Week 3 - Math Assignment

Question 2.34

2.34 Card game. Consider the following card game with a well-shuffled deck of cards. If you draw a red card, you win nothing. If you get a spade, you win $5. For any club, you win $10 plus an extra $20 for the ace of clubs.

	Red Card	Spade	Club (Not Ace)	Ace of Clubs
Count	26	13	12	1
Prize	0	$5	$10	$30
Probability	$\frac{26}{52} = 0.5$	$\frac{13}{52} = 0.25$	$\frac{12}{52} = 0.2308$	$\frac{1}{52} = 0.0192$

(a) Create a probability model for the amount you win at this game. Also, find the expected winnings for a single game and the standard deviation of the winnings.

Expected winning = 0.5 * 0 + 0.25 * 5 + 0.2308 * 10 + 0.0192 * 30
= **4.134**

Standard Deviation = $\sqrt{(0-4.134)^2 * 0.5 + (5-4.134)^2 * .25 + (10-4.134)^2 * 0.2308 + (30-4.134)^2 * 0.0192}$
= **5.433235**

(b) What is the maximum amount you would be willing to pay to play this game? Explain.
Given that the expected winnings for the game is about $4 (i.e 4.134) I would be willing to pay $3 and would expect to win about $1 in average per game.

Question 2.40

2.40 Baggage fees. An airline charges the following baggage fees: $25 for the first bag and $35 for the second. Suppose 54% of passengers have no checked luggage, 34% have one piece of checked luggage and 12% have two pieces. We suppose a negligible portion of people check more than two bags.

	No luggage	One luggage	Two luggage
%	54%	34%	12%
$	0	25	60

(a) Build a probability model, compute the average revenue per passenger, and compute the corresponding standard deviation.

Answer : 15.7, 19.95

Average revenue = 0 * .54 + 25 *.34 + 60 *.12
= 15.7

Standard Deviation = $\sqrt{(0-15.7)^2 * .54 + (25-15.7)^2 * .34 + (60-15.7)^2 * .12}$
= $\sqrt{398.01} = 19.95$

(b) About how much revenue should the airline expect for a flight of 120 passengers? With what standard deviation? Note any assumptions you make and if you think they are justified.
Answer: 1884, 218.54

Revenue for flight of 120 passenger = 15.7 * 120 = **1,884**
Standard Deviation = $\sqrt{Total\ Variance} = \sqrt{Variance\ per\ passenger\ * 120}$
= $\sqrt{398.01 * 120} = \mathbf{218.54}$
Assumption: The proportion of passenger checking in zero, one or two luggage is same. Given the number of passengers (i.e 120) the assumption may not hold good.

Question 2.42
2.42 Selling on Ebay. Marcie has been tracking the following two items on Ebay:

- A textbook that sells for an average of $110 with a standard deviation of $4.
- Mario Kart for the Nintendo Wii, which sells for an average of $38 with a standard deviation of $5.

(a) Marcie wants to sell the video game and buy the textbook. How much net money (profits - losses) would she expect to make or spend? Also compute the standard deviation of how much she would make or spend.
Answer: Spend $72 with standard deviation of 6.4

Net Money = $38 - $110 = ($72.00)
Marcie is expected to spend $72.
Standard deviation = $\sqrt{Total\ Variance} = \sqrt{4^2 + 5^2} = 6.4$

(b) Lucy is selling the textbook on Ebay for a friend, and her friend is giving her a 10% commission (Lucy keeps 10% of the revenue). How much money should she expect to make? With what standard deviation?
Answer: Expected to make $11 with standard deviation of 1.265

Expected commission = 110 * 10% = 11
Standard deviation = $\sqrt{Variance} = \sqrt{4^2 * 10\%} = 1.265$

Question 2.46

2.46 **Income and gender.** The relative frequency table below displays the distribution of annual total personal income (in 2009 inflation-adjusted dollars) for a representative sample of 96,420,486 Americans. These data come from the American Community Survey for 2005-2009. This sample is comprised of 59% males and 41% females.[73]

(a) Describe the distribution of total personal income.

(b) What is the probability that a randomly chosen US resident makes less than $50,000 per year?

(c) What is the probability that a randomly chosen US resident makes less than $50,000 per year and is female? Note any assumptions you make.

(d) The same data source indicates that 71.8% of females make less than $50,000 per year. Use this value to determine whether or not the assumption you made in part (c) is valid.

Income	*Total*
$1 to $9,999 or loss	2.2%
$10,000 to $14,999	4.7%
$15,000 to $24,999	15.8%
$25,000 to $34,999	18.3%
$35,000 to $49,999	21.2%
$50,000 to $64,999	13.9%
$65,000 to $74,999	5.8%
$75,000 to $99,999	8.4%
$100,000 or more	9.7%

(a) Describe the distribution of total personal income.
The total personal income seems to follow normal distribution (which is a continuous probability distribution)

(b) What is the probability that a randomly chose US resident makes less than $50,000 per year?
Answer: 0.622
Probability of < $50,000 = .022 + .047 + .158 + .183 + .212 = **0.622**

(c) What is the probability that a randomly chosen US resident makes less than $50,000 per year and is female? Note any assumptions you make.

Probability that a randomly chosen US resident makes less than $50000 and is female
= Probability of income < 50000 * Probability of selecting female
= 0.622 * 0.41 = **0.255**
Assumption: The income distribution is same for male and female.

(d) The same data source indicates that 71.8% of females make less than $50,000 per year. Use this value to determine whether or not the assumption you made in part (c) is valid
The assumption made in (c) is **not valid**. As the assumption made was that the income distribution is same for female and male, only 62.2% females should make less than $50000 as per the assumption.