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Question 5:

- If we schedule the production of the chemicals in order $C_1, C_2, C_3, \dots, C_i, \dots, C_n$.
- Then, at the end of 1st day, the total amount of chemical is:
$$W_{total} = W_1(1-p)$$
Because on the first day, we only produce W_1 kilograms of chemical C_i and the amount of evaporate is
$$p * W_1$$

  At the end of 2nd day, the total amount of chemicals is:
$$W_{total} = (W_1(1-p) + W_2)(1-p) = W_1(1-p)^2 + W_2(1-p)$$
Because on the second day, we produce W_2 kilograms of chemical C_2, and the amount before evaporation happens is
$$W_{produce} = W_1(1-p) + W_2$$
So, at the end of second day, $W_{total} = W_{produce} * (1-p)$

  Similarly, at the end of nth day, the amount of total amount of chemicals is:
$$W_{total} = W_1(1-p)^n + W_2(1-p)^{n-1} + W_3(1-p)^{n-2} + \cdots + W_n(1-p)$$
- Therefore, in order to produce total extra weight of all chemicals needed to produce to compensate for the evaporation loss is as small as possible the W_{total} must be maximum.
- We claim that, if the chemicals are ordered in an increasing order of weights, the total weight W_{total} will be maximised.
- Proof:
  - Assume $W_1 < W_2 < W_3 < \cdots < W_k < W_{k+1} < \cdots < W_n$
  - Let us see what happens if we swap to adjacent chemicals W_k and W_{k+1}
  - The expected total weight before the swap and after swap are, respectively,
$$W_{total} = W_1(1-p)^n + W_2(1-p)^{n-1} + \cdots + W_k(1-p)^{n-(k-1)} + W_{k+1}(1-p)^{n-k} + \cdots + W_n(1-p)$$
$$W'_{total} = W_1(1-p)^n + W_2(1-p)^{n-1} + \cdots + W_{k+1}(1-p)^{n-(k-1)} + W_k(1-p)^{n-k} + \cdots + W_n(1-p)$$
- Thus,
$$W_{total} - W'_{total} = (1-p)^{n-(k-1)}(W_k - W_{k+1}) + (1-p)^{n-k}(W_{k+1} - W_k)$$
$$W_{total} - W'_{total} = (1-p)^{n-k}((1-p)(W_k - W_{k+1}) + W_{k+1} - W_k)$$
$$W_{total} - W'_{total} = (1-p)^{n-k}(pW_{k+1} - pW_k)$$
$$W_{total} - W'_{total} = p(1-p)^{n-k}(W_{k+1} - W_k) > 0 \; because \; W_{k+1} > W_k$$
Therefore,
$$W_{total} > W'_{total}$$
- Consequently, $W_{total} > W'_{total}$, if and only if $W_k < W_{k+1}$, which means that the swap decreases the expected total weight just in case $W_k < W_{k+1}$.
- Consequently, the maximum total weight is obtained when there are no inversions.
- When we have the maximum total weight after nth day, which means the amount of chemicals we need to reproduce will be minimum.