Lecture_12

February 28, 2024

1 The Solar System

In the last lecture, we briefly looked at the orbital angular momentum and rotational angular momentum of the Sun and the Earth. For the Earth, we found that the orbital angular momentum is significantly higher than its orbital angular momenutm. This is true for all of the planets - their rotational angular momentum scales with the square of their radius, while their orbital angular momentum scales with the squrea of their distance to the Sun. For all of these, $d >> R$, meaning the orbital angular momentum is always much, much higher than the rotational angular momentum.

With that knowledge let's focus on the orbital angular momentum:

$$J_{\text{orb}} = md^2\omega$$

The gas giants have much larger masses, and much larger distances, and similar orbital periods, so we should expect a large fraction of the angular momentum of the solar system to lie with them.

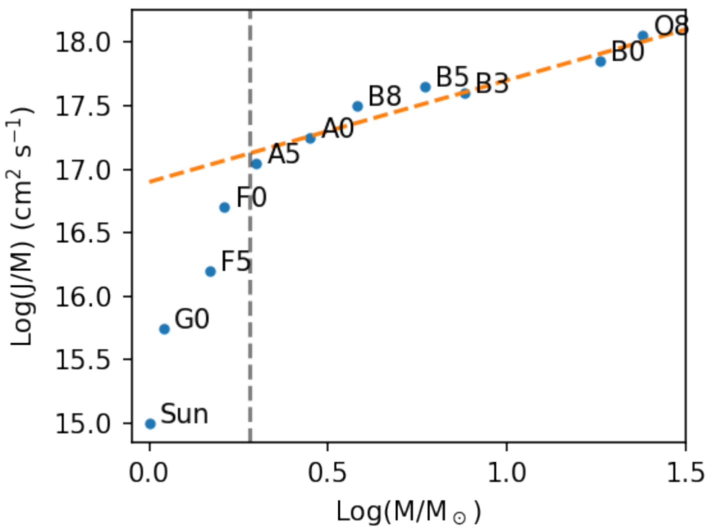
So if you go and calculate the orbital angular momenta for all of the gas giants, you'll get that the total is $J_{\text{orb,giants}} = 3.07 \times 10^{43}$ J s. This accounts for 96% of the total angular momentum of the solar system.

`<b>Summary</b>`

The orbital motion of the Gas Giants contain 96.5% of the total angular momentum of the solar system, and yet the planets only contain 0.13% of the solar systems mass.

1.1 The Solar System's angular momentum in context

So if we look at the specific angular momentum for various star types (where the specific angular momentum $= J/M$, we get the following plot. This shows that for spectral types earlier than F, a nice linear relationship is followed, while for spectral types later than F, there appears to be missing angular momenta from the stars.



So what's going on here? We'll discuss this in later lectures, but young stars rotate quite a lot faster than the Sun (in fact, the stars with labels A-O are young stars in the above), so this suggests that stars lose angular momentum over their life time. Let's see if we can estimate how long this takes

First, we know the Sun loses material through the solar wind. So let's focus on a particle of mass m which leaves the Sun's atmosphere at a radius of $R_\odot$. The angular momentum of this particule is $J = mR_\odot\omega$. Thus the rate of change of the Suns angular momentum should then be

$$\frac{\mathrm{d}J}{\mathrm{d}t} = \frac{\mathrm{d}m}{\mathrm{d}t} r^2 \omega = \dot{M}_\odot R_\odot^2 \omega_\odot$$

Given that we can calculate the orbital angular momentum of the Sun, J, from the last lecture, and we can measure the rate of mass loss from the Sun $\dot{m} \sim 10^{-14}$ $\mathrm{M}_\odot/\mathrm{yr}$, we can estimate how long it would take the Sun to lose all of it's current angular momentum via mass loss as

$$\tau = \frac{J}{\mathrm{d}J/\mathrm{d}t} = \frac{2}{5}\frac{M_\odot}{\dot{M}_\odot} \approx 4 \times 10^{13}\ \mathrm{yr}$$

2 Solar System Formation

The general picture of formation of the solar system suggests that it was formed from the collapse of a giant dust cloud (called a protosolar nebula, more on this in 3rd year) which was mostly composed of hydrogen. A key feature of this model is that the total angular momentum of the system must be conserved.

There is some direct observational evidence for this feature:

- The direction of the Sun's rotation, and of the rotation and orbital motion of (nearly) all the planets coincide ("prograde" direction).
- The rotation axes of nearly all planets are within 25-30$^{\mathrm{o}}$ of being perpendicular to their orbital planes
- The planets orbital planes all coincide to within 7$^{\mathrm{o}}$ (most are even within 1-3$^{\mathrm{o}}$)

So, since the angular momentum for the Sun's rotation, the planet's rotation, and the planet's orbital motion all have the same direction, then is was likely the angular momentum for the protosolar nebula.

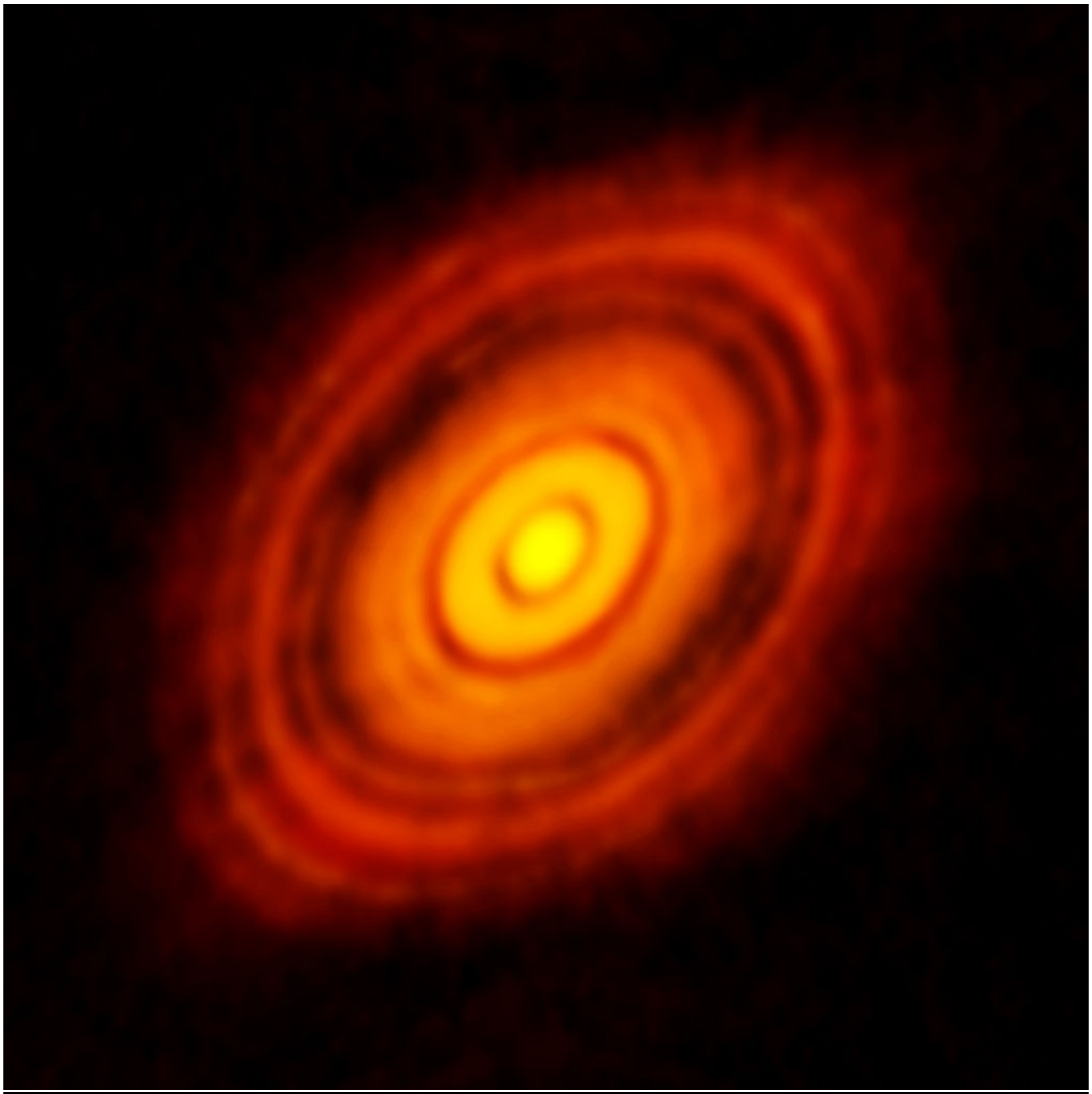
Now, not all observational evidence does support this model:

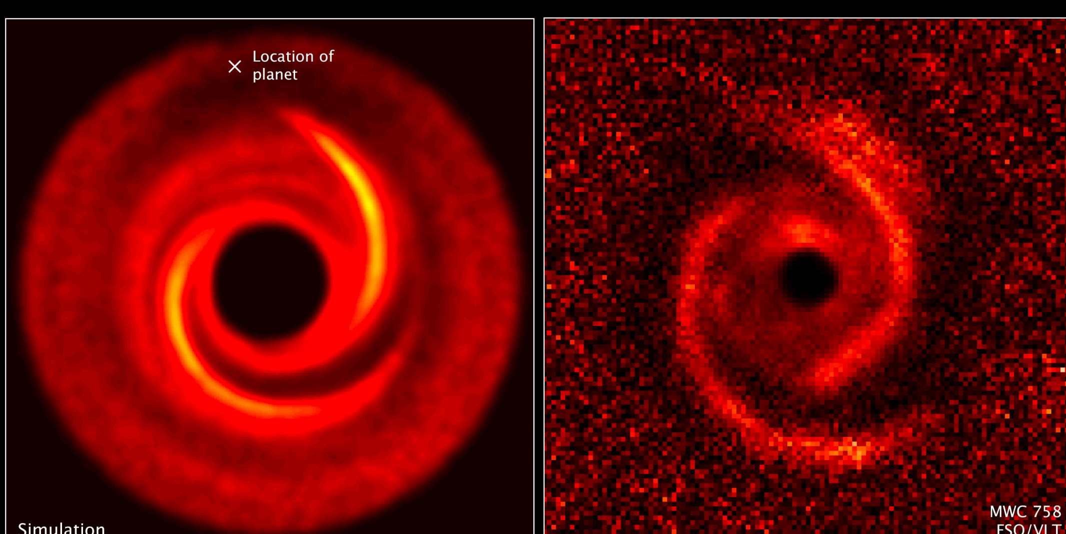
- Venus and Uranus' rotations are "retrograde" (that is, they rotate opposite to everything else)
- Uranus' rotational axis is inclined 98^o to it's orbital plane (that is, it's rotating on it's side)
- Earth, Mars, Saturn, and Neptune have angles between their orbital and rotational planes of 23-29^o

A proposed solution to all of these issues is that these planets collided with planetesimals during formation.

2.1 Protoplanetary discs

Below, I've included some real/simulated observations of protoplanetary discs.





Protoplanetary Disk ▪ Simulated Spiral Arm vs. Observational Data
ESO/VLT

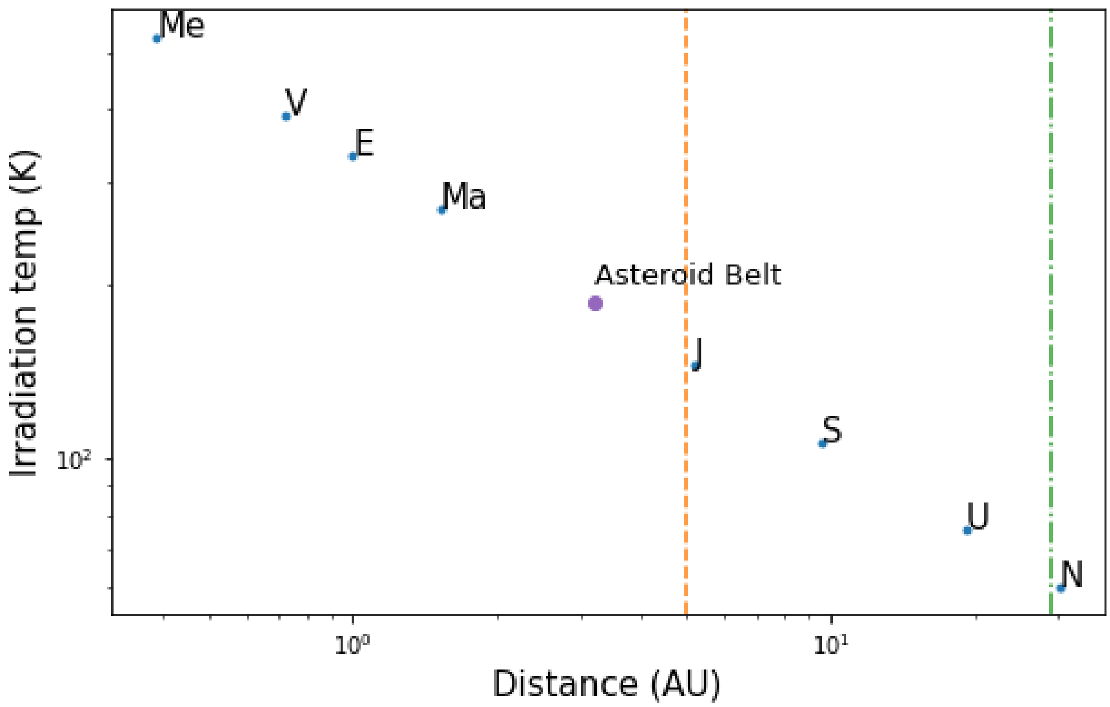
NASA, ESA, and ESO

STScI-PRC15-40a

2.2 Terrestrial versus Gas Giant Planets

So why do we have these two types of planets? And how does this feed into a model of forming the solar system? First, let's propose a model.

- As the protosolar nebula contracted and formed a disc, heavier elements condensed out first.
- Water does not condense when terrestrial planest formed. It's suspected most water on Earth comes from collisions with cometary nuclei (although the question here is whether Earth formed in situ or migrated, talk to Prof. Bitsch!)
- Water could condense beyond some distance from Sun, near orbit of Jupiter (as shown below, where we've calculated the irradiation temperature for different distances from the Sun, and found where the snow line (condensation temperature for water = 150 k) lies.



3 Quirks of planets

3.1 Mercury

- Its rotation is studied using rader signals transmiteed by the Arecibo radio telescope (which has since collapsed) in 1965, using the Doppler effect. The rotational period is $P_{\rm rot} = 58.65$ d.
- This is exactly 2/3 of the orbital period of the planet ($P_{\rm orb} = 87.97$ d). Since Mercury feels the strongest tidal force at perihelion (point of closest approach to the Sun), this means that the tidal bulges always line up with the Sun at this time (as there is 1.5 rotations per orbit, see figure).
- Gravitational influence of other planets causes the perihelion to shift in a counter clockwise

rotation by 574“/century. Newtonian gravity can only explain 531”/century - the remainig 43”/century explained using Eintein’s theory of general relativity.

3.2 Mars and the asteroid belt (between Mars and Jupiter)

- Mars has a low mass of 0.107 $M_{\oplus}$.
- To explain this, it may be that Jupiter became very massive before Mars finished forming, which perturbed orbits of planetesimals near Mars.
- This would meant the “missing mass” of Mars has endde up in the asteroid belt.
- Numerical simulations suggest mars rotational axis may vary chaotically with tilts between 11-49^{o} on timescales as short as a few million years (due to its low mass and gravitational influence of other planets).

3.3 Venus

- Also studied using radar signals sent from Earth, but also using Doppler shifts of reflected sunlight.
- Venus rotates in a retrograde direction, with $P_{\text{rot}} = 243$ d and $P_{\text{orb}} = 224.7$d.
- Main gas in atmosphere is C02 (96.4%). At the base of its atmosphere, it has $T = 740$ K and a pressure of 90 atm.
- This temperature is 2-3 times higher than the irradiation temperature we get when we use the formula from last weeek - due to the greenhouse effect.
- Surface accurately mapped by the Magellan spacecraft. The frequency of the radio signals sent to Earth were Doppler shifted depending on the speed of the spacecraft as it passed over regions of higher/lower mean density.
- It also potentially has traces of phosphine in its upper atmosphere (which **may** be due to life).

3.4 Gas Giants

All planets produce heat due to the decay of radioactive nuclei, including the terrestrial planets, but the Gas Giants show excess heat on top of this. Ti figure out why Gas Giants show this, and terrestrial planets do not, let’s think about the energy that was released by the gravitational collapse of the disc in to the planets, and how long it should take for bodies to dissipate this heat:

$$\tau_{\text{cool}} = \frac{\text{total energy}}{\text{energy loss rate}} \sim \frac{V}{A} \sim \frac{r^3}{r^2} \sim r$$

So, the bigger the planets radius, the longer it takes for it dissipate all of the heat due to contraction. This is the Kelvin-Helmoholtz mechanism. Given this, we would expect Jupiter to still be contracting due to the it’s formation.

4 Quirks of planets

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$$< K >= -\frac{1}{2} < U >$$

$$< E >= \frac{1}{2} < U >= \frac{3}{5}\frac{GM^2}{r}$$

Now, let's look at the time derivative of this:

$$\frac{dE}{dt} = \frac{1}{2}\frac{dU}{dt} = \frac{1}{2}\frac{dU}{dr}\frac{dr}{dt}$$

$$\frac{dE}{dt} = -\frac{3GM^2}{10r^2}\frac{dr}{dt}$$

Given that $\frac{dE}{dt} = P$, we can solve for $\frac{dr}{dt}$:

$$\frac{dr}{dt} = -\frac{10r^2 P}{3GM^2}$$

So if Jupiter's measured power output per unit area is 7.5 W/m^2, then we get a change in radius of 1mm/year, which we are not sensitive to!

Jupiter's heat production can be explained in this way - that is, the excess we see matches what we'd expect from this cooling time for an object of Jupiter's radius. However, Saturn's heat production is too high. - The most likely explanation is that the excess heat in Saturn is generated through the action of the virial theorem continus as He sinks slowly through the H_2 atmosphere.