THE ALLIANCE HIGH SCHOOL

---MATH OLYMPIAD---

Instructions

TIME: $[90]^2$ SECS

JUNIOR TEST

1. Read all questions **carefully**
2. All calculators are not allowed
3. Mathematical tables are not allowed
4. Geometrical sets and green pens allowed
5. Only use a pencil on the answer sheet
6. Give the exact answers only i.e., $\mathbf{8\pi}$, $\mathbf{3\sqrt{6}}$
7. Do not write any marks on the question paper
8. Crying is allowed but silently
9. Do not open the question paper until told to do so
10. In **section I** each problem is worth **5 points**, in **section II** each problem is worth **8 points** and in **section III** each question is worth **10 points**

Mathemagics Club Committee

1.	Collins Kerama	Chairman
2.	Vic Koech	Vice Chairman
3.	Dalton Omondi	Secretary
4.	Michael Mumo	Team Leader
5.	Victor Nthiwa	The Organizing Secretary
6.	Stephen Adika	Treasurer
7.	Godwins Oloo	Form Three Representative

- ✓ Revision of the paper will be done in 3Q on Tuesday during the club meeting
- ✓ The committee reserves the right to nullify any results of any house that commits any form of malpractices.

SETER – MUKOYA KHISA

SECTION I {40 POINTS}

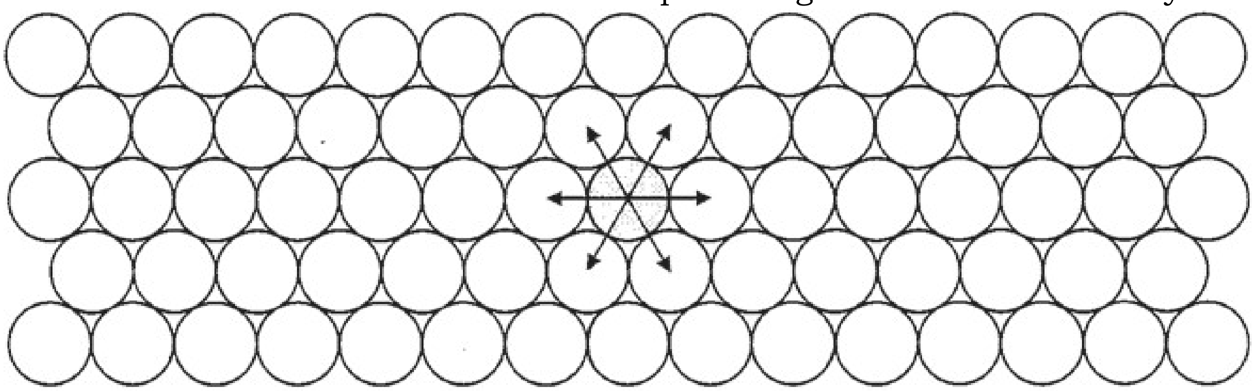
1. Given that **x** and **y** are two real numbers satisfying the following equations; $x + xy + y = 2 + 3\sqrt{2}$ *and* $x^2 + y^2 = 6$, determine the value of $x + y$.

2. Determine the exact remainder when $7^{2022} + 9^{2023} + 8^{2021}$ is divided by **128**.

3. Consider a triangle **ABC**. **D** is the midpoint of **AB**; **E** is the midpoint of **BC** and **F** is the midpoint of **AE**. Determine the ratio of the area if triangle DEF to the area of triangle **ABC**.
4. Identify all the possible ordered pairs (P, Q, R) that satisfy the equation $\mathbf{P^2 + Q^2 = R^2}$ where P, Q *and* R are positive real numbers.

5. List down all the possible values of x that satisfy the equation $x^2(2 - x)^2 = 1 + 2(1 - x)^2$

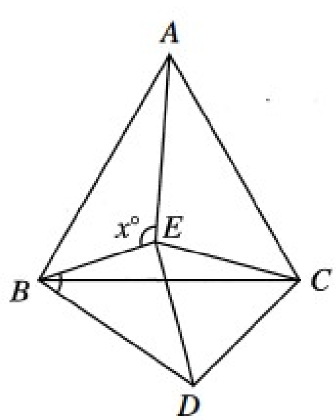
6. Coins of the same size are arranged on a large table (the infinite plane) such that each coin touches six other coins. Then find the percentage of the area covered by the coins.



7. Determine exact sum of the value of;

$$\left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{4}\right) \times \dots \times \left(1 - \frac{1}{2021}\right) \times \left(1 - \frac{1}{2022}\right)$$

8. In the diagram, ΔABC and ΔCDE are equilateral triangles. Given that $\angle EBD = 62°$ and $\angle AEB = x^0$, what is the value of x?

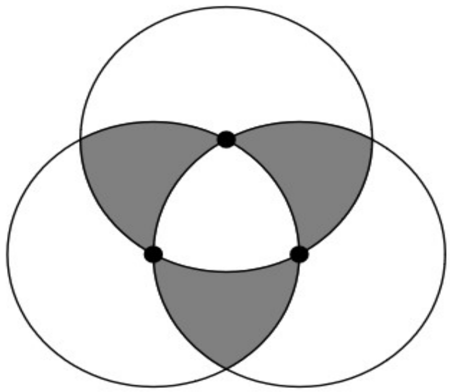


SECTION II {40 POINTS}

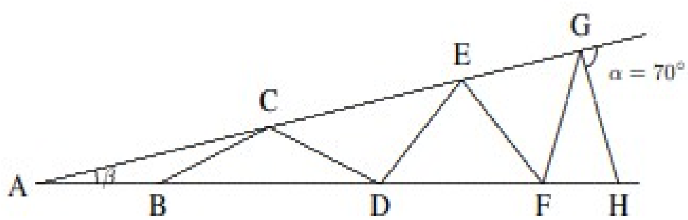
1. Determine the exact value of;

$$\frac{\mathbf{1}}{\mathbf{1+\sqrt{2}}}+\frac{\mathbf{1}}{\mathbf{\sqrt{2}+\sqrt{3}}}+\frac{\mathbf{1}}{\mathbf{\sqrt{3}+\sqrt{4}}}+\ldots+\frac{\mathbf{1}}{\mathbf{\sqrt{2020}+\sqrt{2021}}}+\frac{\mathbf{1}}{\mathbf{\sqrt{2021}+\sqrt{2022}}}$$

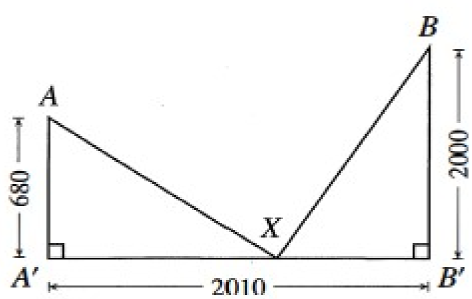
2. Three circles with radius **2** are drawn in such a way that each time one of the points of intersection of two circles is identical with the Centre of the third circle. Determine the area of the shaded part.



3. As shown in the figure, **AB = BC = CD = DE = EF = F G = GH**, ∠**α = 70**◦. Find the size of ∠β in degrees.



4. Let **AA'** and **BB'** be some two line segments which are perpendicular to **A' B'**. The lengths of **AA', BB'** and **A' B'** are **680, 2000** and **2010** respectively. Find the minimal length of **AX + XB** where **X** is a point between **A'** and **B'**.



5. Find the maximum positive integer k, such that

$$\frac{1000\times1001\times1002\times\ldots\times2021\times2022}{11^{K}}$$

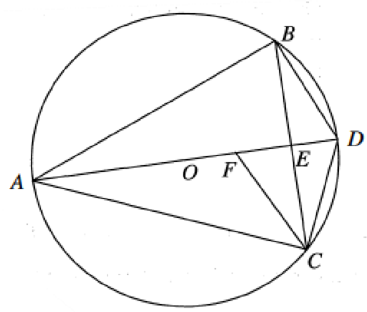
SECTION III {40 POINTS}

1. Determine the number of positive integral values $\{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}\}$ satisfying

$$\frac{\mathbf{1}}{\boldsymbol{a}}+\frac{\mathbf{1}}{\boldsymbol{b}}+\frac{\mathbf{1}}{\boldsymbol{c}}+\frac{\mathbf{1}}{\boldsymbol{d}}=\mathbf{1} \quad \text{Where } \boldsymbol{a}<\boldsymbol{b}<\boldsymbol{c}<\boldsymbol{d}$$

2. A census man went to the home of Khisa and asked the number of children he had and their exact ages. Khisa said that he has three daughters and the product of their ages is 36. The poor census man still pleaded for the precise information. Khisa then said that even if he gave the census man the sum of their ages he will still be stumped and confused but what he should note is that her eldest daughter loves biscuits. Determine the exact ages of the three daughters.

3. In the figure below, **ABC** is an isosceles triangle inscribed in a circle with center **O** and diameter **AD**, with **AB = AC**. **AD** intersects **BC** at **E**, and **F** is the midpoint of **OE**. Given that **BD** is parallel to **FC** and **BC** =$2\sqrt{5}$, find the length of **CD**.



4. How many three element subsets drawn from the set $\boldsymbol{s}=\{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \ldots, \mathbf{88}, \mathbf{89}, \mathbf{90}\}$ are such that the sum of the three elements is a multiple of three?

THE END