CONCORDIA UNIVERSITY
ENGR 244 – Mechanics of Materials

Experiment 5: Deflection of Beams

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Objectives:

The deflection of beams test is an experiment that evaluates the relationship between the applied load and the deflection of a simply supported or cantilever beam. Composed of two parts, a simply supported beam, and a cantilevered beam, the results of the two experiments will help estimate experimental values of the modulus of elasticity and the deflection behaviour of the three samples tested.

Introduction:

The simply supported beam is one of the most common practical applications of beams across structural applications. The small-scale experiment we've conducted informs some critical information about the materials, through two tests. The first is testing a simply supported beam which consists of a beam supported on both its ends under a transverse load applied on its middle that generates a bending moment. The second test is testing a cantilevered beam under a transverse load applied on its free edge to evaluate its deflection [1]. The analysis of the deflection of the beam will allow us to determine a variety of properties such as the elastic moduli of the materials as well as a useful mathematical modelling of the deflection parameters and curvature.

To put this experiment in historical context, beam analysis has always been of interest since ancient times due to its usefulness in construction. A more structured study of stress analysis started with European philosophers like Da Vinci and Galileo, and later Euler, Hooke and Cauchy who derived the fundamental theories of the behaviour of beams under various loads [2]. There aren't many variations of the simply supported beam test and cantilever beam test due to their simplicity. The incentive behind our choice is the reliability of the test in reporting accurate information about the deflective behaviour of the material, as well as the availability of the testing fixture.

Prior to the experiment, certain assumptions are to be noted:

- The machine used has been properly calibrated to output the exact loading shown digitally but hasn't been ASTM certified.
- Based on our measurements, the samples have been machined to have uniform dimensions along their respective lengths.
- The testing samples are uniformly made of steel for the first, aluminum, and brass.

The analysis of the simply supported and cantilevered beams yields very practical information that allow the designer to make and informed choice of material, geometry and dimensions to best accommodate their application. As mentioned before, we especially see its practicality in structures, where a building or civil engineer would need to have the proper data, such as modulus of elasticity, maximum deflection, yield and ultimate strength to design safe and practical structures.

Procedure:

In the two parts of this experiment, we used a bending fixture locally made, and a simple cantilever setup, both seen in figures 1 and 2 bellow. The testing is done on six samples of three metals for the two tests, steel, aluminum and brass.

Figure 1: Cantilever beam testing fixture

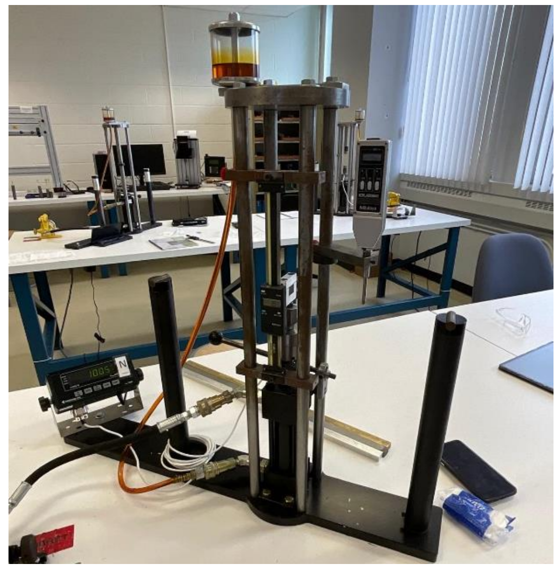

Figure 2: Bending beam testing fixture

I. <u>Point load on a simply supported beam</u>

The first testing fixture for bending the beam comprises of a platform fixed at the workbench that holds two vertically standing rods at a distance L=455mm, as well as a vertically standing pulling mechanism fixed on the platform at an equal distance from both rods, see figure 1. The standing rods act as simple supports for the beam, while the mechanism applies a transverse pulling force on the mid-point of beam. There are two deflection sensors placed on two points of the beam and are always in contact to measure the value of deflection as the load is applied, the two sensors are placed respectively at L/2 and L/4. The test starts by setting the sample on the fixture and resetting the sensors. The pump is actuated manually to slowly increase the load in increments of 200N, up to a maximum load P=1000N, the load is then held steady to record the corresponding deflection for each of the two sensors.
We proceed using the same steps for all three samples, steel, aluminum and brass. The full dimensions of the samples including the base "b", the height "h" and the moment of inertia "I" are shown in table 1 bellow. Note the dimensions have been averaged based on 3 collected locations of the beams.

Dimensions	BRASS	STEEL	ALUMINUM
Base b (mm)	18.99	18.98	19.01
Height h (mm)	12.64	12.61	12.66
Inertia I (mm^4)	3195.84	3171.47	3214.42

Table 1: Sample dimensions for bending test

II. Point load on a cantilever beam

The second testing fixture for the cantilever beam comprises of a simple standing structure that supports a beam on one end only, leaving the second end hanging free, which essentially creates a cantilever. A hanger is placed at the edge of the free end which allows us to hang a set of different weights to mimic a transverse load that generates a loaded cantilever. There are two deflection sensors placed on two points of the beam and are always in contact to measure the value of deflection as the load is applied, the two sensors are placed respectively at the middle and the free edge end of the beam. To initiate the test, it is important to ensure the sensors do not output any initial value of deflection, once that is done, the smallest weight of the set is attached to the hanger and the two values of deflection are recorded, then the team moves up to the next weight until all deflection values for all weights are recorded, refer to table 3 for the weight values and their corresponding loads. Note the weights includes the contribution of the hanger. We proceed using the same steps for all three samples, steel, aluminum and brass. The full dimensions of the samples including the base "b", the height "h" and the moment of inertia "I" are shown in table 2 bellow. Note the dimensions have been averaged based on 3 collected locations of the beams.

Dimensions	BRASS	STEEL	ALUMINUM
Base b (mm)	18.86	18.98	19.13
Height h (mm)	3.05	3.09	3.28
Inertia I (mm^4)	44.59	46.66	55.98

Table 2: Sample dimensions for cantilever test

There are 5 masses used in the cantilever test each weigh 100g up to 500g, the hanger and the supports on the fixture have variable weights for each material since we used three different stations. In table 3, we compiled the total weight that includes the support and the hanger for each test.

Weight number	BRASS		STEEL		ALUMINUM	
	Mass (g)	Load (N)	Mass (g)	Load (N)	Mass(g)	Load (N)
1	119.81	1.17	119	1.17	119.4	1.17
2	218.76	2.15	219	2.15	218.3	2.14
3	319.58	3.13	320.3	3.14	319	3.13
4	422.89	4.15	421.42	4.13	422.2	4.14
5	521.96	5.12	523.2	5.13	523.09	5.13

Table 3: Weights and corresponding loads for cantilever test

Results:

I. Point load on a simply supported beam

Based on the output of the sensors at L/2 and L/4, we have compiled the results of the deflection at every 200N of load up to a maximum of 1000N in table 4 bellow.

Applied load (N)	BRASS		STEEL		ALUMINUM	
	x=L/2	x=L/4	x=L/2	x=L/4	x=L/2	x=L/4
200	-1.43	-0.99	-0.63	-0.43	-1.57	-1.08
400	-3.00	-2.05	-1.27	-0.86	-3.37	-2.32
600	-4.57	-3.12	-1.93	-1.32	-5.18	-3.56
800	-6.17	-4.22	-2.59	-1.77	-7.00	-4.81
1000	-7.79	-5.35	-3.25	-2.24	-8.82	-6.07

Table 4: Experimental beam deflection values

Let's attempt to derive an equation of the elastic curve of the simply supported beam. We know that within the elastic range the curvature of the neutral axis can be expressed as:

$$\frac{1}{\rho} = \frac{M(x)}{EI} \quad \text{and} \quad \frac{1}{\rho} = \frac{d^2y}{dx^2}$$

Equating both equations give us the differential equation of the elastic curve as follows:

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

Integrating this formula twice will yield the following:

$$\frac{dy}{dx}EI = \frac{Px^2}{4} + c_1$$

$$EIy = \frac{Px^3}{12} + c_1x + c_2$$

To determine the values of the constants c_1 and c_2, we can use the boundary conditions of a the simply supported beam. If points A and B are the supports of the beam then $y_A=0$ and $x_A=0$, $y_B=0$ and $x_B=L$, $dy/dx=0$ and $x=L/2$. Substituting these values in the previous equation yields:

$$c_1 = \frac{-PL^2}{16} \quad \text{and} \quad c_1 = 0$$

So out final equation of the elastic curve is as follows:

$$EIy = \frac{P}{12}x^3 - \frac{PL^2}{16}x$$

Using the elastic curve expression above and the published values of the elastic modulus, we can calculate the theoretical defection values at x=L/2 and x=L/4, the elastic modulus values use are:

E_S=200GPa ; E_A=70GPa ; E_B=105GPa

Here is an example of the calculation of the deflection of brass at x=L/2 for a load P=200N:

$$\mathbf{EIy = \frac{P}{12}x^3 - \frac{PL^2}{16}x}$$

$$\mathbf{y = \left(\frac{200}{12} \times 227.5^3 - \frac{200 \times 455^2}{16} \times 227.5\right) \times \frac{1}{105 \times 10^9 \times 3195.84} = -1.17mm}$$

The same procedure has been applied to all of the deflection values and compiled in table 5.

Applied load (N)	BRASS		STEEL		ALUMINUM	
	x=L/2	x=L/4	x=L/2	x=L/4	x=L/2	x=L/4
200	-1.17	-0.80	-0.62	-0.42	-1.74	-1.20
400	-2.34	-1.61	-1.24	-0.85	-3.49	-2.40
600	-3.51	-2.41	-1.86	-1.28	-5.23	-3.60
800	-4.68	-3.22	-2.48	-1.70	-6.98	-4.80
1000	-5.85	-4.02	-3.09	-2.13	-8.72	-5.99

Table 5: Theoretical beam deflection values

Using the experimental deflection values, we can estimate the elastic modulus using the elastic curve formula that we derived and compare these values to the published values of E, results have been compiled in table 6.

Applied load (N)	BRASS		STEEL		ALUMINUM	
	x=L/2	x=L/4	x=L/2	x=L/4	x=L/2	x=L/4
200	85.9	85.3	196.4	197.8	77.8	77.7
400	81.9	82.4	194.9	197.9	72.5	72.4
600	80.6	81.2	192.4	193.4	70.7	70.7
800	79.6	80.0	191.1	192.3	69.8	69.8
1000	78.8	78.9	190.4	189.9	69.2	69.1
Avg. E	81.7		193.6		71.9	

Table 6: Elastic modulus values based on experimental deflections

The values of the modulus of elasticity we calculated based on experimental deflection results are slightly different for steel, for which the average result is lower; similarly, the value for aluminum is slightly higher, but both values are within a reasonable range. The average for brass on the other hand is considerably lower than the published value, with almost 24GPa deviation.

The figures bellow shows the theoretical and experimental values of deflection of mid-span of the beam as a function of the load for each of the specimen independently.

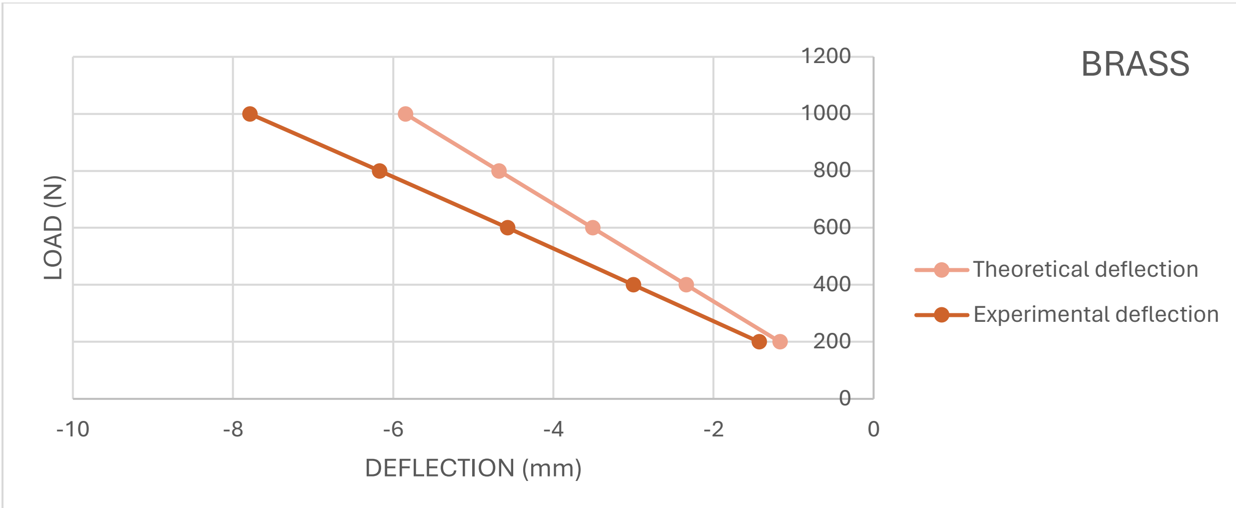

Figure 3: Deflection at mid-span vs. load diagram for Brass

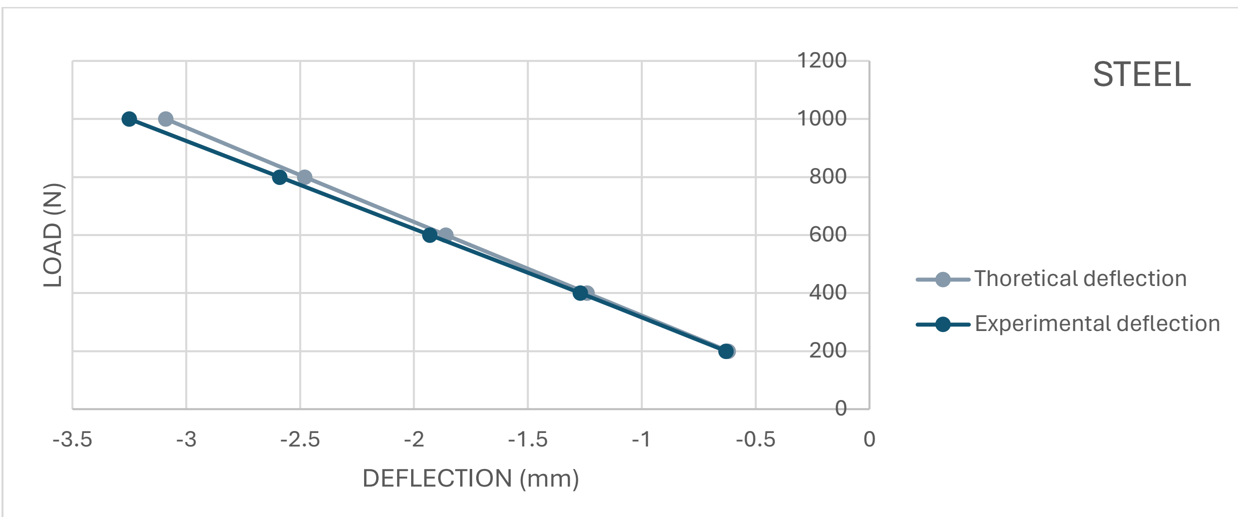

Figure 4: Deflection at mid-span vs. load diagram for Steel

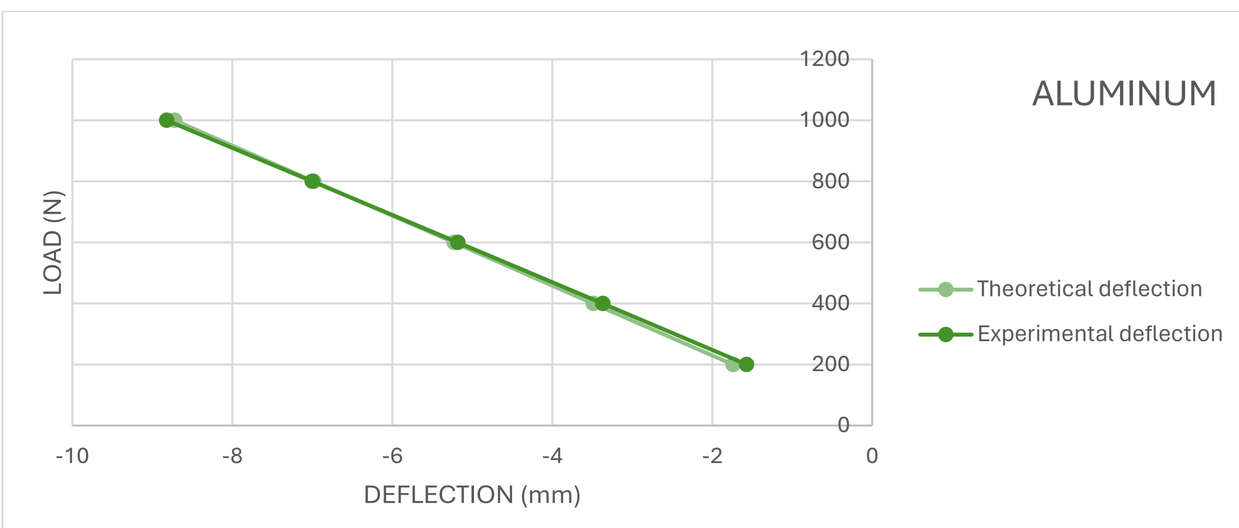

Figure 5: Deflection at mid-span vs. load diagram for Aluminum

In an attempt to visualize the experimental and theoretical deflection values of the beams tested, we collected the results of deflection for all three specimen we previously calculated in figures 3 and 4 and 5 bellow as functions of the load P.

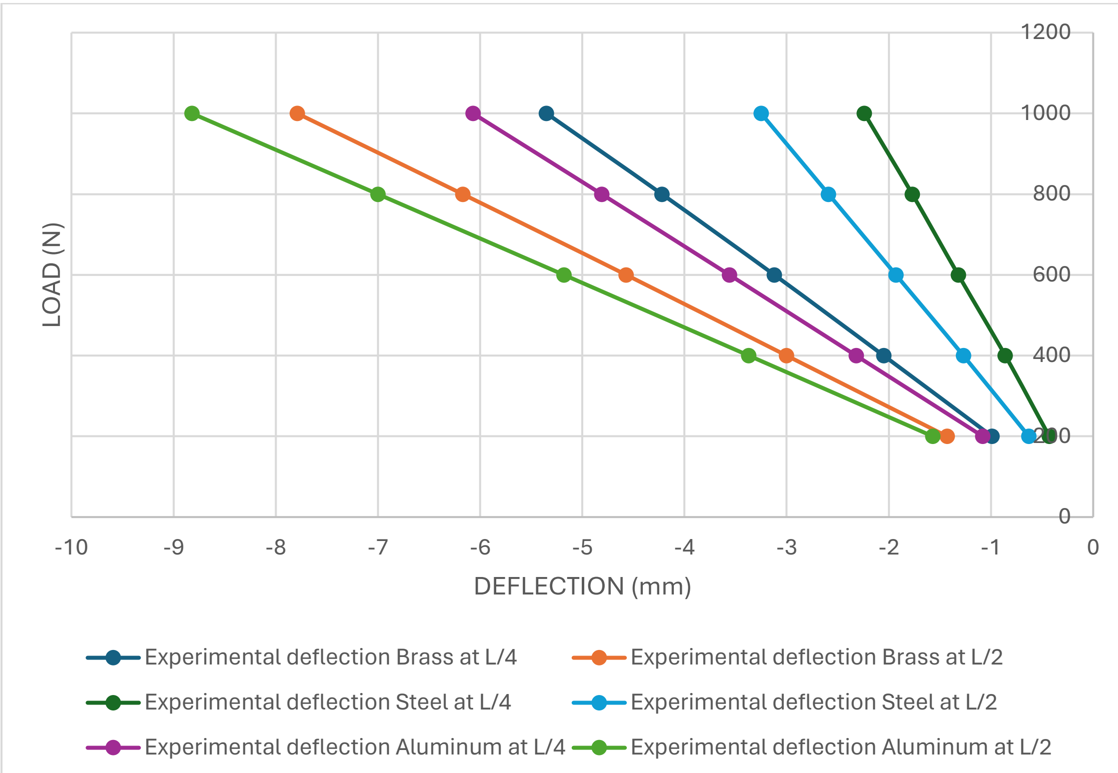

Figure 6: Experimental values of deflection vs. load diagram

II. <u>Point load on a cantilever beam</u>

Based on the output of the sensors at L/2 and L, we have compiled the results of the deflection for each of the three specimens in table 4 bellow (the masses and their corresponding loads are specified in table 3 above).

Weight number	**BRASS**		**STEEL**		**ALUMINUM**	
	x=L/2	x=L	x=L/2	x=L	x=L/2	x=L
1	-0.36	-1.17	-0.20	-0.59	-0.48	-1.44
2	-0.69	-2.25	-0.38	-1.14	-0.90	-2.77
2	-1.01	-3.31	-0.59	-1.78	-1.34	-4.09
4	-1.35	-4.41	-0.78	-2.36	-1.78	-5.46
5	-1.68	-5.47	-0.98	-2.98	-2.22	-6.79

Table 7: Experimental values of deflection of cantilever

Using the elastic curve expression bellow and the published values of the elastic modulus, we can calculate the theoretical defection values at x=L/2 and x=L, the elastic modulus values use are: E_S=200GPa ; E_A=70GPa ; E_B=105GPa

Here is an example of the calculation of the deflection of brass at x=L/2 for the 1st load P=1.09N:

$$\mathbf{y = \frac{P \times (x^3 - 3Lx^2)}{6EI}}$$

$$\mathbf{y = \frac{1.17 \times (125^3 - 3 \times 250 \times 125^2)}{6 \times 105 \times 10^9 \times 44.59} = -0.41mm}$$

The same procedure has been applied to all the deflection values and compiled in table 8.

Weight number	BRASS		STEEL		ALUMINUM	
	x=L/2	x=L	x=L/2	x=L	x=L/2	x=L
1	-0.41	-1.31	-0.20	-0.65	-0.49	-1.56
2	-0.75	-2.39	-0.37	-1.20	-0.89	-2.85
2	-1.09	-3.49	-0.55	-1.75	-1.30	-4.16
4	-1.44	-4.61	-0.72	-2.31	-1.72	-5.50
5	-1.78	-5.70	-0.89	-2.86	-2.13	-6.82

Table 8: Theoretical values of deflection of cantilever

Using the experimental deflection values, we can estimate the elastic modulus using the following formula:

$$\mathbf{E = \frac{P \times (x^3 - 3Lx^2)}{6Iy}}$$

Results of the elastic modulus have been compiled in table 9.

Applied load (N)	BRASS		STEEL		ALUMINUM	
	x=L/2	x=L	x=L/2	x=L	x=L/2	x=L
200	119.2	117.3	203.6	220.9	70.9	75.68
400	113.5	111.4	197.2	210.3	69.2	71.9
600	113.3	110.6	185.8	197.0	67.9	71.2
800	112.2	109.8	184.9	195.5	67.6	70.6
1000	111.2	109.3	182.7	192.2	67.2	70.3
Avg. E	113.9		190.8		68.6	

Table 9: Elastic modulus values based on experimental deflections of cantilever

The figures bellow shows the theoretical and experimental values of deflection of the beam as a function of the load for each of the cantilever specimen independently.

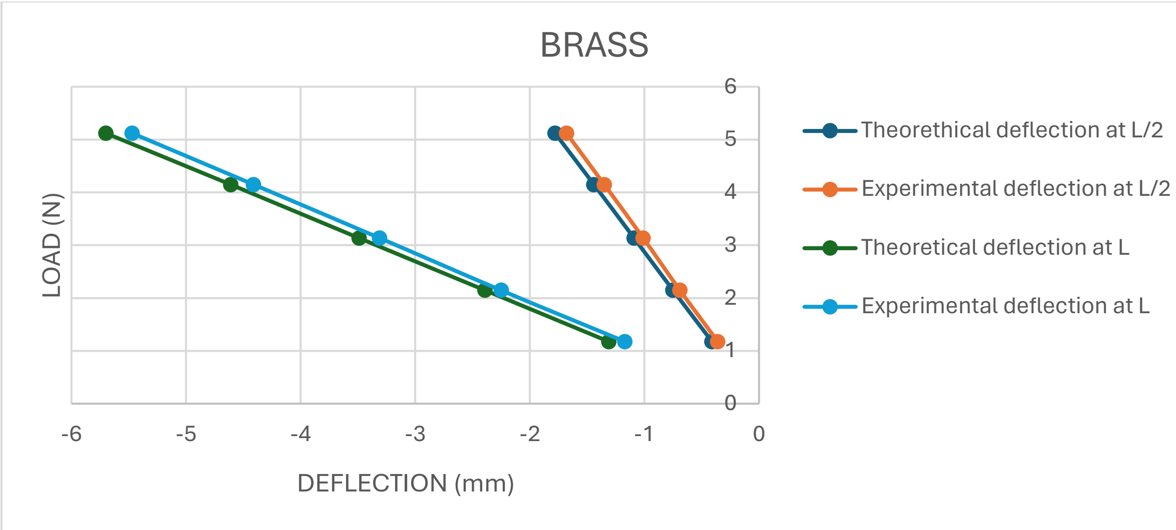

Figure 7: Deflection vs. load diagram for Brass cantilever

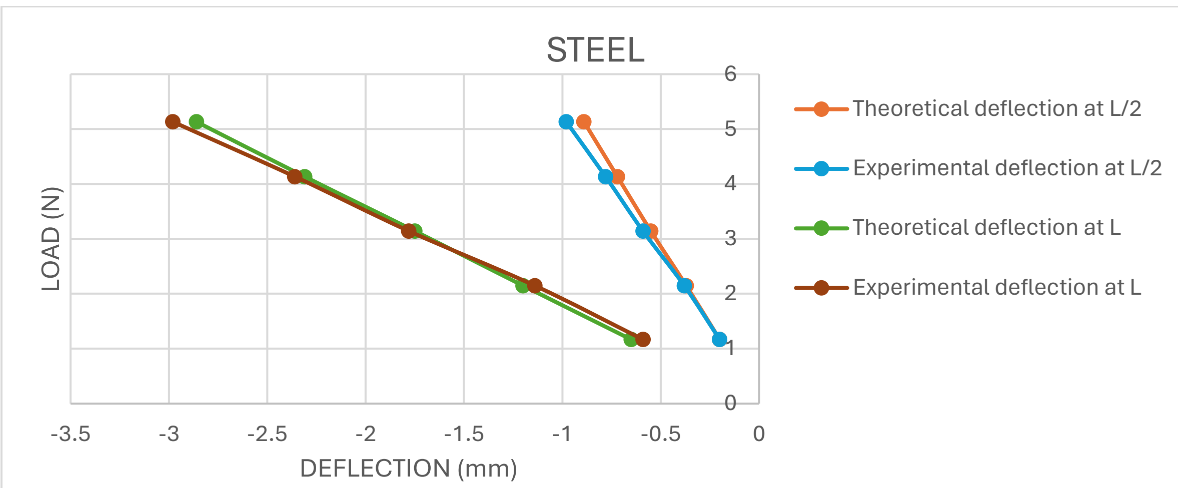

Figure 8: Deflection vs. load diagram for Steel cantilever

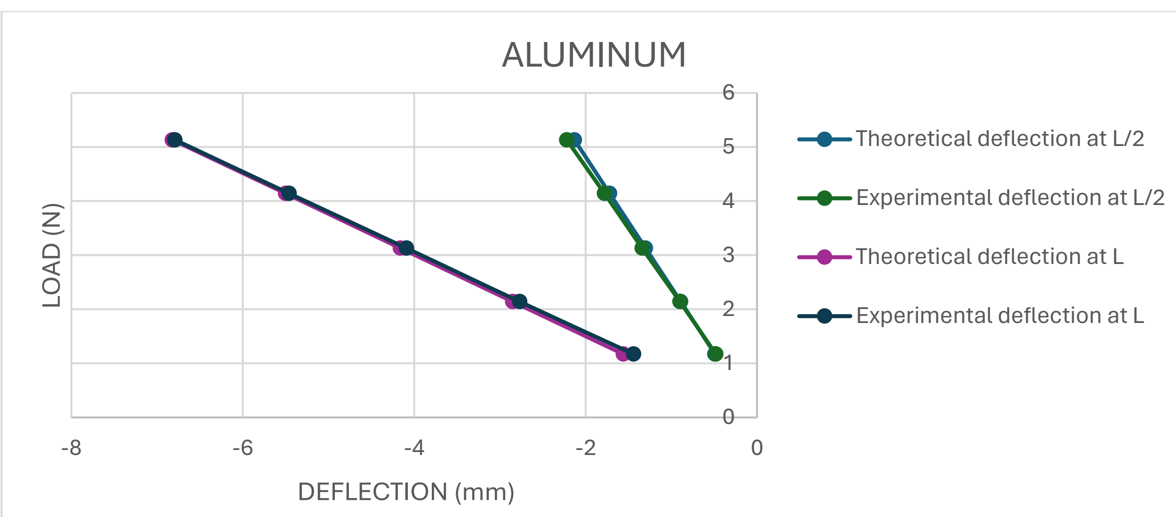

Figure 9: Deflection vs. load diagram for Aluminum cantilever

Discussions:

For both the tests conducted, upon examination of the results, we noticed some slight variations between the experimental and theoretical values. If we calculate the accuracy of the test using: $\%\text{error} = \frac{|\text{theoretical value} - \text{experimental value}|}{|\text{theoretical value}|} \times 100\%$, we end up with a percentage for each load and sensor, we considered the maximum and minimum accuracy for each sample.

Accuracy of the simply supported beam test for:

- Brass: varies between 66.8% and 77.8%
- Steel: varies between 94.8% and 98.8%
- Aluminum: varies between 90% and 99.8%

Accuracy of the cantilever beam test for:

- Brass: varies between 87.8% and 95.8%
- Steel: varies between 89.9% and 100%
- Aluminum: varies between 92.3% and 99.6%

The accuracy of the test depends on various factors based on our results, the most notable is the brass samples which shows a weaker accuracy for both tests compared to steel and aluminum, this must be due to its sensitivity to pickup permanent deformation. Since the samples are reused for multiple times during the experiments, brass is more likely to have the most notable variations due to repetitive testing. As for steel and aluminum, the accuracy is considerably high varying generally above the 90% mark. These deductions support the claim that the type of material might affect the accuracy of the test.

Comparing the results of both tests with each other, we note that the simply supported beam test scored the lowest accuracy among the two with the brass sample. Other than this exception, the level of accuracy of both tests seems to be quite close. The tests are unlikely to have very different accuracy since as we stated previously, the brass sample might fall into the exception for the permanent deformation that the sample might have endured.

Comparing the different locations of the samples, the sensors have shown very close accuracy on all positions. The configuration of the sensors doesn't seem to have a considerable impact on the accuracy of both tests.

Regarding the elastic modulus of both tests, we notice the experimental values are very close to the theoretical ones for the four aluminum and steel samples, with some slight variations, the averages we calculated are within a reasonable range of the published values. On the other hand, the brass samples have shown considerable deviations from the published values of the elastic moduli in both tests. Computing the experimental values for the simply supported beam has yielded an almost 24MPa deviation lower than the published value. In addition, the cantilever test's modulus of elasticity has a smaller deviation, being higher than the published values by almost 9MPa. These deviations further support the claim we argued stating that the brass samples exhibit more sensitivity to repetitive bending tests.

Conclusion:

In this experiment, we conducted two tests generating bending stresses to analyze the behaviour of three metal beams for each test, brass, steel and aluminum. The first test was a simply supported beam under a transverse loading, and the second consisted of a cantilevered beam under one point loading. We recorded the values of the deflection of the beams, and derived formulas to analyze their behaviour and extract some important material properties. We successfully determined the stress distribution and compared the theoretical and experimental values of deflection and elastic moduli. The accuracy of the test is concluded to be high for most of the samples as we noted, except for the brass samples which showed considerable deviations from the theoretical values. Although the testing procedure and equipment isn't without flaw, and several factors might result in faulty measurements or results, the test has some essential practical implication, for example in quality control procedures, as well as in design of structural members. We can also note its importance in the research field where the value of the tests would come through establishing databases for researchers and designers to facilitate the design process, or simply evaluate new materials.

References:

[1] T. H. G. Megson, *Structural and stress analysis*. Butterworth-Heinemann, 2019.

[2] S. Timoshenko 1878-1972., *History of strength of materials : with a brief account of the history of theory of elasticity and theory of structures*, 1 online resource (x, 452 pages) : illustrations vols. New York: Dover Publications, 1983. [Online]. Available: http://app.knovel.com/hotlink/toc/id:kpHSMWBAH3/history-of-strength