

# Lecture\_05

January 29, 2024

## 1 Optics

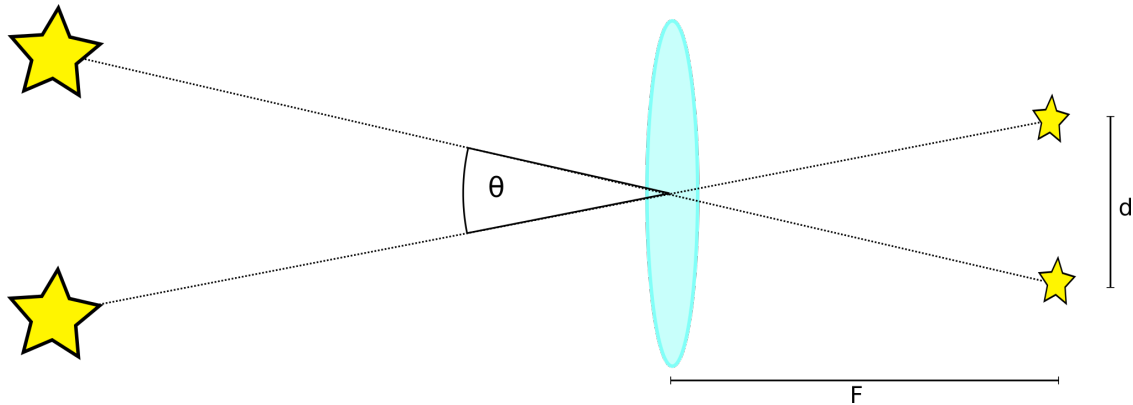
### **Key Question**

What's the minimum distance between two objects which can individually be resolved, and how do optical telescopes work?

### 1.1 Focal length, diameter, f number, plate scale

There are a few key phrases that get used when discussing telescope design that are useful to define. First, we have the focal length ( $F$ ), which is the distance from a lens to where an image is created. Second, we have the diameter of a lens,  $D$ . Often photographers refer to the focal ratio,  $f$ , which is  $F/D$ .

Now imagine we have 2 stars that are separated by an angle of  $\theta$  on the sky, and a distance  $d$  at the focal plane.



If we define the plate scale to be  $s$  and that its units should be arcsec/mm, then the relation to  $\theta$  and  $d$  is

$$\theta[\text{arcsec}] = s[\text{arcsec/mm}] d[\text{mm}]$$

Using the small angle approximation, we have that

$$\theta[\text{rad}] = \frac{d}{F}$$

Converting  $\theta$  from radians to arcsec and combining with the above gives that

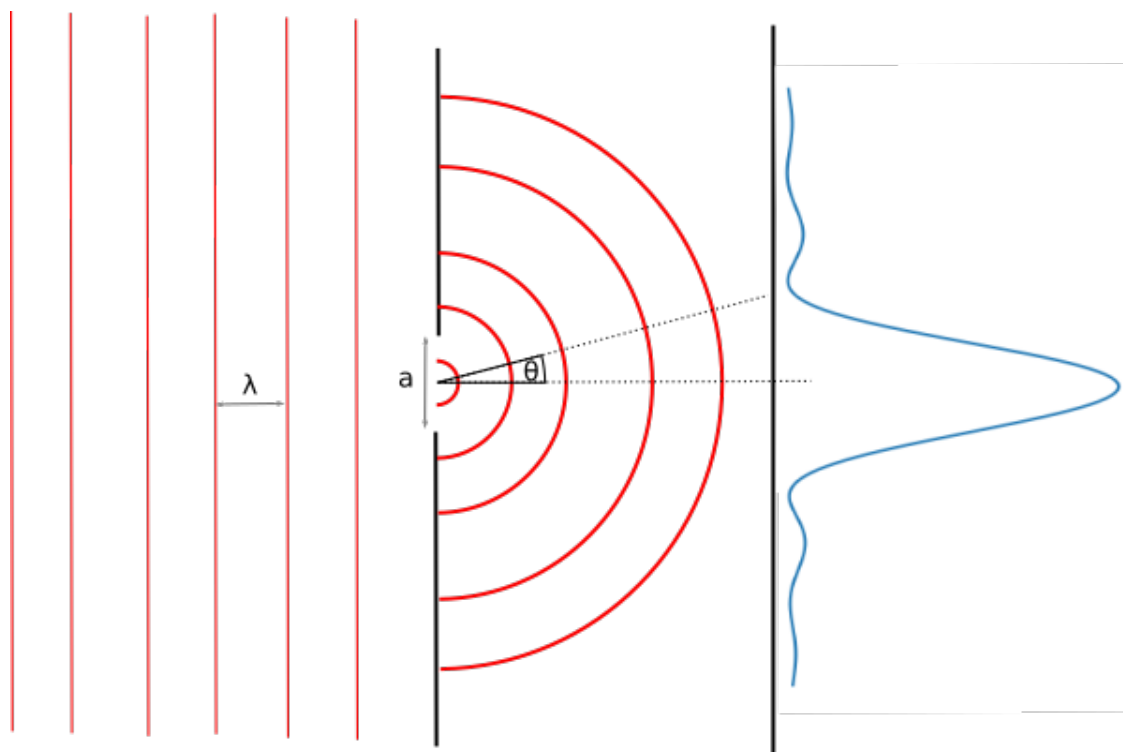
$$s[\text{arcsec/mm}] = \frac{206.265}{F[\text{m}]}.$$

## 1.2 Fraunhofer diffraction

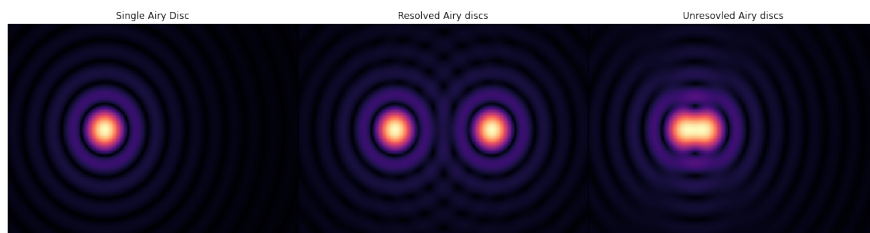
Recall from your optics lecture that a wave with wavelength  $\lambda$  which passes through a slit of width  $a$ , where  $a \gg \lambda$ , will produce a pattern given by:

$$I = I_0 \frac{\sin^2\left(\frac{\pi a \sin(\theta)}{\lambda}\right)}{\left[\frac{\pi a \sin(\theta)}{\lambda}\right]^2}$$

This is shown in the below figure. This pattern has a minima at  $\sin \theta = \frac{m\lambda}{a}$  where  $m$  is an integer. The interesting thing here is to think about what the mathematical relationship between the slit (which is a boxcar function) and the Intensity is. The Fourier transform of a boxcar is a sinc, and our function is a  $\text{sinc}^2$ . This is a branch of optics known as Fourier optics, which allows us to use Fourier methods to establish what various optical elements are doing.



If the slit is 2 dimensional, like the aperture of a telescope, then the pattern produced is this function rotated around its axis, and is called an Airy Disc. In this case, the difference between the minima can still be given by  $\sin \theta = \frac{m\lambda}{a}$ , but  $m$  is no longer integer. The first minimum occurs when  $m = 1.22$  (the derivation is tough and beyond the scope of this current module).



So, if we have 2 sources in our image, they will both produce Airy Discs. In order to not have their primary peaks overlapping, we want their angular separation to be:

$$\sin \theta > \frac{1.22\lambda}{a}$$

or, for small angles

$$\theta > \frac{1.22\lambda}{a}.$$

This is the Rayleigh criteria, and is very important when considering what wavelength to observe your source at.

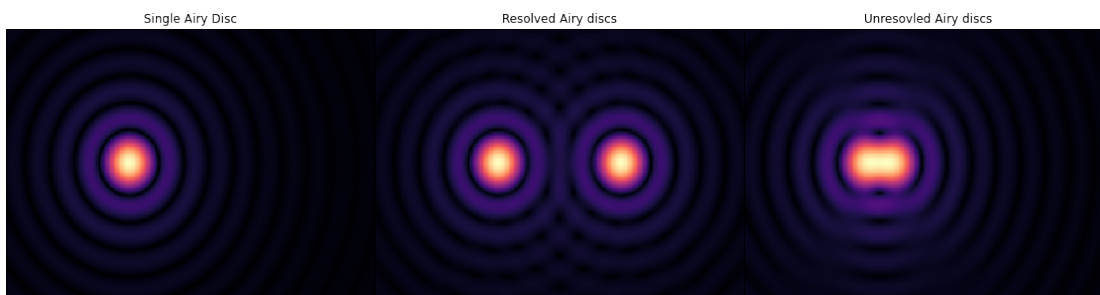
**Example:** A 10m optical telescope

For a 10m aperture optical telescope (observing at  $\lambda = 5000\text{\AA}$ ) we find that

$$\theta = \frac{1.22 \times 0.5\mu\text{m}}{10\text{m}} = 0.01''$$

Again, this is much smaller than the seeing we're used because of Earth's atmosphere. As such, it makes sense to put optical telescopes into space where they can make full use of their resolution.

For example, consider the HST. Its mirror is 2.5 meters in diameter so  $\theta = 0.05''$ .

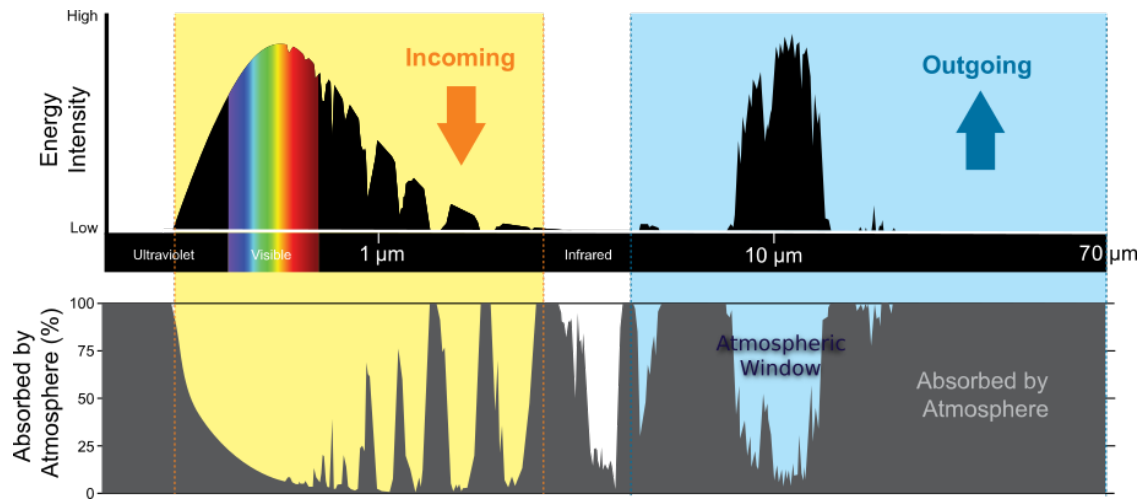


### 1.3 4 branches of Astronomy

Now, consider the following three wavelength regimes:

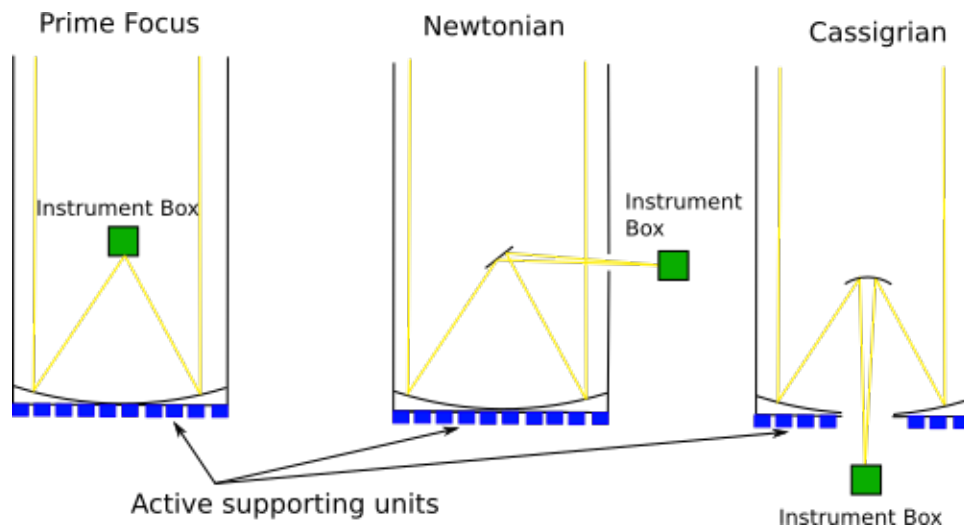
- Radio (10m-mm). Take a 40m telescope and  $\lambda=1\text{m}$ , then  $\theta = 0.8^\circ$ .
- IR (30 m-1 m). Take a 6.5m telescope and  $\lambda=10\text{ m}$ , then  $\theta = 0.4''$ .
- Optical (300 nm - 1 m). Take a 10m telescope and  $\lambda=500\text{ nm}$ , then  $\theta = 0.01''$ .
- X-ray (0.12 nm - 4.1 nm). Take a 1 m telescope and  $\lambda=1\text{ nm}$ , then  $\theta = 0.0002''$ .

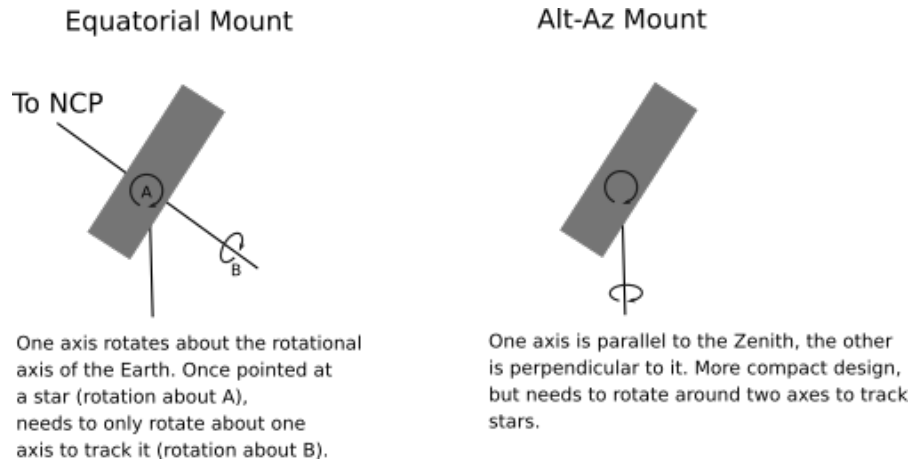
So, single dish radio telescopes have very poor resolution (we'll get to this next lecture), while X-ray imaging should have outstanding resolution. From our viewpoint on the Earth's atmosphere, we have access to a limited window, covering the optical window and the radio window (image from <https://www.weather.gov/jetstream/absorb>)



## 1.4 Telescope Designs

So how do we design optical telescopes to maximise their effectiveness? There are three optical designs which telescopes follow: Prime focus, Newtonian, and Cassigrian. Each of them is capable of having active supporting units under their primary mirrors, which allows mirrors to keep their shape when at an angle to Earth's gravitational field.



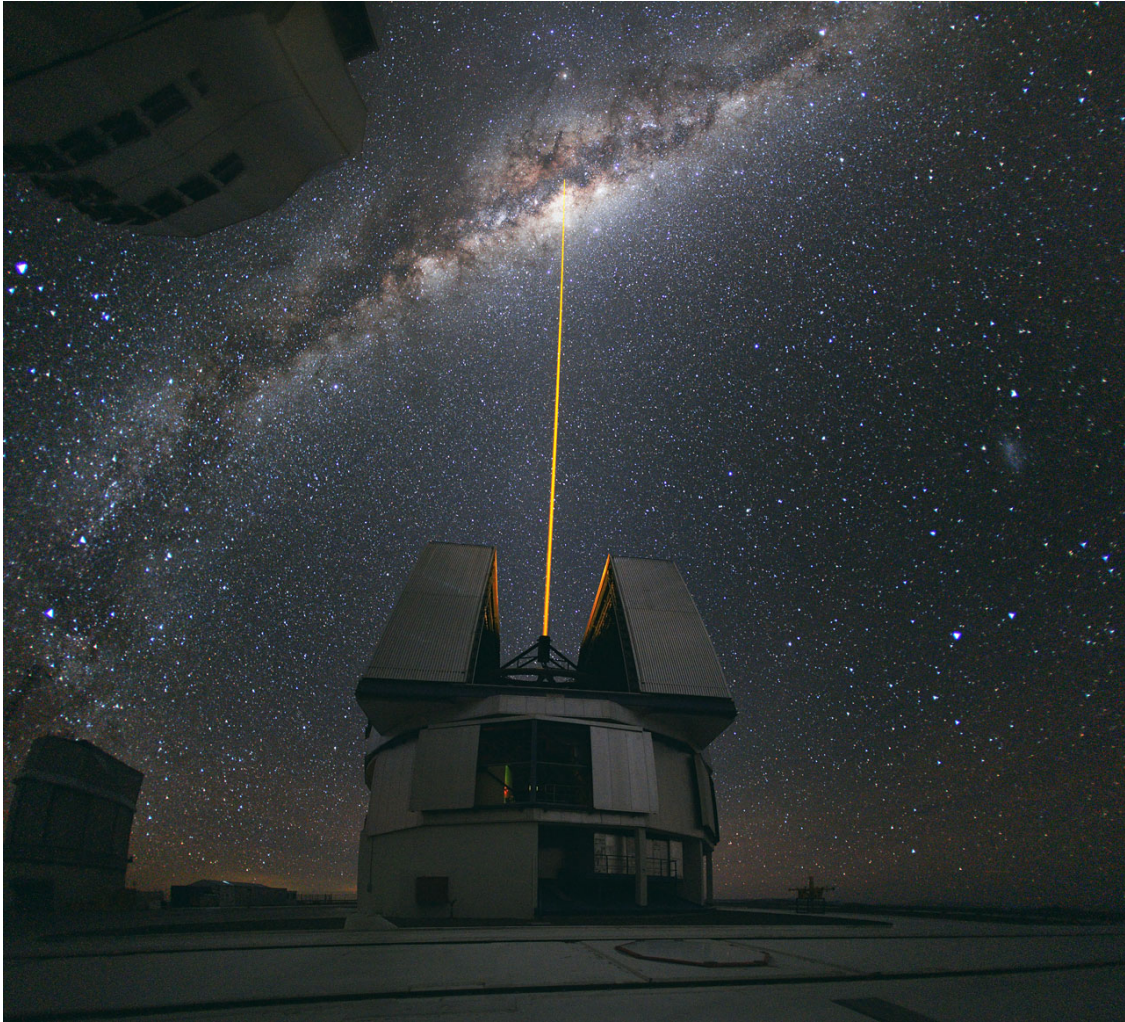


In combination to the optical design, there is also the question of mount design. There are two primary designs: equatorial, and Altitude-Azimuth design (Alt-Az). For equatorial telescopes, one of the axes is aligned parallel to the Earth's rotational axis. This means that when the telescope is pointed at a star, it then only requires movement around one axis in order to track stars. Very common for small, commercial telescopes, but for large telescopes, they require huge counterweights, and so becomes expensive. The Hale telescope for example: <https://sites.astro.caltech.edu/palomar/about/telescopes/hale.html>. It's a 5m telescope, but requires a Dome which is 41 m tall and 42 m in diameter.



Alt-Az telescopes are significantly more compact in design. One axis is perpendicular to the Zenith, the other is parallel to it. As such, they require motion around 2 axes in order to track stars. See, for example, the VLT (<https://www.eso.org/public/images/potw1036a/>). These 10m telescopes only need a dome which is 30 m tall and 28 m in diameter.

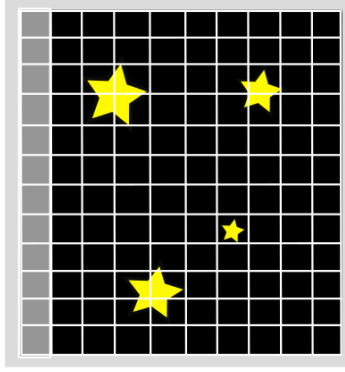




## 1.5 Optical

The human eye has a very low quantum efficiency (QE) of about 1% (this means around 1 in every 100 photons is actually detected). At optical wavelengths, we typically use CCDs, or charged-coupled devices, for observations. This is due to their very high quantum efficiency, meaning their response is nearly linear - that is, the number of counts you observe is directly proportional to the intensity of light.

They also have very high dynamic ranges - each pixel in a CCD is capable of registering up to ~65,000 counts accurately. Typically, CCD's are Si based. As a photon interacts with a given pixel, it causes excitation of an electron into a conduction band. When finished observing, the accumulation of charge within a pixel can be shifted to an adjacent pixel. As such, an image is read out pixel-by-pixel, as shown below.



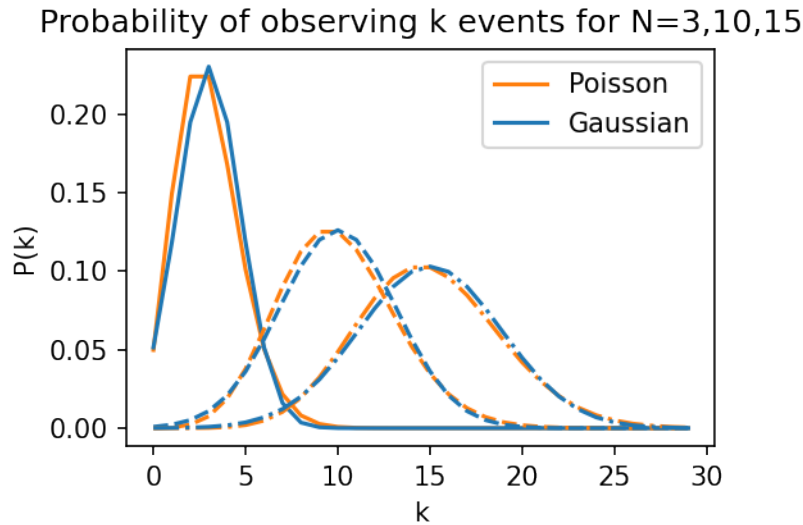
Now, assume the number of photons reaching our detector over a fixed interval of time is  $N$ , but the arrival time of each photon is randomly distributed. The probability of detecting  $k$  photons over a fixed time interval is then given by a Poisson distribution

$$P(k) = \frac{N^k e^{-N}}{k!}$$

It is easy enough to show that for such a distribution, the standard deviation is given by  $\sigma = \sqrt{N}$ . For large  $N$  ( $>10$ ), this becomes a Gaussian with  $\sigma = \sqrt{N}$  (see below)

$$P(k) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(k-N)^2/2\sigma^2}$$

As such, any process which involves a large number of events has an ideal error on it of  $\sqrt{N}$  - this is called shot noise.



Now imagine we wish to measure the signal-to-noise(SNR) ratio of a star. This is given by:

$$\text{SNR} = \frac{\text{Signal}}{\text{Noise}}$$

For a CCD, there are 4 main sources of noise: - Shot noise due to  $S_0$  photons from the source,  $\sigma_0 = \sqrt{S_0}$  - Shot noise due to  $S_b$  photons from the background,  $\sigma_b = \sqrt{S_b}$  - Shot noise due to  $S_d$  counts caused by the thermal properties of the CCD,  $\sigma_d = \sqrt{S_d}$ . - A time independant readout noise,  $\sigma_R = R$ . This is not a square root, and is the standard deviation in the number of electrons measured at the readout step.

Let's assume all of these processes are independant. We are also going to assume they all involve high enough numbers that they are Gaussians. As such, the variance of the sum is the sum of the variances,  $\sigma_{\text{Total}}^2 = \sigma_0^2 + \sigma_b^2 + \sigma_d^2 + \sigma_R^2$ . As such, the noise term,  $N = \sigma_{\text{Total}}$ , is given by

$$N = \sqrt{S_0 + S_b + S_d + R^2}$$

and thus the Signal-to-Noise ratio is given by

$$\text{SNR} = \frac{S_0}{\sqrt{S_0 + S_b + S_d + R^2}}.$$

This is the most basic form of this equation. Note that  $S_0$ ,  $S_b$ , and  $S_d$  will all scale linearly with exposure time.

There are 3 limiting cases for the basic form of the equation.

1. **Object limited:**  $S_0 \gg S_b, S_d, R^2$ . In this case, the equation simplifies to  $\text{SNR} = \frac{S_0}{\sqrt{S_0}} = \sqrt{S_0}$ . Since the number of counts detected,  $S_0$ , is proportional to time, then  $\text{SNR} \propto \sqrt{t}$ . This means you eventually get diminishing returns on increasing your exposure time. Additionally,  $S_0$  is proportional to  $D^2$ , where D is the aperture of the telescope. As such,  $\text{SNR} \propto D$ .
2. **Background limited:**  $S_b \gg S_0, S_d, R^2$  In this case, we get  $\text{SNR} = \frac{S_0}{\sqrt{S_b}}$ . Both  $S_0$  and  $S_b$  scale the same way with exposure time and telescope aperture, and the SNR scales the same as in the above. For fixed  $S_0$ , the SNR scales with the square root of the background signal - so if your background increases (because for example the moon rises), the SNR drops. This is important in determining when to observe your targets - can they withstand a bright moon, or do you need no moon at all?
3. **Read noise limited:**  $R^2 \gg S_0, S_b, S_d$  Read noise is independant of exposure time, so this typically only occurs for short exposures.  $\text{SNR} = \frac{S_0}{R}$ . In this regime, since  $R$  is time independant, SNR scales linearly with time and with the square of the aperture.