Week 2 - Math Assignment

Question 2.16

PB & J. Suppose 80% of people like peanut butter, 89% like jelly, and 78% like both.
Given that a randomly sampled person likes peanut butter, what's the probability that he also likes jelly?

Answer: 0.975

78 out of 80 people who like peanut also likes jelly. So given the person likes peanut butter, the probability that he also likes jelly is $\frac{78}{80} = 0.975$

Question 2.18

2.18 Weight and health coverage, Part II. Exercise 2.14 introduced a contingency table summarizing the relationship between weight status, which is determined based on body mass index (BMI), and health coverage for a sample of 428,638 Americans. In the table below, the counts have been replaced by relative frequencies (probability estimates).

		Weight Status			
		Neither overweight nor obese (BMI $<$ 25)	Overweight ($25 \leq$ BMI < 30)	Obese (BMI ≥ 30)	Total
Health Coverage	Yes	0.3145	0.3306	0.2503	0.8954
	No	0.0352	0.0358	0.0336	0.1046
	Total	0.3497	0.3664	0.2839	1.0000

(a) What is the probability that a randomly chosen individual is obese?

Answer: 0.2839

Total probability of obese people from table is 0.2839

(b) What is the probability that a randomly chosen individual is obese given that he has health coverage?

Answer: 0.2795399

% People obese in health coverage "Yes" group $= \frac{0.2503}{0.8954} = 0.2795399$

(c) What is the probability that a randomly chosen individual is obese given that he doesn't have health coverage?

Answer: 0.3212237

% of people obese in health coverage "No" group $= \frac{0.0336}{0.1046} = 0.3212237$

(d) Do being overweight and having health coverage appear to be independent?

Answer: The probability of being overweight given the person has health coverage (0.3692205) is not far away from the probability of being overweight given the person has no health coverage (0.3422562). We may need more information to do hypothesis testing to determine if the difference is statistically significant or not. **But, given the very small difference it <u>does appear</u> that being overweight and having health coverage is independent.**

Question: 2.20

2.20 Assortative mating. Assortative mating is a nonrandom mating pattern where individuals with similar genotypes and/or phenotypes mate with one another more frequently than what would be expected under a random mating pattern. Researchers studying this topic collected data on eye colors of 204 Scandinavian men and their female partners. The table below summarizes the results. For simplicity, we only include heterosexual relationships in this exercise.[69]

		Partner (female)			
		Blue	Brown	Green	Total
Self (male)	Blue	78	23	13	114
	Brown	19	23	12	54
	Green	11	9	16	36
	Total	108	55	41	204

(a) What is the probability that a randomly chosen male respondent or his partner has blue eyes?

Answer: $\frac{\mathbf{144}}{\mathbf{204}} = \mathbf{0.7058824}$

$$Probability\ of\ male\ respondent\ or\ his\ partner\ has\ blue\ eyes$$
$$= Probability\ of\ male\ respondent\ have\ blue\ eyes + Probability\ of\ male's\ partner\ have\ blue\ eyes$$
$$- Probability\ of\ both\ male\ and\ his\ partner\ have\ blue\ eyes$$
$$= \frac{114}{204} + \frac{108}{204} - \frac{78}{204} = \frac{144}{204} = 0.7058824$$

(b) What is the probability that a randomly chosen male respondent with blue eyes has a partner with blue eyes?

Answer: $\frac{\mathbf{78}}{\mathbf{114}} = \mathbf{0.6842105}$

$$Probabililty\ of\ a\ blue\ eyed\ male\ having\ a\ blue\ eyes\ partner = \frac{blue\ eyes\ male\ with\ blue\ eyes\ partner}{total\ blue\ eyes\ male}$$
$$= \frac{78}{114} = 0.6842105$$

(c) What is the probability that a randomly chosen male respondent with brown eyes has a partner with blue eyes? What about the probability of a randomly chosen male respondent with green eyes having a partner with blue eyes?

$$Probability\ of\ a\ brown\ eyes\ male\ having\ blue\ eyes\ partner = \frac{brown\ eyes\ male\ with\ blue\ eye\ partner}{total\ brown\ eyes\ male}$$
$$= \frac{\mathbf{19}}{\mathbf{54}} = \mathbf{0.3518519}$$

$$Probability\ of\ a\ green\ eyes\ male\ having\ blue\ eyes\ partner = \frac{green\ eyes\ male\ with\ blue\ eyes\ partner}{total\ green\ eyes\ male}$$

$$= \frac{\mathbf{11}}{\mathbf{36}} = \mathbf{0.3055556}$$

(d) Does it appear that the eye colors of male respondents and their partners are independent? Explain your reasoning.
Answer: The eye colors of male respondent and their partner are **not independent**. The actual value of male having matching eye color partner deviates significantly from expected value. For example there are 52.94% blue eyed partners. If we assume random pattern then 52.94% of males that is about 60 males should have partner with blue eye. But the actual value is 78. Similarly many other combinations deviate from expected value. (If we conduct a chi-square test, the test would confirm that the eye color of male and their partner are not independent at 5% significant level)

Question 2.26
2.26 Twins. About 30% of human twins are identical, and the rest are fraternal. Identical twins are necessarily the same sex { half are males and the other half are females. One-quarter of fraternal twins are both male, one-quarter both female, and one-half are mixes: one male, one female. You have just become a parent of twins and are told they are both girls. Given this information, what is the probability that they are identical?
Answer: $\mathbf{\frac{15}{32.5}}$ **= 0.4615385**

There are 32.5% twins who are both girls (Half of 30% identical twin and one-quarter of 70% non-identical twins)
There are 15% twins who are both girls and identical
So, the probability of a twin who are both girls, being identical $= \frac{15}{32.5} = 0.4615385$