Lecture_2

January 25, 2022

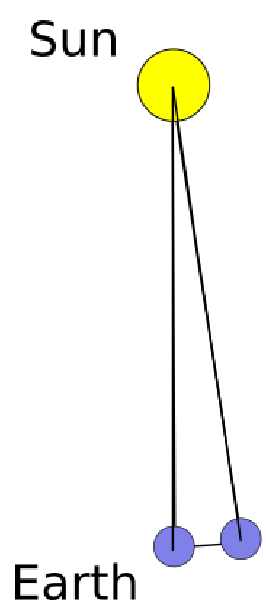
Key Question How do we convey the positions of stars in the night sky to people who are observing from a different place on the Earths surface, or who live at different time to us?

1 Sidereal Day

In the last lecture, we found that the relationship between the synodic period (S) and the sidereal period (P) is given by:

$$\frac{1}{S} = \frac{1}{P_{\oplus}} - \frac{1}{P}(P > P_{\oplus})\frac{1}{S} = \frac{1}{P} - \frac{1}{P_{\oplus}}(P < P_{\oplus})$$

Now consider the duration of a day. If you start a stop watch when the Sun is directly overhead, then stop it when it is overhead again, 24 hours will have passed. However, in this time, the Earth has moved in it's orbit around the Sun, and so it has rotated ~361° in order for the Sun to be over head again. As such, we can use the above equation to solve for the true period of the Earth. Here, S=24 hours. So, using the second equation (as the period we are interested in is less than 1 year), we find that P= 23 hours and 56 minutes. This is a **sidereal day**.

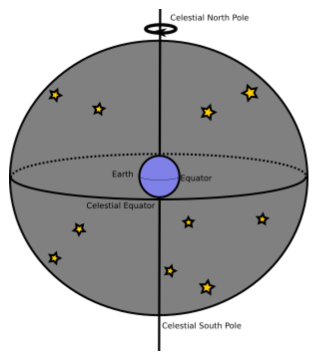


2 Positional Astronomy

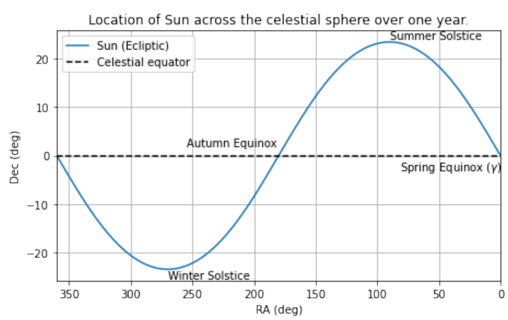
2.0.1 Returning to the Celestial Sphere

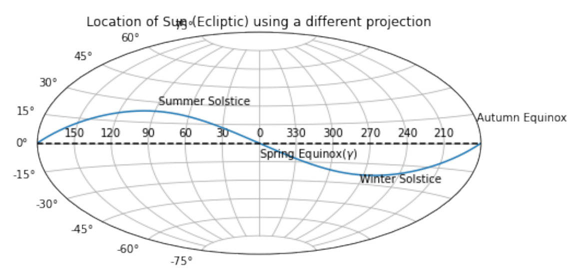
Now that we've explored the various models for the solar system, and convinced ourselves that the Heliocentric model is the way to go, let's discuss positional astronomy - that is, assignging co-ordiantes to objects which are understandable from various locations on the Earths surface and at different times. There are several things we need to account for, such as the rotation of the Earth, the orbit of the Earth around the Sun, and the precession of the Earths axis.

To start, we're going to use a geocentric frame as shown below.



In this coordinate system, and due to the Earths tilt of 23.5 degrees relative to the orbital plane, the sun follows a path throughout the year known as the **ecliptic**. The below piece of code shows the position of the sun on the celestial sphere when viewed from Earth throughout the year. Note that the x-axis is inverted. This is because RA is measured Eastwards from the vernal equinox.

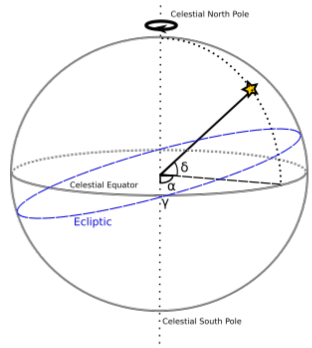




2.0.2 The Equatorial Coordinate System

So, for any given object, its position on the sky is given as: - **Declination** (δ) with respect to the celestial equator (equivalent to "Latitude") - **Right Ascension** with (α) respect to the Spring Equinox γ (equivalent to "Longitude")

This is known as the equatorial coordinate system, with Right ascension measured Eastwards from the location of the vernal equinox (γ). This system is independant of the rotation of the Earth.



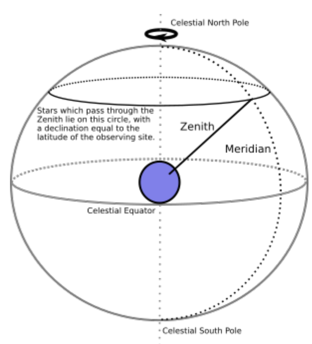
Declination is measured in degrees (°), minutes('), and seconds(") of arc, and goes between $-90^{o} \leq \delta \leq 90^{o}$, with negative declination meaning the object is below the celestial equator.

Right ascension is a bit more complicated and is measured in time, and is given in hour angle (hr), minute (min), and seconds. α= 0hr 0min 0sec corresponds to γ, while α= 12hr 0min 0sec corresponds to 180° away from γ. The reasons for this system will be made clear in the next lecture

2.1 Observing from the Earths Surface

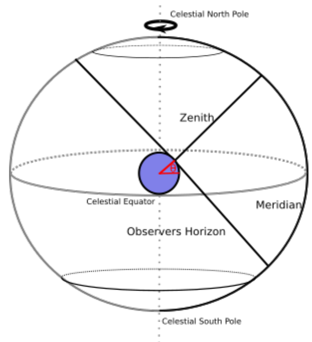
2.1.1 Declination

Consider an observer standing on the surface of the Earth. The point directly above the observer is known as the zenith, and the line which joins the North celestial pole with the celestial equator that passes directly over head is known as the meridian. Stars are easiest to observe as they pass through the Meridian, as at that point in time, there is a minimum in the atmospheric depth the starlight must pass through to reach the observer.



Also, any star whose declination matches the latitude of the observing site will pass through the Zenith. So, for Cork (a latitude of 52^o), any star that has a declination of $+52^o$ will be directly overhead at some stage during the day/night.

The observing site also sets a relative horizon for the observer. Consider the image below (where we've moved the observer to the limb of the Earth for ease of drawing). The observer is at an latitude of θ. Any star which has a declination greater $90^o - \theta$ will never set below the observers horizon (meaning they're always visible), while any star with a declination less than (-$90^o + \theta$) will never rise (meaning they're never visible).



Example Distance from Sirius to the Zenith

What is the minimum angular distance between Sirius and the zenith as viewed from Cork (latitude 52^o)? What altitude above the horizon will it be at this time? Sirius is (approximately) located at - α = 06 hr 45 min - δ = -16^o 42'

It will therefore be closest to the zenith when passing through the meridian ($\Delta\alpha$=0). At this time, the angle between the zenith and Sirius will be $52^o - (-16^o 42') = 68^o 42'$. Since the angle between the zenith and the horizon is 90^o, then the altitude of Sirius will be $90^o - 68^o 42' = 21^o 18'$.

2.1.2 Right Ascension (Time)

Solar (civil) dat is the time between successive passages of the Sun cross the meridian, and is 24 hours. In this time, the Earth rotates 361 degrees (the extra degree comes from the movement of Earth around the Sun throughout the day).

The sidereal day is the same, but for successive passages of stars across the meridian. This is the **true** rotation of the Earth, and is 23 hrs and 56 mins.

So a civil day (from now on, just referred to as day) lasts for 24 hours. However, the sidereal day lasts for 23 hours and 56 minutes. This means stars will rise and set 4 minutes earlier every day. Also, as stars move across the night sky, telescopes must track them at the sidereal rate.

The right ascension of an object (α) is defined relative to the position of the Sun on the Vernel equinox (γ in previous diagrams). The below code shows that this happens between 22:57:29 and 22:57:30 on the 20th of March this year.

```
[359d59m59.98109124s 0d00m00.0190618s]
```

Now, from Greenwich, the sun passes through the meridian at noon. As such, any other star which lies on the meridian at noon from Greenwich has $\alpha = 0$ hrs. Furthermore, at midnight (12 hours later) any star lying on the meridian as observed from Greenwich will have $\alpha = 12$ hrs. This is the ideal time to observe a source with $\alpha = 12$, as it is at it's closest point to the Zenith, minimising the atmosphereic effects which hinder observations.

One month later (towards the end of April), stars which are passing through the meridian at midnight will have a difference in right ascension of

$$\alpha = \frac{4\text{min}}{60\text{min}} \times 30 + 12\text{hr} = 14\text{hr}$$

It's also important to note that when a star is on the meridian from Greenwich (longitude of 0^o), it **is not** from other observing sites. Take Cork (longitude=8.5^o West) as an example. A star will pass through our meridian

$$\frac{8.5}{360} \times 24 \times 60 = 34\,\mathrm{min}$$

later than when it passed through the meridian from Greenwich.

Exercise: Work out this difference between Dublin and Cork.

The time between equinoxes is ~ 365.25 days. Because of this, the time of the equinox shifts by ~ 6 hours every year. As such, the equinox next year will occur at 2023-03-21 05:10:07.

```
[359d59m59.96453645s 0d00m00.00253351s]
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This date has always been important in religions. For example, Easter is the first Sunday after the first full moon after the Spring equinox. Irish monks played an important role in developing the "Computus", which calculated the date of Easter with respect to the equinox.

2.1.3 Reference Epochs

While this system is independant of the rotation of the Earth because we are defining everything relative to a fixed point (γ, the location at which the ecliptic intersects the celestial equator at the spring equinox), it is **not** independant of the precession of the Earth's axis. So, we must specify an epoch which the coordiantes are correct for. The most common epoch is the J2000 epoch, which uses a reference date of noon at Greenwich on January 1 2000. Whenan object coordinates are given with this epoch, they can then be precessed to match the current observation date. The other common reference epoch is B1950, which is midnight on January 1, 1950.

2.1.4 Reference Times

The time at which an astronomical event (a supernova, an eclipse, a gamma-ray burst) usually occurs is typically given in Julian days (JD). This is the time which has lapsed, in days, since noon at Greenwich on January 1, 4713 BC. There are variants such as the Modified Julian Date (MJD), which is the time in days since midnight, November 17 1858. MJD is typically used instead of JD simply because it removes a very large constant. The below code shows todays JD and MJD (as computed when I last evaluated this cell).

```
The current time is: 2022-01-20T08:30:35.669
The current time in JD is: 2459599.854579499
The current time in MJD is: 59599.354579498904
```