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A NUMERICAL METHOD FOR COMBINED SPIN TRACKING AND RAYTRACING OF CHARGED PARTICLES

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Abstract

A numerical method for the resolution of the differential equation $\vec{S}' = \vec{S} \times \vec{\omega}$ that governs the spin motion of a particle in a magnetic field is presented. This method has been straightforwardly implemented in the raytracing program Zgoubi which solves in a similar way the equation of motion of the particle, $\vec{u}' = \vec{u} \times \vec{b}$. Examples are given : a comparison with analytical calculations by S. Nurushev ; the calculation of spin transfer matrices of some classical optical elements ; the depolarization in the QDD spectrometer SPES 2 of Saclay, and finally a study of the depolarization near the resonance $\gamma G = \nu_z$ in the synchrotron Saturne 2.

I. INTRODUCTION

The depolarization of a particle beam travelling in a magnetic field $\vec{B}$ takes its origin in the spin precession undergone by each particle. This motion of the spin $\vec{S}$ is governed by the Thomas-BMT first order differential equation [1] :

$$\frac{d\vec{S}}{dt} = \frac{q}{\gamma m}\vec{S} \times \vec{\Omega} \tag{1}$$

where

$$\vec{\Omega} = (1+\gamma G)\vec{B} + G(1-\gamma)\vec{B}_{/\!/} \tag{2}$$

q, m, γ and G are respectively the charge, rest mass, Lorentz relativistic factor, and anomalous magnetic moment of the particle. $\vec{B}_{/\!/}$ is the component of $\vec{B}$ which is parallel to the velocity $\vec{v}$ of the particle.

A more practical notation may be introduced by normalizing equation (1) in the following way. Let $B = \left\|\vec{B}\right\|$ and $v = \|\vec{v}\|$; $ds = vdt$ is the differential path, $\frac{\gamma m v}{q} = B\rho$ is the rigidity of the particle ; $\vec{S}' = \frac{d\vec{S}}{ds} = \frac{1}{v}\frac{d\vec{S}}{dt}$ is the derivative of the spin with respect to the path.

Introducing also $\vec{b} = \frac{\vec{B}}{B\rho}$ and $\vec{b}_{/\!/} = \frac{\vec{B}}{B\rho}$, and

$$\vec{\omega} = \frac{\vec{\Omega}}{B\rho} = (1+\gamma G)\vec{b} + G(1-\gamma)\vec{b}_{/\!/} \tag{3}$$

equation (1) can be re-writen in a normalized way

$$\vec{S}' = \vec{S} \times \vec{\omega} \tag{4}$$

From the values of the magnetic factor $\vec{\omega}(M_0)$ and the spin $\vec{S}(M_0)$ of the particle at position M_0 of its trajectory, the spin $\vec{S}(M_1)$ at position M_1, following a displacement ds, is given by the Taylor expansion :

$$\vec{S}(M_1) = \vec{S}(M_0) + \frac{d\vec{S}}{ds}(M_0)ds + \frac{d^2\vec{S}}{ds^2}(M_0)\frac{ds^2}{2} + ... \tag{5}$$

The derivatives $\vec{S}^{(n)} = \frac{d^n\vec{S}}{ds^n}$ of $\vec{S}$ at M_0 are obtained by differentiating equation (4) :

$$\begin{aligned} \vec{S}' &= \vec{S} \times \vec{\omega} \\ \vec{S}'' &= \vec{S}' \times \vec{\omega} + \vec{S} \times \vec{\omega}' \\ \vec{S}''' &= \vec{S}'' \times \vec{\omega} + 2\vec{S}' \times \vec{\omega}' + \vec{S} \times \vec{\omega}'' \\ &etc... \end{aligned} \tag{6}$$

where the derivatives $\vec{\omega}^{(n)}$ are obtained from equation (3) (assuming that $\vec{b}$ is known, as is the case when raytracing).

The last point consists in getting $\vec{b}_{/\!/}$ and its derivatives from the knowledge of $\vec{b}$. This can be done in the following way. Let $\vec{u} = \frac{\vec{v}}{v}$ be the normalized velocity of the particle, then,

$$\begin{aligned}
\vec{b}_{/\!/} &= (\vec{b}.\vec{u})\vec{u} \\
\vec{b}'_{/\!/} &= (\vec{b}'.\vec{u} + \vec{b}.\vec{u}')\vec{u} + (\vec{b}.\vec{u})\vec{u}' \\
\vec{b}''_{/\!/} &= (\vec{b}''.\vec{u} + 2\vec{b}'.\vec{u}' + \vec{b}.\vec{u}'')\vec{u} + 2(\vec{b}'.\vec{u} + \vec{b}.\vec{u}')\vec{u}' + (\vec{b}.\vec{u})\vec{u}'' \\
& etc...
\end{aligned} \tag{7}$$

II. NUMERICAL TRACKING OF POLARIZED CHARGED PARTICLES

The method described previously for the resolution of the - normalized - Thomas-BMT differential equation (4)

$$\vec{S}' = \vec{S} \times \vec{\omega}$$

is similar to that used in the raytracing code Zgoubi [2] to solve the - normalized - Lorentz equation

$$\vec{u}' = \vec{u} \times \vec{b} \tag{8}$$

Namely, from the vector position $\vec{R}(M_0)$ and unit velocity $\vec{u}(M_0)$ at point M_0 of the particle trajectory, the vector position at M_1 following a displacement ds is obtained by Taylor expansion

$$\vec{R}(M_1) = \vec{R}(M_0) + \vec{u}(M_0)ds + \vec{u}'(M_0)\frac{ds^2}{2} + ... \tag{9}$$

where the derivatives $\vec{u}^{(n)} = \frac{d^n\vec{u}}{ds^n}$ are obtained by differenciations of equation (8), similarly to what was done in equation (6) for the spin $\vec{S}$.

The practical meaning of this is that the quantities $\vec{b}, \vec{b}_{/\!/}$, $\vec{u}$ and their derivatives $\vec{b}^{(n)}$ and $\vec{u}^{(n)}$ as involved in equations (6-7) for the resolution of $\vec{S}' = \vec{S} \times \vec{\omega}$, can be picked up directly from the step by step numerical resolution of (8).

This results in a straightforward implementation of this method of spin tracking in the raytracing program Zgoubi.

III. SPIN TRANSFER IN A LONG QUADRUPOLE

As a first illustration of this numerical simulation of spin motion, we refer to the note by S. Nurushev [3], in which the case of a quadrupole is studied.

The caracteristics of the quadrupole are

Gradient	=	21.57 T/m
Effective length	=	2.99 m

and the momentum of the proton beam is p = 300 GeV/c (hence, rigidity is $B\rho = \frac{p}{q} = 1000.69$ T.m, $\gamma G = 573.24$).

For a straightforward comparison, the raytracing is done assuming a sharp edge quadrupole, together with the following initial coordinates (at entrance of the magnet) :

$$x_0 = 2.8 \text{ mm} \ \ , x_0' = 0.49 \text{ mrd}, \quad z_0 = 3 \text{ mm}, \quad z_0' = 0.24 \text{ mrd}$$

(s = longitudinal axis, (sx) = horizontal plane, (sz) = vertical plane).

Three particles having the same initial conditions, but three different initial spins : $\vec{S} \equiv \vec{S}_x$, $\vec{S} \equiv \vec{S}_z$ and $\vec{S} \equiv \vec{S}_s$ are raytraced in order to get the spin transfer matrix $[M_R]$. This results in a very good agreement with the matrix $[M_N]$ stated by S. Nurushev, as can be seen below :

$$[M_N]^{(*)} = \begin{bmatrix} 0.992 & 0.008 & -0.127 \\ 0.008 & 0.992 & 0.127 \\ 0.127 & -0.127 & 0.997 \end{bmatrix} \ ; \ [M_R] = \begin{bmatrix} 0.992 & 0.008 & -0.126 \\ 0.008 & 0.992 & 0.127 \\ 0.126 & -0.127 & 0.984 \end{bmatrix}$$

A second step in this publication was to study the dependence of the depolarization on the beam dimensions. Similar initial conditions have been reproduced for the raytracing with Zgoubi :

x_0' and z_0' uniformely distributed in the range -1 mrd, +1 mrd
x_0 and z_0 uniformely distributed in the range $\pm\frac{Q}{2}$, with Q = 2,4,6,8 or 10 cm.

For the calculation of each of the mean values of the three components $\langle S_x \rangle$, $\langle S_z \rangle$ and $\langle S_s \rangle$, three sets of 200 particles each have been raytraced, each set having an initial polarization parallel to, respectively, x (transverse), z (vertical) or s (longitudinal) axis.

The resulting mean polarizations are given in table 1, together with the values published by S. Nurushev, for comparison.

$^{(*)}$A numerical check by means of formulae (19) of reference [3] showed that the values of terms M_{13}, M_{23}, M_{31} and M_{32} as published had to be corrected by a factor of 10. The value of 0.997 coult not be established from the formula given for M_{33}.

The agreement is very good for beam diameters $Q \leq 4$ cm. It appears to deteriorate strongly for $Q \geq 8$ cm ; this may certainly be interpreted in terms of lack of precision of formulae (22-24) of reference [3] for large values of the parameters σ_{x_0} and σ_{z_0}.

Taking the fringe field of the quadrupole into account is straightforward with the raytracing procedure. This has been done with a fringe fall off of 15 cm (gradient raising from 0 to 21.57 T.m). No neat effect appears from this and it can be concluded that the fringe field has negligible effects in this configuration.

Q (cm)		*From reference* [3]	*From raytracing*
	$\langle S_x \rangle$	0.978	0.978
2	$\langle S_z \rangle$	0.975	0.976
	$\langle S_s \rangle$	0.953	0.954
	$\langle S_x \rangle$	0.914	0.914
4	$\langle S_z \rangle$	0.902	0.910
	$\langle S_s \rangle$	0.815	0.824
	$\langle S_x \rangle$	0.807	0.816
6	$\langle S_z \rangle$	0.780	0.810
	$\langle S_s \rangle$	0.586	0.626
	$\langle S_x \rangle$	0.657	0.697
8	$\langle S_z \rangle$	0.609	0.690
	$\langle S_s \rangle$	0.266	0.387
	$\langle S_x \rangle$	0.463	0.571
10	$\langle S_z \rangle$	0.390	0.564
	$\langle S_s \rangle$	0.015 (*sic*)	0.135

Table 1 : Components $\langle S_x \rangle$, $\langle S_z \rangle$ and $\langle S_s \rangle$ of the mean polarisation at the exit of the quadrupole, calculated for initial polarizations, respectively (1,0,0), (0,1,0), (0,0,1). Raytracing has been performed with sets of 200 particles, having the following (random and uniform) distributions :

-1 mrd $\leq x'_0,\ z'_0 \leq +1$ mrd and $-\frac{Q}{2} \leq x_0, z_0 \leq +\frac{Q}{2}$ (column 2). S. Nurushev's values are given in column 1 for comparison.

IV. ROTATIONS IN DIPOLES AND SOLENOIDS

An elementary effect of dipoles and solenoids is the rotation of the spin with respect to a privileged axis. For simplification we consider idealized optical elements : sharp edge field, uniform field.

In a dipole magnet, the spin precesses around the field vector $(1+\gamma G)\vec{B}_z + G(1-\gamma)\vec{B}_{/\!/}$ (2), which identifies to a precession around $\vec{B}_z$ as long as the particle moves orthogonally to $\vec{B}_z$. The angle of rotation of the spin around the $z-$axis in the moving frame is (fig. 1)

$$\Delta\theta_z = \frac{q\Omega}{\gamma m}\frac{\rho\alpha}{\beta c} - \alpha = \gamma G\alpha \tag{10}$$

where α is the deviation angle of the particle in the dipole magnet.

In a solenoid, the spin precesses around the vector $(1+\gamma G)\vec{B}_s + G(1-\gamma)\vec{B}_{/\!/}$, which identifies to a precession around $\vec{B}_s$ if the particle velocity is parallel to $\vec{B}_s$. In that case the rotation angle in a solenoid of length L is (figure 2)

$$\Delta\theta_s = \frac{q\Omega}{\gamma m}\frac{L}{\beta c} = (1+G)\frac{B_s L}{B\rho} \tag{11}$$

These formulae have been "tested" for a proton ($G = 1.792847$) of rigidity $B\rho = 2.5$ T.m (hence, $\gamma G = 2.294677$) travelling on the ideal optical axis of the element.

Raytracing in a dipole of total deviation $\alpha = 45°$ (i.e. $B_z = 1.25$ T and $\rho\alpha = \pi/2$) gives

$$\Delta\theta_z = 103.265°$$

with the following spin transfer matrix, which is that of a pure rotation around the z (vertical) axis :

$$\begin{bmatrix} -0.2294 & 0 & +0.9733 \\ 0 & 1 & 0 \\ -0.9733 & 0 & -0.2294 \end{bmatrix}$$

while equation (10) leads to

$$\gamma G\alpha = 103.260°,\ \cos(\gamma G\alpha) = -0.22938,\ \sin(\gamma G\alpha) = 0.97334.$$

Raytracing in a solenoid with longitudinal field $B_s = 0.25$ T and effective length $L = 2$ m gives

$$\Delta\theta_s = 32.005°$$

and the following spin transfer matrix (pure rotation around the s (longitudinal) axis) :

$$\begin{bmatrix} 0.8480 & 0.5300 & 0 \\ -0.5300 & 0.8480 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

while equation (11) gives

$$(1+G)\frac{B_S L}{B\rho} = 32.004^\circ, \ \cos\left[(1+G)\frac{B_S L}{B\rho}\right] = 0.84801, \ \sin\left[(1+G)\frac{B_S L}{B\rho}\right] = 0.52997.$$

It is worth mentioning that the raytracing in such uniform fields is very stable with respect to the numerical integration step size ds (equation 5) which can be very large, therefore leading to rather fast computation. The reason is that in such fields, all the derivatives of $\vec{b}$ involved in equations (6-7), as well as in the resolution of $\vec{u}' = \vec{u} \times \vec{b}$ (eq. 8) are zero.

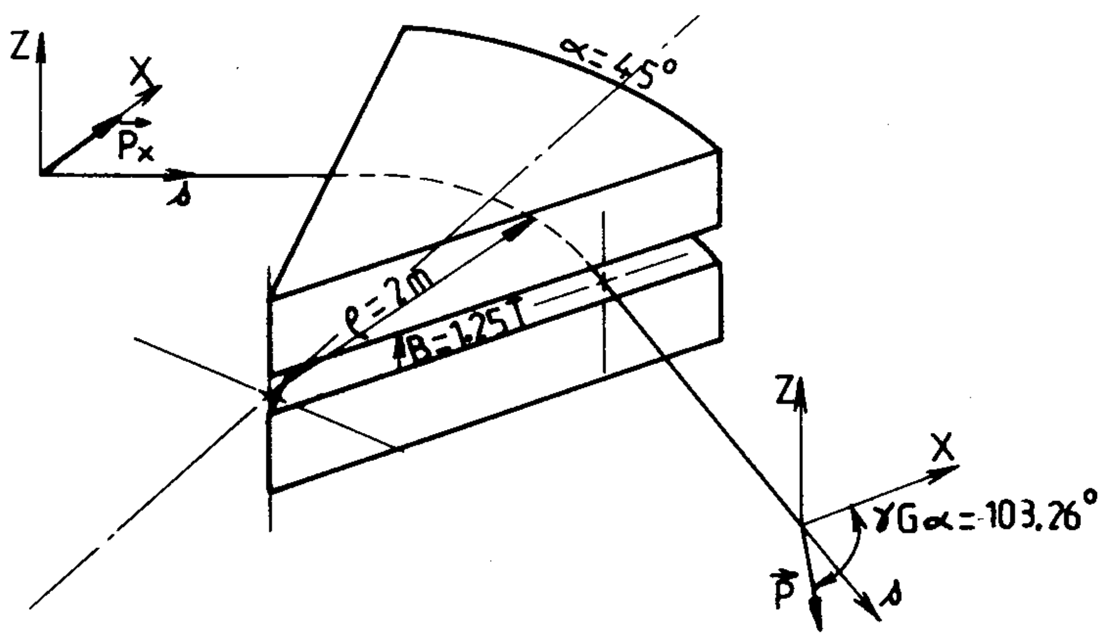

Figure 1 : Particle deviation and spin rotation in a dipole magnet.

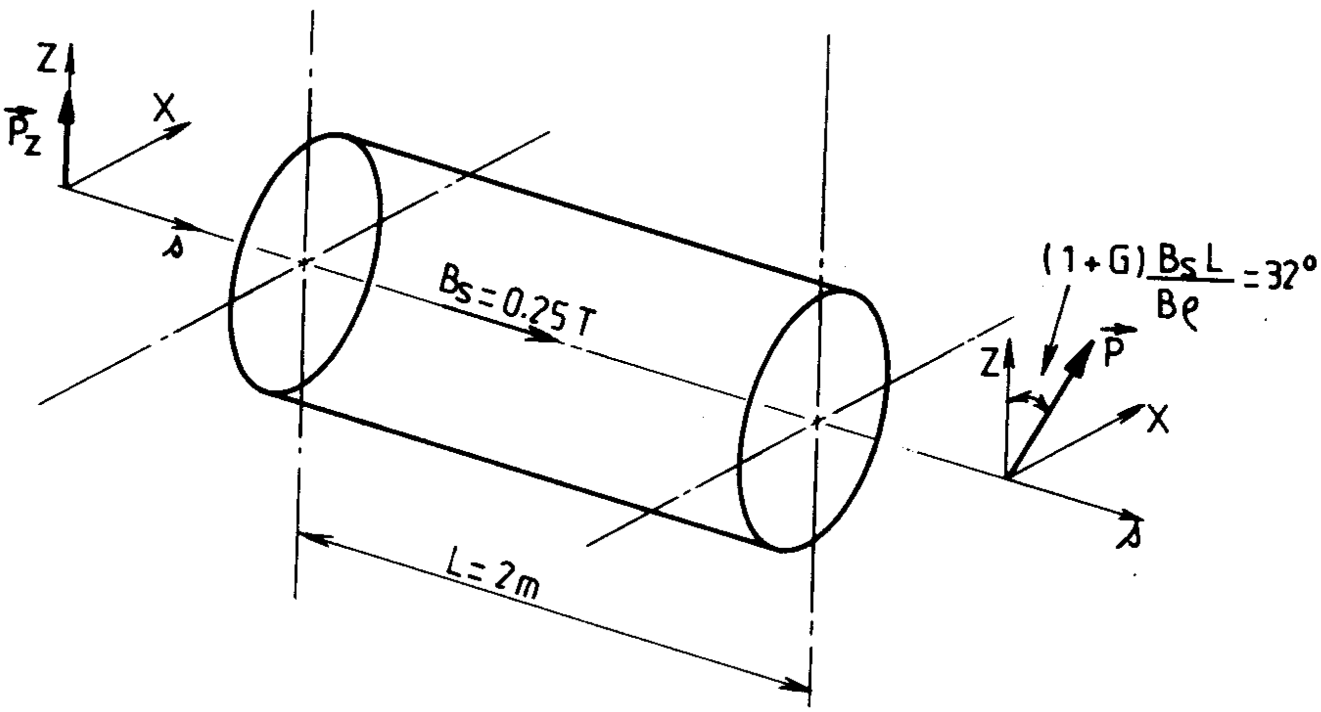

Figure 2 : Spin rotation in a solenoid.

V. SPIN TRANSFER IN THE SPECTROMETER SPES 2

In this example, we produce some figures related to the spin motion of polarized protons in the QDD spectrometer SPES 2 of Laboratoire National Saturne at Saclay [4].

The spectrometer is represented in figure 3, and its caracteristics are the following :

Momentum acceptance	$\pm$ 18 %
Focal plane width	$\simeq$ 2 m
Solid angle	20 msr
Momentum resolution	$5\ 10^{-4}$
Maximum momentum	830 MeV/c at 1.8 Tesla

For this simulation of spin tracking and raytracing through SPES 2, we use the measured field maps (i.e. 2D median plane maps) of each of the three magnets, CONCORDE, A1 and A2.

Two series of calculations are performed. A first series provides the spin transfer matrices (from target to focus) of the reference trajectory corresponding to each of the three momenta $p_0 = 700$ MeV/c and $p_0 \pm 18$ % (table 2). A second series provides the depolarization undergone at p_0 and $p_0 \pm 18$ %, for beams that fill the whole acceptance of the spectrometer (table 3). Detailed comments of the results are provided in the table captions.

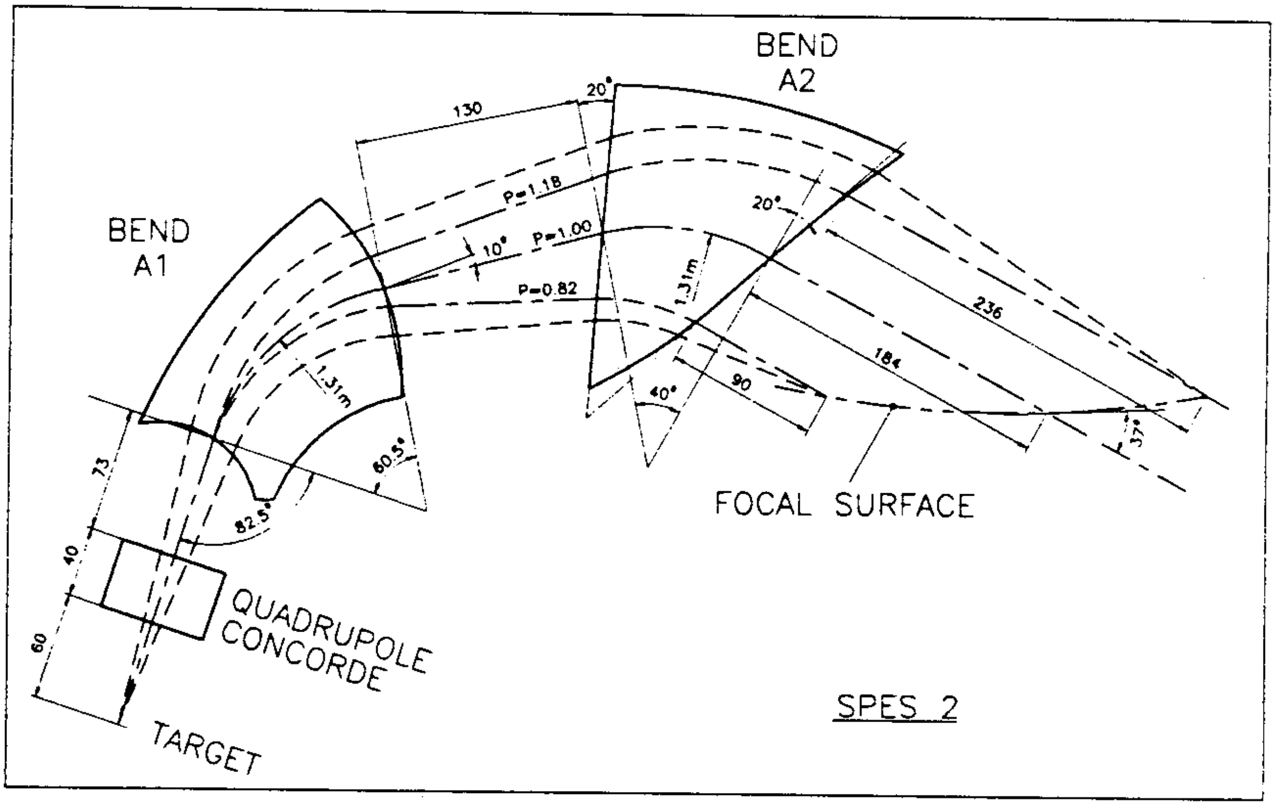

Figure 3 : Spectrometer SPES 2. The total deviation $\alpha \simeq 105°$ is almost independent of the momentum p because the dispersion coefficient $R_{26} = \frac{\delta\theta}{\delta p/p}$ of the transfer matrix is close to zero. Therefore the spin rotation $\Delta\theta_z = \gamma G \alpha$ is practically proportional to the Lorentz factor $\gamma = \sqrt{1 + \frac{p^2c^2}{E_0^2}}$.

Momentum (MeV/c)	*Spin transfer matrix*	*Equivalent rotation of the spin (deg)*	$\gamma G\alpha$ *(deg)*	*Particle deviation (from raytracing)* α *(deg)*
700 *(central momentum)* ($\gamma G = 2.2369$)	$\begin{matrix} -0.576 & 0 & 0.817 \\ 0 & 1 & 0 \\ -0.817 & 0 & -0.576 \end{matrix}$	234.82	235.10	105.10
826 ($\gamma G = 2.3887$)	$\begin{matrix} -0.327 & 0 & 0.945 \\ 0 & 1 & 0 \\ -0.945 & 0 & -0.327 \end{matrix}$	250 93	250.88	105.03
574 ($\gamma G = 2.1018$)	$\begin{matrix} -0.796 & 0 & 0.605 \\ 0 & 1 & 0 \\ -0.605 & 0 & -0.796 \end{matrix}$	217.24	224.30	106.72

Table 2 : Spin transfer matrix (from target to focus point) for a particle starting with $x_0 = x'_0 = z_0 = z'_0 = 0$, and $p_0 = 700$ MeV/c (central momentum of the spectrometer) or 700 MeV/c $\pm$ 18 % . The reference frame at the focus point has its s axis in the direction of the particle velocity. It can be seen that the matrix is that of a pure rotation around z (vertical) axis. The corresponding rotation angle is indicated in column 2. Column 3 gives the expected value for that angle, $\Delta\theta_z = \gamma G\alpha$ (eq. 10), where α is provided by the raytracing calculations (column 4). The comparison between columns 2 and 3 shows the very good agreement between analytical predictions and the numerical spin tracking, at $p_0 = 700$ MeV/c and $p_0 = 700$ MeV/c + 18 % . A slight difference ($\simeq$ 3 %) appears at $p_0 = 700$ MeV/c$-$18 % : it can be seen from figure 3 that low momentum particles travel mostly in non uniform field regions (close to the inward lateral field boundaries of the dipoles), which results in a strongly non-linear transport, therefore deteriorating the validity of the approximation $\Delta\theta_z = \gamma G\alpha$.

Momenta	*Mean polarization [S] – with respect to reference particle axis*	*Mean polarization – with respect to axis rotated by angle A (from table 3, column 2)*
700 (*central momentum*)	$\begin{matrix} -0.540 & -0.008 & -0.784 \\ 0.003 & 0.993 & -0.01 \\ 0.788 & -0.006 & -0.534 \end{matrix}$	$(A = 234.82)$ $\begin{matrix} 0.951 & -0.008 & 0.01 \\ 0.006 & 0.993 & 0.008 \\ -0.02 & -0.006 & 0.952 \end{matrix}$
826	$\begin{matrix} -0.291 & -0.008 & -0.897 \\ 0.002 & 0.994 & -0.008 \\ 0.902 & -0.003 & -0.289 \end{matrix}$	$(A = 250.93)$ $\begin{matrix} 0.943 & 0.002 & -0.02 \\ 0.008 & 0.994 & 0.004 \\ -0.02 & -0.008 & 0.947 \end{matrix}$
574	$\begin{matrix} -0.751 & -0.006 & -0.566 \\ 0.003 & 0.993 & -0.01 \\ 0.570 & -0.009 & -0.744 \end{matrix}$	$(A = 217.24)$ $\begin{matrix} 0.940 & -0.006 & -0.003 \\ 0.004 & 0.993 & 0.01 \\ -0.003 & -0.009 & 0.937 \end{matrix}$

Table 3 : Column 1 of the table gives the average polarization matrix [S] corresponding to the whole acceptance of the QDD spectrometer : -50 mrd $\leq x'_0 \leq +50$ mrd, and -50 mrd $\leq z'_0 \leq +50$ mrd, such that

$$\begin{bmatrix} \langle S_x \rangle \\ \langle S_z \rangle \\ \langle S_s \rangle \end{bmatrix}_{Focus} = [S] \begin{bmatrix} S_{x0} \\ S_{z0} \\ S_{s0} \end{bmatrix}_{target}$$

[S] is calculated by raytracing 200 particles starting with $x_0 = z_0 = 0$, and random uniform distribution in x'_0 and z'_0.

For easier understanding, it is more practical to express [S] in the reference frame which is rotated with respect to the previous one by the angle A given by the elementary spin transfer matrix (column 2 of table 2). This is what is shown in column 2. It clearly appears that the three axis undergo negligible coupling. Some depolarization occurs, stronger for lower momenta (more than 6 % depolarization for initially longitudinal spin, at 574 MeV/c).

VI. DEPOLARIZATION NEAR γG= ν_z IN SATURNE 2

The Saturne 2 synchrotron accelerates polarized protons from 47 MeV ($\gamma G = 1.8826$) up to 3 GeV ($\gamma G = 7.5257$). Regular vertical tunes are within the range $3 < \nu_z < 4$. Therefore both imperfection and intrinsic resonances are to be crossed during acceleration, or may be neighboured during extraction [5].

In this example we show a numerical simulation of the depolarization effects in the static case, in the vicinity of the resonance $\gamma G = \nu_z$ at 968 MeV.

Table 4 gives the transfer matrix of one period for the particular tuning (of the 4-period machine, see figure 4) $\nu_x = 3.620$ and $\nu_z = 3.643$. With such conditions, and assuming a vertical emittance $\frac{\varepsilon_z}{\pi} = 96\ 10^{-6}$ m.rd, the depolarization resonance is found to be $|\varepsilon| \simeq 5.05\ 10^{-2}$ [6].

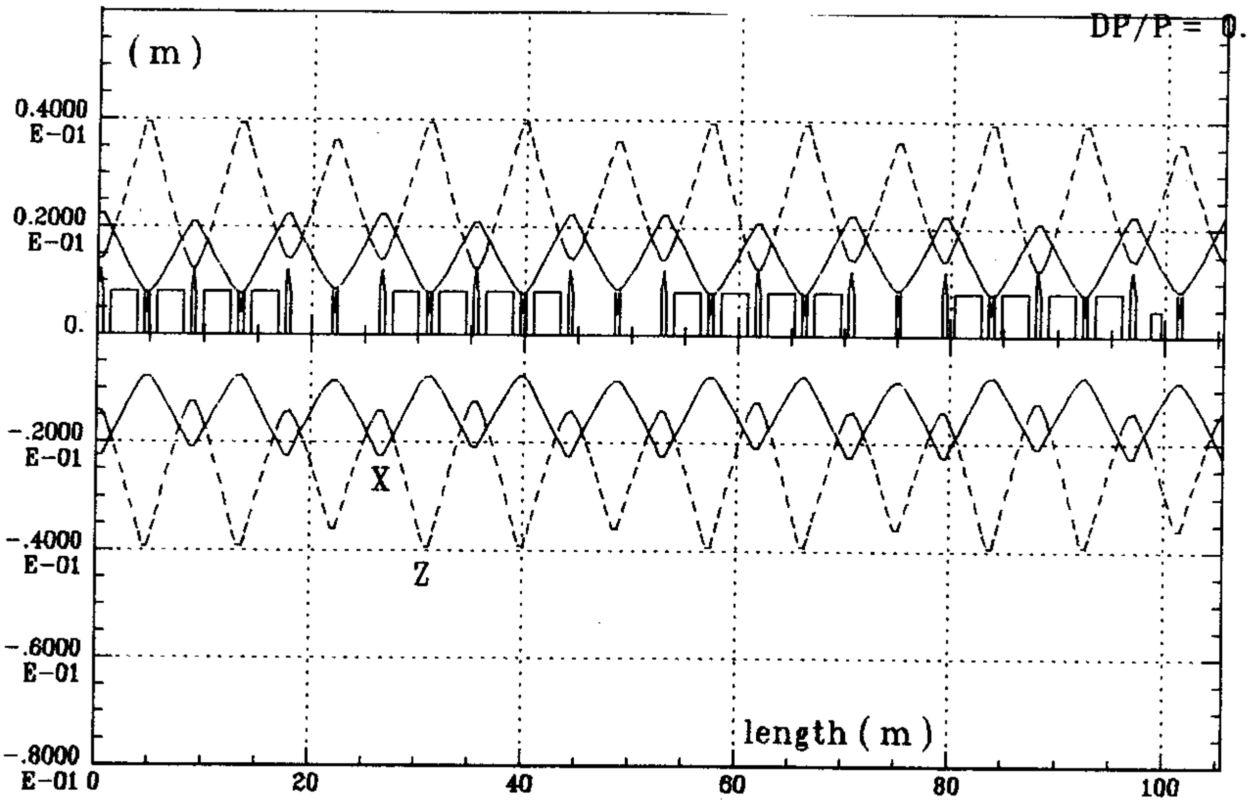

Figure 4 : A scheme of Saturne 2 and the horizontal (solid lines) and vertical (dashed lines) beam envelops for $\frac{\varepsilon_x}{\pi} = 33\ 10^{-6}$ m.rd and $\frac{\varepsilon_z}{\pi} = 96\ 10^{-6}$ m.rd.

First order transfer coefficients (MKSA units)

2.39590	−8.55135	0	0	0	−2.97459
0.32543	−0.74397	0	0	0	−0.85843
0	0	0.64490	−1.08639	0	0
0	0	0.29777	1.04894	0	0
−1.08887	5.12878	0	0	1.00000	3.87078
0	0	0	0	0	1.00000

Table 4 : First order transfer coefficients of a period of the synchrotron Saturne 2. The corresponding tuning of the machine is $\nu_x = 3.620$, $\nu_z = 3.643$.

The raytracing program Zgoubi is operated so as to perform multiturn tracking of one particle over a period. The initial conditions of that particle are taken on the $\frac{\varepsilon_z}{\pi} = 96.10^{-6}$ m.rd Courant invariant. Figure 5 shows a plot of the tracking in the z, z' phase-space, at the beginning of the cell.

The spin motion of the particle starting with vertical spin $\vec{S} \equiv \vec{S}_z$ is observed at the same point ($s = 0$, figure 4) for different energies : close to the resonance $\gamma G = \nu_z$, and at several other dissonances $\Delta = \gamma G - \nu_z \neq 0$. The results are summarized in figures 6 and 7 from which the very good agreement with theoretical expectations [6] may be concluded.

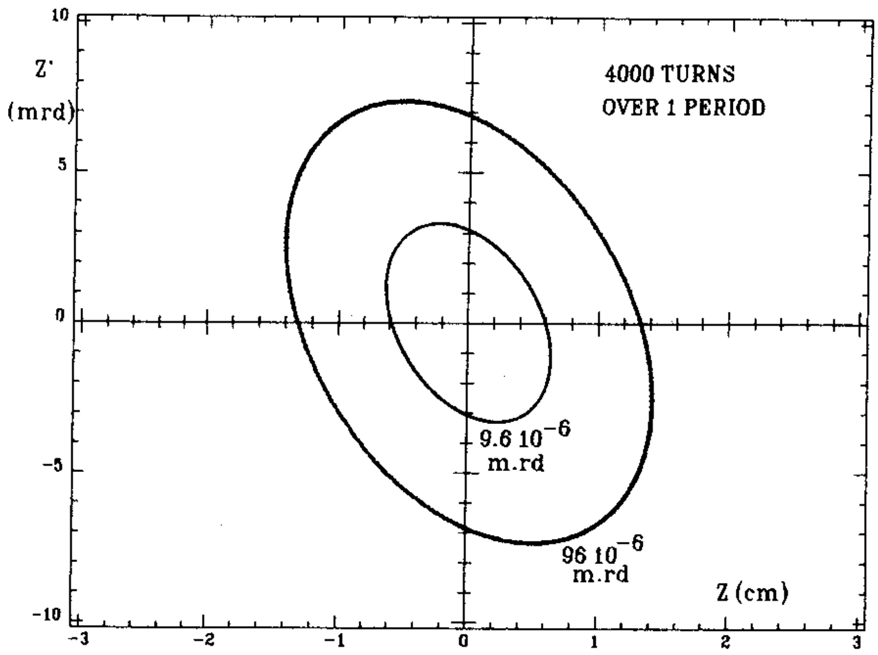

Figure 5 : Plot of the vertical phase ellipses (at $s = 0$) of two particles undergoing 4000 turns in one cell (equivalent to 1000 turns in Saturne 2), with initial conditions z_0 and z_0' taken on the invariants $\frac{\epsilon_z}{\pi} = 96\ 10^{-6}$ m.rd and $\frac{\epsilon_z}{\pi} = 9.6\ 10^{-6}$ m.rd (while $x_0 = x_0' = 0$). This plot shows the effective stability of the numerical tracking over a large number of turns : the ellipses have a negligible spread $\frac{\Delta\epsilon_z}{\epsilon_z}$.

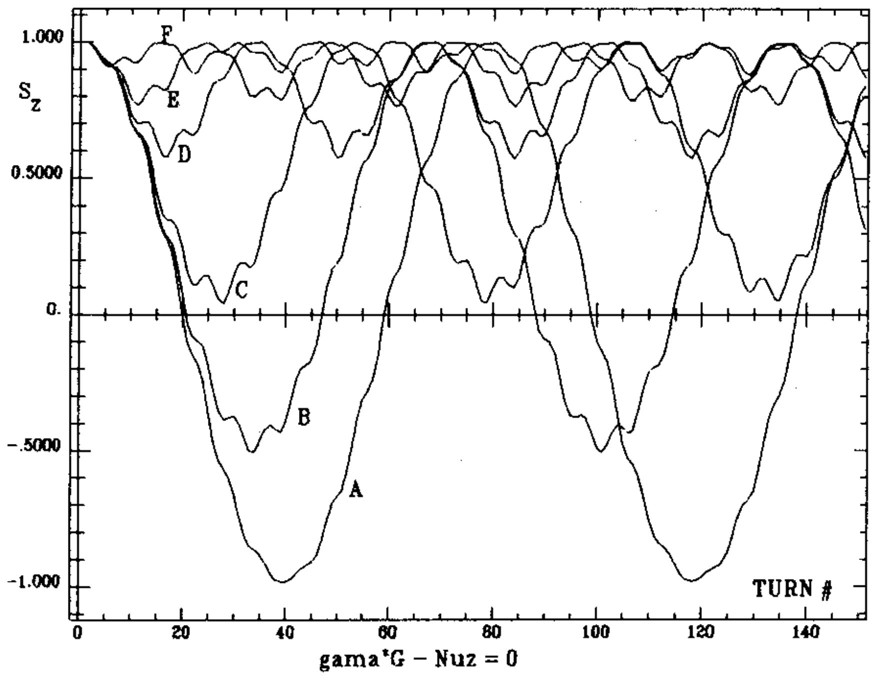

Figure 6 : Motion of the z−component of the vector polarization $\vec{S}$ over 150 turns, observed at the begining of the cell ($s = 0$, figure 4), for a particle starting with $\vec{S} \equiv \vec{S}_z$ (vertical), and for the following dissonances : A) $\Delta = \gamma G - \nu_z = 0.69\ 10^{-2}$, B) $\Delta = 3.23\ 10^{-2}$, C) $\Delta = 5.77\ 10^{-2}$, D) $\Delta = 10.85\ 10^{-2}$, E) $\Delta = 15.93\ 10^{-2}$ and F) $\Delta = 26.09\ 10^{-2}$.

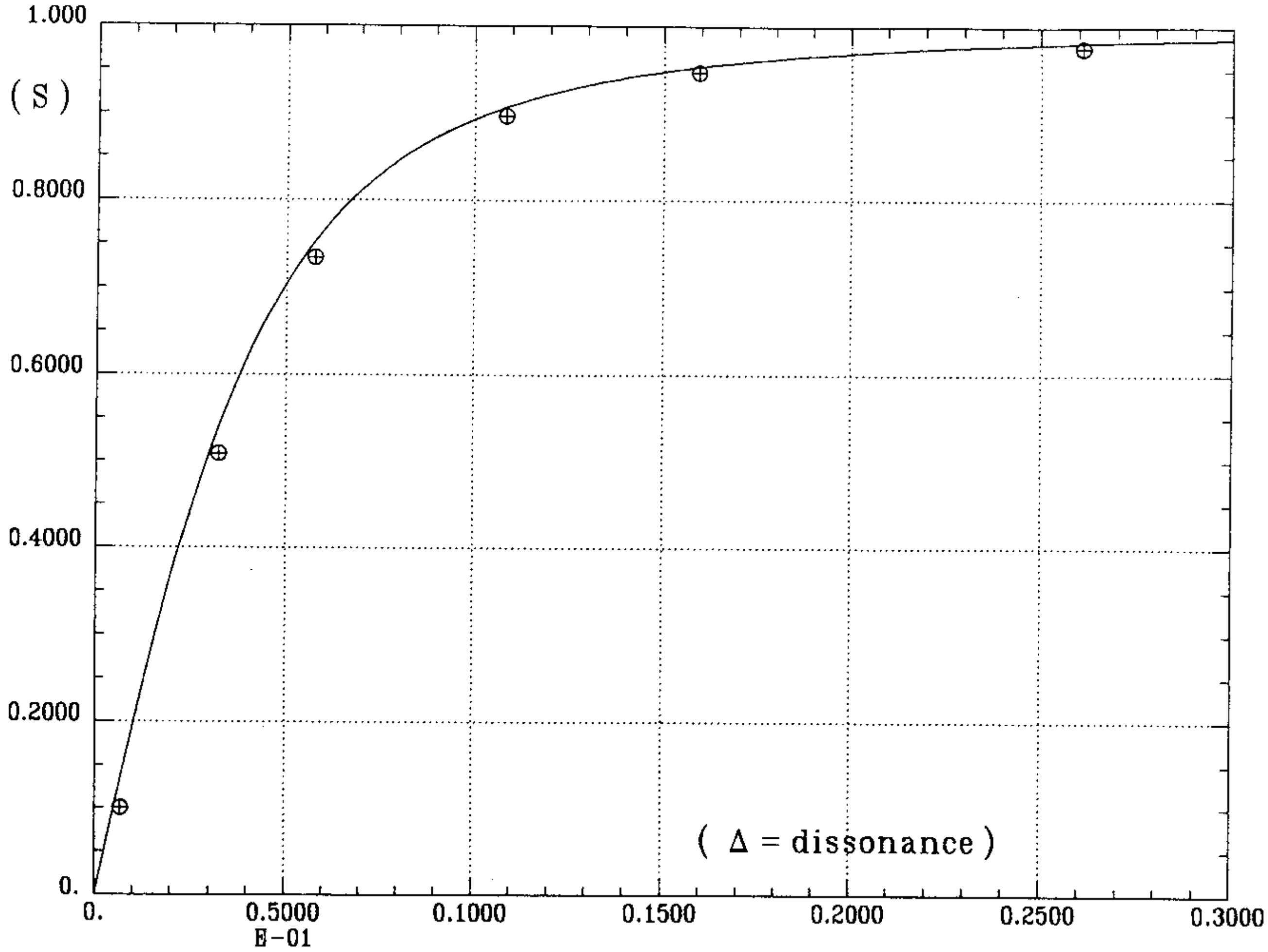

Figure 7 : A comparison between the theoretical depolarization in the static case (solid line), given by $S = \sqrt{\frac{\Delta^2}{|\varepsilon|^2+\Delta^2}}$ [6], with $|\ \varepsilon\ | =$ resonance strength $= 5.05\ 10^{-2}$ and $\Delta =$ dissonance $= \gamma G - \nu_z$, and the results of the numerical raytracing (dots), obtained from the tracking of S_z (see figure 6) by $\langle S_z \rangle =$ mean value of the projection of S on the vertical axis $= S^2$.

VII. CPU TIME CONSUMPTION

It can be seen from formulae 5-7 that only a few more equations need to be added to the existing code Zgoubi in order to track the spin meanwhile step by step raytracing. The consequence is that the increase in CPU time consumption is low when the spin tracking option is requested.

Checks have been performed during the raytracing simulations presented in the previous sections. They are summarized in the following table which gives the ratio $\frac{CPU\ time\ with\ spin\ tracking}{CPU\ time\ without\ spin\ tracking}$

Type of simulation	$\frac{CPU\ time\ with\ spin\ tracking}{CPU\ time\ without\ spin\ tracking}$	*Total CPU time for spin tracking on a microvax 2 (seconds)*
1000 *particles in a quadrupole with fringe fields (section III)*	1.17	830
1000 *particles in a dipole with fringe fields*	1.19	755
One particle, 75 *turns in Saturne* 2, *including plotting (section VI)*	1.15	775

VIII. CONCLUSION

The efficiency of the numerical method proposed for the resolution of the Thomas-BMT equation of spin motion has been proven, by means of its use in the raytracing program Zgoubi. Several classical results have been found back by raytracing, and discrepancies have been pointed out where the analytical first order spin transfer formalism reaches its intrinsic limits. It has been shown how this tool can be used for the detailed study of the depolarization in large acceptance spectrometers, as well as beam lines, and the study of resonant depolarizations in periodic machines.

Further developments

The use of this program for more sophisticated simulations in periodic machines (such as synchrotron motion) is worked at. As mentioned in another publication [7], Zgoubi is a very fast code. It should therefore be possible to perform multiturn tracking, eventually involving large numbers of particles, with reasonable CPU time consumption.

This brings major interests such as the ability of simulating realistic fields, fringe fields and high order multipoles, amidst the numerous advantages of raytracing procedures, compared to matrix transport methods.

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