MATH 6205 - Numerical Methods for Financial Derivatives
Fall 2018

Purpose:

The objective of this Python program is to compute the values of American Call and Put options using Monte Carlo simulation. Geometric Brownian Motion as the underlying diffusion process for the Stock price. Since Weiner process is a continuous process, we used Euler discretization to discretize it and used it to simulate. In order to calculate the prices of American style of options, we need to calculate the discounted risk neutral expectations. Also, American options can be exercised at any point of time, so we need to calculate the optimal stopping time as well in order to compute the discounted expectation. The Monte Carlo simulation along with the regression technique can be used to compute the prices of American options. A linear regression is applied at each stopping time which in our case is the time step of the discretization. The independent variable (x) will be our stock values generated using the Euler discretization and the dependent variable (y) will be the discounted payoff values from the immediate next time step. This process is applied to all the time steps and then the average of initial time step (t0) gives us the price of the American option at time, t0. We have used a third order polynomial for the regression and the Least Squares method is used to find the optimal coefficients.

Output:

```
In [110]: runfile('/Users/khan/Desktop/Sem-III/MATH 6204/hw4_Mohammed_Ameer_Khan.py', wdir='/Users/khan/
Desktop/Sem-III/MATH 6204')
Reloaded modules: hw4_Mohammed_Ameer_Khan

In [111]: runfile('/Users/khan/Desktop/Sem-III/MATH 6204/main1.py', wdir='/Users/khan/Desktop/Sem-III/
MATH 6204')

Monte Carlo Regression method I values of American Call

Time step  MC_value
0.010      43.213215
0.001      43.405263

Monte Carlo Regression method I values of American Put

Time step  MC_value
0.010      35.053112
0.001      45.763221

In [112]:
```

Analysis:

The Monte Carlo simulated using regression values of American Call improves slightly when we increase the time steps from 100 to 1000. But in case of American put, the option price increases by almost a value of 10 when we increase the time steps from 100 to 1000. We have kept our number of simulations fixed at 1000 and used Euler discretization method to discretize the stock price SDE.

The accuracy of the option prices can be improved by increasing the degree of polynomial but again there will be a slight difference even with the increment of the degree of the polynomial used in regression. The regression method also introduces the additional bias apart from the discretization bias, this leads to drastic difference between in the American put option values when we increase the time steps from 100 to 1000. Nevertheless, this technique is a break-through in computing the American option prices.