Homework 9

MATH 6204 (8204) - 090

Fall 2018

100 Points

Reading Assignment: Lecture 11

Due: Tuesday, November 20, 11:59pm   Student Name: ___________________________

Instruction. On the due date, you should email your completed homework with a single zipped folder named “hw9_YourName. The folder should at least include a driver file named “main.py” and an implementation file named “thomas.py for your Python program.

Problem. The Thomas algorithm is a simplified form of Gaussian elimination that can be used to solve tridiagonal systems of equations:

$$\begin{bmatrix} \alpha_0 & \beta_0 & & & 0 \\ \gamma_1 & \alpha_1 & \beta_1 & & \\ & \ddots & \ddots & \ddots & \\ & & \gamma_{N-2} & \alpha_{N-2} & \beta_{N-2} \\ 0 & & & \gamma_{N-1} & \alpha_{N-1} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \\ x_N \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{N-1} \\ b_N \end{bmatrix}$$

Essentially, this algorithm involves two core substitutions based on a forward loop and a backward loop, as indicated below:

1. Step 1 (forward loop): change γ_i to zero.
   - Set $\widehat{\alpha}_0 = \alpha_0$, $\widehat{b}_0 = b_0$
   - For $i = 1, \ldots, N-1$:

     $$\widehat{\alpha}_i = \alpha_i - \beta_{i-1}\left(\frac{\gamma_i}{\widehat{\alpha}_{i-1}}\right),\ \widehat{b}_i = b_i - \widehat{b}_{i-1}\left(\frac{\gamma_i}{\widehat{\alpha}_{i-1}}\right)$$

2. Step 2: (backward loop):
   - Set $x_{N-1} = \widehat{b}_{N-1}/\widehat{\alpha}_{N-1}$
   - For $i = N-2, ..., 0$:

     $$x_i = (\widehat{b}_i - \beta_i x_{i+1})/\widehat{\alpha}_i$$

You are asked to solve the following 10×10 tridiagonal system with the Thomas algorithm.

$$\begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \end{bmatrix}$$

Your Python program must at least include these two files: main.py and thomas.py. The thomas.py function should serve as as a general tridiagonal matrix solver. That is, it can be used to solve any tridiagonal system, and should not be limited to the 10×10 system example.

To enhance the speed of the Thomas algorithm, you can apply just-in-time compiling from *numba*'s @jit.

Note that when your Python program is executed, it should be able to generate console outputs that are well tabulated. It is also required that your Python program should include remarks specifying the homework's purpose, algorithms, and author (i.e., your name) on top of your main.py function. To make your code reader-friendly, you should add remarks when necessary.