

# Lecture\_12

March 4, 2024

## 1 The Solar System

In the last lecture, we briefly looked at the orbital angular momentum and rotational angular momentum of the Sun and the Earth. For the Earth, we found that the orbital angular momentum is significantly higher than its rotational angular momentum. This is true for all of the planets - their rotational angular momentum scales with the square of their radius, while their orbital angular momentum scales with the square of their distance to the Sun. For all of these,  $d \gg R$ , meaning the orbital angular momentum is always much, much higher than the rotational angular momentum.

With that knowledge let's focus on the orbital angular momentum:

$$J_{\text{orb}} = md^2\omega$$

The gas giants have much larger masses, and much larger distances, and similar orbital periods, so we should expect a large fraction of the angular momentum of the solar system to lie with them.

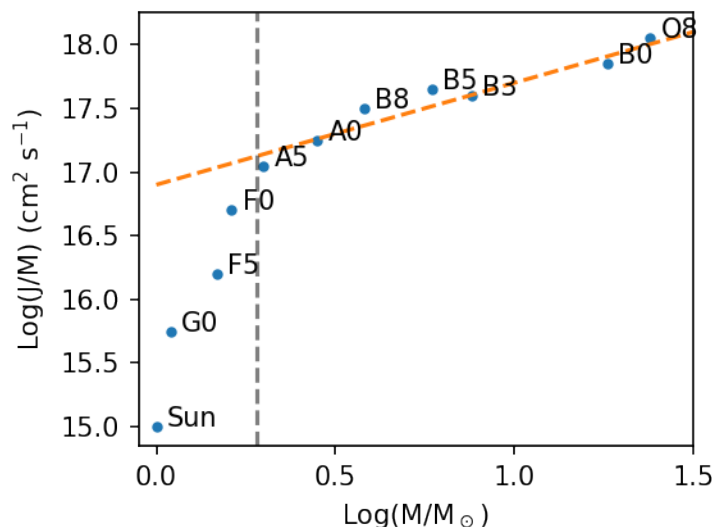
So if you go and calculate the orbital angular momenta for all of the gas giants, you'll get that the total is  $J_{\text{orb,giants}} = 3.07 \times 10^{43}$  J s. This accounts for 96% of the total angular momentum of the solar system.

### **Summary**

The orbital motion of the Gas Giants contain 96.5% of the total angular momentum of the solar system, and yet the planets only contain 0.13% of the solar systems mass.

### 1.1 The Solar System's angular momentum in context

So if we look at the specific angular momentum for various star types (where the specific angular momentum =  $J/M$ , we get the following plot. This shows that for spectral types earlier than F, a nice linear relationship is followed, while for spectral types later than F, there appears to be missing angular momenta from the stars.



So what's going on here? We'll discuss this in later lectures, but young stars rotate quite a lot faster than the Sun (in fact, the stars with labels A-O are young stars in the above), so this suggests that stars lose angular momentum over their life time. Let's see if we can estimate how long this takes

First, we know the Sun loses material through the solar wind. So let's focus on a particle of mass  $m$  which leaves the Sun's atmosphere at a radius of  $R_\odot$ . The angular momentum of this particle is  $J = mR_\odot\omega$ . Thus the rate of change of the Sun's angular momentum should then be

$$\frac{dJ}{dt} = \frac{dm}{dt} r^2 \omega = \dot{M}_\odot R_\odot^2 \omega_\odot$$

Given that we can calculate the orbital angular momentum of the Sun,  $J$ , from the last lecture, and we can measure the rate of mass loss from the Sun  $\dot{m} \sim 10^{-14} M_\odot/\text{yr}$ , we can estimate how long it would take the Sun to lose all of its current angular momentum via mass loss as

$$\tau = \frac{J}{dJ/dt} = \frac{2}{5} \frac{M_\odot}{\dot{M}_\odot} \approx 4 \times 10^{13} \text{ yr}$$

## 2 Solar System Formation

The general picture of formation of the solar system suggests that it was formed from the collapse of a giant dust cloud (called a protosolar nebula, more on this in 3rd year) which was mostly composed of hydrogen. A key feature of this model is that the total angular momentum of the system must be conserved.

There is some direct observational evidence for this feature:

- The direction of the Sun's rotation, and of the rotation and orbital motion of (nearly) all the planets coincide ("prograde" direction).
- The rotation axes of nearly all planets are within 25-30° of being perpendicular to their orbital planes
- The planets orbital planes all coincide to within 7° (most are even within 1-3°)

So, since the angular momentum for the Sun's rotation, the planet's rotation, and the planet's orbital motion all have the same direction, then it was likely the angular momentum for the protosolar nebula.

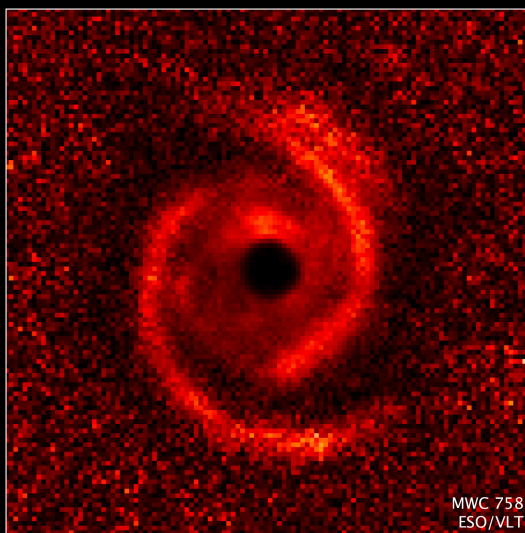
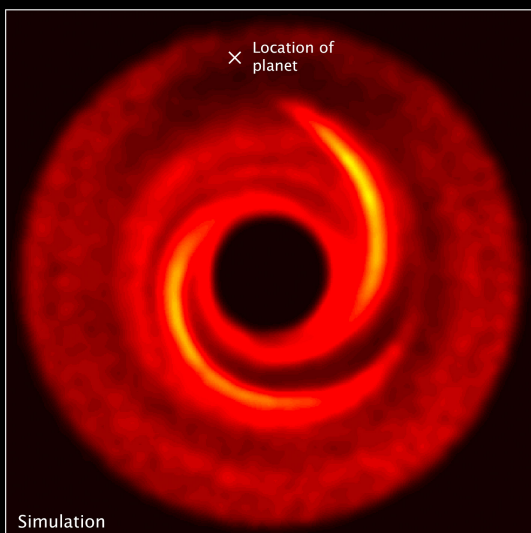
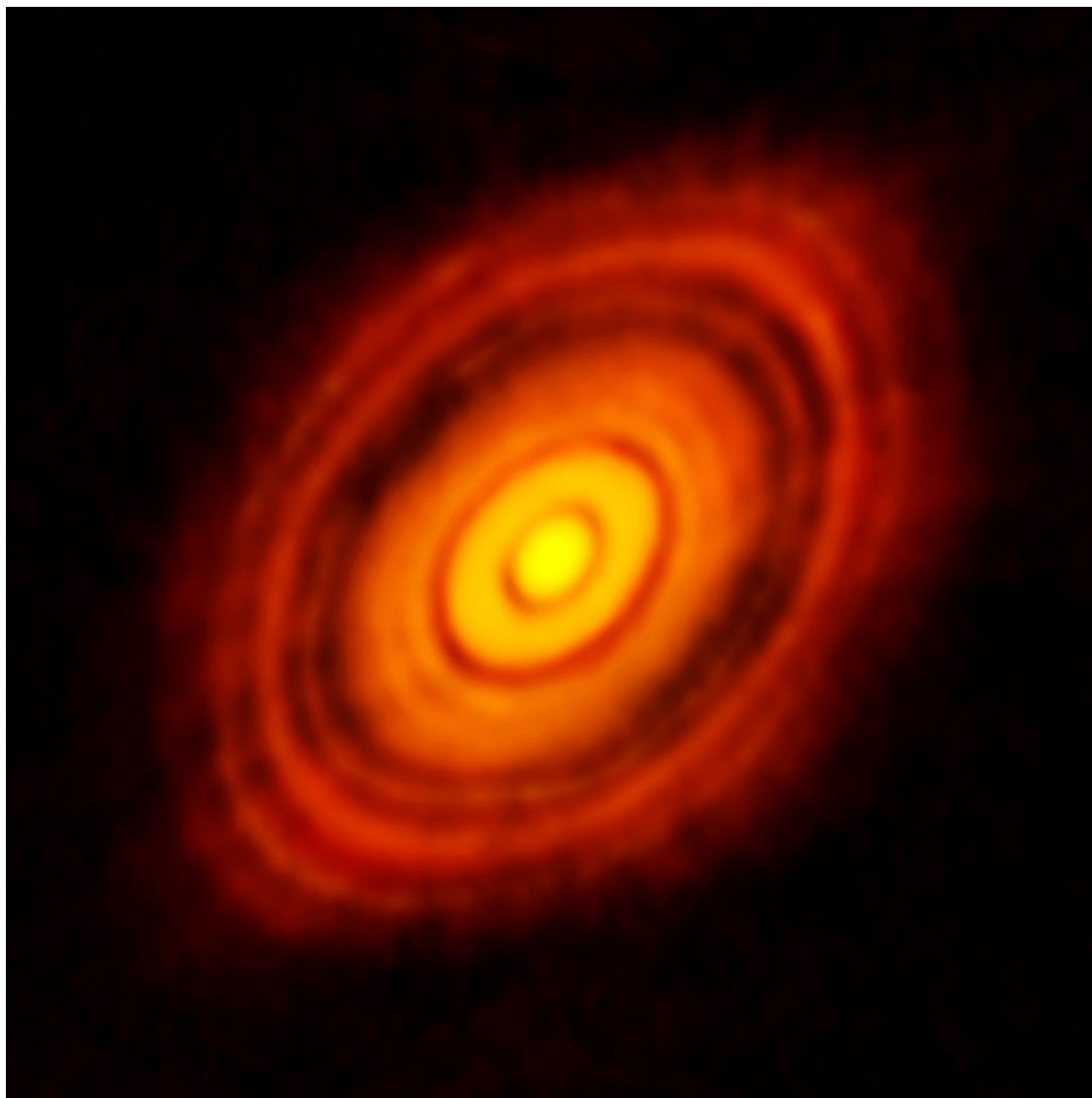
Now, not all observational evidence does support this model:

- Venus and Uranus' rotations are "retrograde" (that is, they rotate opposite to everything else)
- Uranus' rotational axis is inclined  $98^\circ$  to its orbital plane (that is, it's rotating on its side)
- Earth, Mars, Saturn, and Neptune have angles between their orbital and rotational planes of  $23\text{-}29^\circ$

A proposed solution to all of these issues is that these planets collided with planetesimals during formation.

## 2.1 Protoplanetary discs

Below, I've included some real/simulated observations of protoplanetary discs.

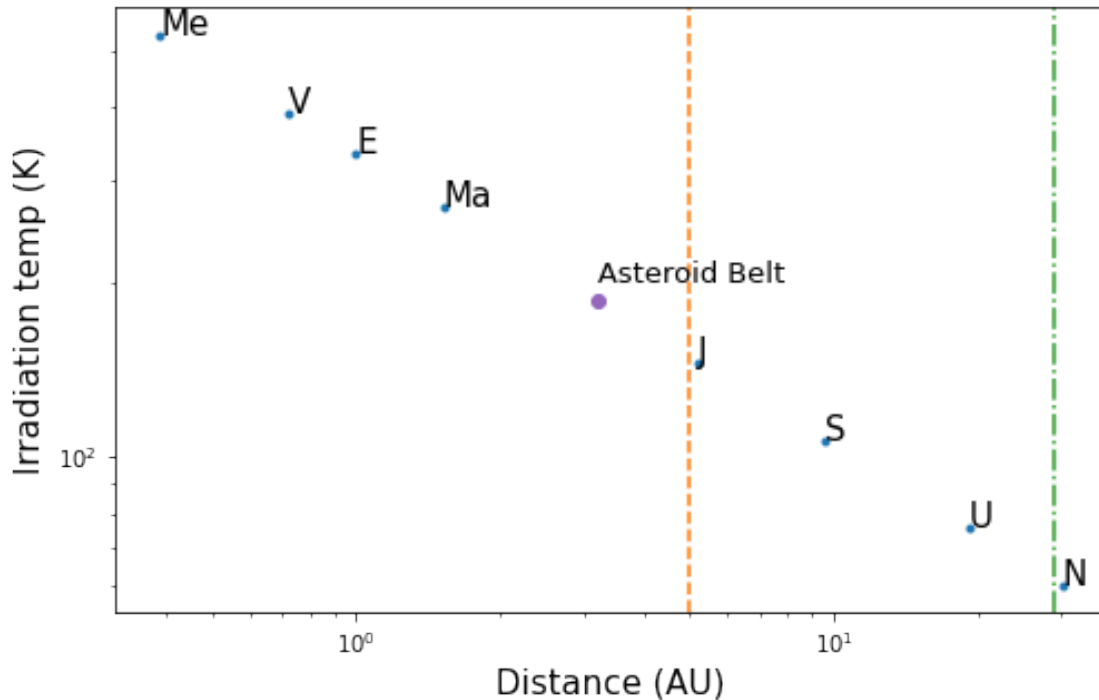


Protoplanetary Disk ■ Simulated Spiral Arm vs. Observational Data  
ESO/VLT

## 2.2 Terrestrial versus Gas Giant Planets

So why do we have these two types of planets? And how does this feed into a model of forming the solar system? First, let's propose a model.

- As the protosolar nebula contracted and formed a disc, heavier elements condensed out first.
- Water does not condense when terrestrial planet formed. It's suspected most water on Earth comes from collisions with cometary nuclei (although the question here is whether Earth formed in situ or migrated, talk to Prof. Bitsch!)
- Water could condense beyond some distance from Sun, near orbit of Jupiter (as shown below, where we've calculated the irradiation temperature for different distances from the Sun, and found where the snow line (condensation temperature for water = 150 K) lies.



## 3 Quirks of planets

### 3.1 Mercury

- Its rotation is studied using radar signals transmitted by the Arecibo radio telescope (which has since collapsed) in 1965, using the Doppler effect. The rotational period is  $P_{\text{rot}} = 58.65$  d.
- This is exactly  $2/3$  of the orbital period of the planet ( $P_{\text{orb}} = 87.97$  d). Since Mercury feels the strongest tidal force at perihelion (point of closest approach to the Sun), this means that the tidal bulges always line up with the Sun at this time (as there is 1.5 rotations per orbit, see figure).
- Gravitational influence of other planets causes the perihelion to shift in a counter clockwise

rotation by 574"/century. Newtonian gravity can only explain 531"/century - the remaining 43"/century explained using Einstein's theory of general relativity.

### 3.2 Mars and the asteroid belt (between Mars and Jupiter)

- Mars has a low mass of  $0.107 M_{\oplus}$ .
- To explain this, it may be that Jupiter became very massive before Mars finished forming, which perturbed orbits of planetesimals near Mars.
- This would mean the "missing mass" of Mars has ended up in the asteroid belt.
- Numerical simulations suggest Mars rotational axis may vary chaotically with tilts between 11-49° on timescales as short as a few million years (due to its low mass and gravitational influence of other planets).

### 3.3 Venus

- Also studied using radar signals sent from Earth, but also using Doppler shifts of reflected sunlight.
- Venus rotates in a retrograde direction, with  $P_{\text{rot}} = 243$  d and  $P_{\text{orb}} = 224.7$  d.
- Main gas in atmosphere is CO<sub>2</sub> (96.4%). At the base of its atmosphere, it has  $T = 740$  K and a pressure of 90 atm.
- This temperature is 2-3 times higher than the irradiation temperature we get when we use the formula from last week - due to the greenhouse effect.
- Surface accurately mapped by the Magellan spacecraft. The frequency of the radio signals sent to Earth were Doppler shifted depending on the speed of the spacecraft as it passed over regions of higher/lower mean density.
- It also potentially has traces of phosphine in its upper atmosphere (which **may** be due to life).