CONCORDIA UNIVERSITY
ENGR 244 – Mechanics of Materials

Experiment 4: Stress Analysis of Beams Using Strain Gauges

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Experiment Date:

Mai 30th, 2024

Lab Section:

ENGR 244 AJ-X

Objective:

In this experiment we attempt to observe the behaviour of a metal specimen under a specific transverse load that generates a bending moment and analyze the stress distribution and strain produced from bending. Based on our results we will determine some important material properties such as Poisson's ratio, the elastic and shear modulus, and the stress distribution.

Introduction:

The bending stress analysis on a simply supported beam using strain gauges is a test used to analyse the distribution of stresses along a beam and deduce the corresponding properties. In terms of mechanics of materials, bending is the application of a load perpendicular to the longitudinal axes of a member that causes it to deform. The deformation is a result of the stresses and strains induced by the bending moment created by the load; this deformation is a set of point that form an arc resulting from the deflection of the beam with a curvature ρ from its neutral position [1].

The stresses caused by a planar bending moment on a beam induce compression on the inside diameter of the curvature, and tensile on the outside diameter.

The study of structural analysis has long been of interest considering its importance in construction of structures, early studies of beam stress analysis conducted by European scientists like Galileo Galilei, and later Thomas Young, Robert Hooke and others who later developed the fundamental theory behind stress analysis, failure, elasticity and structural engineering [2].

The strain gauge test with four loading points has been chosen for this experiment due to its accuracy and reliability in providing accurate data as well as the availability of the testing material.

There exist various other methods of analyzing stress distribution in beams such as three loading points test that's quite similar to our test of choice in using strain gauges, the cantilever beam testing where only two loading points are used to analyze bending moments, and digital image correlation using advanced optical techniques and software to measure the displacement due to bending.

Prior to the experiment, certain assumptions are to be noted:

- The gauges used has been properly calibrated to output the exact loading shown digitally.
- The fixture hasn't been ASTM certified.
- The testing sample is uniform and contains only steel.
- The geometry of the sample has been machined to have smooth flat surfaces all around, with a straight longitudinal axis.

Testing materials for bending is a crucial part of stress analysis, as it helps inform the behaviour of materials under different types of loads. In more practical terms, part of a structural engineer's responsibility is to evaluate if the material used for their structures will withstand the different loads applied to it. Analyzing the stresses produced by bending moments applied to every member of the structure is a direct application of our experiment. Similar applications exist across the board in all engineering fields, like automotive and aerospace.

Procedure:

In this experiment we used a bending fixture to analyze the stresses generated by bending a beam uniformly made of steel, the dimensions of the member are as follow, width b=18.77mm, height h=31.90mm. The testing fixture comprises of a platform fixed at the workbench that holds two vertically standing rods at a distance L=455mm, as well as a vertically standing pulling mechanism fixed on the platform at an equal distance from both rods, see figure 1. The standing rods act as simple supports for the beam, while the mechanism applies a pulling force on two points of beam with an equal force P/2. The two loading points are at an equal distance c=45mm from the middle of the beam, see figure 2.

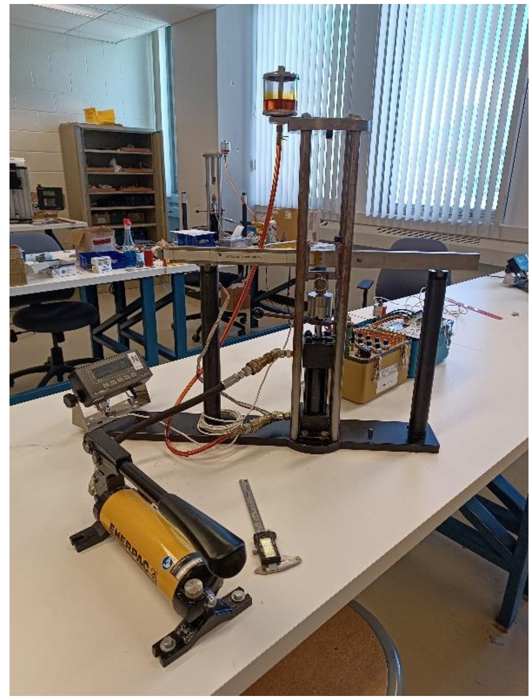


Figure 1: Bending testing fixture

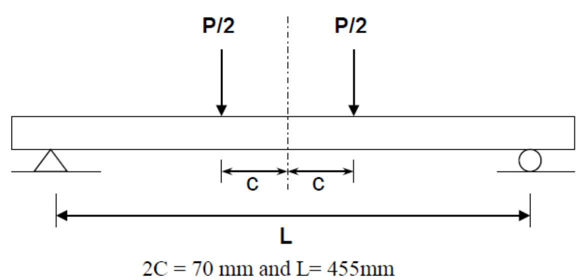


Figure 2: Free body diagram of the beam [3]

Figure 3: Strain indicator and recorder

The collection of data is done through strain gauges, the gauges act as sensors and measure the strain that the beam experiences. There is a total of six gauges placed on the different location of the beam as seen in figure 4 bellow. The gauges collect the data during the test and displays it on the strain indicator as seen in figure 3.

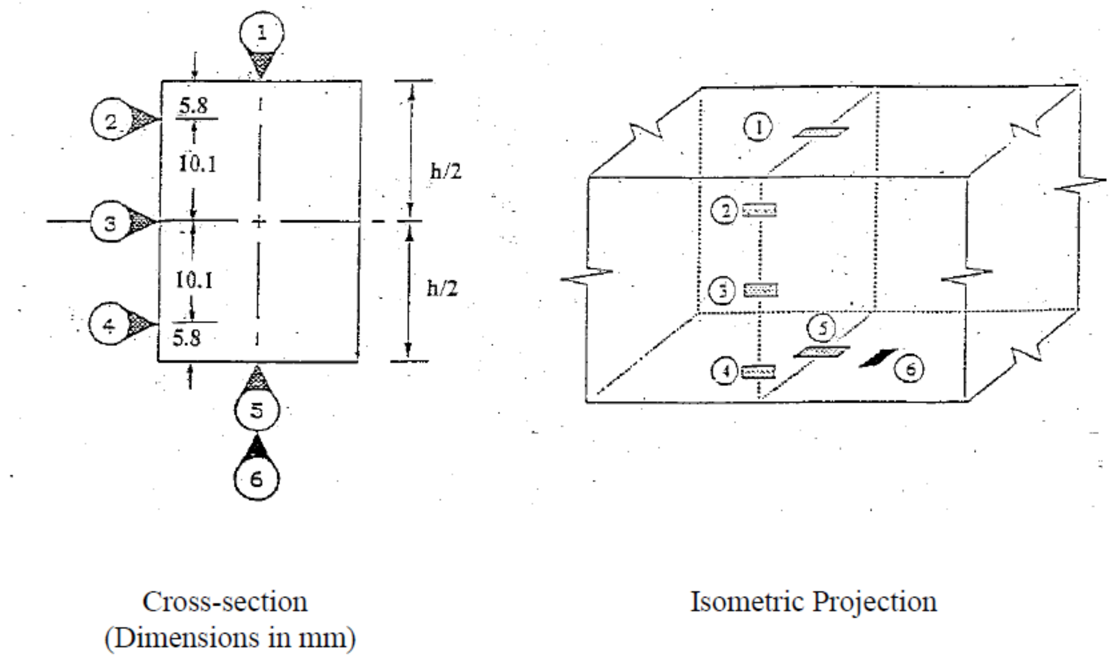


Figure 4: Location and orientation of strain gauges [3]

The test starts by setting the sample on the fixture and calibrating the gauges using the gauge selector. The pump is actuated manually to slowly increase the load on the beam, and for every 1000N, the load is held steady to record the corresponding strain for each of the six gauges. We record the strain at every point of the loading curve, and at the end of the test we record the unloaded beam strain values.

Results:

The results of the strains recorded during the test have been compiled in table 1 for each of the gauges on the beam for a load interval of 1000N.

Load P (N)	Strain ($\times 10^{-6}$)					
	Gauge 1	Gauge 2	Gauge 3	Gauge 4	Gauge 5	Gauge 6
1000	-142	-91	-3	84	146	-40
2000	-287	-182	-5	171	290	-80
3000	-431	-274	-5	259	437	-123
4000	-575	-365	-5	348	581	-164
5000	-719	-455	-4	437	727	-205
Unload	-1	1	-7	-10	-2	-1

Table 1: Strain indicator readings per load

Let us determine the moment of inertia of the beam; to do so we can use the following equation:

$$\mathbf{I} = \frac{\mathbf{bh^3}}{\mathbf{12}} = \frac{18.77 \times 31.90^3}{12} = 50775.60mm^4$$

In figure 2, we can see the distribution of the shear force, and the bending moment of the beam.

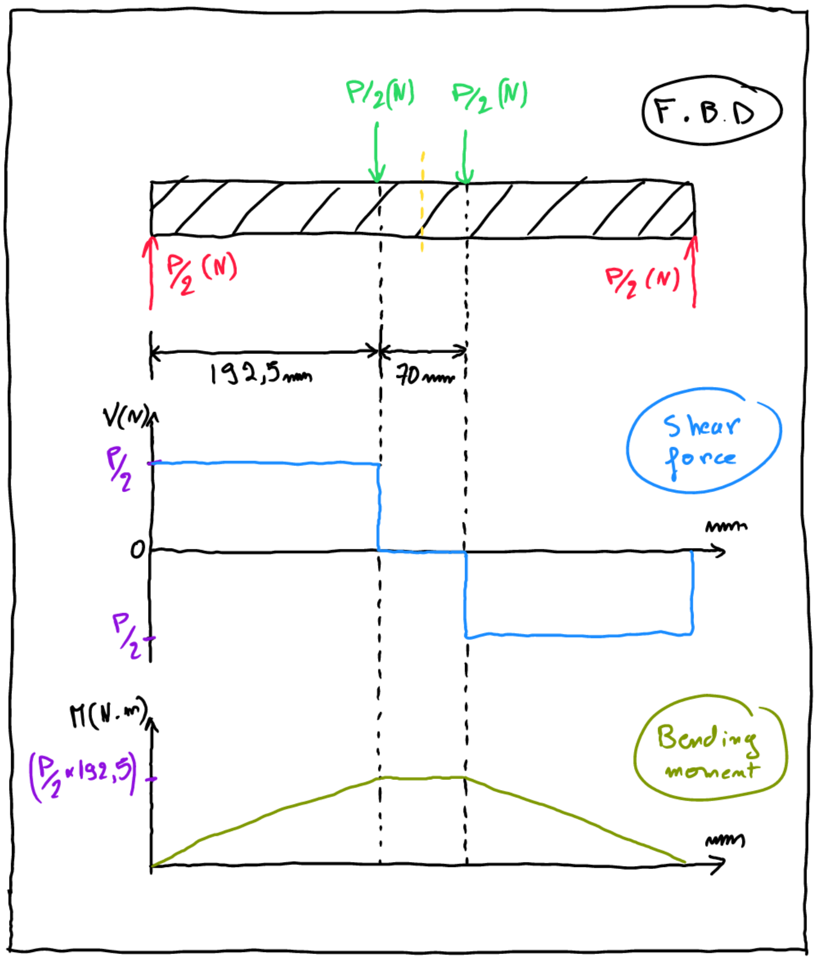


Figure 2: Shear force, bending moment, and free body diagrams

In order to calculate the bending stresses at strain gauges points for each loading stage, we can use the following formula:

$$\boldsymbol{\sigma} = \frac{\mathbf{My}}{\mathbf{I}} \qquad (1)$$

M: the moment applied to the beam (N.m)
y: the distance from the neutral axis (mm)
I: the moment of inertia of the beam (mm^2)

To calculate the moment at any point we can use the following equation:
$\mathbf{M = r \times P}$
r: the perpendicular distance from the line of action of the load (mm)

Here is an example of the calculation of the bending stress at strain gauge #1 at P=1000N:

$$\sigma = \frac{-15.9 \times 1000 \times 192.5/2}{50775.60} = -30.14 \text{ MPa}$$

The results of the calculation of the bending stress for all points and loads is compiled in table 2.

Load P (N)	Stress (MPa)					
	Gauge 1	Gauge 2	Gauge 3	Gauge 4	Gauge 5	Gauge 6
1000	-30.14	-19.15	0	19.15	30.14	30.14
2000	-60.28	-38.29	0	38.29	60.28	60.28
3000	-90.42	-57.44	0	57.44	90.42	90.42
4000	-120.56	-76.58	0	76.58	120.56	120.56
5000	-150.70	-95.73	0	95.73	150.70	150.70

Table 2: Bending moment stress by loads on each strain gauge using equation (1)

Figure 3 represents the strain-stress curves of the beam based on results from channels 1 and 5.

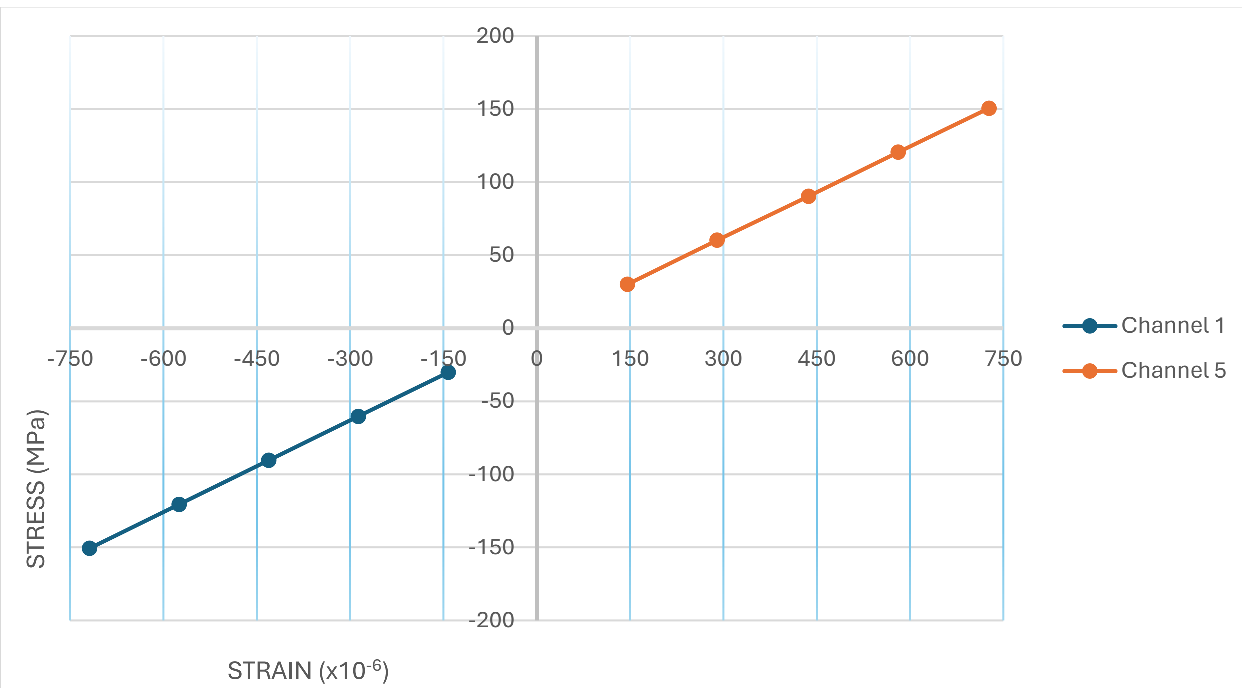


Figure 3: Stress-Strain diagram (channel 1 and 5)

In order to find the elastic modulus, we can use the following equation:

$$\mathbf{E} = \frac{\boldsymbol{\sigma}}{\boldsymbol{\varepsilon}} = 209 \text{ MPa}$$

Poisson's ratio is defined as the absolute value of the ratio of the transverse strain to the axial strain, it is basically the ratio of the slopes of curves given by channels 5 and 6, refer to figure 4.

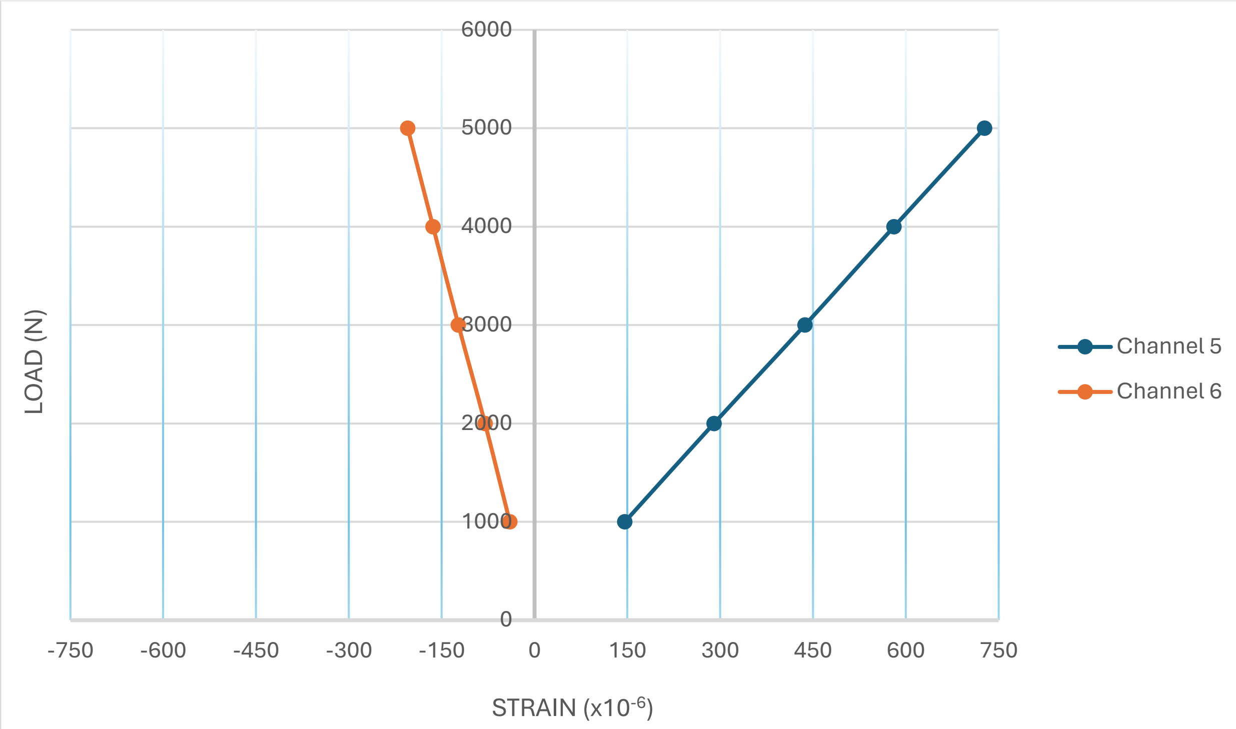


Figure 4: Load-Strain diagram (Channel 5 and 6)

In order to calculate Poisson's ratio, we can use the following equation:

$$\boldsymbol{\nu} = \frac{\boldsymbol{\Delta\varepsilon_t/\Delta P}}{\boldsymbol{\Delta\varepsilon_l/\Delta P}} = \frac{(205 - 40) \times 10^{-6}/(5000 - 1000)}{(727 - 146) \times 10^{-6}/(5000 - 1000)} = 0.28$$

Table 3 contains the stress at each gauge using a published value of the elastic modulus **E=200GPa** along with the experimental values for the strains through the following equation:

$$\boldsymbol{\sigma = E \times \epsilon} \qquad (2)$$

Load P (N)	Stress (MPa)					
	Gauge 1	Gauge 2	Gauge 3	Gauge 4	Gauge 5	Gauge 6
1000	-28.4	-18.2	0	18.2	28.4	27.4
2000	-57.4	-36.4	0	36.4	57.4	57.4
3000	-86.2	-54.8	0	54.8	86.2	86.2
4000	-115.0	-73.0	0	73.0	115.0	115.0
5000	-143.8	-91.0	0	91.0	143.8	143.8

Table 3: Bending moment stress by loads on each strain gauge using equation (2)

In order to calculate the shear modulus, we can use the following equation:

$$\mathbf{G} = \frac{\mathbf{E}}{[\mathbf{2} \times (\mathbf{1} + \boldsymbol{\nu})]} = \frac{209 \times 10^9}{[2 \times (1 + 0.28)]} = 81.64 \text{ GPa}$$

Considering that the published value of the shear modulus is G=77 MPa, the value that we calculated experimentally G=81.64 GPa is slightly higher but within a reasonable range.

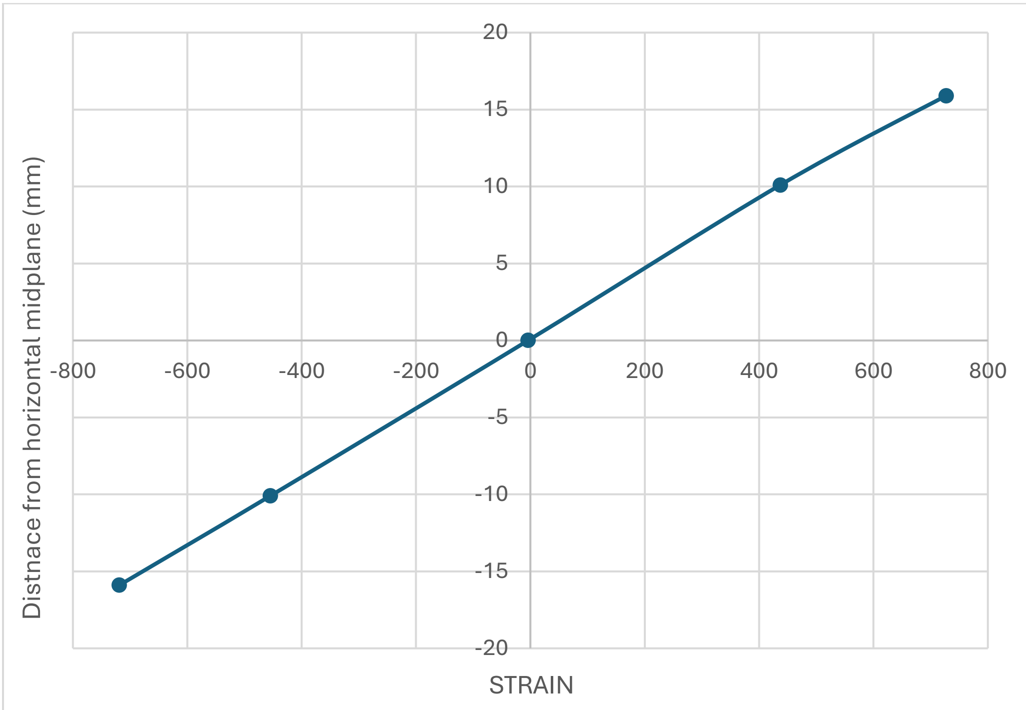


Figure 5: Strain vs the distance from the horizontal midplane of the beam for P=5000N

Based on our analysis of the curve on figure 5, we note that the experimental location of the neutral axis is almost the same as the theoretical one. Considering that the neutral axis is defined as the axis where the strain is null, so little to no variation in the recorded strain indicate the location of the neutral axis.

Load P (N)	**Results**	**Stress (MPa)**					
		Gauge 1	Gauge 2	Gauge 3	Gauge 4	Gauge 5	Gauge 6
1000	Experimental	-30.14	-19.15	0	19.15	30.14	30.14
	Theoretical	-28.4	-18.2	0	18.2	28.4	27.4
2000	Experimental	-60.28	-38.29	0	38.29	60.28	60.28
	Theoretical	-57.4	-36.4	0	36.4	57.4	57.4
3000	Experimental	-90.42	-57.44	0	57.44	90.42	90.42
	Theoretical	-86.2	-54.8	0	54.8	86.2	86.2
4000	Experimental	-120.56	-76.58	0	76.58	120.56	120.56
	Theoretical	-115.0	-73.0	0	73.0	115.0	115.0
5000	Experimental	-150.70	-95.73	0	95.73	150.70	150.70
	Theoretical	-143.8	-91.0	0	91.0	143.8	143.8

Table 3: Experimental vs. theoretical bending stress values

Based on the tabulated experimental and theoretical results of the stresses at each gauge in table 3, we notice that the experimental values are slightly higher than the theoretical ones across the board, with some slight variations. For example, at gauge 1 which is the furthest from the neutral axis, the experimental value is less than 2 MPa higher than the theoretical one. Based on the direction of the bending, and observing the generated moment, location of gauges 1 and are in compression, gauge 3 sits on the neutral axis therefore experiences no stress due to bending. And the three gauges that are left, 4 and 5 and 6 all experience tension.

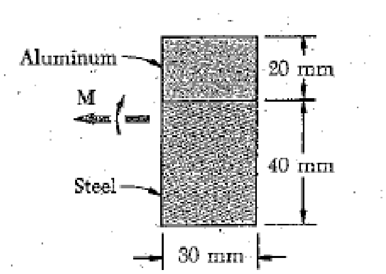
Discussions:

An analysis of the strain-distance from N.A shown in graph 5 can answer an important question; after bending, does the section of a beam remain plane? The section returns to its pre-bending state after removing the load, and this is indicated by the linearity of the graph, it informs that the strain is directly proportionate to the distance from the horizontal mid-plane of the beam. Our observation aligns with the Bernoulli-Euler beam theory which is the basis of the assumption where any plane of any perpendicular section to the N.A. of the beam will remain perpendicular after bending.

If we compare the experimental and theoretical results of the stresses due to the bending moment on the beam, we notice some slight variations as noted in table 3. There are many factors that can contribute to amplification of these discrepancies. The most notable ones are human and machine error. First, the results depend critically on the caution of the experimenter, since any misalignment or an improper loading position, or misuse of the tools, would impact the results. The test also depends greatly on the accuracy of measurements, in the case of variations like we noticed in our results, the tools used to measure the strains at each gauge location may not output accurate readings. Some other factors may include inaccurate geometry of the beam, or as specimen exhibiting some anisotropic behaviour.

Analysis of a composite beam

Let us solve a problem to demonstrate a different type of bending behaviour with a composite beam composed of aluminum and steel. In this problem we will use a transformation to change one of the materials into and equivalent of the second.



In order to do so, we can use the ratio of the modulus of elasticity of the two materials to determine a factor “n” that we can use as a multiplier for the dimensions to scale up.

E_S=200 GPa ; E_A=69 MPa

$$n = \frac{E_S}{E_A} = \frac{200}{69} = 2.898$$

The change in dimension of steel must only be applied to the width "b" since the transformation happens in a direction parallel to the neutral axis.

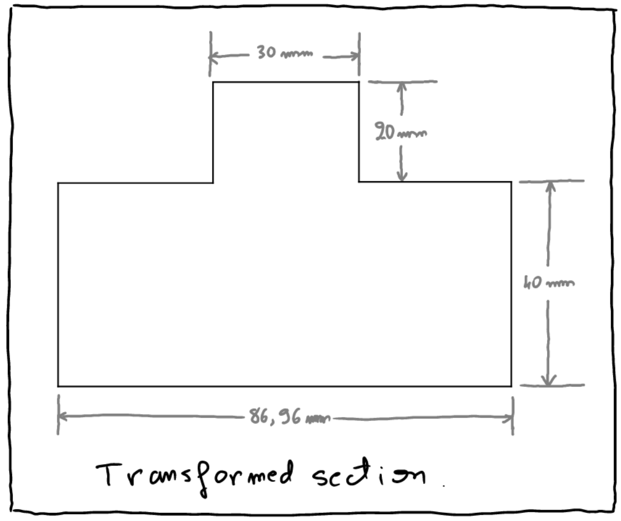


Figure 6: Aluminum transformed section

We must now determine the centroid of the section and its moment of inertia.

$$\overline{\mathbf{Y}} = \frac{\sum(\mathbf{y} \times \mathbf{A})}{\sum \mathbf{A}} = \frac{50 \times (30 \times 20) + 20 \times (86.96 \times 40)}{(30 \times 20) + (86.96 \times 40)} = 24.41\text{mm}$$

$$\mathbf{I} = \sum(\frac{\mathbf{bh^3}}{\mathbf{12}} + \mathbf{Ad^2}) = \frac{30 \times 20^3}{12} + (30 \times 20)(25.59)^2 + \frac{86.96 \times 40^3}{12} + (86.96 \times 40)(4.41)^2$$
$$= 944.34 \times 10^3\text{mm}^4$$

We can no apply equation 1 to find the stresses at the upper and lower edges of the section, knowing that the upper edge is in compression, and the lower in tension:

$$\boldsymbol{\sigma_{Up}} = \frac{\mathbf{My}}{\mathbf{I}} = \frac{-2000 \times 10^3 \times 35.59}{944.34 \times 10^3} = -75.39 \text{ MPa}$$

$$\boldsymbol{\sigma_{Lo}} = \frac{\mathbf{My}}{\mathbf{I}} = \frac{2000 \times 10^3 \times 24.41}{944.34 \times 10^3} = 51.71 \text{ MPa}$$

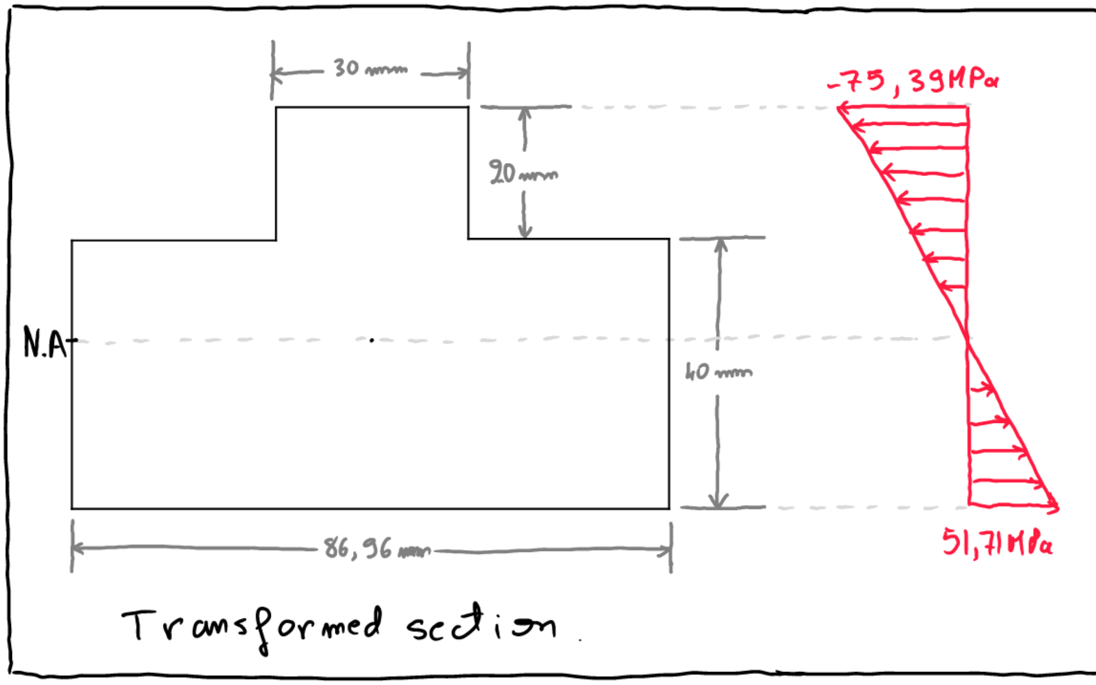


Figure 7: Stress distribution of the aluminum beam

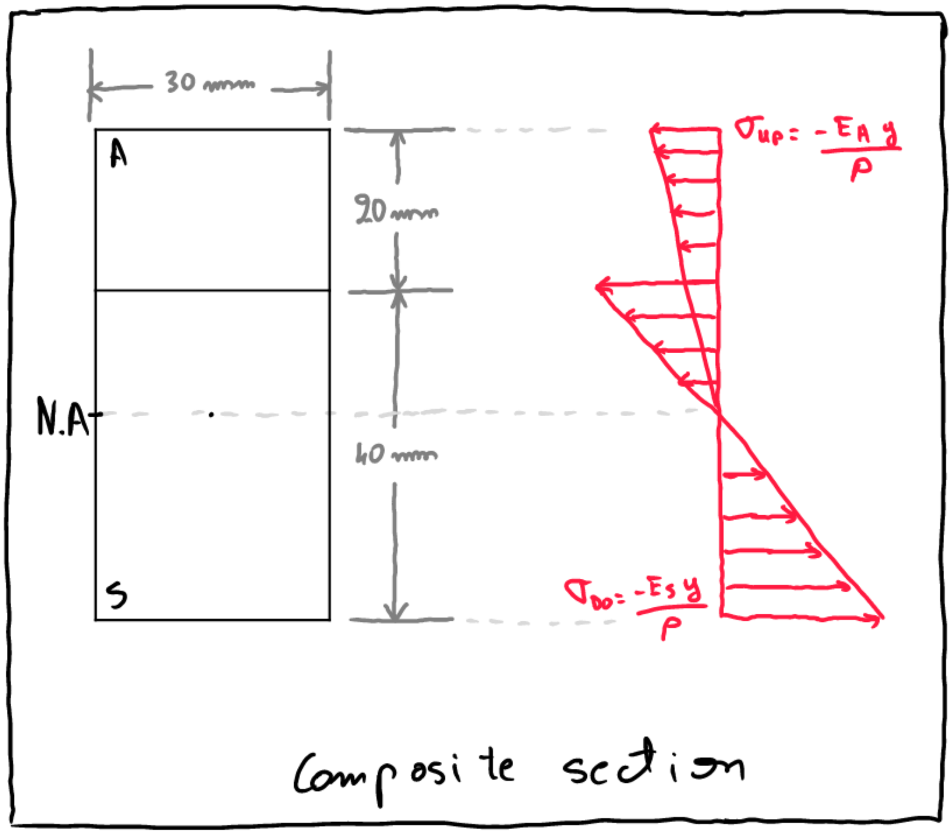


Figure 8: Stress distribution of composite beam

Conclusions:

In this experiment we tested the behaviour of a steel beam under a bending moment, and we analyzed the stresses at different optimal locations along its cross section. The experimental results collected during the experiment were compared to the theoretical expectations, and we found some slight variation in the results. These variations were justified by the margin of error during the experiment due to several factors, human mistake, faulty equipment, or sample issues. We also confirmed the theory that the stress distribution does indeed have a linear profile along the cross section of the beam using observations based on our resulting strains. In conclusion, the experiment successfully demonstrated the behaviour of a well used metal under bending effect. From this experiment we can deduce the practicality of the bending test in research and development, this test is an essential part of the stress analysis done by designers to ensure the safety and applicability of a certain material within their design. The extensive use of beam in the construction field for example, is a great incentive to utilize the bending test to analyse the capacity of a structure in the design, the usefulness of the information deduced based on this experiment extends all engineer practices.

References:

[1] J. P. Den Hartog, *Strength of materials*. Courier Corporation, 2012.

[2] S. Timoshenko 1878-1972., *History of strength of materials : with a brief account of the history of theory of elasticity and theory of structures*, 1 online resource (x, 452 pages) : illustrations vols. New York: Dover Publications, 1983. [Online]. Available: http://app.knovel.com/hotlink/toc/id:kpHSMWBAH3/history-of-strength

[3] “ENGR 244 Lab manual_2020.” Departement of Building, Civil and Enviromental Engineering.