CONCORDIA UNIVERSITY
ENGR 244 – Mechanics of Materials

Experiment 6: Behaviour of Columns Under Axial Load

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Objective:

This test aims to understand the behaviour of beams under axial loading. It covers materials undergoing buckling under this type of load, to help inform the critical loads, stresses, and overall stability of the member.

Introduction:

In this experiment, we will be testing columns of different lengths under different support conditions. This test consists of subjecting a material to a controlled axial compressive loading. Through recordings of the critical loads for each specimen, we can determine various material properties. Subjecting materials to this testing method helps engineers define their behaviour in practical applications, and more importantly their capacity, limitations, and points of failure [1]. A brief insight of the experiment's historical context, inevitably leads us to the 18th century with the important contributions of the Swiss physicist Leonhard Euler to the theory of buckling, and particularly with the Euler buckling formula that determines the critical load based on the specimen's geometry and material [2].

The buckling test is among a series of stress analysis testing procedures that inform the behaviour of materials under various loading conditions. Since in structural members, buckling is a realistic and common type of failure. In addition, the test has been chosen due to its reliability in reporting valuable data on this type of loading condition.

Prior to the experiment, certain assumptions are to be noted:

- The machine used has been properly calibrated to output the exact loading shown digitally but hasn't been ASTM certified.
- Based on our measurements, the samples have been machined to have proper straightness along their gauge length, and accurate length.
- The testing samples are uniformly made of aluminum.

This experiment has some important practical applications, since in the design process, particularly in structural members, buckling under axial compressive loading is a critical phenomenon. This requires the designer to be informed about the critical loads that a members or structure can withstand, and design accordingly by modifying the geometry of the member or opting for another material. This testing method is also applicable to research and development sector where important data can be collected to be accessible to designers, such as the most commonly used American standard and wide flanged beams used in structures.

Procedure:

In this experiment, we used buckling testing fixtures locally made to test aluminum specimens with different lengths under different supporting conditions. We used five hollow specimens, and three solid specimens, ranging in three lengths 75mm, 125mm and 225mm.

The fixture used is comprised of 4 vertically standing rods fixed at the work bench and holds a stationary upper platform that houses a support, see figure 1 bellow. On the lower side of the fixture, a second support is placed axially aligned with the upper one. The fixtures use two different types of supports, either a pin support that allows rotation, or a fixed support that prevents rotation. The loading conditions comprise of three cases, pin-pin support, pin-fixed support, and fixed-fixed support. The sample is set on both its sides into the supports. Through a hydraulic pump actuated by a simple hand lever, the single axes moving lower platform is pushed vertically upwards, therefore applying a controlled axial compressive load.

Figure 1: Testing fixture

The testing procedure starts by setting a sample on the corresponding supports into the testing fixture. Once the digital load monitor is set to zero and the setup is complete, the pump is actuated manually to steadily increase the compressive load, the operator keeps increasing the load until buckling happens, and the sample is plastically deformed. At this point the test is complete, and the buckling load, which is equivalent to the critical load is recorded. We perform the same procedure on all the different samples and collect the corresponding data.

Results:

The sample's inner and outer diameters have been measured using Vernier caliper on different locations and averaged, the data have been collected in table 1 bellow, along with the effective lengths, based on the supporting conditions, the cross-sectional area, the moment of inertia, and the radius of gyration of each sample.

Type of column	Column length L (mm)	End condition	Effective length factor K	Effective length L_{eff} (mm)	Inner diameter d_{in} (mm)	Outer diameter d_{out} (mm)	Cross sectional area A (mm^2)	Moment of inertia I (mm^4)	Radius of gyration r (mm)
Hollow	75	Pin-Pin	1	75	4.63	6.32	14.53	55.76	1.96
	125	Pin-Pin	1	125	4.38	6.39	17.00	63.77	1.93
	225	Pin-Pin	1	2:25	4.55	6.30	14.91	56.29	1.94
	225	Pin-Fixed	0.707	159.07	4.58	6.34	15.09	57.71	1.95
	225	Fixed-Fixed	0.5	112.5	4.61	6.36	15.08	58.14	1.96
Solid	225	Pin-Pin	1	225	N.A	6.33	31.47	78.81	1.58
	225	Pin-Fixed	0.707	159.07	N.A	6.32	31.37	78.31	1.58
	225	Fixed-Fixed	0.5	112.5	N.A	6.34	31.57	79.31	1.58

Table 1: Test sample dimensions

As a measure of comparing our experimental values, we can calculate the critical load P_{cr} and critical stress σ_{cr} using the data in table 1 above along with the published value of the elastic modulus for aluminum E=70GPa. The equations of the critical load and stress are as follows:

$$\mathbf{P_{cr}} = \frac{\boldsymbol{\pi^2 \times E \times I}}{\mathbf{L_{eff}}^2} \quad ; \quad \boldsymbol{\sigma_{cr}} = \frac{\boldsymbol{\pi^2 \times E}}{(\mathbf{L_{eff}/r})^2}$$

The use of Eulers formula for P_{cr} is based on the following assumptions:

- The load is centric (P acts along the centroidal axis of the column).
- The ends of the column are pinned.
- The column behaves elastically.

Here is an example of the critical load for the 75mm hollow pin-pin supported specimen:

$$\mathbf{P_{cr}} = \frac{\boldsymbol{\pi^2 \times 70 \times 10^9 \times 55.76 \times 10^{-12}}}{(\mathbf{75 \times 10^{-3}})^2} = \mathbf{6848.54\ N}$$

Here is an example of the critical stress for the 75mm hollow pin-pin supported specimen:

$$\boldsymbol{\sigma_{cr}} = \frac{\boldsymbol{\pi^2 \times 70 \times 10^9}}{(\mathbf{75/1.96})^2} = \mathbf{471.83\ MPa}$$

Real columns used in the test do not meet the assumptions used in deriving the Euler's formula, in order to take into account these discrepancies, for 6061-T6 aluminum alloy, we used some design formulas from the Aluminum association, based on different values of the slenderness ratio **λ=L/r** as follows:

- σ_{All} = 131 MPa for λ < 9.5
- σ_{All} = 139 - 0.868 (λ) MPa for 9.5 < λ < 66
- σ_{All} = (351 x 103)/λ2 MPa for λ > 66

Here is an example of the allowable stress for the 75mm hollow pin-pin supported specimen:

$$\boldsymbol{\sigma_{All}} = \mathbf{139 - 0.868 \times (38.26) = 105.79\ MPa}$$

After running the calculations for all specimen, we compiled the theoretical and experimental results in table 2 bellow.

Type of column	Column length L (mm)	End condition	Slendern-ess ratio **λ**	Allowable stress **σ_{All}** (MPa)	Theoretical critical load P_{Cr} **(N)**	Theoretical critical stress **σ_{Cr}** (MPa)	Experimental critical load P_{Cr} **(N)**	Experimental critical stress **σ_{Cr}** (MPa)
Hollow	75	Pin-Pin	38.26	105.79	6848.54	471.33	4189	288.30
	125	Pin-Pin	64.77	82.77	2819.64	165.86	2513	147.82
	225	Pin-Pin	115.98	26.09	768.18	51.52	703	47.15
	225	Pin-Fixed	81.57	52.75	1575.64	104.41	1899	125.84
	225	Fixed-Fixed	57.40	89.18	3173.71	210.45	2322	121.96
Solid	225	Pin-Pin	142.40	17.30	1075.51	34.17	1225	38.92
	225	Pin-Fixed	100.68	34.62	2138.15	68.16	2355	75.07
	225	Fixed-Fixed	71.20	69.24	4329.33	137.13	2742	86.85

Table 2: Experimental and theoretical results for critical loads and stresses

We used the theoretical, experimental and allowable results of stress to plot the curves in figure 2 bellow to better compare the values.

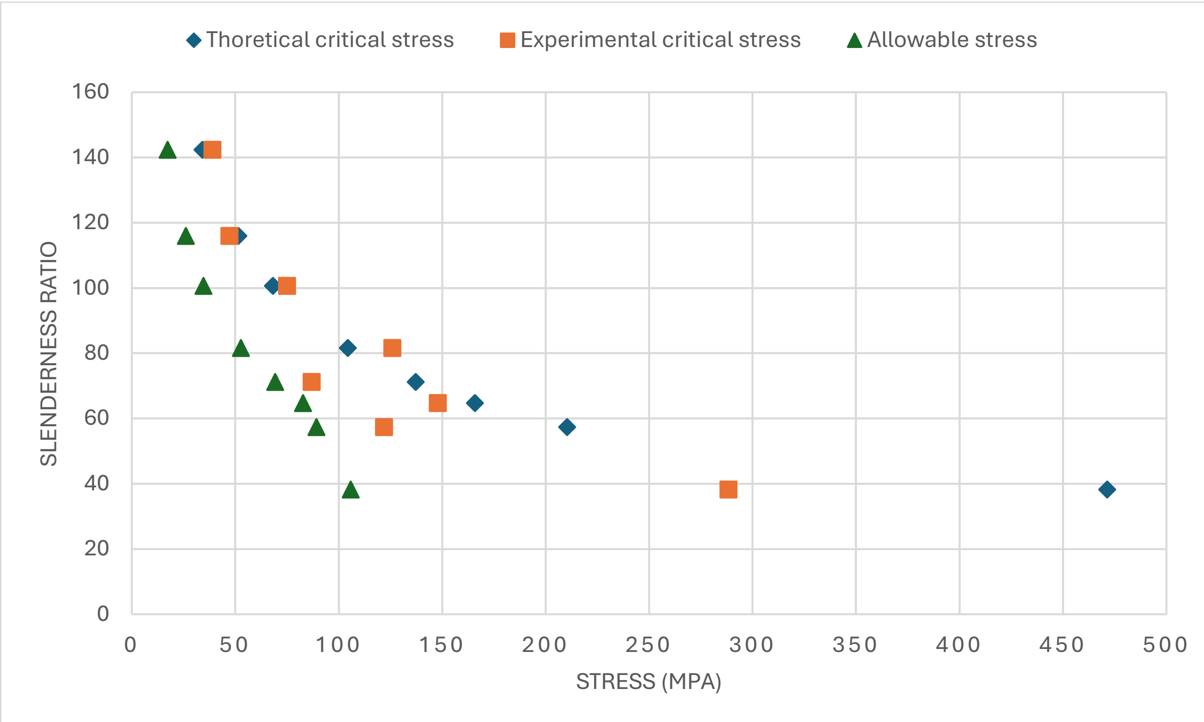

Figure 2: Theoretical, experimental critical stress and allowable stress

Discussions:

There are some trends we notice in figure 2 in the results section above when comparing the theoretical critical stresses obtained with the Euler formula and the experimental results. The values for the experimental stresses of the solid 225mm pin-pin, hollow 225mm pin-pin, and solid 225mm pin-fixed samples are almost identical to the theoretical ones, although we see a considerable difference in the smallest size of 75mm pin-pin hollow sample where the experimental stress is much lower than the theoretical one, as well as the hollow 225mm fixed-fixed sample. These discrepancies can be amplified by numerous factors, such as imperfections in the geometry of the samples, and the supports. The boundary conditions are one of the most influential factors, the pin supports may have rotational resistance as the load is increased, and the fixed support might allow slight movement, this directly affects the effective length factor K, and consequently the critical stress.

On the other hand, the allowable stresses values calculated using the Aluminum Association's recommendation for design stresses are overall lower than the experimental and theoretical results we obtained. The trend includes all specimens no matter their slenderness ratio, this is not an unexpected outcome, since the allowable designs stresses must be lower than the critical values to keep a margin of safety in the design process.

As we've seen, along with the supporting conditions, slenderness ratio are some important values to consider in the design process. Both properties are related to the geometry of the beam and its cross-sectional area. The supporting conditions directly impact the effective length, as a result, fixed beams resist buckling better than pin supported ones, since they experience lower critical stresses. The slenderness ratio, in its part, impacts the buckling capacity, it is related to the effective length as well as the radius of gyration, which is a function of the area and its distribution related to the neutral axes of the cross section. So, the geometry of the beam is a very influential aspect of the buckling strength of beams.

Another trend we notice in our results is the higher buckling capacity of the circular bars as opposed to the circular tubes, this further confirms the effect of the geometry of the cross-sectional area of the beams. The higher area of the bars seems to help resist the buckling effect. This can be seen directly in Euler's theorem of buckling since the radius of gyration, along with the effective length and the moment of inertia which define the sample's geometry and mass distribution relative to the neutral axis, these three properties form the slenderness ration. The higher this ratio gets, the higher the critical load, the more buckling a column can resist.

Conclusions:

In this experiment we tested various aluminum bars and tubes with different lengths in a buckling experiment under different supporting conditions to evaluate the buckling capacity of the material. We recorded the experimental values of critical loads and deducted the values of critical stresses. Using Euler's theorem of buckling, we computed the theoretical values of critical stresses and loads and compared the results with the experimental. The accuracy of the test was overall close to the expected theoretical expected values, with some slight variations due to the testing fixture's limitations related to the support conditions. Based on our results, we successfully determined that the length, geometry of the cross section and supporting conditions affect the buckling capacity of beams. Although the testing procedure and equipment isn't perfect, the test has some essential practical implication, particularly in the design of structural members. We might see its importance in construction where an informed choice of material, and geometry leads to a safer, more economical design. Any application where buckling might be a suspected type of failure requires the methods used in our experiment to determine a material's appropriate use.

References:

[1] L. Anand, K. Kamrin, and S. Govindjee, "482 Elastic buckling of columns," in *Introduction to Mechanics of Solid Materials*, L. Anand, K. Kamrin, and S. Govindjee, Eds., Oxford University Press, 2022, p. 0. doi: 10.1093/oso/9780192866073.005.0003.
[2] S. Timoshenko 1878-1972., *History of strength of materials : with a brief account of the history of theory of elasticity and theory of structures*, 1 online resource (x, 452 pages) : illustrations vols. New York: Dover Publications, 1983. [Online]. Available: http://app.knovel.com/hotlink/toc/id:kpHSMWBAH3/history-of-strength