Homework 1

MATH 6204 (8204) - 001

Fall 2018

100 Points

Reading Assignment: Lectures 1 and 2

Due: Tuesday, September 11, 7:00 pm Student Name: ____________________________

Problem. The classical Black-Scholes equation,

$$\frac{\partial V(S_t,t)}{\partial t}+\frac{1}{2}\sigma^2 S_t^2\frac{\partial^2 V(S_t,t)}{\partial S_t^2}+rS_t\frac{\partial V(S_t,t)}{\partial S_t}-rV(S_t,t)=0,$$

has a closed-form solution for European-style options. For the European call, its time-t value is determined by

$$V_C^{eur}(S_t,t)=S_te^{-\delta(T-t)}F(d_1)-Ke^{-r(T-t)}F(d_2),$$

while for the European put, its time-t value is determined by

$$V_P^{eur}(S_t,t)=-S_te^{-\delta(T-t)}F(-d_1)+Ke^{-r(T-t)}F(-d_2),$$

where T is the maturity date, δ is the time-continuous dividend yield, r is the risk-free interest rate, K is the strike price, $F(.)$ is the standard Normal cumulative distribution,

$$F(x)=\int_{-\infty}^{x}f(x)dx,\quad \text{with}\ \ f(x)=\frac{1}{\sqrt{2\pi}}e^{-x^2/2},$$

and d_1, d_2 are given by

$$d_1=\frac{\ln(\frac{S}{K})+(r-\delta+\frac{\sigma^2}{2})\times(T-t)}{\sigma\sqrt{T-t}},$$

$$d_2=d_1-\sigma\sqrt{T-t}.$$

Based on the parameter values ($K=100$, $S_0=100$, $t=0$, $T=1$, $\delta=0.025$, $r=0.05$, $\sigma=0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, 1.00$), you are asked to write a Python program to compute:

1. $V_C^{eur}(S_0,0)$
2. $V_P^{eur}(S_0,0)$

Your Python program must include remarks on top that specify (1) its purpose, (2) the numerical methods it uses, (3) its author and date of completion, and (4) the files included. Note that you need to indicate what the driver file is if there are multiple program files. Also, your code must be made as reader-friendly as possible by adding remarks elsewhere when necessary, and when it is executed, it can generate console outputs that are well tabulated.

In your Python code, you must approximate the standard Normal cumulative distribution, respectively, using *scipy.stats.norm.cdf* and the following formula,

$$F(x) = 1 - f(x) * z * ((((a_5 z + a_4)z + a_3)z + a_2)z + a_1), \; 0 \leq x < \infty,$$
$$F(x) = 1 - F(-x), \quad x < 0,$$

where

$$z = \frac{1}{1 + 0.2316419x}$$
$$a_1 = 0.319381530,$$
$$a_2 = -0.356563782,$$
$$a_3 = 1.781477937,$$
$$a_4 = -1.821255978,$$
$$a_5 = 1.330274429.$$

The above information is available in the textbook; see Appendix A4 (page 270 - 273) and D2 (page 307 - 309). To be certain, using either the scipy function or the above formula to calculate the normal cumulative function, your results should be the same.

Note that **no duplicated homework from other students' is allowed**. You must turn in your completed homework by email (hwlin@uncc.edu) on the due date (no later than 6:30 pm). Your homework must include your console outputs (this must be a direct copy of from your computer screen) and Python code. Your Python code can be either a single source file (hw1_YourName.cpp) or multiple files. If your code includes multiple files, **you must email me with a single zipped folder (hw4_YourNalme).**