Computational exercise: The electric field of a continuous charge distribution
PH 220

Objective: Write a computer code that reads in the location and size of a cube with a uniform charge distribution, and then returns the components of the electric field at various points in space.

Background: In the last computational exercise, we found the electric field at some arbitrary point in space due to several point charges. This exercise is similar, but instead of point charges, we want to consider a continuous charge distribution.

The numerical process for this task is similar to what you did for multiple point charges. We simply need to divide the continuous charge distribution into many small pieces, and then treat each piece as a point particle. In the limit where the size of those pieces approaches zero, we are effectively evaluating the integral

$$\vec{E} = \int_{vol} \frac{k\, dq}{r^2} \hat{r}$$

So, really, the only thing we need to do is have some way of knowing how "small" is "small enough" when it comes to dividing up the charge distribution. One approach is to select a small size and complete the calculation, then reduce the size by a factor of two and complete the calculation again. If the two results are within some desired tolerance of each other, then you can be confident that your calculation is "good enough". This process can either be completed manually (where you run your code, edit it, and run it again), or can be built into the code itself (i.e. it will start with a certain size, and then keep reducing that size by a factor of two until the desired precision is achieved.

Exercise requirements: Write a computer code that does the following:

1. Reads a file specifying the total charge, length of a side, and location of the center of a cube. For simplicity, we are going to assume that the faces of the cube are parallel to the x-y, y-z, and x-z planes. Additionally, the code should read in the number of points at which to find the electric field, and the coordinates of those points. This input file will be formatted as follows. The first line of the input will consist of five floating point numbers representing the total charge of the cube (in nC), the length of a side (in cm), and the x, y, and z, coordinates of the center of the cube (in cm). This line is followed by a single integer specifying how many points we are finding the electric field for. This is followed by a line for each of those points, giving the x-coordinate in cm, the y-coordinate in cm, and the z-coordinate in cm. The following is a sample input file:

```
145.0 6.00 -1.50 4.00 3.00
4
    0.00    0.00    0.00
   10.00    2.00   20.00
    5.00  -12.00    0.00
    0.00   15.00  -10.00
```

2. Calculates the electric field components at each of the requested points.
3. Writes out the coordinates of the requested points, in order, and the resulting electric field components, formatted as follows:

```
  1: (  0.000,  0.000,  0.000) E=< 119971.549,-395832.258,-272403.332> N/C
  2: ( 10.000,  2.000, 20.000) E=<  17036.062,  -2956.665,  25191.179> N/C
  3: (  5.000,-12.000,  0.000) E=<  15677.117, -38618.043,  -7223.400> N/C
  4: (  0.000, 15.000,-10.000) E=<   3906.084,  28625.563, -33825.528> N/C
```

Validation: You can verify whether your code is working correctly or not by feeding it the example input file above, and seeing whether it reproduces the example output.

Grading rubric: Your grade on this exercise, out of 50 points total, will be calculated based on the following criteria:

- Your code accurately finds the electric field for a test input file that I provide. This test file will include ten test coordinates, for a total of 30 electric field components. Each component is worth one point. I will allow a 5% error on each of the components, so you might want to make sure your code is calculating the electric field to within 0.5%. ***Note: depending on how small the divisions of the cube need to be, it may take your code a while to arrive at the answer!*** (30 points)
- Your code is well organized and “pretty”. (3 points)
- The output is formatted as required. (2 points)
- You have included all of the following comments in your code:
    - A header that describes what the code does and how to use it. (5 points)
    - A description of each of the variables in your code. (5 points)
    - Frequent descriptions of what your code is doing, including any numerical methods that you are employing. (5 points)