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Question 3:

- Let A is the sequence on $100n$ numbers and N' be the net sequence N in the reverse order.
- Let compute the convolution of the two sequences we get sequence $C = A * N'$
- Consider the $(k+n)-th$ element in the sequence C: (this element represents the number of fish when we place the net at the position $k-th$ from the left end.
$$C_{k+n} = A_k N'_n + A_{k+1} N'_{n-1} + \cdots + A_{k+n} N'_0$$
Hence, $0 \leq C_{k+n} \leq \sum_{j=0}^{k+n} A_{k+j}$.
- Our goal is to find the maximum element in the sequence C.
- Using FFT algorithm to find the convolution of the two sequences $C = A * N'$

$$A = \langle A_0, A_1, \dots, A_{100n-1} \rangle \qquad N' = \langle N'_0, N'_1, \dots, N'_{n-1} \rangle$$
$$\Downarrow O(n) \qquad \Downarrow O(n)$$
$$P_A(x) = A_0 + A_1 x + \cdots + A_{100n-1} x^{100n-1} \qquad P_{N'}(x) = N'_0 + N'x + \cdots + N'_{n-1} x^{n-1}$$
$$\Downarrow O(n \log n) \qquad \Downarrow O(n \log n)$$
$$\{P_A(1), P_A(\omega_{101n-1}), \dots, P_A(\omega_{101n-1}^{101n-2})\} \qquad \{P_{N'}(1), P_{N'}(\omega_{101n-1}), \dots, P_{N'}(\omega_{101n-1}^{101n-2})\}$$

$$\Downarrow multiplication\ O(n)$$
$$\{P_A(1)P_{N'}(1), P_A(\omega_{101n-1})P_{N'}(\omega_{101n-1}), \dots, P_A(\omega_{101n-1}^{101n-2})P_{N'}(\omega_{101n-1}^{101n-2})\}$$

$$\Downarrow IDFT\ O(n \log n)$$

$$P_C(x) = \sum_{j=0}^{101n-2} \left(\underbrace{\sum_{i=0}^{j} A_i B_{j-i}}_{C_j} \right) x^j$$
$$\Downarrow$$
$$C = \langle \sum_{i=0}^{j} A_i B_{j-i} \rangle_{j=0}^{j=101n-2}$$

$$\Downarrow O(n)$$
$$C_{max} = \max(C)$$

- Time complexity: $O(n \log n)$

$$N' = \langle N'_0, N'_1, \ldots, N'_{n-1} \rangle$$

$$\Downarrow O(n)$$

$$P_{N'}(x) = N'_0 + N'x + \cdots + N'_{n-1}x^{n-1}$$

$$\Downarrow O(n \log n)$$

$$\{P_{N'}(1), P_{N'}(\omega_{101n-1}), \ldots, P_{N'}(\omega_{101n-1}^{101n-2})\}$$