Item Response Theory - Final Essay

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Contents

submitted to:

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Statutory Declaration: I hereby declare that I composed the present paper independently and that I have used no other resources than those indicated. The text passages which are taken from other works in wording or meaning have been identified as such. I also declare that this work has not been partly or completely used in another examination.

1 Introduction

2 Data preparation

The dataset consisted of 3376 observations, the variables being the ten items of the SCS, the sum score, gender and age. From the age variable, three cases where the reported age was 100 years or higher were set to missing values. The remaining cases had a mean age of 30.9 years (median 28 years, range [14, 85]). From the gender variable, 13 values were missing and 15 cases where the reported gender was "3" were set to missing values. Of the remaining cases, 2295 (68.5%) reported gender "1" and 1053 (31.4%) reported gender "2". The SCS data contained at least one missing value for 133 cases.

The pattern of missing SCS items is shown in Figure **??**. It can be seen that item Q9 was missing most often, though not by a large margin (Q9: 27 missing values, Q5: 13 missing values). It can be seen that the majority of cases with missing values (118 cases / 88.7%) had only a single item missing, while there were no prominent patterns of items that tended to be jointly missing. Eight cases where more than two SCS items were missing were excluded from all further analyses. For the remaining 3368 cases, the probability of missing values at each SCS variable was modeled as a function of the values in all other SCS variables using a logistic regression model: $P(M_{i,q} = 1|X_{i,q'}) = \sigma(X_{i,q'}\hat{\beta})$, where $M_{i,q}$ is 1 if the i^{th} person has a missing value at item $q \in \{Q1, Q2, ...Q10\}$, $X_{i,q'}$ denotes the item values of all other items, σ is the logistic function, and $\hat{\beta}$ are the estimated regression weights (Guan and Yusoff (2011)). Note that each variable's pattern of missing values could only be predicted based on the observations without missing values in any other variable, since those cases were excluded by the logistic model by default of the implementation. Since the majority of cases had either no or only one variable missing, however, this should not bias the overall picture very much.

```
## Loading required package: ggplot2

## Loading required package: ggthemes

## Loading required package: reshape2

## Loading required package: readxl

## Loading required package: VIM

## Loading required package: colorspace

## Loading required package: grid

## VIM is ready to use.

## Suggestions and bug-reports can be submitted at: https://github.com/statistikat/VIM/issues

##
## Attaching package: 'VIM'

## The following object is masked from 'package:datasets':
##
##     sleep

## Loading required package: mice

##
## Attaching package: 'mice'

## The following object is masked from 'package:stats':
##
##     filter

## The following objects are masked from 'package:base':
##
##     cbind, rbind
```

```
## Loading required package: dplyr
##
## Attaching package: 'dplyr'
## The following objects are masked from 'package:stats':
##
##     filter, lag
## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
## Loading required package: tidyr
##
## Attaching package: 'tidyr'
## The following object is masked from 'package:reshape2':
##
##     smiths
## Loading required package: psych
##
## Attaching package: 'psych'
## The following objects are masked from 'package:ggplot2':
##
##     %+%, alpha
## Loading required package: ggcorrplot
## Loading required package: eRm
##
## Attaching package: 'eRm'
## The following object is masked from 'package:psych':
##
##     sim.rasch
## Loading required package: lme4
## Loading required package: Matrix
##
## Attaching package: 'Matrix'
## The following objects are masked from 'package:tidyr':
##
##     expand, pack, unpack
## Loading required package: lavaan
## This is lavaan 0.6-11
## lavaan is FREE software! Please report any bugs.
##
## Attaching package: 'lavaan'
## The following object is masked from 'package:psych':
##
##     cor2cov
```

```
##
##    0    1    2    3
##   13 2295 1053   15
##  [1]  41  50  23  42  36  29  24  35  26  43  21  39  37  64  28  46  34  31  47
## [20]  22  61  16  40  33  30  56  49  51  18  20  45  32  15  27  25  59  58  19
## [39]  14  38  48  44  55 100  65  17  77  57  60  52  53  62  71  78  54  63  67
## [58]  68  72 999  85  69  70  66  84 123  73
```

```
## Warning in plot.aggr(res, ...): not enough vertical space to display frequencies
## (too many combinations)
```

```
##
##  Variables sorted by number of missings:
##  Variable Count
##         Q9    27
##         Q3    22
##         Q4    22
##         Q7    22
##         Q8    21
##         Q6    20
##        Q10    17
##         Q2    16
##         Q1    15
##         Q5    13
##
##  iter imp variable
##   1   1  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   1   2  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   1   3  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   1   4  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   1   5  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   2   1  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   2   2  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   2   3  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   2   4  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   2   5  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   3   1  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   3   2  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   3   3  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   3   4  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   3   5  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   4   1  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   4   2  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   4   3  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   4   4  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   4   5  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   5   1  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   5   2  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   5   3  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   5   4  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   5   5  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
```

```
## Warning in biserialc(x[, j], y[, i], j, i): For x = 1 y = 1 x seems to be
## dichotomous, not continuous
```

```
## Warning in biserialc(x[, j], y[, i], j, i): For x = 2 y = 2 x seems to be
```

```
## dichotomous, not continuous
```

```
## Warning in biserialc(x[, j], y[, i], j, i): For x = 3 y = 3 x seems to be
## dichotomous, not continuous
```

```
## Warning in biserialc(x[, j], y[, i], j, i): For x = 4 y = 4 x seems to be
## dichotomous, not continuous
```

```
## Warning in biserialc(x[, j], y[, i], j, i): For x = 5 y = 5 x seems to be
## dichotomous, not continuous
```

```
## Warning in biserialc(x[, j], y[, i], j, i): For x = 6 y = 6 x seems to be
## dichotomous, not continuous
```

```
## Warning in biserialc(x[, j], y[, i], j, i): For x = 7 y = 7 x seems to be
## dichotomous, not continuous
```

```
## Warning in biserialc(x[, j], y[, i], j, i): For x = 8 y = 8 x seems to be
## dichotomous, not continuous
```

```
## Warning in biserialc(x[, j], y[, i], j, i): For x = 9 y = 9 x seems to be
## dichotomous, not continuous
```

```
## Warning in biserialc(x[, j], y[, i], j, i): For x = 10 y = 10 x seems to be
## dichotomous, not continuous
```

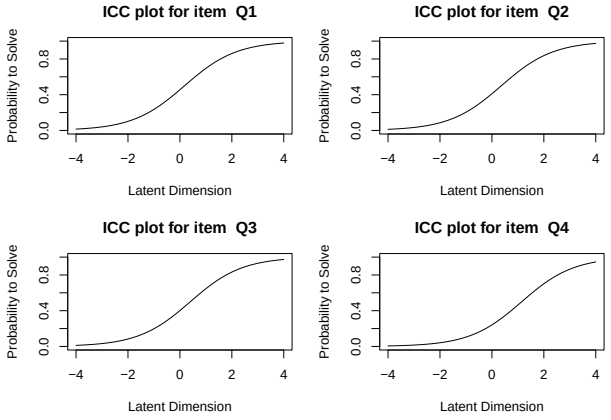


Figure 1: Pattern of missing SCS values.

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.436828 (tol = 0.002, component 1)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, : Model is nearly unide
##  - Rescale variables?
```

```
## lavaan 0.6-11 ended normally after 15 iterations
##
##   Estimator                                         DWLS
##   Optimization method                              NLMINB
##   Number of model parameters                           20
##   Number of equality constraints                        9
##
##   Number of observations                             3368
##
## Model Test User Model:
```

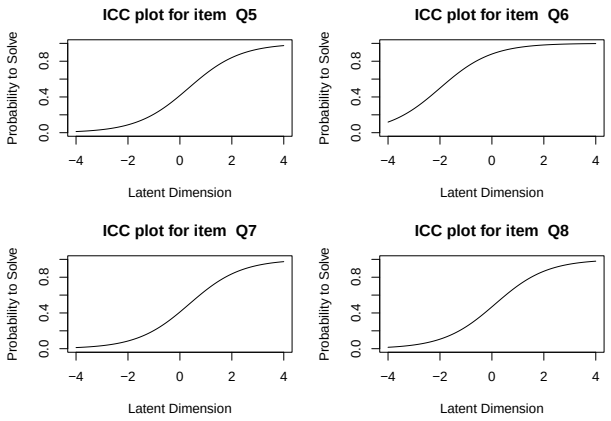


Figure 2: Pattern of missing SCS values.

```
##                                                Standard      Robust
##   Test Statistic                               1426.970    1345.917
##   Degrees of freedom                             44          44
##   P-value (Chi-square)                          0.000       0.000
##   Scaling correction factor                                 1.068
##   Shift parameter                                           9.179
##        simple second-order correction (WLSMV)
##
## Model Test Baseline Model:
##
##   Test statistic                             39187.849   24353.080
##   Degrees of freedom                             45          45
##   P-value                                       0.000       0.000
##   Scaling correction factor                                 1.610
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                   0.965       0.946
##   Tucker-Lewis Index (TLI)                      0.964       0.945
##
##   Robust Comparative Fit Index (CFI)                           NA
##   Robust Tucker-Lewis Index (TLI)                              NA
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                         0.097       0.094
##   90 Percent confidence interval - lower        0.092       0.089
##   90 Percent confidence interval - upper        0.101       0.098
##   P-value RMSEA <= 0.05                         0.000       0.000
##
##   Robust RMSEA                                                 NA
##   90 Percent confidence interval - lower                       NA
##   90 Percent confidence interval - upper                       NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                          0.100       0.100
```

```
##
## Weighted Root Mean Square Residual:
##
##   WRMR                                           5.094       5.094
##
## Parameter Estimates:
##
##   Standard errors                             Robust.sem
##   Information                                   Expected
##   Information saturated (h1) model        Unstructured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   SCS =~
##     Q1      (ldng)    1.218    0.022   55.949    0.000    1.218    0.773
##     Q2      (ldng)    1.218    0.022   55.949    0.000    1.218    0.773
##     Q3      (ldng)    1.218    0.022   55.949    0.000    1.218    0.773
##     Q4      (ldng)    1.218    0.022   55.949    0.000    1.218    0.773
##     Q5      (ldng)    1.218    0.022   55.949    0.000    1.218    0.773
##     Q6      (ldng)    1.218    0.022   55.949    0.000    1.218    0.773
##     Q7      (ldng)    1.218    0.022   55.949    0.000    1.218    0.773
##     Q8      (ldng)    1.218    0.022   55.949    0.000    1.218    0.773
##     Q9      (ldng)    1.218    0.022   55.949    0.000    1.218    0.773
##     Q10     (ldng)    1.218    0.022   55.949    0.000    1.218    0.773
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     SCS               0.000                              0.000    0.000
##    .Q1                0.000                              0.000    0.000
##    .Q2                0.000                              0.000    0.000
##    .Q3                0.000                              0.000    0.000
##    .Q4                0.000                              0.000    0.000
##    .Q5                0.000                              0.000    0.000
##    .Q6                0.000                              0.000    0.000
##    .Q7                0.000                              0.000    0.000
##    .Q8                0.000                              0.000    0.000
##    .Q9                0.000                              0.000    0.000
##    .Q10               0.000                              0.000    0.000
##
## Thresholds:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     Q1|t1             0.378    0.035   10.888    0.000    0.378    0.240
##     Q2|t1             0.489    0.035   14.002    0.000    0.489    0.310
##     Q3|t1             0.511    0.035   14.631    0.000    0.511    0.324
##     Q4|t1             0.964    0.037   26.095    0.000    0.964    0.611
##     Q5|t1             0.474    0.035   13.622    0.000    0.474    0.301
##     Q6|t1            -0.914    0.036  -25.504    0.000   -0.914   -0.580
##     Q7|t1             0.489    0.035   14.005    0.000    0.489    0.310
##     Q8|t1             0.346    0.035   10.019    0.000    0.346    0.220
##     Q9|t1             0.025    0.034    0.723    0.470    0.025    0.016
##     Q10|t1           -0.032    0.034   -0.931    0.352   -0.032   -0.020
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
```

```
##     SCS               1.000                                1.000    1.000
##    .Q1                1.000                                1.000    0.403
##    .Q2                1.000                                1.000    0.403
##    .Q3                1.000                                1.000    0.403
##    .Q4                1.000                                1.000    0.403
##    .Q5                1.000                                1.000    0.403
##    .Q6                1.000                                1.000    0.403
##    .Q7                1.000                                1.000    0.403
##    .Q8                1.000                                1.000    0.403
##    .Q9                1.000                                1.000    0.403
##    .Q10               1.000                                1.000    0.403
##
## Scales y*:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     Q1               0.634                                0.634    1.000
##     Q2               0.634                                0.634    1.000
##     Q3               0.634                                0.634    1.000
##     Q4               0.634                                0.634    1.000
##     Q5               0.634                                0.634    1.000
##     Q6               0.634                                0.634    1.000
##     Q7               0.634                                0.634    1.000
##     Q8               0.634                                0.634    1.000
##     Q9               0.634                                0.634    1.000
##     Q10              0.634                                0.634    1.000
##
## R-Square:
##                   Estimate
##     Q1               0.597
##     Q2               0.597
##     Q3               0.597
##     Q4               0.597
##     Q5               0.597
##     Q6               0.597
##     Q7               0.597
##     Q8               0.597
##     Q9               0.597
##     Q10              0.597
```

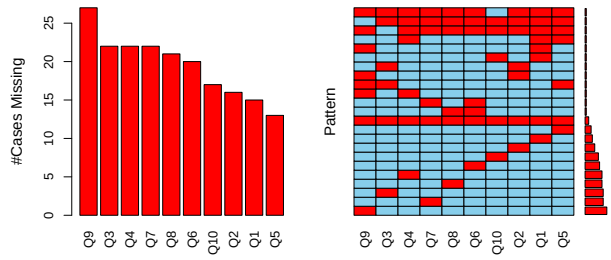


Figure 3: Pattern of missing SCS values.

For dichotomization of the item data, I considered two options, namely, thresholding each of the 10 items at its own median, to ensure an even distribution of observations into both categories for each item, or finding a common threshold for all items. Since the items have only four levels each, a median split would

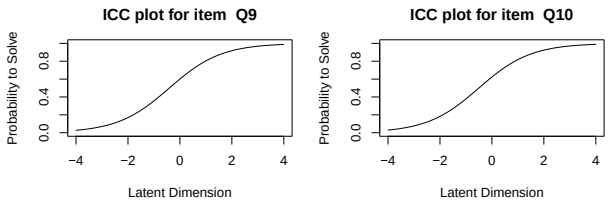


Figure 4: Pattern of missing SCS values.

not necessarily lead to a very balanced dichotomization. Furthermore, the item levels are designed to have the same meaning across all items, therefore I decided to dichotomize at a common threshold of 2, i.e., the dichotomous items $D_q \in \{D_1, D_2, ..., D_{10}\}$ were defined such that

$$D_{i,q} = \begin{cases} 0 \text{ if } Q_{i,q} \in \{1, 2\}, \\ 1 \text{ if } Q_{i,q} \in \{3, 4\}, \end{cases}$$

The distribution of the dichotomized items is shown in 2 Since most variables' median was 2, this was not much different from an item-wise median threshold (see 1). Subsequently, I calculated biserial correlations between all pairs of dichotomized items. Moreover, I calculated item discrimination, i.e., each items ability to discriminate between high- and low-scoring individuals, using the adjusted item-total correlation method (Reynolds and Livingston (2021)), i.e., by calculating biserial correlation coefficients between each (dichotomized) item's scores and the sum of all other (dichotomized) items.

#TODO make tables smaller (too wide) #TODO make captions and table referencing work

Next, I estimated a Rasch model using three different software implementations.

The Rasch model...

The first method was the one implemented in the R package `eRm` (Mair and Hatzinger (2007))

The second method was `lme4` (Doran et al. (2007)) #discuss items as random or fixed effect

The thirs method was `lavaan` (Rosseel (2012), Templin (2022))

3 Results

Descriptive characteristics of the 10 SCS items are shown in 1, the proportions of 'correct' responses, i.e., responses greater than 2, are shown in 2.

Item intercorrelations are shown in Figure 5 #TODO discuss

Table 1: Descriptive item statistics (mean, median and range before dichotomization)

stat	Q1	Q10	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9
max	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0
mean	2.3	2.5	2.2	2.2	1.9	2.2	3.1	2.2	2.3	2.5
median	2.0	3.0	2.0	2.0	2.0	2.0	3.0	2.0	2.0	2.0
min	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Table 2: Distribution and discrimination of dichotomized items

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
percent in category 1	40.5	37.8	37.3	27	38.2	71.9	37.8	41.3	49.4	50.8
number of cases in category 1	1365	1274	1256	911	1286	2422	1274	1391	1663	1711
discrimination	0.45	0.45	0.44	0.34	0.29	0.26	0.42	0.37	0.31	0.36

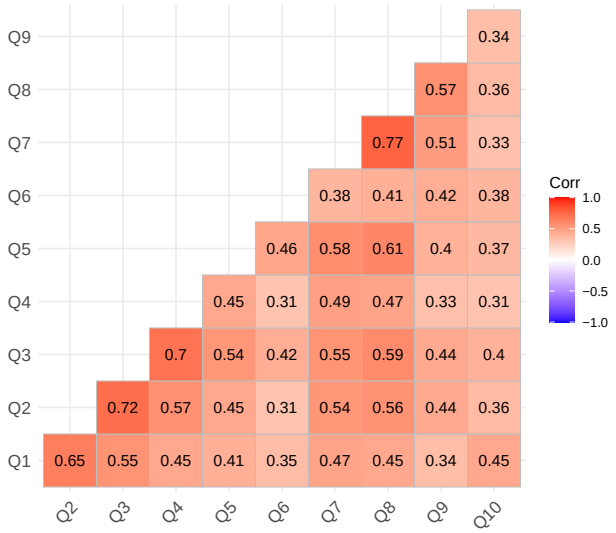


Figure 5: Pattern of missing SCS values.

4 Analysis code

In the following, the complete analysis code and its output are shown.

```
require(ggplot2)
require(ggthemes)
require(reshape2)
require(readxl)
require(VIM)
require(mice)
require(dplyr)
require(tidyr)
require(psych)
require(ggcorrplot)
require(eRm)
require(lme4)
require(lavaan)


#####
#part 1: data preparation, descriptive analyses
#####
{
df = read_xlsx("SCS_data.xlsx")
SCS_vars = names(df)[1:10]
#set missing values
print(table(df$gender))
df$gender[df$gender == 3] = NA
df[df==0] = NA

print(unique(df$age))
df$age[df$age >= 100] = NA
mean(df$age,na.rm=T)
median(df$age,na.rm=T)
min(df$age,na.rm=T)
max(df$age,na.rm=T)

sprintf("%i cases are incomplete",sum(!complete.cases(df)))
sprintf("%i cases have incomplete SCS data",sum(!complete.cases(df[,SCS_vars])))


#missing data motifs
# and missing proportion per item
pdf("missingplot.pdf",width = 8, height = 4)
aggr(df[!complete.cases(df[,SCS_vars]),SCS_vars],
     numbers=TRUE, sortVars=TRUE,prop=FALSE,
     labels=SCS_vars,
     ylab=c("#Cases Missing","Pattern"))
dev.off()

nmissing = rowSums(is.na(df[,SCS_vars]))
table(nmissing[nmissing!=0])
prop.table(table(nmissing[nmissing!=0]))
```

```r
#missing-at-random analysis
#(check whether missing data points in each variable
#can be jointly predicted by all the other variables)
pvals = data.frame(matrix(ncol = length(SCS_vars), nrow=0))
colnames(pvals) = SCS_vars
for (var in SCS_vars){
  formula = sprintf("I(is.na(%s)) ~ .", var)
  formula0 = sprintf("I(is.na(%s)) ~ 1", var)
  m = summary(glm(formula, data=df[,1:10]))$coefficients
  pvals[var,rownames(m)[2:10]] = m[2:10,"Pr(>|t|)"]
}
min(p.adjust(unlist(pvals), method="fdr"),na.rm=T)


#-> missing at random can be assumed
#remove cases where more than two SCS variables are missing

#15 cases removed
df_clean = df[rowSums(is.na(df[,SCS_vars])) <= 2,]

#use multiple imputation for remaining data
df_clean = complete(mice(df_clean))


#descriptives
df_clean[,1:10] %>% summarise_all(list(mean=mean, median = median, min = min, max = max)) %>%
  round(1) %>%
  gather(variable, value) %>%
  separate(variable, c("var", "stat"), sep = "\\_") %>%
  spread(var, value) -> descriptives

#re-calculate sum score
df_clean$score = rowSums(df_clean[,1:10])

#dichotomization
dich = df_clean
dich[,1:10] = data.frame(lapply(df_clean[,1:10], function (x) as.numeric(x > 2)))
dich$score = rowSums(dich[,1:10])

}
```

```
##
##    0    1    2    3
##   13 2295 1053   15
##  [1]  41  50  23  42  36  29  24  35  26  43  21  39  37  64  28  46  34  31  47
## [20]  22  61  16  40  33  30  56  49  51  18  20  45  32  15  27  25  59  58  19
## [39]  14  38  48  44  55 100  65  17  77  57  60  52  53  62  71  78  54  63  67
## [58]  68  72 999  85  69  70  66  84 123  73
```

```
## Warning in plot.aggr(res, ...): not enough vertical space to display frequencies
## (too many combinations)
```

```
##
##  Variables sorted by number of missings:
##  Variable Count
```

```
##         Q9    27
##         Q3    22
##         Q4    22
##         Q7    22
##         Q8    21
##         Q6    20
##        Q10    17
##         Q2    16
##         Q1    15
##         Q5    13
##
##  iter imp variable
##   1   1  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   1   2  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   1   3  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   1   4  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   1   5  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   2   1  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   2   2  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   2   3  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   2   4  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   2   5  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   3   1  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   3   2  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   3   3  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   3   4  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   3   5  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   4   1  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   4   2  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   4   3  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   4   4  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   4   5  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   5   1  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   5   2  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   5   3  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   5   4  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
##   5   5  Q1  Q2  Q3  Q4  Q5  Q6  Q7  Q8  Q9  Q10  gender  age
```

```
#####
#part 2: CTT-style item analysis
#####
{

  #biserial correlations
  biserial_cor = biserial(dich[,SCS_vars],dich[,SCS_vars])
  ggcorrplot(biserial_cor, type = "lower", lab = TRUE)
  ggsave("biserial_cor_mat.pdf",width = 6, height = 6)
  #dichotomous item statistics (percent and N correct, discriminativity)
  dich.distro = rbind(as.character(round(100*unlist(lapply(dich[,SCS_vars], mean)),1)),
                      as.character(as.integer(unlist(lapply(dich[,SCS_vars], sum)))))
  rownames(dich.distro) = c("percent in category 1", "number of cases in category 1")

  discrimination = c()
  for (item in 1:10){
```

```
    itemname = SCS_vars[item]
    discrimination[itemname] = as.character(round(biserial(rowSums(dich[,-item]),dich[,item]),2))
  }

  dich.stats = rbind(dich.distro, discrimination)
}
```

```
## Warning in biserialc(x[, j], y[, i], j, i): For x = 1 y = 1 x seems to be
## dichotomous, not continuous
```

```
## Warning in biserialc(x[, j], y[, i], j, i): For x = 2 y = 2 x seems to be
## dichotomous, not continuous
```

```
## Warning in biserialc(x[, j], y[, i], j, i): For x = 3 y = 3 x seems to be
## dichotomous, not continuous
```

```
## Warning in biserialc(x[, j], y[, i], j, i): For x = 4 y = 4 x seems to be
## dichotomous, not continuous
```

```
## Warning in biserialc(x[, j], y[, i], j, i): For x = 5 y = 5 x seems to be
## dichotomous, not continuous
```

```
## Warning in biserialc(x[, j], y[, i], j, i): For x = 6 y = 6 x seems to be
## dichotomous, not continuous
```

```
## Warning in biserialc(x[, j], y[, i], j, i): For x = 7 y = 7 x seems to be
## dichotomous, not continuous
```

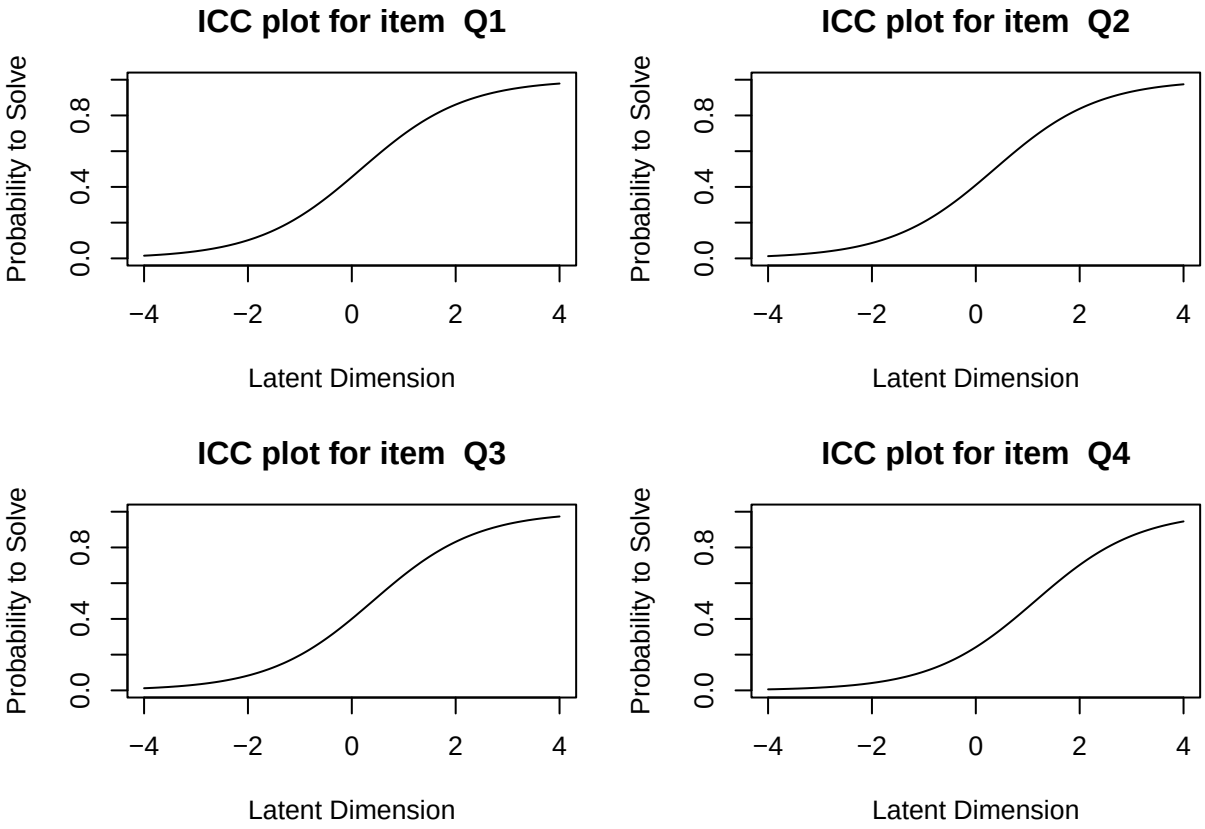
```
## Warning in biserialc(x[, j], y[, i], j, i): For x = 8 y = 8 x seems to be
## dichotomous, not continuous
```

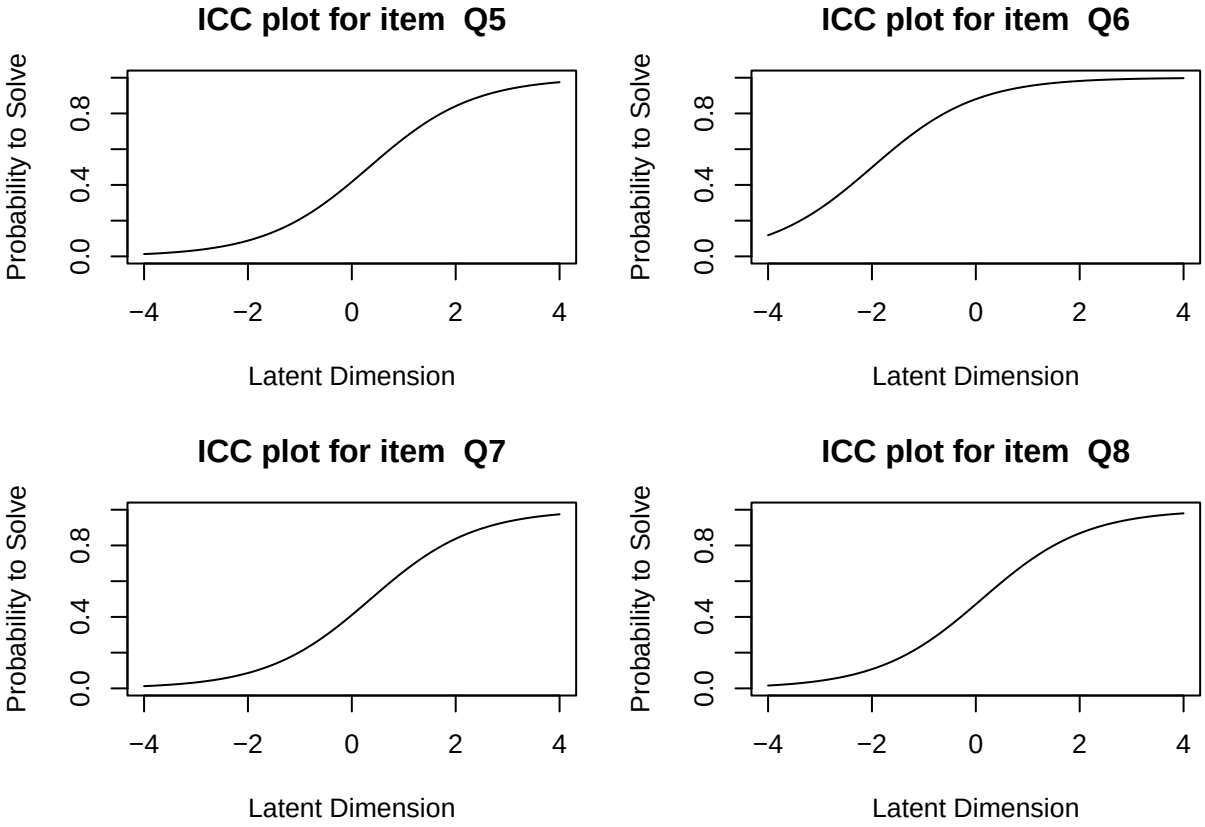
```
## Warning in biserialc(x[, j], y[, i], j, i): For x = 9 y = 9 x seems to be
## dichotomous, not continuous
```

```
## Warning in biserialc(x[, j], y[, i], j, i): For x = 10 y = 10 x seems to be
## dichotomous, not continuous
```

```
#####
#part 2a: estimate Rasch model
#####
{
  #approach 1: eRm
  #prepare data for eRm estimation
  #(just item data in wide format)
  data_for_eRm = dich[,1:10]
  rasch_model_1 = RM(data_for_eRm)
  plotICC(rasch_model_1)


  #approach 2: lme4
  #prepare data for lme4 estimation
  #(item and subject data in long format)
  data_for_lme4 = dich[,1:10]
  data_for_lme4$id = 1:nrow(data_for_lme4)
  data_for_lme4 = melt(data_for_lme4, id.vars = "id")
  rasch_model_2 = glmer(value~0+variable+(1|id), data = data_for_lme4,
                        family = binomial)
  #TODO check: how to interpret parameters?
  #aproach 3: lavaan
```

```
  #modified copy from https://jonathantemplin.com/wp-content/uploads/2022/02/EPSY906_Example05_Binary_I
  lavaansyntax = "

    # loadings/discrimination parameters:
    SCS =~ loading*Q1 + loading*Q2 + loading*Q3 + loading*Q4 + loading*Q5 + loading*Q6 + loading*Q7 + l

    # threshholds use the | operator and start at value 1 after t:
    Q1 | t1; Q2 | t1; Q3 | t1; Q4 | t1; Q5 | t1; Q6 | t1; Q7 | t1; Q8 | t1; Q9 | t1;Q10 | t1;

    # factor mean:
    SCS ~ 0;

    # factor variance:
    SCS ~~ 1*SCS

    "

  rasch_model_3 = sem(model = lavaansyntax, data = dich[,SCS_vars], ordered = SCS_vars,
                      mimic = "Mplus", estimator = "WLSMV", std.lv = TRUE, parameterization = "theta
  summary(rasch_model_3, fit.measures = TRUE, rsquare = TRUE, standardized = TRUE)


  #make ICC plot function
  ICC_plot = function(betas){
    df = data.frame(x=seq(-5,5,.01))
    for (i in 1:length(betas)){
      df[[SCS_vars[i]]] = logistic(df$x, betas[i])
    }

    df = melt(df, id.vars = "x")
    colnames(df)[2] = "item"
    plt=ggplot(df, aes(x = x, y = value, color = item, label = item)) + geom_line() +
      theme_clean() + xlab("Person parameter") + ylab("P(item solved)")
    return(directlabels::direct.label(plt, "last.qp"))
      }

}
```


ICC plot for item Q1
Probability to Solve
0.0
0.4
0.8
−4
−2
0
2
4
Latent Dimension
ICC plot for item Q2
Probability to Solve
0.0
0.4
0.8
−4
−2
0
2
4
Latent Dimension
ICC plot for item Q3
Probability to Solve
0.0
0.4
0.8
−4
−2
0
2
4
Latent Dimension
ICC plot for item Q4
Probability to Solve
0.0
0.4
0.8
−4
−2
0
2
4
Latent Dimension




```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.434651 (tol = 0.002, component 1)
```

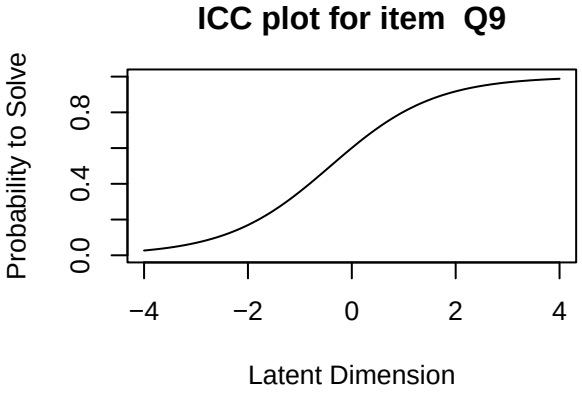
```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, : Model is nearly unide
##  - Rescale variables?
```

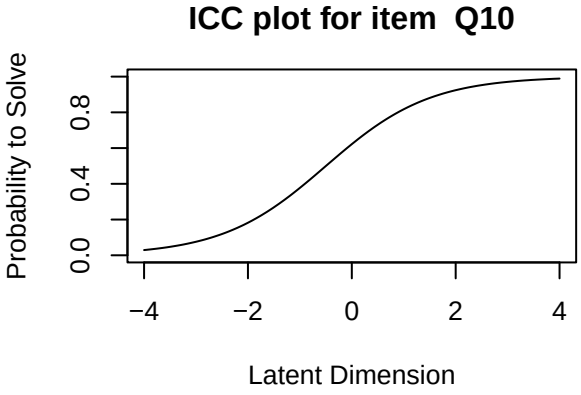
```
## lavaan 0.6-11 ended normally after 15 iterations
##
##   Estimator                                         DWLS
##   Optimization method                             NLMINB
##   Number of model parameters                          20
##   Number of equality constraints                       9
##
##   Number of observations                            3368
##
## Model Test User Model:
##                                              Standard      Robust
##   Test Statistic                             1423.644    1343.659
##   Degrees of freedom                               44          44
##   P-value (Chi-square)                          0.000       0.000
##   Scaling correction factor                                 1.067
##   Shift parameter                                           9.150
##         simple second-order correction (WLSMV)
##
## Model Test Baseline Model:
##
##   Test statistic                            38904.014   24194.283
```

```
##   Degrees of freedom                                45          45
##   P-value                                        0.000       0.000
##   Scaling correction factor                                  1.609
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.964       0.946
##   Tucker-Lewis Index (TLI)                       0.964       0.945
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.097       0.094
##   90 Percent confidence interval - lower         0.092       0.089
##   90 Percent confidence interval - upper         0.101       0.098
##   P-value RMSEA <= 0.05                          0.000       0.000
##
##   Robust RMSEA                                                  NA
##   90 Percent confidence interval - lower                        NA
##   90 Percent confidence interval - upper                        NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.100       0.100
##
## Weighted Root Mean Square Residual:
##
##   WRMR                                           5.088       5.088
##
## Parameter Estimates:
##
##   Standard errors                             Robust.sem
##   Information                                   Expected
##   Information saturated (h1) model          Unstructured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   SCS =~
##     Q1      (ldng)    1.215    0.022   55.937    0.000    1.215    0.772
##     Q2      (ldng)    1.215    0.022   55.937    0.000    1.215    0.772
##     Q3      (ldng)    1.215    0.022   55.937    0.000    1.215    0.772
##     Q4      (ldng)    1.215    0.022   55.937    0.000    1.215    0.772
##     Q5      (ldng)    1.215    0.022   55.937    0.000    1.215    0.772
##     Q6      (ldng)    1.215    0.022   55.937    0.000    1.215    0.772
##     Q7      (ldng)    1.215    0.022   55.937    0.000    1.215    0.772
##     Q8      (ldng)    1.215    0.022   55.937    0.000    1.215    0.772
##     Q9      (ldng)    1.215    0.022   55.937    0.000    1.215    0.772
##     Q10     (ldng)    1.215    0.022   55.937    0.000    1.215    0.772
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     SCS               0.000                               0.000    0.000
```

```
##    .Q1                0.000                                0.000    0.000
##    .Q2                0.000                                0.000    0.000
##    .Q3                0.000                                0.000    0.000
##    .Q4                0.000                                0.000    0.000
##    .Q5                0.000                                0.000    0.000
##    .Q6                0.000                                0.000    0.000
##    .Q7                0.000                                0.000    0.000
##    .Q8                0.000                                0.000    0.000
##    .Q9                0.000                                0.000    0.000
##    .Q10               0.000                                0.000    0.000
##
## Thresholds:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     Q1|t1            0.378    0.035   10.921    0.000    0.378    0.240
##     Q2|t1            0.489    0.035   14.037    0.000    0.489    0.311
##     Q3|t1            0.515    0.035   14.767    0.000    0.515    0.327
##     Q4|t1            0.962    0.037   26.091    0.000    0.962    0.611
##     Q5|t1            0.473    0.035   13.623    0.000    0.473    0.301
##     Q6|t1           -0.912    0.036  -25.478    0.000   -0.912   -0.579
##     Q7|t1            0.489    0.035   14.041    0.000    0.489    0.311
##     Q8|t1            0.341    0.035    9.882    0.000    0.341    0.217
##     Q9|t1            0.025    0.034    0.723    0.470    0.025    0.016
##     Q10|t1          -0.029    0.034   -0.862    0.389   -0.029   -0.019
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     SCS              1.000                                1.000    1.000
##    .Q1               1.000                                1.000    0.404
##    .Q2               1.000                                1.000    0.404
##    .Q3               1.000                                1.000    0.404
##    .Q4               1.000                                1.000    0.404
##    .Q5               1.000                                1.000    0.404
##    .Q6               1.000                                1.000    0.404
##    .Q7               1.000                                1.000    0.404
##    .Q8               1.000                                1.000    0.404
##    .Q9               1.000                                1.000    0.404
##    .Q10              1.000                                1.000    0.404
##
## Scales y*:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     Q1               0.635                                0.635    1.000
##     Q2               0.635                                0.635    1.000
##     Q3               0.635                                0.635    1.000
##     Q4               0.635                                0.635    1.000
##     Q5               0.635                                0.635    1.000
##     Q6               0.635                                0.635    1.000
##     Q7               0.635                                0.635    1.000
##     Q8               0.635                                0.635    1.000
##     Q9               0.635                                0.635    1.000
##     Q10              0.635                                0.635    1.000
##
## R-Square:
##                   Estimate
##     Q1               0.596
```

```
##      Q2                0.596
##      Q3                0.596
##      Q4                0.596
##      Q5                0.596
##      Q6                0.596
##      Q7                0.596
##      Q8                0.596
##      Q9                0.596
##      Q10               0.596
```





5 References

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