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Question 4:

(a) Let define sequence $A = \langle 1, \underbrace{0, 0, \ldots, 0}_{k}, 1 \rangle$

- Our goal is to find the convolution of $A * A$
- So, the corresponding polynomial of A is $P_A(x) = 1 + x^{k+1}$
- As we know, the convolution of $A * A$ is the sequence of coefficient of the polynomial
$$P_C(x) = P_A(x) * P_A(x)$$
$$P_C(x) = (1 + x^{k+1}) * (1 + x^{k+1})$$
$$P_C(x) = 1 + 2x^{k+1} + x^{2(k+1)}$$
- Therefore the convolution of $A * A$ is:
$$\langle 1, \underbrace{0, 0, \ldots, 0}_{k}, 2, \underbrace{0, 0, \ldots, 0}_{k}, 1 \rangle$$

(b) Let define sequence $A = \langle 1, \underbrace{0, 0, \ldots, 0}_{k}, 1 \rangle$. So, the corresponding polynomial is $P_A(x) = 1 + x^{k+1}$

$$DFT(A) = \langle P_A(\omega_{k+2}^0), P_A(\omega_{k+2}^1), P_A(\omega_{k+2}^2), \ldots, P_A(\omega_{k+2}^{k+1}) \rangle$$
$$DFT(A) = \langle (1 + \omega_{k+2}^0), (1 + \omega_{k+2}^{1*(k+1)}), (1 + \omega_{k+2}^{2*(k+1)}), \ldots, (1 + \omega_{k+2}^{(k+1)*(k+1)}) \rangle$$
$$DFT(A) = \langle (1 + 1), (1 + \omega_{k+2}^{(k+1)}), (1 + \omega_{k+2}^{2(k+1)}), \ldots, (1 + \omega_{k+2}^{(k+1)^2}) \rangle$$