A note for the "artificial potential field" variant of the Quadrotor Swarm system

Alex Devonport

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In order to make the dynamics of the quadrotor swarm system more dense, we will add an "artificial potential field"-based controller to the system. This controller applies a force to the quadrotors that seeks to minimize a *artificial potential* U that depends on the position of all of the quadrotors. The potential applied to each quadotor is intended to repel it from the other quadrotors, and to attract it towards the origin. We will actually use three potential fields for each quadrotor, which separately consider the x-, y-, and z-axis positions of the quadrotors. This has the advantage of simplifying some expressions, and allows us to ensure that the forces resulting from the potential fields are contained in an interval set.

The potential fields for the i^{th} quadrotor are

$$U_{i,x} = F_r e^{-min_{j\neq i}|x_i - x_j|} + F_a|x_i| \tag{1}$$
$$U_{i,y} = F_r e^{-min_{j\neq i}|y_i - y_j|} + F_a|y_i| \tag{2}$$
$$U_{i,z} = F_r e^{-min_{j\neq i}|z_i - z_j|} + F_a|z_i|. \tag{3}$$

The forces applied to the system in each direction are just the negative derivatives of the each direction's potential field. Therefore, we have

$$F_{i,x} = -\frac{\partial U_{i,x}}{\partial x_i} = F_r \mathrm{sgn}(x_i - x_{j^*}) e^{-|x_i - x_{j^*}|} - F_a \mathrm{sgn}(x_i) \tag{4}$$
$$F_{i,y} = -\frac{\partial U_{i,y}}{\partial y_i} = F_r \mathrm{sgn}(y_i - y_{j^*}) e^{-|y_i - y_{j^*}|} - F_a \mathrm{sgn}(y_i) \tag{5}$$
$$F_{i,z} = -\frac{\partial U_{i,z}}{\partial z_i} = F_r \mathrm{sgn}(z_i - z_{j^*}) e^{-|z_i - z_{j^*}|} - F_a \mathrm{sgn}(z_i) \tag{6}$$

where $j^* = \mathrm{argmin}_{j\neq i}|x_i - x_j|$ in the $F_{i,x}$ equation, and is defined analogously for the other two. Fach force is bounded by the interval $[-(F_r + F_a), F_r + F_a]$.

There isn't as nice of an expression for the second derivatives of the potential functions, but we can see that they will be bounded by the interval $[-F_r, F_r]$ almost everywhere. The second derivatives show up in the system Jacobian: as long as we only need Jacobian bounds and not the Jacobian itself, this information will suffice.