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# Law of Large Graphs

# 1 Introduction

OCP/HCP/FCP connectome setting examples of  $\bar{A}$  being used

Looking for better estimators of mean graph by making assumption that many vertices will behave similarly

[1] [3] [4] [5]

[8]

# 2 Model

## 2.1 MLE

Bernoulli says use ABar

- 2.2 RDPG
- 2.3 Model Assumption: SBM
- 2.4 ASE: PHat

dimension reduction, helps reduce variance, explanation that it is asymp unbiased.

### 2.5 Performance Evaluation: ARE

Defn and importance of asymp unbiased [7] [9]

## 3 Results

## 3.1 Main Result

RE Eqn from var(PHat) eqn

#### 3.2 Validation with simulated data

showing variance and RE simulations against expectations.

#### 3.3 Performance on Real Data

cross-validation different dimensions of PHat vs. ABar Perhaps permutation tests Law of Large Graphs 2

# 4 Discussion

Given the popularity of large N connectome datasets that have known vertex correspondence, perhaps ASE should be chosen over ABar when estimating a group averaged graph. NMF may be even better estimate than ASE. [6]

## 5 Methods

- 5.1 Algorithm
- 5.2 Choosing Dimension

Zhu and Ghodsi

## 5.3 Dealing with Diagonal

Scheinerman Iterations

#### 5.4 Dataset Description

CoRR

#### 5.5 Source code and data

# 5.6 Proof of Var(PHat)

We are assuming that it should be condensed. Perhaps reference full proof uploaded to arxiv?

## References

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- [4] Donniell E Fishkind, Daniel L Sussman, Minh Tang, Joshua T Vogelstein, and Carey E Priebe. Consistent adjacency-spectral partitioning for the stochastic block model when the model parameters are unknown. 2012.
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- [8] Roberto Imbuzeiro Oliveira. Concentration of the adjacency matrix and of the Laplacian in random graphs with independent edges. page 46, 2009.
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