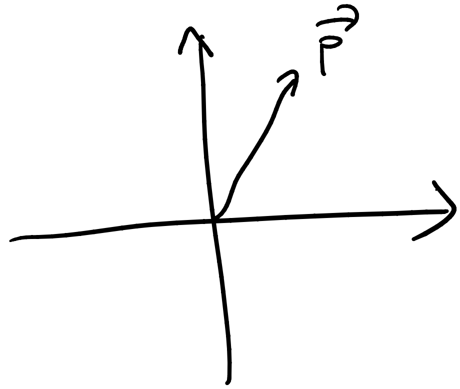
Notes

Monday, July 15, 2019 4:40 PM

$\vec{p} = (1, 2)$



This is a **2-dimensional** vector because it uniquely specifies a point in 2-dimensional space (x-y).
A 3-dimensional vector would have an x, y, and z component, for example.

NumPy and linear algebra have different definitions of dimensionality:

NumPy arrays represent spaces.
Vectors point to specific locations in spaces.

E.g.

$$\begin{bmatrix} 0, & 1, & 2 \\ 3, & 4, & 5 \\ 6, & 7, & 8 \end{bmatrix} \Rightarrow \text{2D space (requires 2 indices)}$$

$$\Downarrow$$

$$[i, j] \Rightarrow \text{2D vector}$$

$$\begin{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \\ \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \\ \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \end{bmatrix} \Rightarrow \text{3D space}$$

Index with $[i, j, k] \Rightarrow$ 3D vector

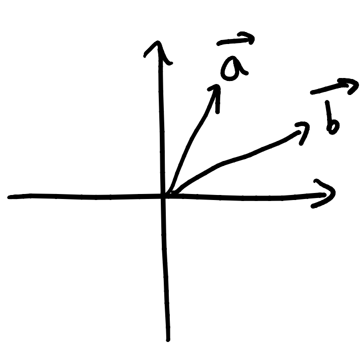
So, $(x, y, z) \Rightarrow$ vector in 3D space

vs. $([x, y, z]) \Rightarrow$ 1-D array of length 3

So, $(x, y, z) \Rightarrow$ vector in 3D space

np.array([x, y, z]) $\Rightarrow$ 1-D array of length 3

If we have two vectors, we might then want to know how similar these two vectors are.
We can determine this via their dot product.



$$\vec{a} = (a_x, a_y)$$
$$\vec{b} = (b_x, b_y)$$

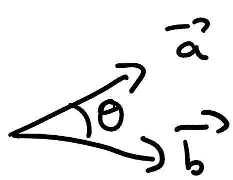
DOT PRODUCT:

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$$
$$= \|\vec{a}\| \|\vec{b}\| \cos\theta$$

↳ geometric interpretation

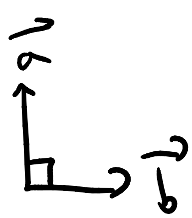
theta θ = smaller angle between vectors

If theta is small, then the vectors are closer together in angle, or overlap more in other words. We can compute the dot product of **a** and **b** using the first expression, and use the geometric expression to calculate theta (or perhaps more simply the cosine of theta). This then informs us how much these two vectors overlap.

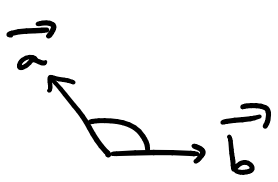


$\Rightarrow$ acute angle $\Rightarrow \hat{a} \cdot \hat{b} > 0$

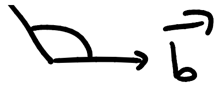
unit vector ↓ ($\hat{a}$)

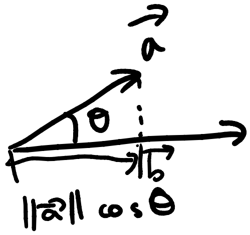


$\Rightarrow$ right angle $\Rightarrow \hat{a} \cdot \hat{b} = 0$



$\Rightarrow$ obtuse angle $\Rightarrow \hat{a} \cdot \hat{b} < 0$





$$b = \|\vec{a}\| \cos\theta$$

THESE CONCEPTS ALL SCALE TO N-DIMENSIONS

$$\vec{a} \cdot \vec{b} = \sum_{i=0}^{N-1} a_i b_i$$

$$\|\vec{a}\| = \sqrt{\sum_{i=0}^{N-1} a_i^2}$$

So if we had a database of n-dimensional vectors (e.g. faces in a database), we can compare two of them by taking their dot products (after normalizing each vector so that they are of unit length). If their values are close to 1, they are more similar; if they are close to 0, they are less similar. (Note that different vectors shouldn't be close to -1 as that is the antiparallel vector, it is very specific compared to the range of vectors allowed by 0.)