

1. Determine which of the following is a proper topology for the integers \mathbb{Z} . If the set is not a proper topology, then give the additional elements to make the set a proper topology and provide justification why the given elements should be added.

(a) Let \mathbb{Z}_O denote the odd integers and \mathbb{Z}_E denote the even integers. $\mathbb{T} = \{\emptyset, \mathbb{Z}_O, \mathbb{Z}_E, \mathbb{Z}\}$

(b) Let $k \in \mathbb{N}$ and \mathbb{Z}_k denote the integers that whose absolute value is less than k (i.e. $|x| < k$).
 $\mathbb{T} = \{\mathbb{Z}_k \mid k \in \mathbb{Z}\}$

2. Let (\mathbb{Z}, \mathbb{T}) be a topological space. Suppose that $\{\{n, n+1\} \mid n \in \mathbb{Z}\} \subset \mathbb{T}$. Prove that \mathbb{T} is the discrete topology.

3. Let $X = \{1, 2, 3, 4\}$. Now consider two different topologies for X , $\mathbb{T} = \{\emptyset, \{2\}, \{1, 2\}, \{3, 4\}, X\}$ and $\mathbb{S} = \{\emptyset, \{1\}, \{2, 3\}, \{1, 4\}, X\}$. Show that (X, \mathbb{T}) and (X, \mathbb{S}) are homeomorphic.

4. (Python Question)

- (a) Create a python script that intake a list of vertices and outputs a random simplicial complex from the given vertices.
- (b) Modify your script to compute the Euler Characteristic for the generated complex.
- (c) Generate 100 complexes from a list of 10 vertices and record each Euler Characteristic.
- (d) Find the average Euler Characteristic from part d and plot the results in a histogram and a box plot. Discuss your findings.