB}}$

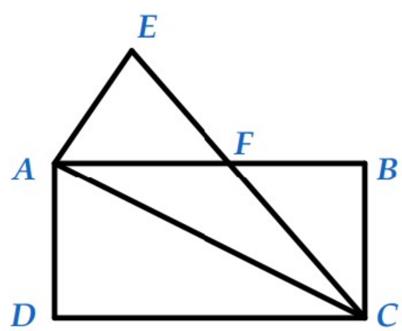
2. How many terms are there in the expansion of;

$$(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}+\boldsymbol{d}+\boldsymbol{e})^{\boldsymbol{2023}}$$

3. Determine the least positive integer **s** such that the number below is a perfect square.

$$\boldsymbol{2018}\times\boldsymbol{2020}\times\boldsymbol{2024}\times\boldsymbol{2026}+\boldsymbol{n}$$

4. ABCD is a rectangle with vertices **A, B, C, D**. It is given that $\boldsymbol{AB}=\boldsymbol{8cm}$ and $\boldsymbol{BC}=\boldsymbol{4cm}$. **E** is a point outside the rectangle such that the line segment **CE** intersects **AB** at a point **F**. Suppose that **AE = AD, CE = CD** and the area of the ΔAFC is $\mathbf{x\ cm^2}$. Find the value of **x**.



5. Mukoya has **2023** cartons such that the first carton has 5 pencils, the second carton has 7 pencils, the third carton has 10 pencils and in general, the $\boldsymbol{k^{th}}$ carton has exactly **k** pencils more than the number of pencils in $(\boldsymbol{k-1})^{\boldsymbol{th}}$ carton. Determine the exact total number of pencils in all the **2023** cartons Mukoya had.

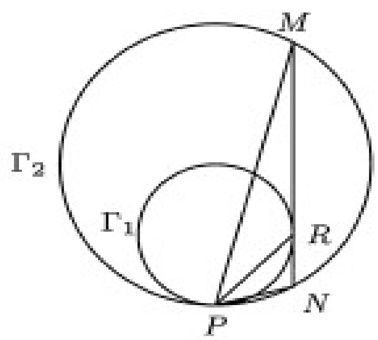
SECTION III [40 POINTS]

1. Using **Nesbitt's inequality**, prove that indeed $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\geq\frac{3}{2}$ given that $\boldsymbol{a,b,c}\in\mathbb{R}^{+}$.

2. A census man visited Mukoya, he asked him, "How many children do you have and what are their exact ages?" Mukoya replied, "I have a son and a daughter." The census man said, "proceed". Mukoya replied, "The last digit of the product of their ages is equal to my beautiful wife's favourite number." The census man still pleaded for the exact ages of the children. Mukoya said, "I'm sorry but even if I gave you the sum of their ages you will still be confused but note that at one point one of my kids who loves biscuits was $\boldsymbol{k}$ times older than the other kid. Given that $\boldsymbol{k}$ is Mukoya's lucky number, determine the least possible age of Mukoya's son and daughter and his lucky number.

   −Mukoya's Census Taker Problem −

3. Two circles $\boldsymbol{T_1}$and $\boldsymbol{T_2}$ with radii $\boldsymbol{r_1}$and $\boldsymbol{r_2}$ respectively, touch internally at the point **P**. A tangent parallel to the diameter through **P** touches $\boldsymbol{T_1}$ at **R** and intersects $\boldsymbol{T_2}$ at **M** and **N**. Prove that **PR** bisects $<\boldsymbol{MPN}$.



4. There are 6 black balls and 8 grey balls in a bag. Five balls are drawn out at random and placed in a black box. The remaining 9 balls are put in a grey box. What is the probability that the number of black balls in the grey box plus the number of grey balls in the black box is not a prime number?

SETER – MUKOYA KHISA {Head of the Examination and Analysis Department}

THE END.