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0.1 Introduction

0.1.1 Historical retrospective

The reductionistic idea that all the countless variety of matter types that surrounds us could be in fact brought to a combination of much fewer substances has been around at least since the time of Ancient Greece. A thought that you can construct everything you see around out of one or few (e.g. fire, earth, water and air) indivisible elements ($\alpha\tau o\mu o\zeta$ in Greek) is simple, logical and therefore conceptually attractive. Knowing all about these elements could potentially grant us profound understanding of nature. But it wasn't before the XIX century when this idea has become something more than a philosophical concept and obtained solid scientific evidence.

The composition of the periodic table of elements in 1860s [1] was a tremendous step forward, reducing the number of elements to O(100). The elements of the periodic table resembled the ancient Greek concept so much, that they were christened atoms. But the periodic character of the table and strong correlation of atom position in the table with its chemical properties was insinuating on a certain inner structure of the atoms, a possibility for them to be composed out of even smaller objects. The discovery of isotopes in 1913 [2] left little room for other explanation.

Further evidences in favour of atomistic views kept coming in late XIX and early XX centuries from theoretical and experimental sides. The molecular kinetic theory has been heavily criticized throughout the XIX century, but the explanation of the Brownian motion [3] has secured its dominance from there on lying a foundation for what is to become the statistical physics. Of particular importance was the discovery of the first subatomic particle in 1897, which was called the electron [4].

Further studies of radioactive materials have allowed to compose a seemingly consistent understanding of what matter is composed of. By the time of neutron discovery in 1932 [5] the list of what was called elementary particles was reasonably short: an electron, a proton, and a neutron. It was still left to figure out how these elements interact forming the known atoms, molecules and all the matter around. That required additional efforts on the theoretical side, including resolving the inconsistencies between the two new branches of physics supposed to describe the microworld and the fields, namely the quantum theory and the field theory.

To move forward the physicist have made use of another source of elementary particles - the cosmic rays. Cosmic rays contained particles of much higher energies comparing to the radioactive materials. Cosmic ray experiments have led to the discovery of the first known antiparticle - the positron [6], confirming the theoretical predictions by Dirac [7]. Further discoveries of the muon [8], pion [9], kaon [10] and Λ_0 [11] have shown that the list of elementary particles was still far from being completed.

The second half of the XX century has pronounced a new era in particle physics with the extensive use of particle accelerators. Accelerators have become the main experimental tool in the discovery of new particles and investigation of their properties. Comparing to the cosmic rays, accelerators could offer higher energies and better control over the experimental conditions. Thanks to these new tools by the end of 1960s the number of newly discovered particles has exceeded one hundred and kept growing,

apparently taking away the reductionistic dream of having a reasonably small number of elementary particles.
On the other hand, the properties of the newly discovered particles (sometimes called "the particle zoo") had provided enough experimental data for theorists to make further assumptions. The particles, if grouped by their properties, have formed patterns - a situation resembling the old story with the atoms of the periodic table. This observation has allowed to assume the existence of even smaller fundamental particles with a fractional charge that would make up all the visible hadrons. These particles were eventually called quarks [12], [13]. By the late 1960s hypothesizing the existence of only three quarks was enough to explain all the visible particles and successfully predict new ones [14]. Since then three more quarks were discovered and as of now all the experimental evidence suggests that the quarks are truly fundamental particles being indivisible in the Ancient Greek sense.
At the same time serious theoretical efforts were taken in order to describe the interactions between fundamental particles, taking into account the known fundamental forces. In the mid-1970s a theory called The Standard Model was finalized. It included three out of four known fundamental forces (excluding the gravity) and predicted a number of particles which were not discovered by that time. All the key predictions of the theory were successfully confirmed by further experiments, making it a dominant theory in particle physics. The theory was able to describe all the surrounding matter with only 12 fundamental fermions (and their antiparticles) and 5 bosons. The SM is described in more detail in the Chapter 1.
Theoretical efforts aimed to further simplify the list of fundamental particles are ongoing, but up to the time of this thesis writing none of them were confirmed experimentally.

0.1.2 Actual challenges

The establishment of the Standard Model was a colossal step forward in understanding of the microworld physics. Nevertheless despite its great success and very good agreement with vast majority of the experimental data there is a number inconsistencies and lacunae in the theory, which do not allow to think of the SM as of the final theory. Here are most notable of these problematic questions:

1. A number of neutrino experiments have established that the neutrinos have a tiny though non-zero mass. The minimal Standard Model assumes neutrinos to be massless and does not allow to provide mass to the neutrinos.

2. Astrophysical and cosmological evidences confirm the existence of the dark matter which does not correspond to any of the SM particles.

3. Cosmological observations show a substantial disproportion between observed matter and anti-matter in favour of the former. The SM does not provide an explanation how such an imbalance could have been formed.

4. The discovery of the gravitational waves in 2016 had confirmed the existence of the graviton - the mediator of the gravitational force. The gravitational force is not represented in any way in the SM.

5. No explanation is provided to the vastly different magnitude of the fundamental forces, i.e. why the gravity is 10^{24} times weaker than the weak force.

In order to attack these and other problems numerous efforts have been taken to either modify the SM or to replace it with a more fundamental theory, but so far none of these Beyond Standard Model (BSM) theories were ever confirmed experimentally. The SM is still a source of most accurate predictions for any physical process that involves elementary particle interactions. Description of the BSM theories goes beyond the scope of current thesis.
The SM depends on the list of 18 free parameters (to be described in more detail in Chapter 1). These parameters can not be calculated intrinsically and must be measured experimentally. The more precisely we know the values of these parameters - the better is the accuracy of the SM prediction. Precise knowledge of the SM input parameters can also give hints on where to look for a more fundamental theory.
The LHC experiments have already contributed greatly by discovering the last missing piece of the SM, the Higgs boson. This has ended the era of SM particle discoveries but at the same time started the era of LHC precision measurements. The LHC experiments were capable to measure some parameters of the SM for the first time (like the mass of the Higgs boson), but also could improve the existing measurements, boosting the predictive power of the SM.
This thesis is a part of an ongoing effort at the ATLAS experiment to improve the precision of the W boson mass, which is also among the SM free parameters. The mass of the W boson was first measured at Large Electron-Positron (LEP) after its discovery in 1983. The precision of the measurement was further improved by the experiments at Tevatron collider. The only LHC result performed so far was published by ATLAS collaboration in 2018.
Hadron colliders are a challenging environment for the W boson-related measurements, the precision is highly impacted by a number of factors one of them being the pile-up. Current analysis is based on the data collected during two special LHC runs with low pile-up, taken in 2017 and 2018.

0.1.3 Thesis composition

The first chapter contains the description of the Standard Model, its constituents and input parameters. Chapter 2 is dedicated to W boson and its properties. Chapter 3 tells about the Large Hadron Collider (LHC) and its operations. ATLAS detector is described in Chapter 4. Chapter 5 is dedicated to the description of the shower shapes reweighting. And so on and so forth...

1 The Standard Model

"Potentielle citation sans aucun rapport avec le sujet"

— Personne inconnue, *contexte à déterminer*

The SM of particle physics is a quantum field theory that postulates the existence of three generations of quarks and leptons interacting through three fundamental forces: electromagnetic, weak and strong. From the mathematical point of view the SM is a gauge quantum field theory that has internal symmetries of the unitary product group $SU(3) \times SU(2)_L \times U(1)$. The fourth fundamental force, namely the gravity, is not included in the SM. Nevertheless, since the magnitude of the gravity interaction is negligible on the microscopic scale, it has little to no effect on the precision of the SM predictions. The model has 18[1] free input parameters - the physical constants that can not be predicted from within the theory and must be measured experimentally. Evidently, the SM predictions are based on these parameters, so the better we know them - the better we can predict how nature behaves on the micro level. The free parameters of the SM are briefly described in section 1.1

A comprehensive description of the quantum field theory formalism goes beyond the scope of current dissertation and can be found in the corresponding textbooks [1], [2], [3], [4], [5], [6]. In the following sections a brief overview of key SM features and constituent parts is provided.

1.1 General composition and key parameters

In this section I will describe the fields that enter the SM. Their existence and interactions result in the three fundamental forces that are taken into account by the theory. The quanta of these fields are also called fundamental particles and possess a number of properties like mass, charge (or charges), spin etc (see figure 11). The fundamental particles are divided into two groups based on their spin: particles with integer spin are called fermions and those with half-integer spin are bosons.
Let's start from the fermion sector. According to the Pauli exclusion principle[7] two fermions can not occupy the same quantum numbers. This in turn, has a consequence that the fermions must occupy a

[1] There are SM extensions that take into account the non-zero neutrino mass. Then the model gets 7 additional parameters, so their total number reaches 25. Although current thesis only considers the SM where neutrinos are massless.

finite volume in space-time and as a result make up matter. Half of the fundamental fermions have colour charge and therefore take part in strong interaction - they are called quarks. The other six fermions do not have colour charge and are called leptons (from Greek "$\lambda\epsilon\pi\tau o\sigma$" meaning "little", as they are lighter than the quarks of the same generation). Different types of quarks and leptons are also called flavours, so there are 6 flavours of quarks and 6 flavours of leptons.
For some reason which is yet unknown the twelve elementary fermions make three generations. Particles in the second and third generations have exactly the same charge and spin as the particles of the first generation, but are heavier and also unstable. Normally the particles of higher generations quickly decay down to their lighter kin of the first generation and can only be observed in cosmic rays and particle accelerators. That means all the matter that surrounds us consists of four fundamental fermions of the first generation[2](the first column in Fig. 11).
The two quarks of the first generation are called up-quark and down-quark (or u-quark and d-quark for short). All the nuclei of the ordinary matter we see around are built out of these two types of quarks. Quarks are capable of interacting through all three SM forces: electromagnetic, weak and strong. Electrons, muons and tau-leptons are sensitive to electromagnetic and weak interaction, while neutrinos can interact (and therefore be detected) only through weak force. For this reason in particle physics the term "leptons" is sometimes used in a narrow sense referring to electron-like particles only. For all quarks and electron-like particles the antiparticles were observed as well as the corresponding annihilation phenomena. It is still not clear if neutrinos annihilate
From our experience we know that matter interacts with matter. But within the SM fermions do not interact with each other immediately. The interaction is mediated by boson-type particles. The SM includes five types of bosons: four vector bosons serving as force carriers for electromagnetic, weak and strong interactions, and a spinless Higgs boson whose role would be described in more detail in the corresponding subsection 1.4.1. The Higgs boson along with W and Z bosons are massive, while photos and gluons are massless.
The masses of the fundamental particles make 12 out of 18 free parameters of the SM[3].
As it was mentioned, bosons interact with fermions through fundamental interactions. The interaction depends on the charge of the interacting particles and on the type of the interaction itself. Each type interaction has a coupling constant that defines the scale of the interaction. Hence two more parameters to the SM: the strong and electromagnetic coupling constants (the latter is also called the fine structure constant). Weak coupling constant is redundant since it can be obtained from other parameters.
And the remaining four parameters are coming from the CabibboKobayashiMaskawa matrix (CKM matrix), that contains information on the strength of the flavour-changing weak interaction. [8].

An important feature of the Quantum Field Theory (QFT) is that particles also interact with physical vacuum. For instance, a charged particle polarizes the physical vacuum, so the vacuum screens the

[2]Strictly speaking we already know that this is not completely true for the neutrinos, as they oscillate between the flavours due to their tiny mass. But in the SM neutrinos are assumed massless.

[3]The masses of W and Z bosons can be replaced by other parameters, e.g. weak mixing angle θ_W and Higgs potential vacuum expectation value (v. e. v.).

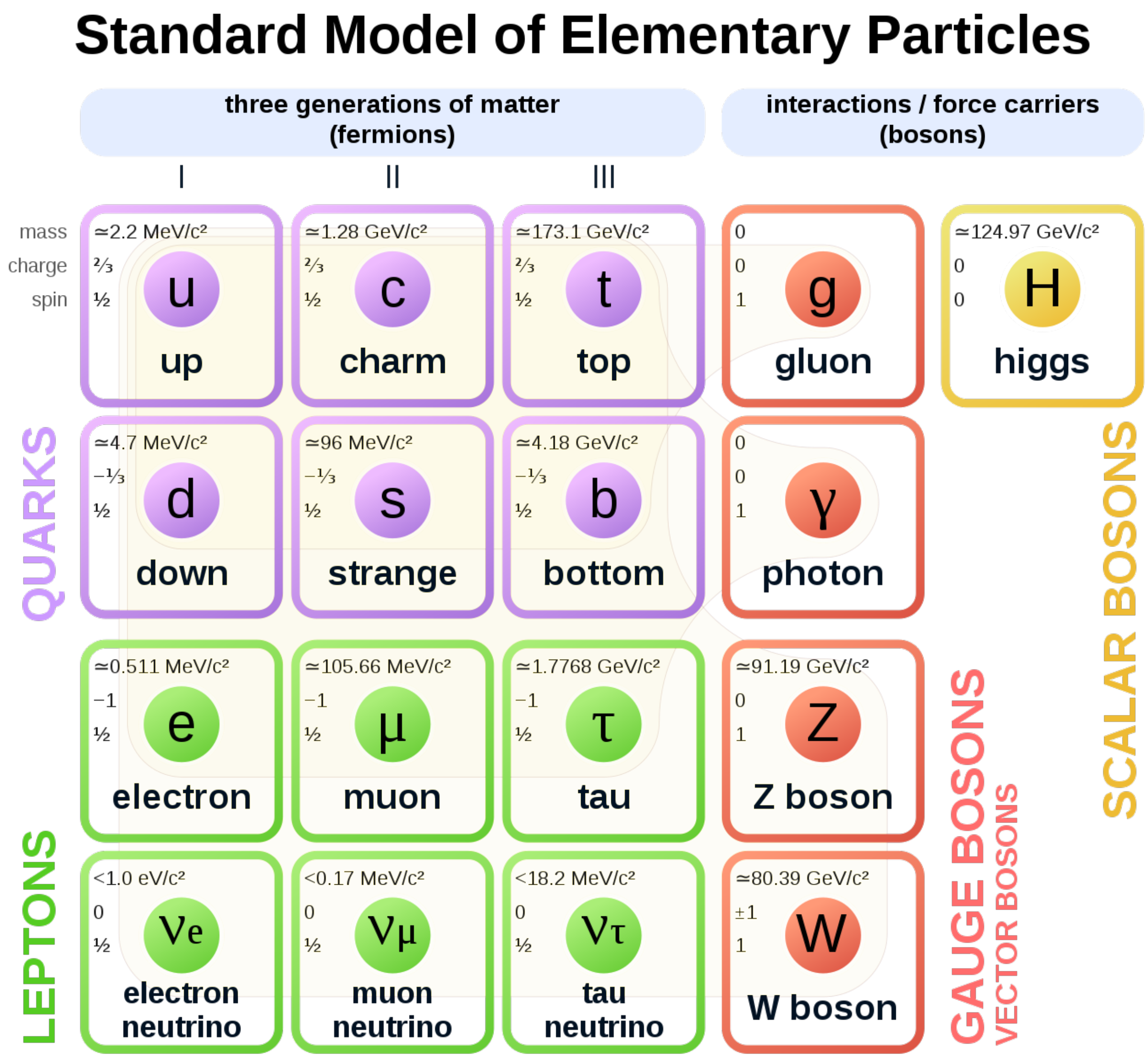


Figure 11: The list of particles that enters the SM[9].

charge of the particle[10].This interaction with virtual particles depends on the energy scale and so do the observed quantities like charge, mass etc. The SM is able to predict parameter evolution, so if the value of a certain input parameter q_0 is known at the energy Λ_0 then it is possible to predict its measurable value q at the energy Λ. This changing of physical parameters is an integral part of the QFT and is called *renormalisation* [2], [11]. In the picture 12 the dependence of the SM coupling constants on the energy is shown.

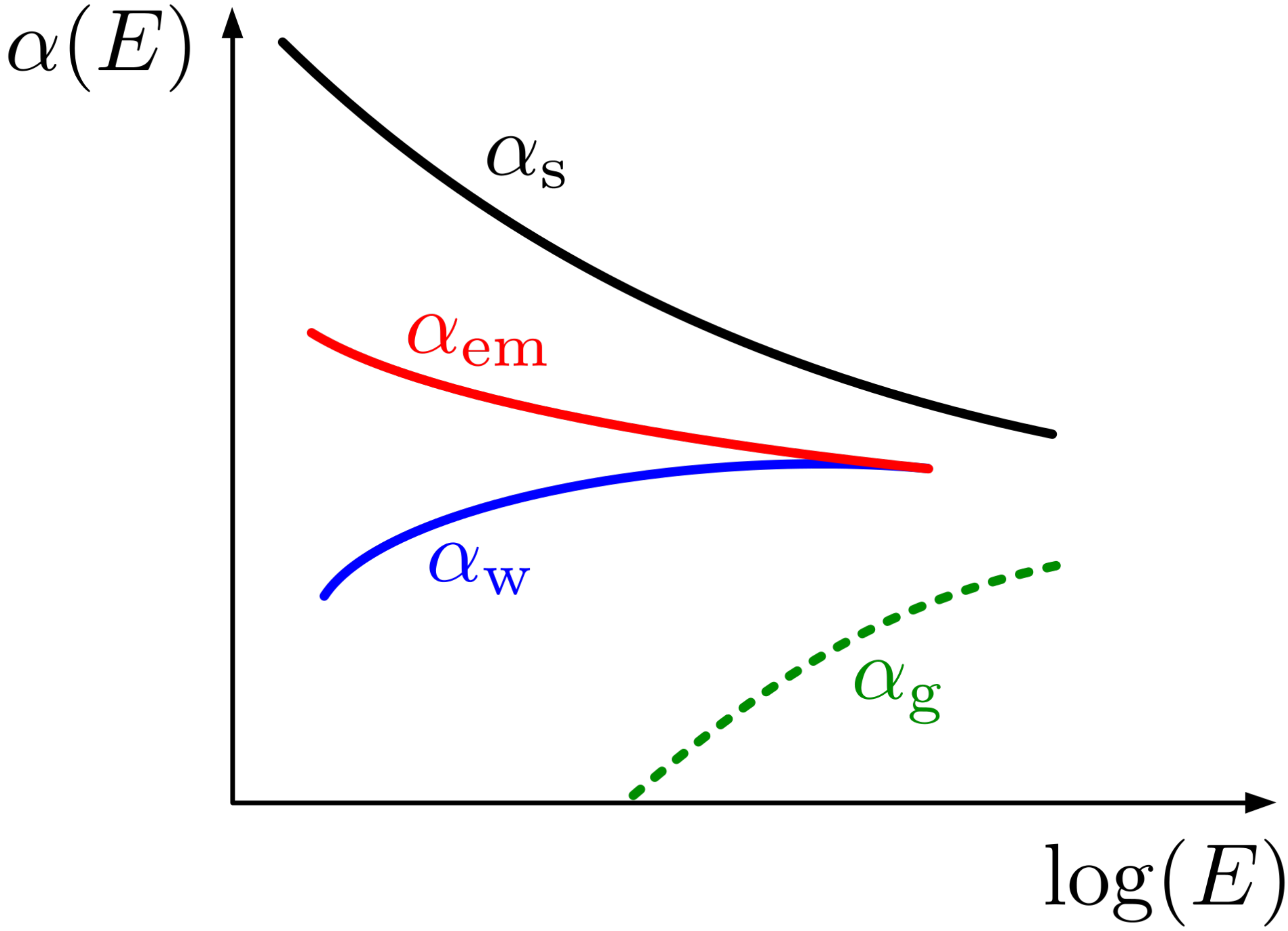


Figure 12: The evolution of the SM running coupling constants [12].

As we can see from picture 12 the strong coupling constant is getting smaller with the energy. This phenomena is called *the asymptotic freedom* [13], [14], [15].

1.2 Classical fields and gauge invariance principle

A consistent mathematical description of fields appears to be more challenging task compared to the description of physical objects that have definite size and shape. The derivation of Maxwell's equations has been a great success and allowed to obtain the first equations of motion of relativistic fields. It has also subseqently led to understanding of special relativity [16], [17], [18]. Although for a more general case of fields other than electromagnetic it would be very useful to adopt a more systematic approach like that of Lagrangian or Hamiltonian in classical mechanics.

It has turned out that for the relativistic case Hamiltonian approach was not quite convenient, as the dedicated role of time over other degrees of freedom was in discord with relativistic space-time unification. However it was found possible to describe the fields within the Lagrangian approach. In the classic mechanics the action of a mechanical system of i mechanical objects is defined as:

$$S = \int L dt = \int \left(\sum_i T_i - U_i \right) dt,$$

where T_i and U_i are the kinetic and potential energies of the i^{th} object. Considering that by definition a field exists in every point of space-time, we need to define the Lagrangian density such that $L = \int \mathcal{L}(\phi, \partial_k \phi, \dot{\phi}) d^3 x$, where ϕ is a field and $\partial_k \phi = \nabla \phi$ - the field gradient, $\partial_k = \frac{\partial}{\partial x^k}$, k = 1, 2, 3. Here and further Latin indices run through (1, 2, 3) and are used to denote spacial coordinates, while Greek indices denote space-time coordinates and run though (0, 1, 2, 3). So the action would look like:

$$S = \int L dt = \int \mathcal{L}(\phi, \partial_\mu \phi, \dot{\phi}) d^4 x, \tag{1.1}$$

Now we may use the principle of least action to obtain the equations of motion using the Euler-Lagrange formalism. Let's check it with the example of electromagnetic fields. The Lagrangian density of electromagnetic fields in a vacuum can be written like:

$$S = -\frac{1}{4} \int F^{\mu\nu} F_{\mu\nu} d^4 x. \tag{1.2}$$

Electromagnetic tensor can be defined in terms of electric and magnetic field intensities: $F_{i0} = -F_{0i} = E_i$, $F_{ij} = \epsilon_{ijk} H_k$, where ϵ_{ijk} - anti-symmetric Levi-Civita symbol. Alternatively $F_{\mu\nu}$ can be defined in terms of 4-potential A_μ:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{1.3}$$

Now we can safely apply the variational principle and putting $\delta S = 0$ obtain the Maxwell equations in a vacuum:

$$\partial_\mu F_{\mu\nu} = 0. \tag{1.4}$$

Noticing the symmetries of the system and using the Noether's theorem[19] we can find the invariants of electromagnetic field. For example, translational symmetry in time and space ensures conservation of energy and momentum. Let's now consider a symmetry of a different kind. The field potential can be shifted by a gradient of an arbitrary function $\alpha = \alpha(x^\mu)$:

$$\begin{aligned} A_\mu(x) &\to A'_\mu(x) = A_\mu(x) + \partial_\mu \alpha(x) \\ F_{\mu\nu} &\to F'_{\mu\nu} = \partial_\mu (A_\nu(x) + \partial_\nu \alpha(x)) = \partial_\mu A_\nu - \partial_\nu A_\mu = F_{\mu\nu}. \end{aligned} \tag{1.5}$$

Let's now consider the electromagnetic theory in the presence of charges and currents:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + j^{\mu}A_{\mu}. \tag{1.6}$$

Now we have an interaction of a field potential A_μ with 4-current $j^\mu = (-\rho, j^i)$. It turns out to be a general property of the field theories: the only form of interaction allowed is between a gauge field and a current. After applying the gradient field transformation and the least action principle we can obtain the corresponding conservation law:

$$\partial_\mu j^\mu = 0. \tag{1.7}$$

So this gradient symmetry[2] or as it is called more often gauge symmetry leads to the conservation of electric current. If a theory is invariant under gauge transformations then it is called a gauge invariant theory. As we have just seen electrodynamics is the simplest example of such a theory. Taking gauge symmetries into consideration [20] has played a huge role in the development of the SM.
Gauge degree of freedom can be constrained in arbitrary way by applying additional conditions on the gauge function. This is called fixing the gauge and becomes necessary after quantization. Any physical result must be gauge-invariant, i.e. must not depend on the gauge.

1.3 Quantum electrodynamics

Quantum Electrodynamics (QED) is a theory of interaction between light and electrically charged particles. Historically it was the first quantum field theory to reach good agreement between quantum mechanics and special relativity. QED vacuum has zero expectation value. Nowadays it is considered to be one of the most precise physical theories ever: theory predictions and experiment results agree up to $O(10^{-8})$. It has also served as a model for composition of the subsequent parts of the SM, describing other fundamental interactions.
Let's consider free Dirac field based Lagrangian:

$$\mathcal{L} = \bar{\psi}(x)(i\partial\!\!\!/ + m)\psi(x), \tag{1.8}$$

where ψ and $\bar{\psi}$ are Dirac wave function and its complex conjugate respectively, $\partial\!\!\!/ \equiv \gamma_\mu\partial^\mu$, γ_μ is one of the four gamma-matrices and m is the mass of the Dirac field. Such a theory, though, would not be physically consistent. This reflects the fact the quantum nature of spin and spinor fields have to be treated as quantum fields. For instance, an attempt to calculate the energy of a Dirac field would lead to a contradiction: the energy would not be positively defined, as some spinors would have negative energies.
This Lagrangian has an internal symmetry to the U(1) transformation: $\psi \to e^{-i\alpha(x)}\psi$, $\bar{\psi} \to e^{i\alpha(x)}\bar{\psi}$. According to Noether's theorem this symmetry implies current conservation: $j^\mu = \bar{\psi}\gamma^\mu\psi$. Now let's get the combined Lagrangian of electromagnetic and Dirac fields, adding the interaction term:

$$\mathcal{L} = \mathcal{L}_{Dirac^{free}} + \mathcal{L}_{EM^{free}} + \mathcal{L}_{Interaction} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(x)(i\partial\!\!\!/ + m)\psi(x) - q\bar{\psi}\gamma^\mu A_\mu\psi, \tag{1.9}$$

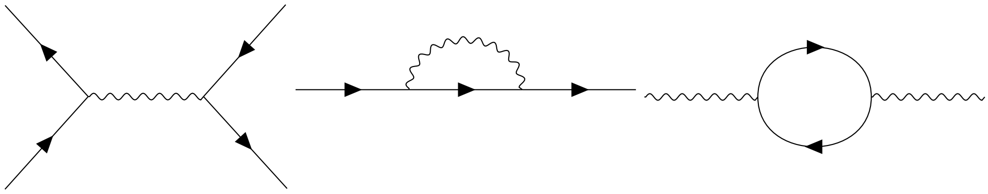


Figure 13: The QEQ diagrams: Compton scattering, electron self-energy, photon self-energy.

where q represents the elementary electric charge. This Lagrangian above is gauge invariant and can be rewritten in a more convenient form:

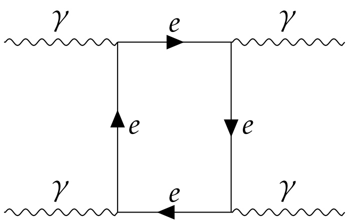
$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(x)(i\not{D} + m)\psi(x), \tag{1.10}$$

where $D_\mu = \partial_\mu + iqA_\mu$ is a covariant derivative. If one considers space-time in the presence of a field as curved, then A_μ would play a role of connectivity. It must be noted that values like m and q meaning electron mass and charge[4] are the SM input parameters mentioned in 1.1.

Further calculations are to be performed by the means of the quantum field theory formalism that treats interaction terms like a perturbation to the free fields, making power series expansion in the coupling constant. In the case of electrodynamics the coupling constant is quite small so good precision is reached soon. Since the photons do not directly interact with other photons, QED allows only one type of vertex - with two electron lines and one photon line.

Although the tree-level processes and diagrams were well understood by 1930th, the loop diagrams were properly explained only by the end of the 1940th making it possible obtain numerical results of the higher orders of power series expansion and achieve higher precision predictions for QED processes[21], [10], [22], [23], [24], [25], [26], [27].

It must be noted that although direct photon-photon interaction is impossible, light-by-light scattering is still possible through loops:



This process was theoretically described in 1936 [28] and experimentally observed 83 years after in heavy ion collisions at the LHC [29].

[4] Charge of the electron is related to the electromagnetic coupling constant.

1.4 Electroweak theory and the Higgs mechanism

All the fermions of the standard model are subject to weak interaction, so its importance for physical processes can not be underestimated. At low energy weak interaction manifests itself mainly through flavour-changing decays like beta-decay and muon decay. The electroweak theory was created in the end of 1950s[11] [5] [30] thanks to numerous experimental results that allowed to shape its properties. The theory assumed that the electromagnetic and weak fundamental forces are actually manifestation of the same field that has a gauge symmetry $SU(2)_L \quad U(1)$ with massive charged and neutral bosons. A few years later the structure of electroweak vacuum was explained along with the mechanism that has allowed the bosons to gain mass [31], [32]. Assuming this the Lagrangian of the electroweak theory must consist of three parts[33]:

- Gauge fields that would mediate the interaction.
- Fermions that interact with gauge fields
- A scalar Higgs field with non-zero vacuum energy that breaks the SU(2) symmetry and couples to the fermions.

$$\mathcal{L}_{EW} = \mathcal{L}_{Gauge} + \mathcal{L}_{Higgs} + \mathcal{L}_{Fermions} \tag{1.11}$$

1.4.1 Electroweak gauge fields

As it was already pointed our before, knowing the symmetries of a physical system allows one to compose the gauge fields Lagrangian. The part with U(1) symmetry would look like the electromagnetic field from 1.2 having the hypercharge Y, a vector potential B_μ and a gauge coupling g_1. The SU(2) field would have 3 vector components $W_\mu^{1,2,3}$, three isospin operators I_1,I_2,I_3 and a gauge coupling g_2. We can pick the Pauli matrices σ^i as the representation of generators of the SU(2) group, then the structure constants are ϵ_{abc} - Levi-Civita symbol.

$$\begin{aligned} &\mathcal{L}_{\mathcal{G}} = -\tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} - \tfrac{1}{4}W^a_{\mu\nu}W^{\mu\nu,a}B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \\ &W^a_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + g_2\epsilon_{abc}W^b_\mu W^c_\nu, \end{aligned} \tag{1.12}$$

where the term $g_2\epsilon_{abc}W^b_\mu W^c_\nu$ appears due to the non-Abelian nature of the SU(2) group (the generators don't commute).

1.4.2 Fermion sector

Each fundamental fermion generation expressed as left-handed doublets and right-handed singlets is a fundamental representation of the group $SU(2) \times U(1)$:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \begin{pmatrix} e_R \end{pmatrix}, \begin{pmatrix} \mu_R \end{pmatrix}, \begin{pmatrix} \tau_R \end{pmatrix}, \tag{1.13}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} b \\ t \end{pmatrix}_L, \begin{pmatrix} u_R \end{pmatrix}, \begin{pmatrix} d_R \end{pmatrix}, \begin{pmatrix} c_R \end{pmatrix}, \begin{pmatrix} s_R \end{pmatrix}, \begin{pmatrix} t_R \end{pmatrix}, \begin{pmatrix} b_R \end{pmatrix}. \tag{1.14}$$

Their quantum states are classified using the following quantum numbers: weak isospin I_3, I, weak hypercharge Y. Their electric charge can be obtained using the Gell-Mann-Nishijima relation:

$$Q = I_3 + \frac{Y}{2}. \tag{1.15}$$

The fermions are divided by their chirality: only the left-handed particles take part in weak interaction. The left-handed fermion fields of each lepton and quark generation j

$$\psi_j^L = \begin{pmatrix} \psi_{j+}^L \\ \psi_{j-}^L \end{pmatrix} \tag{1.16}$$

make SU(2) doublets, with indices $\sigma = \pm$, while the right-handed fermions can be written as singlets:

$$\psi_j^R = \psi_{j\sigma}^L. \tag{1.17}$$

Like in the the electromagnetic case we can define the covariant derivative that would ensure the gauge invariance of the Lagrangian:

$$D_\mu = \partial_\mu - i g_2 I_a W_\mu^a + i g_1 \frac{Y}{2} B_\mu, \tag{1.18}$$

with $I_a \equiv \frac{\sigma_a}{2}$, then fermion Lagrangian takes the following form:

$$\mathcal{L}_{Fermions} = \sum_f \bar{\psi}_j^L i \gamma^\mu D_\mu \psi_j^L + \sum_{f,\sigma} \bar{\psi}_{f,\sigma}^R i \gamma^\mu D_\mu \psi_{f,\sigma}^R. \tag{1.19}$$

1.4.3 Higgs fields breaking the symmetry

The Higgs field is represented by single complex scalar doublet field $\Phi(x)$, that has 4 independent components. It spontaneously breaks the $SU(2) \times U(1)$ gauge symmetry, leaving the $U(1)_{EM}$ symmetry intact. The Higgs field doublet has the hypercharge $Y = 1$:

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}. \tag{1.20}$$

The Higgs field Lagrangian with non-zero vacuum expectation value:

$$\mathcal{L}_{Higgs} = (D_\mu \Phi)^+ (D_\mu \Phi) - V(\Phi) + \mathcal{L}_{Yukawa}. \tag{1.21}$$

The gauge invariance of the Higgs Lagrangian is ensured in the traditional way by using the covariant derivative:

$$D_\mu = \partial_\mu - i g_2 I_a W_\mu^a + i\frac{g_1}{2} B_\mu. \tag{1.22}$$

Higgs potential contains the mass term and quartic self-interaction:

$$V(\Phi) = -\mu^2 \Phi^+ \Phi + \frac{\lambda}{4}\partial_\mu (\Phi^+\Phi)^2. \tag{1.23}$$

Valuum expectation value $< \Phi >$ does not vanish:

$$< \Phi(x) >= \frac{1}{\sqrt(2)}\begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = \frac{2\mu}{\sqrt(\lambda)}. \tag{1.24}$$

Applying the unitarity gauge [34] we can constraint three out of four degrees of freedom of the Higgs field and rewrite the Higgs doublet in the following way:

$$\Phi(x) = \frac{1}{2}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \tag{1.25}$$

which leaves us with a physical real neutral scalar field $H(x)$ with

$$M_H = \sqrt(2)\mu. \tag{1.26}$$

This real field would couple to itself forming triple and quartic self-coupling vertices, to the gauge fields through the covariant derivatives and to the charged fermions, giving them mass. Yukawa term in Lagrangian the unitary gauge:

$$\mathcal{L}_{Yukawa} = -\sum_f m_f \bar{\psi}_f \psi_f - \sum_f \frac{m_f}{v} \bar{\psi}_f \psi_f H, \tag{1.27}$$

where

$$m_f = g_f \frac{v}{\sqrt(2)} = \sqrt(2)\frac{g_f}{g_2} M_W. \tag{1.28}$$

Higgs coupling constants to the corresponding fermion flavour are denoted as g_f. This relation between the Higgs coupling and the mass of the W boson illustrates how much the SM parameters are intertwined and particularly underlines the importance of the M_W measurement.

1.4.4 Physical interpretation of gauge fields and parameters

Higgs coupling to the gauge fields results in the following terms in the Lagrangian:

$$\frac{1}{2}\frac{g_2}{2} v (W_1^2 + W_2^2) + \frac{v^2}{4}(W_\mu^3, B_\mu)\begin{pmatrix} g_2^2 & g_1 g_2 \\ g_1 g_2 & g_1^2 \end{pmatrix}\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}. \tag{1.29}$$

In order to get the physical meaning of this expression let us make a transition to the basis of physical fields:

$$\begin{array}{rcl} W^{\pm}_{\mu} & = & \frac{1}{\sqrt{2}}(W^{\mp}_{\mu} \mp i W^{\mp}_{\mu}) \\ \begin{pmatrix} Z_{\mu} \\ A_{\mu} \end{pmatrix} & = & \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & cos\theta_W \end{pmatrix} \begin{pmatrix} W^3_{\mu} \\ B_{\mu} \end{pmatrix}, \end{array} \tag{1.30}$$

where θ_W is called the weak mixing angle or the Weinberg angle. In the new basis expression 1.29 has transparent physical sense:

$$M^2_W W^+_{\mu} W^{-\mu} + \frac{1}{2}(A_{\mu}, Z_{\mu}) \begin{pmatrix} 0 & 0 \\ 0 & M^2_Z \end{pmatrix} \begin{pmatrix} A_{\mu} \\ Z_{\mu} \end{pmatrix}, \tag{1.31}$$

with

$$\begin{array}{rcl} M_W & = & \frac{1}{2} g_2 v \\ M_Z & = & \frac{1}{2}\sqrt{g^2_1 + g^2_2}\, v. \end{array} \tag{1.32}$$

The mixing angle θ_W also has a very clear physical meaning:

$$\cos\theta_W = \frac{g_2}{g^2_1 + g^2_2} = \frac{M_W}{M_Z}. \tag{1.33}$$

With A_{μ} having a sense of electromagnetic potential its coupling to the electron must have a physical meaning of the electric charge $e = \sqrt{4\pi\alpha}$ we can express e in terms of gauge couplings:

$$e = \frac{g_1 g_2}{g^2_1 + g^2_2}, \quad g_2 = \frac{e}{\sin\theta_W}, \; g_1 = \frac{e}{\cos\theta_W}. \tag{1.34}$$

Thus the demonstrated Weinberg rotation fully replaces the original parameters g_1, g_2, λ, μ^2, g_f by another set of measurable values e, M_W, M_Z, M_H, m_f which are the input parameters of the SM.

1.5 Chromodynamics

The Quantum Chromodynamics (QCD) is a non-Abelian gauge theory that describes strong interaction. The QCD is symmetric under unbroken SU(3) colour symmetry, so the interaction scheme is built in the same way as electromagnetic and electroweak theories. To preserve the gauge invariance the gauge field of gluons is introduced with 8 components, since SU(N) group has $\frac{N^2-1}{2}$ independent elements. The gluons are massless vector bosons like the photons, although because of the non-Abelian nature of the gauge group they couple not only to the fermions but also to the other gluons. The gauge invariant QCD Lagrangian with kinetic term containing covariant derivative would look like:

$$\begin{array}{rcl} \mathcal{L}_{QCD} & = & -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \bar{\psi}_a (i(\gamma^{\mu} D_{\mu})^{ab} - m\delta^{ab})\psi_b, \\ F^a_{\mu\nu} & = & \partial_{\mu} A^a_{\nu} - \partial_{\nu} A^a_{\mu} + g_s f^{abc} A^b_{\mu} A^c_{\nu}, \\ D_{\mu} & = & \partial_{\mu} + i g_s A^a_{\mu} t_a. \end{array} \tag{1.35}$$

with ψ being the quark field, m is the mass of the quark, a,b = 1, 2, ..., 8 are the colour indices, g_s is the strong coupling constant, f^{abc} are the structure constants of the SU(3) group and t_a are the generators

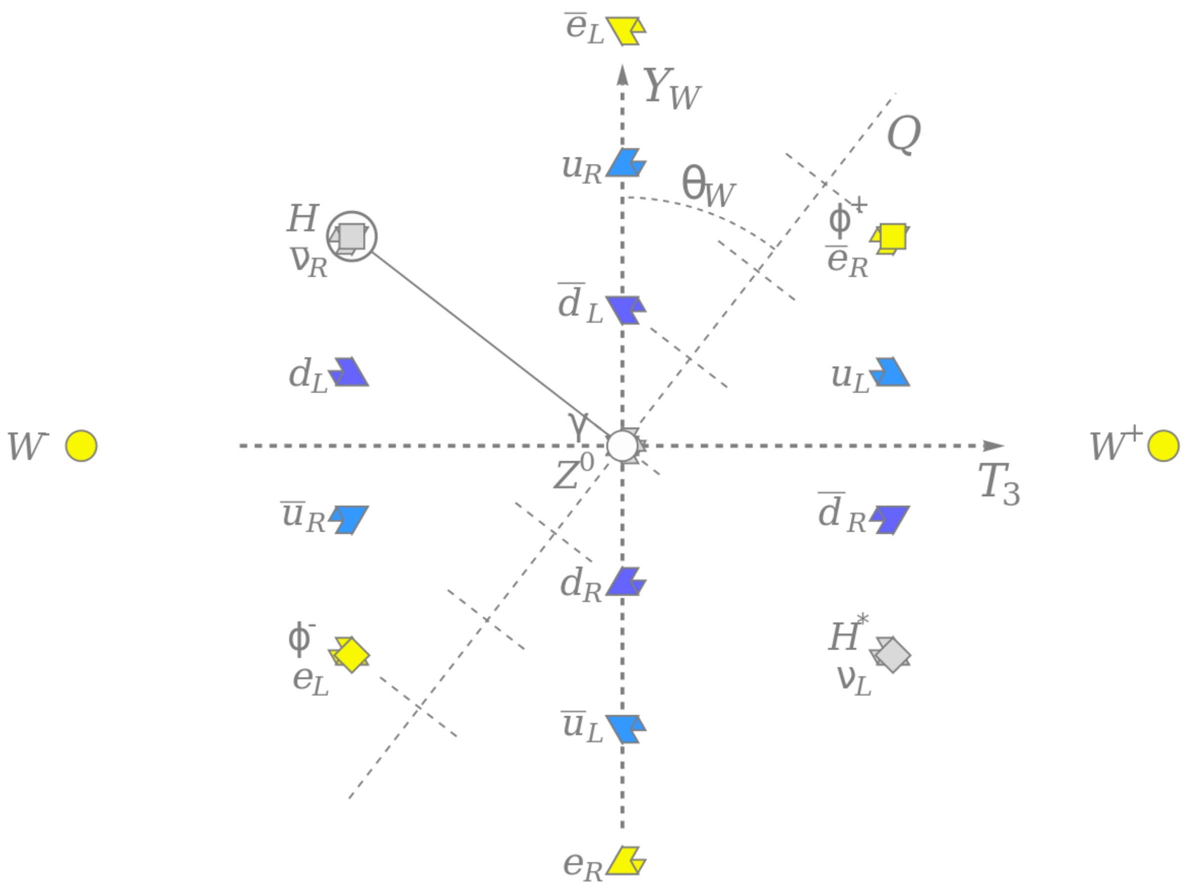


Figure 14: Electroweak sector and the Weinberg rotation [12].

of the SU(3) group.

As it was already mentioned in 1.3 quantitative calculations in QFT treat particle interaction as a perturbation to the free field theory. Coupling constant is considered to be a small parameter so every next power of the coupling constant is much smaller than the previous. Thanks to the asymptotic freedom α_s becomes small at higher energies and allows perturbative calculations. But at certain energy scale called $\Lambda_{QCD} \approx 200$ MeV, QCD becomes non-perturbative. It means we may no longer assume that interaction is a small perturbation of the free fields. This phenomena is known as the *colour confinement*.

Because of the colour confinement we can only observe colourless objects like baryons and mesons, but not quarks and gluons. If a high-energetic parton gets torn out of a hadron then it creates an avalanche-like process creating quark-antiquark pairs until fully hadronizes (see pic. 15) confining its colour. Such an avalanche is called a hadronic jet.

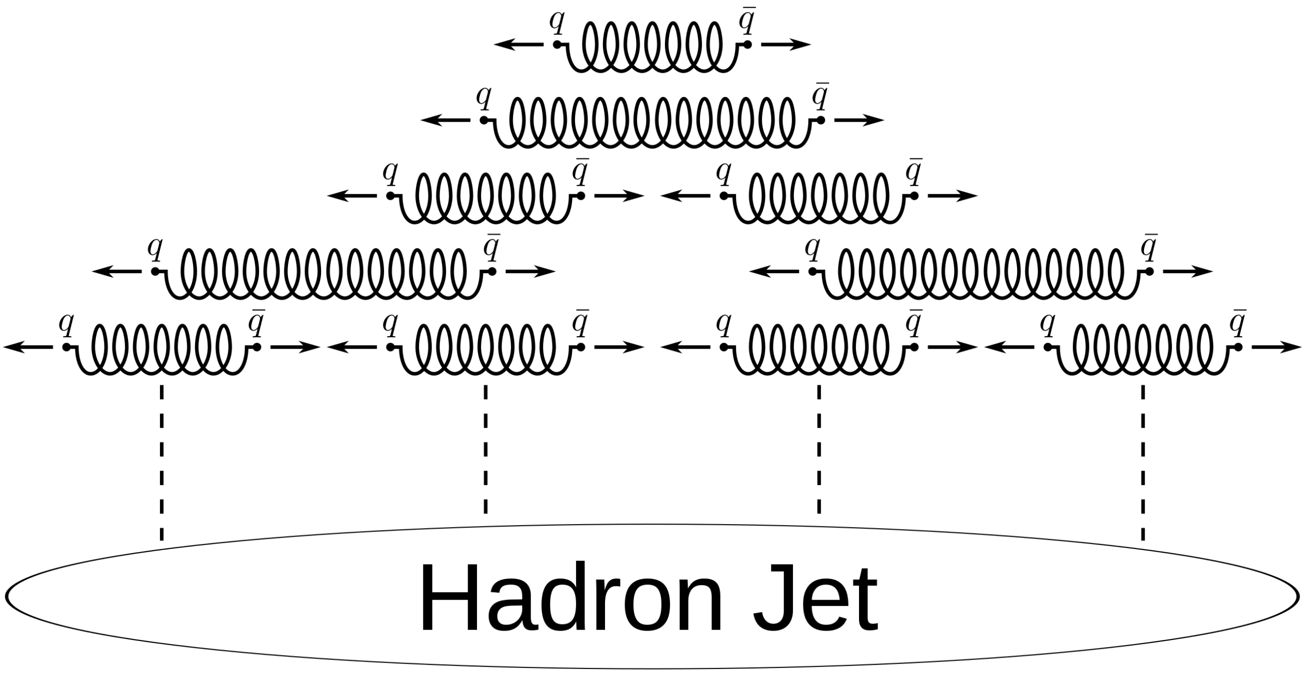


Figure 15: The formation of a jet [35].

Currently there is no viable physical theory that would describe QCD vacuum and low-energy behaviour of quarks and gluons. This also means that although nuclear forces are evidently residuals of the QCD interaction of partons within the baryons, there is no continuity between the QCD and nuclear physics. Confinement and low-energy QCD remain to be an unsolved problem of modern physics.

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