Lecture_14

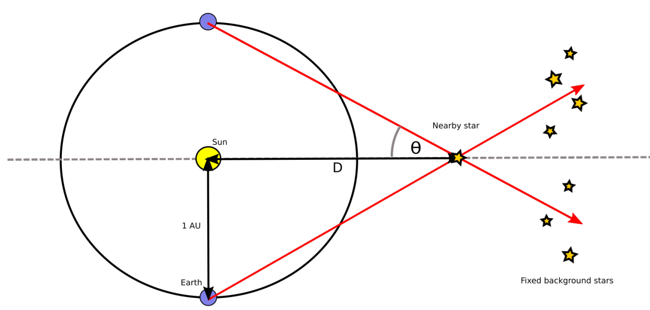
March 4, 2024

1 Stars

In this lecture, we are going to start discussing how we convert observable properties of stars (parallax, flux, colour) into intrinsic physical properties (distance, luminosity, temperature). Let's start with parallax.

1.1 Parallax

The next section of the course is briefly going cover the basic properties of stars. Before proceeding, an important ingredient we need in order to understand the stellar population in the Milky Way is the distance to any star. To determine the distance, consider the setup below, where an astronomer observes a star at two different times, separated by 6 months. During this time, the Earth has moved in its orbit.



The change in the position of the star relative to some fixed, background stars, can then be used to estimate the distance to the star, as we know that Earth lies 1 astronomical unit ($1 \text{ AU} = 1.49 \times 10^{11}$ m) away from the Sun. As such, an object which exhibits a parallax of $\theta = 1$" has to be a distance

$$D = \frac{1.49 \times 10^{11} \text{ m}}{\tan(\theta)} \approx \frac{1.49 \times 10^{11} \text{ m}}{\theta} = 3.0856 \times 10^{16} \text{m} \equiv 1\text{pc}.$$

A much easier expression to remember is then that

$$D = \frac{1}{\theta}$$

when θ is given in arcseconds, and D will be given in parsecs.

1.2 Luminosity

With this tool in hand, we can now discuss how bright we expect stars to be. The flux arriving at the Earth from a star located at a distance d from us will be

$$F = \frac{L}{4\pi D^2}$$

Thus, if we measure both the flux and distance to an object, we can calculate its luminosity via

$$L = 4\pi D^2 F$$

For example, Sirius exhibits a parallax of 0.374”, and the measured flux from it is $F = 1.2 \times 10^{-7}$W m^{-2}. The distance is then

$$d = \frac{1}{\theta} = 2.67\text{pc}$$

meaning the luminosity of Sirius is

$$L = 1.02 \times 10^{28}\text{W}$$

Given that the Sun has a luminosity of $L_{\odot} = 3.828 \times 10^{26}$ W, this means that the luminosity of Sirius is

$$L = 26.7 L_{\odot}.$$

Instead of fluxes, optical astronomers tend to use magnitudes when discussing how bright objects are in the sky.

1.2.1 Apparent Magnitude

The term apparent magnitude comes from Hipparchus, who catalogued the visible stars. He assigned the brightest stars a magnitude of 1, and the faintest stars as a magnitude of 6. So, what does a magnitude actually represent? The conversion between flux and magnitude is

$$m = -2.5 \log_{10}\left(\frac{F}{C}\right)$$

where C is a normalising constant. There’s no real way of working this out from first principles, except it has to do with the response of the human eye. Now consider an an object which is 100 brighter than another object ($F_2 = 100F_1$). If we calculate $m_1 - m_2$, then the above expression becomes

$$m_1 - m_2 = -2.5 \log_{10}\left(\frac{F_1}{F_2}\right) = -2.5 \log_{10}\left(\frac{1}{100}\right)$$

we find that it is 5. This means that an object which is 100 times brighter will have a magnitude which is 5 less - the magnitude system is an inverted scale (the more positive a magnitude is, the fainter it is).

The Sun has a magnitude of -27, while the faintest objects observable in the night sky have a magnitude of $\sim +30$. The naked eye can see as faint as magnitude +6 if conditions are exceptional. **Apparent magnitudes** are typically written as m.

1.2.2 Absolute Magnitude

A useful quantity that can be derived using the above equation is the absolute magnitude of a source, which typically written as M. This is the magnitude a source would have if it were at a distance of 10 pc from the Sun. To calculate what M is, we use:

$$m - M = -2.5 \log_{10}\left(\frac{F}{C}\right) + 2.5 \log_{10}\left(\frac{F_{10}}{C}\right)$$

where F_{10} is the flux we would see if the object were at 10pc. This simplifies to

$$m - M = -2.5 \log_{10}\left(\frac{F}{F_{10}}\right)$$

Finally, remembering that $F = \frac{L}{4\pi D^2}$, so $F_{10} = \frac{L}{4\pi 10^2}$, we can simplify this further to

$$m - M = 5 \log_{10}\left(\frac{D(\text{pc})}{10(\text{pc})}\right)$$

So, if we know the distance to a source and its apparent magnitude, we can immediately calculate its absolute magnitude.

`<b>Example: Absolute magnitude of the Sun</b>`

The Sun has an apparent magnitude of -26.74, and lies 1.49×10^{11} m ($= 4.848 \times 10^{-6}$ pc) from Earth. What is its absolute magnitude?

Using the above formula, we find

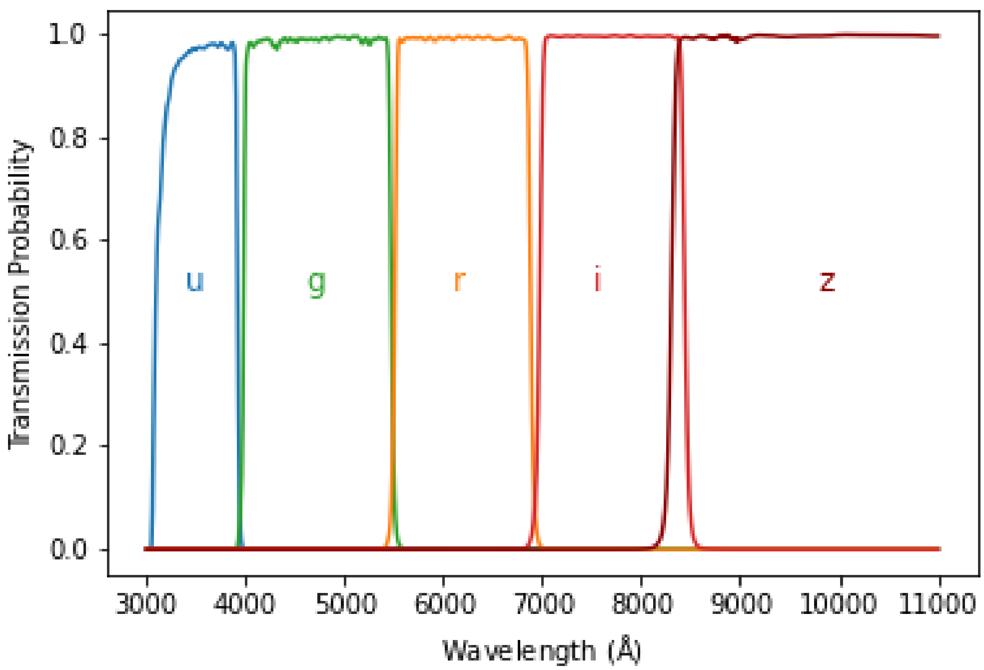
$$-26.74 - M = 5 \log_{10}\left(\frac{4.848 \times 10^{-6}}{10}\right) M = 4.83$$

1.2.3 Observing Filters

Before going any further, we need to discuss the constant in the equation.

$$m = -2.5 \log_{10}\left(\frac{F}{C}\right)$$

If we measued the flux ($F_{\text{bol}} = \int_0^\infty F_\lambda d\lambda$) coming from a star across the entire electromagnetic spectrum, the magnitudes we calculate are called **bolometric magnitudes**. You can already imagine that carrying out such a flux measurement is very tricky. Instead, stars are typically observed using different filters, which cover different parts of the spectrum. The below figure shows the sensitivity curves (S) for the SDSS (Sloan Digital Sky Survey) filters.



These transmission curves tell us which part of the spectrum we are sensitive too when using them. For example, when using the u filter, we are sensitive to light from $\sim 3000 - 4000$ Å.

As such, the u-band magnitude of the source, $m_u \equiv u$, is given by

$$u = -2.5 \log_{10} \left(\frac{\int_0^\infty S_u F_\lambda d\lambda}{C_u} \right) u = -2.5 \log_{10} \left(\int_0^\infty S_u F_\lambda d\lambda \right) + C_u$$

Here, C_u is a constant for the u band filter. Similar constants (C_g,C_r,C_i,C_z) exist for each band (g,r,i,z). For the **AB** system, which SDSS uses, the constants are chosen such that a star with magnitude 0 has a flux density of 3.631×10^{-20} erg cm^{-2} s^{-1} Hz^{-1}. In Carroll and Ostlie, they use the Johnson set of filters. For this system, the constants use the **Vega** system, in which Vega has a magnitude of 0 in each band.

If the distance to the source is known, and m_u measured, then the absolute magnitude in the u-band M_u can be calculated.

1.2.4 Colours

Consider a blackbody which is observed using the same filters as above, and which we know the distance to. Each filter is sensitive to a different part of the spectrum, meaning the same source will appear with different fluxes in each band. The different in absolute magnitudes between the g and i bands will be

$$g - i = -2.5 \log_{10} \frac{F_{\mathrm{g}} C_{\mathrm{i}}}{F_{\mathrm{i}} C_{\mathrm{g}}}$$

where the Cs in the above equation are the normalising constants for each band above, and for simplicity we have let $F_g = \int_0^\infty S_g F_\lambda d\lambda$ etc. This means

$$g - i = -2.5 \log_{10} \frac{F_{\rm g}}{F_{\rm i}} + C_{\rm g-i}$$

where $C_{\rm g-i} = C_{\rm g} - C_{\rm i}$

So, imagine we can measure both $m_{\rm u}$ and $m_{\rm r}$ for a bunch of stars. What does this tell us? Consider the code below. It produces two Figures - one with a blackbody of temperature 5000 kelvin and one with a temperature of 8000 kelvin.

In the case of 5000 K star, $\frac{F_{\rm g}}{F_{\rm i}}$=1.047, and $g - i$=-0.05 (ignoring the constants). For the 8000 K star, $\frac{F_{\rm g}}{F_{\rm i}}$=2.46, and $g - i$=-0.95. So immediately, this tells us that if we can measure g and i (or another combination of colours), then we can estimate the temperature of the star, and that hotter stars have more negative $g - i$ values.

It is common to write $m_{\rm g}$ as g, or $m_{\rm i}$ as i. If you are observing in the Johnson Filters, which are U, B, V, R, and I, then $m_{\rm U}$ is written as U, $m_{\rm V}$ as V etc. This means the colour could be given by $B - V$ etc. This is how Section 3.6 of Carroll writes everything.