Lecture_08

February 15, 2023

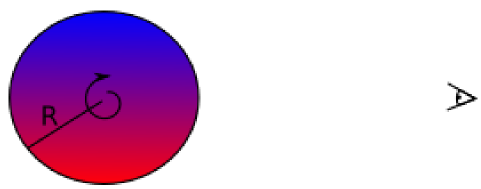
1 Doppler broadening continued

In the previous lecture, we saw that the distribution of velocities of particles in gas broadens the absorption/emission feature, and that the shape of this broadening is a Gaussian with the form

$$I(\nu) = I_0 \exp\left[-\left(\frac{c(\nu-\nu_0)}{\nu_0 v_{\max}}\right)^2\right]$$

where $v_{\max}$ is the velocity which is most probable to occur in a Maxwell-Boltzmann distribution.

There is other types of motion which can give rise to Doppler broadening of a line. This is mainly due to rotation. Imagine we have a body rotating as shown below.



The width of the line due to this rotation is simply given by

$$\frac{\delta\lambda}{\lambda} = \frac{2v_{\mathrm{rot,max}}}{c} = \frac{4\pi R}{Pc}$$

where $v_{\mathrm{rot,max}}$ is the maximum rotational velocity of the object (so at the equator), and P is the rotational period of the object.

Finally, there can also be turbulent broadening of the line, which involves the chaotic, bulk motions of gas rather than random motions within the gas.

2 Collisional/Pressure Broadening

The final effect on absorption/emission features that we are going to talk about is collisional/pressure broadening. This is when we take into account what effect impacts between atoms have on their energy levels (recall the change in effective potential during scattering discussed in PY2101. The difference in the terms are 1. Collisional Broadening is when each individual collision is considered. 2. Pressure Broadening is when the statistical effect of many collisions is considered.

The result of this broadening is very similar to natural broadening. However, a key difference is that this effect depends on the number density of particles in the environment, since this is related to the number of expected collisions which should occur. As such, this effect is stronger in dense environments.

In order to get an estimate of the expected width of a collisional broadened line, we can lean on the result from natural line broadening. First, we are going to assume that every collision results in a particle being removed from an excited state. The mean time between collisions, δt_0, will now replace the time uncertainty we used in Heisenberg's uncertainty principle when considering natural broadening. Hence, we get that

$$\delta\lambda \geq \frac{\lambda^2}{2\pi c \delta t_0}$$

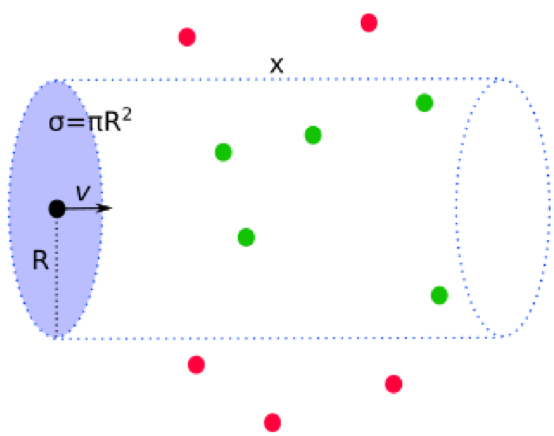
The mean time between collisions is given by

$$\delta t_0 = \frac{l}{v}$$

where l is the mean free path of the particles and v is the average velocity of the particles. Assuming the average velocity from a Maxwell-Boltzmann distribution gives

$$v = \left(\frac{8k_{\mathrm{B}}T}{\pi m}\right)^{1/2}$$

So what is the mean free path, l? Consider a particle which is travelling at a velocity v in a gas, as shown below. If we define the maximum separation at which we expect an interaction to occur between this particle and any other particle to be R, then as the particle travels through the gas, it carves out a cylinder of volume $V = \sigma x$. σ is known as the cross sectional area of the particle, and is given by $\sigma = \pi R^2$. If any particle falls within the area σ as our particle moves, we would expect an interaction. For clarity, I have coloured the particles we expect an interaction to occur with as green, and those for which no interaction is expected as red.



The average length between interactions (mean free path) is then given by

$$l = \frac{x}{N}$$

where N is the total number of particles in the system, and is $N = Vn = x\sigma n$. As such, the mean free path simplifies to

$$l = \frac{1}{n\sigma}$$

As such, the average time between interactions is given by

$$\delta t_0 = \frac{1}{n\sigma}\left(\frac{\pi m}{8k_\mathrm{B}T}\right)^{1/2}$$

and so the expected width of the emission/absorption feature is given by

$$\delta\lambda \geq \frac{\lambda^2 n\sigma}{2\pi c}\left(\frac{8k_\mathrm{B}T}{\pi m}\right)^{1/2}$$

Recalling that for an ideal gas $PV = Nk_\mathrm{B}T$, which means $P = nk_\mathrm{B}T$, then allows us to substitute in the above to get

$$\delta\lambda \geq \frac{\lambda^2 P\sigma}{2\pi c}\left(\frac{8}{\pi k_\mathrm{B}Tm}\right)^{1/2}$$

So, in the end, if we neglect Doppler effects, we would expect any line to be broadened due to two components - the natural broadening, and the pressure broadening, both of which can be used to give us physical insight into what's occuring within the medium that is either producing the absorption or emission features.

3 The continuum of spectra: Black-body radiation

We now know that if we were to look at the light coming from an object within the Universe, we'd expect to see features of particular widths at wavelengths associated with transitions between states of different atoms. However, what the continuum behaviour? That is, what function should describe the light outside of these wavlengths? To understand this, we need to consider black-body radiation.

First, a black-body is a physical body that absorbs all electromagnetic radiation incident on it. "A perfect absorber/emitter of radiation" - which means the efficiency of absorption/emissions is 1. The distribution of the emitted radiation with frequency/wavlenegth is determined by the object's temperature T, accround to the Planck blackbody radiation law (see the website for the full derivation, combines information from PY2102 and PY2104).

$$u(\omega, T)d\omega = \frac{\hbar\omega^3 d\omega}{\pi^2 c^3\left(\exp\left[\frac{\hbar\omega}{kT}\right] - 1\right)}$$

This gives emitted energy density as a function of ω. This function then describes the expected spectrum of an object outside of absorption/emission features. There are a couple of interesting features we can immediately work out. First, the peak of this function occurs at

$$\frac{du}{d\omega} = 0$$

which has the solution

$$(3 - \beta\hbar\omega)\exp(\beta\hbar\omega) = 3$$

where $\beta = 1/kT$. This is a transcendental equation, and must be either solved graphically or numerically. The solution is

$$\frac{\hbar\omega_{\mathrm{peak}}}{kT} = \frac{h\nu_{\mathrm{peak}}}{kT} = 2.822$$

which is the Wien displacement law. This relates the frequency at which a blackbody peaks with the temperature of the object. This peak moves towards higher frequencies for objects with higher temperatures. It can be used to inform the observing strategy for a particular object - cold objets peak at low frequencies (so need Radio/IR observations), while hot objects peak towards the UV/X-ray.

Also, note that the blackbody curve for a higher T lies completely above the blackbody curve for a lower T. Consider

$$\frac{u(\omega, T_1)}{u(\omega, T_2)} = \frac{\left(\exp\left[\frac{\hbar\omega}{kT_2}\right] - 1\right)}{\left(\exp\left[\frac{\hbar\omega}{kT_1}\right] - 1\right)}$$

which is always > 1 if $T_1 > T_2$.