

Lecture_06

February 2, 2023

1 The Signal-to-Noise Equation

 Key Question

How do we estimate whether a telescope will be able to detect a given source?

1.1 Optical

Now imagine we wish to measure the signal-to-noise(SNR) ratio of a star. This is given by:

$$\text{SNR} = \frac{\text{Signal}}{\text{Noise}}$$

For a CCD, there are 4 main sources of noise: - Shot noise due to S_0 photons from the source, $\sigma_0 = \sqrt{S_0}$ - Shot noise due to S_b photons from the background, $\sigma_b = \sqrt{S_b}$ - Shot noise due to S_d counts caused by the thermal properties of the CCD, $\sigma_d = \sqrt{S_d}$. - A time independent readout noise, $\sigma_R = R$. This is not a square root, and is the standard deviation in the number of electrons measured at the readout step.

Let's assume all of these processes are independent. We are also going to assume they all involve high enough numbers that they are Gaussians. As such, the variance of the sum is the sum of the variances, $\sigma_{\text{Total}}^2 = \sigma_0^2 + \sigma_b^2 + \sigma_d^2 + \sigma_R^2$. As such, the noise term, $N = \sigma_{\text{Total}}$, is given by

$$N = \sqrt{S_0 + S_b + S_d + R^2}$$

and thus the Signal-to-Noise ratio is given by

$$\text{SNR} = \frac{S_0}{\sqrt{S_0 + S_b + S_d + R^2}}.$$

This is the most basic form of this equation. Note that S_0 , S_b , and S_d will all scale linearly with exposure time.

There are 3 limiting cases for the basic form of the equation.

1. **Object limited:** $S_0 \gg S_b, S_d, R^2$. In this case, the equation simplifies to $\text{SNR} = \frac{S_0}{\sqrt{S_0}} = \sqrt{S_0}$. Since the number of counts detected, S_0 , is proportional to time, then $\text{SNR} \propto \sqrt{t}$. This means you eventually get diminishing returns on increasing your exposure time. Additionally, S_0 is proportional to D^2 , where D is the aperture of the telescope. As such, $\text{SNR} \propto D$.
2. **Background limited:** $S_b \gg S_0, S_d, R^2$. In this case, we get $\text{SNR} = \frac{S_0}{\sqrt{S_b}}$. Both S_0 and S_b scale the same way with exposure time and telescope aperture, and the SNR scales the same as in the above. For fixed S_0 , the SNR scales with the square root of the background signal -

so if your background increases (because for example the moon rises), the SNR drops. This is important in determining when to observe your targets - can they withstand a bright moon, or do you need no moon at all?

3. **Read noise limited:** $R^2 \gg S_0, S_b, S_d$ Read noise is independent of exposure time, so this typically only occurs for short exposures. $SNR = \frac{S_0}{R}$. In this regime, since R is time independent, SNR scales linearly with time and with the square of the aperture.

2 Infrared Astronomy

Recall Wien's displacement law, which states that

$$\lambda T = \text{constant} = 2898 \mu\text{m K}$$

So stars with a temperature of 2500 Kelvin will peak in Intensity at a wavelength of $1.2 \mu\text{m}$. So if we want to observe the coolest stars (M-dwarfs), around which we've found a lot of exoplanets (which are also very cold objects), then we need to move to infrared wavelengths. This region is also where we're likely to see the optical spectra of highly redshifted Galaxies, making this a very important window for astronomy.

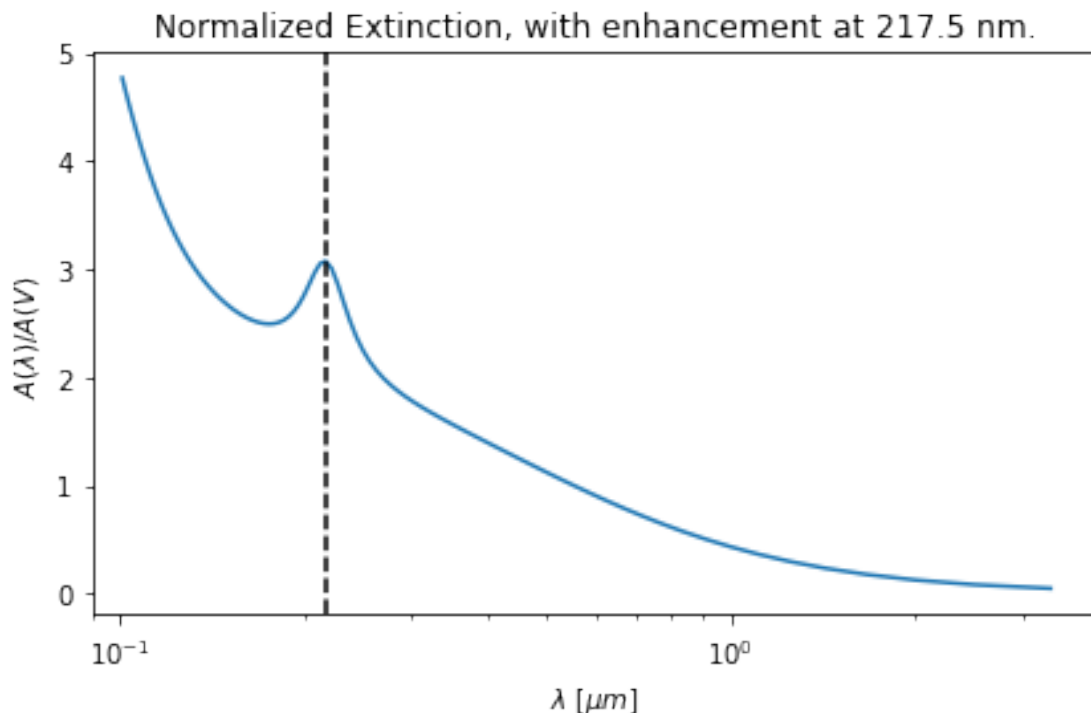
One of the largest issues comes from the thermal radiation produced by the sky and the telescopes themselves. IR instruments are typically cooled to minimise these effects, normally with liquid He. This of course means they have limited lifespans - the helium on the Spitzer space telescope lasted for 5 years.

$1\text{-}5 \mu\text{m}$ observations can be done from the ground, but above this, it is much better to go to space (especially $> 25 \mu\text{m}$).

IR astronomy comes with one other major advantage over Optical astronomy. The effects of interstellar extinction are far weaker at IR wavelengths than at optical wavelengths.

Typically, when discussing interstellar extinction, a value is given as A_V . This represents the magnitude of light absorbed due to interstellar extinction in the Johnson V band. So $A_V = 5$ means 5 mags are absorbed at V band, and so an object will appear 100 times fainter in the night sky than it should. The below figure shows extinction as a function of wavelength - it is stronger at short wavelengths, meaning it affects optical astronomy quite significantly.

Very roughly, there is a linear dependence on A_V with distance, with $A_V = 1$ corresponding to a rough distance of 1 kpc.



Ratio of extinction in K band to V band is: 0.12533034482999322

Example: A star with $A_V = 10$

Imagine we are observing a star through a dust cloud, and that $A_V = 10$ mag. The extinction at K band (which is at $2 \mu\text{m}$) would be:

$$\frac{A_K}{A_V} = 0.1253303 A_K = 10 \times 0.1253303 = 1.2$$

So there's only 1.2 mags of extinction at the K band. Now if we assume that the object is equally bright in both V and K bands, that means the object is ~ 3000 brighter in the K band than in the V band!

So, observing at IR wavelengths allows us to see objects which are heavily obscured by dust at optical wavelengths, such as star forming regions, galactic centres, etc!

3 Radio Astronomy

Karl Jansky observed the first radio waves coming from our Galaxy, when he found evidence for periodic radio emission. He first thought it was from the Sun, but after several months of observations, he found the period to be 23 hr and 56 min, and was coming from the centre of the Milky Way.

The strength of radiation from a radio source is typically given by the Spectral Flux Density, $S(\nu)$, and normally has units of Jy ($1 \text{ Jy} = 1 \times 10^{-26} \text{ W/m}^2/\text{Hz}$). This turns out to be a very bright unit. The most common astrophysical sources are typically in the mJy to μJy range.

Imagine a telescope has efficiency of $f(\nu)$ at ν . The power recieved by the telescope from a source is then given by

$$P = \int_A \int_{\nu} S(\nu) f(\nu) d\nu dA$$

where A is the collecting area of your telescope.

If $S(\nu)$ is constant over the frequency range of interest $\Delta\nu$, and if our efficiency is 1, then

$$P = S A \Delta\nu \text{ (Watts)}$$

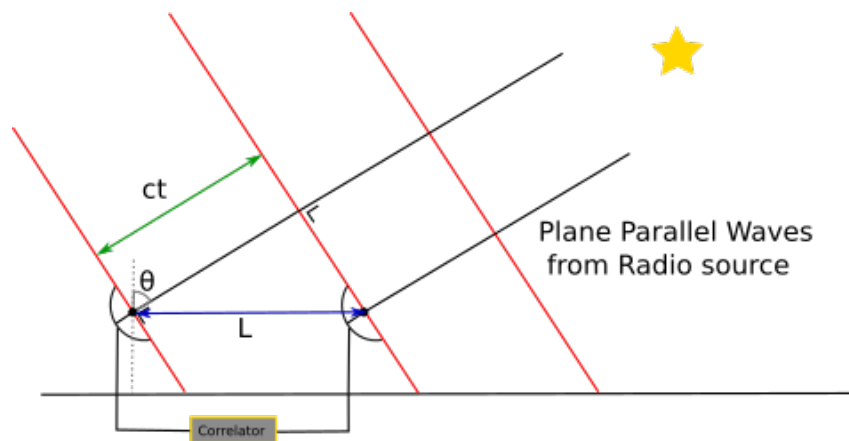
3.1 Interferometry

Now, let's consider the Rayleigh criteria from the last lecture again. Assuming an observing wavelength $\lambda = 10\text{cm}$, and a dish with diameter 50m. The resolution of such a telescope is roughly 8 arcmins.

Now, consider two radio dishes, separated by a distance L, and joined by a correlator. The path difference between the two telescopes for incoming photons, ct , is given by $ct = L \sin(\theta)$. Because of this path difference, the photons arriving at the two dishes are out of phase with each other. By design or through use of time delay, we can ensure that $ct = n\lambda$, where n is an integer. As such, the photons arriving at telescope 1 and 2 can be constructively interfered. This then means that $n\lambda = L \sin(\theta)$. At this stage, the entire setup becomes analogous to Young's Double Slit experiment.

Now assume that the angle θ changes by a small angle $\Delta\theta$. The difference between the new and old time delay, $c\Delta t$, is given by $c\Delta t \sim L\Delta\theta$.

In order for the photons to constuctively interfere again, and for us to see a maximum in intensity in our interference pattern, the change in the path length must at least $c\Delta t \sim \lambda$ - if it is less, the photons will not. Thus, in order to see another peak in the inteference pattern, the minimum angular distance is given by $\Delta\theta \sim \frac{\lambda}{L}$.



This gives the approximate resolution of a interferometer. If L is large, then $\Delta\theta$ is very small.

Very Long Baseline Interferometry (VLBI)

Imagine we had 2 dishes separated by the diameter of the Earth, and observing at 10cm. The resolution of the combined dishes would be 1.6×10^{-3} arcseconds!

We finished our discussion last week by looking at how interferometry works. With this method, the condition required for two sources to be resolved is that the difference in the additional path length photons must travel to reach the second telescope is $c\Delta t \sim \lambda$, such that the photons from the source which is $\Delta\theta$ away from source which is on axis are still arriving in phase and may be combined coherently. We also expressed this condition as $\Delta\theta = \frac{\lambda}{L}$, where L is the separation of the telescopes (also known as the baseline).

We only considered two dishes, but radio interferometers are typically composed of multiple dishes so that the image can be properly sampled, and this phase difference must be measured for every pair of telescopes, which is why the computational cost of this technique quickly increases as you add more baselines to the array.

As examples, consider the Very Large Array - this is an array of 27 dishes, each 25 m in diameter, in New Mexico. The dishes can be repositioned to change the baselines, with the largest baseline being 37 km.

Question: Event Horizon Telescope

Imagine the supermassive black hole at the centre of M87, which lies 16 Mpc away. The Schwarzschild radius for a black hole is given by $2GM/c^2$. If the mass is 2.4×10^9 solar masses, work out what the angular size of the black hole should be for an observer on Earth. If we were carrying out observations at 10 cm, how long would the smallest baseline need to be to resolve the black hole?

Finally, let's ask the question: if this technique can achieve fantastic spatial resolutions, why don't we do it at other wavelengths?

For radio wavelengths (say 10cm), this can be done by accurately time tagging data as it arrives. After all, for $\lambda = 10$ cm, $\Delta t = 0.3$ nanoseconds. Such a high accuracy in timing is achievable using GPS, meaning the data can be recorded, digitised, and combined in a correlator.

However, if we wanted to carry out interferometry at optical wavelengths, things become trickier. Assume a wavelength of $\lambda = 500$ nm. Then $\Delta t = 1.6$ femtoseconds. As such, time tagging data becomes much more difficult. Instead, the handful of optical interferometers which exist (Very Large Telescope Interferometer or the Large Binocular Telescope interferometer) employ mirrors to redirect light from each mirror into a chamber where the photons can be interfered directly.