

# G1

- IMF  $\xi(M) = \frac{dN}{dM}$ , # stars per mass bin. Can inform you about PDMF and depends on formation mechanism of low/high M stars.
- Peak  $\sim 0.2 - 0.3 M_{\odot}$ , low M stars form via fragmentation of cloud, high M by accretion.
- Salpeter =  $M^{-2.35}$ , Chabrier = lognormal  $\xi(M) \propto M^{-2.3}$

## Stellar Initial Mass Function

# stars per mass bin.

- The stellar IMF describes the fractional distribution in mass of a newly formed star system. That is, if a net mass  $M_{\text{net}}$  is converted to new stars (from a molecular cloud), the # of stars in a mass interval  $[M, M+dM]$  is:

$$dN = N_0 \xi(M) dM$$

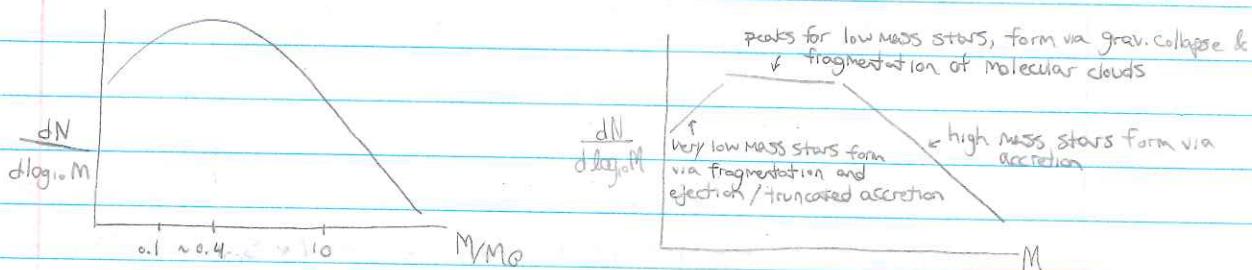
where  $\xi(M)$  is the IMF and is normalized such that  $\int_{M_{\min}}^{M_{\max}} \xi(M) \cdot M dM = 1$ .

- The IMF is usually described by a power law, or piece-wise power laws for different mass intervals.

• Salpeter:  $\xi(M) \sim M^{-2.35}$

• Kroupa:  $\xi(M) \sim \begin{cases} M^{-0.3}, & M < 0.08 M_{\odot} \\ M^{-1.3}, & 0.08 M_{\odot} < M < 0.5 M_{\odot} \\ M^{-2.3}, & M > 0.5 M_{\odot} \end{cases}$

• Chabrier:  $\xi(M) \sim \begin{cases} \frac{1}{M} \exp(\log_{10} M), & M < 1 M_{\odot} \text{ (log-normal distribution)} \\ M^{-2.3}, & M > 1 M_{\odot} \end{cases}$



- Salpeter determined the IMF for the solar neighborhood by first measuring the present-day luminosity function (# stars per L bin)  $\rightarrow$  get M-L relation  $\rightarrow$  get present-day mass function (PDMF)  $\rightarrow$  take into account stellar evolution models, metallicity, stellar winds, etc etc etc  $\rightarrow$  determine IMF.

Hilary

## G2

- Galactic disk = orderly, in plane
- spheroid = NOT.
- ↑ v in both indicate lots of DM past ~15 kpc

### • Stellar orbits in the Galactic Disk and Spheroid

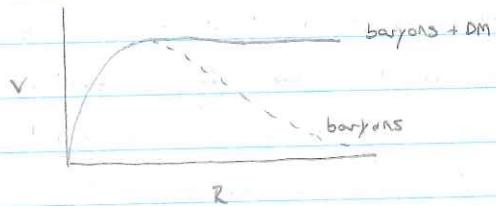
#### • Galactic Disk

- stars' orbits are confined (mostly) to a plane; low inclination
- nearly circular orbits
- orbit in same direction

#### • Galactic Spheroid

- orbits are randomly oriented, not confined to plane; can have high inclination
- orbits can be highly elliptical
- orbits are not in any one direction

• The high orbital speeds of stars in both the disk and spheroid/halo indicate the presence of a large amount of dark matter. Roughly,  $M \sim a^3/P^2$ . For the Sun,  $a \sim 8000 \text{ pc}$ ,  $V \sim 220 \text{ km/s} \rightarrow P \sim 220 \text{ Myr}$ , so  $M \sim 9 \times 10^{10} M_\odot$  interior to Sun.



• but most of the DM lies past our Sun's orbit, and past  $\sim 15000 \text{ pc}$  we'd expect the velocity of stars to decrease b/c there's very little luminous matter. But we don't see this, which implies the presence of a lot of non-luminous DM.

$$M_{\text{tot}} \sim 10^{12} M_\odot$$

## G3

- $\Delta t = \frac{1}{N_* \cdot SFR_{SNH}}$ ,  $N_* = \frac{\int_{M=IMF}^{M_{\text{SN}}} \xi(M) dM}{M_0}$  for Type II SNs,  $SFR_{SNH} = SFR_{MW} \cdot \frac{V_{SNH}}{V_{MW}}$
- $N_* \approx 1/150 M_\odot$ ,  $SFR_{SNH} \approx 10^{-9} M_\odot/\text{yr}$ ,  $\Delta t \approx 150 \text{ Gyr} > t_{\text{universe}}$
- Newborn stars form in active SF regions

### Supernova Frequency

- To determine how long we have to wait to see a SN w/in 3pc of the Sun, we'll need to consider the IMF, i.e. the # of stars formed (per  $M_\odot$ ) capable of going SN, and the SFR of the MW, which we'll assume is a constant  $1 M_\odot/\text{yr}$ .
- let's just consider core collapse SNe, so  $M > 8 M_\odot$ . We could also include close-in binaries capable of WD accretion into Type Ia, but Meh.

- To determine the # of stars formed per solar mass that could become SNe, we can use the Salpeter IMF:  $\xi(M) \sim M^{-2.35}$ :

$$N_* = \frac{\int_{M_{\text{min,SN}}}^{\infty} \xi(M) dM}{\int_{M_{\text{min}}}^{\infty} M \xi(M) dM} = \frac{\int_{8 M_\odot}^{\infty} M^{-2.35} dM}{\int_{1 M_\odot}^{\infty} M \cdot M^{-2.35} dM} = \dots \approx \frac{1}{150} M_\odot^{-1}$$

- The time we'll need to wait b/w SNe is  $\Delta t = 1/f$ , where  $f = N_* \cdot SFR_{\text{solar neighborhood}}$ .

- the star formation rate in the solar neighborhood needs to be scaled down to the volume of the MW we're interested in, i.e. 3pc from Earth:

$$SFR_{SNH} = SFR_{MW} \cdot \frac{V_{SNH}}{V_{MW}} = SFR_{MW} \cdot \frac{\frac{4}{3} \pi R_{3pc}^3}{\pi R_{MW}^2 h_{MW}} = (1 M_\odot/\text{yr}) \cdot \frac{\frac{4}{3} \pi (3 \text{pc})^3}{\pi (10^4 \text{pc})^2 (10^3 \text{pc})}$$

$$\therefore SFR_{SNH} \approx 10^{-9} M_\odot/\text{yr}$$

$$\therefore \Delta t = \frac{1}{(1/150 M_\odot^{-1})(10^{-9} M_\odot/\text{yr})} = 150 \text{ Gyr} \gg t_{\text{universe}}$$

- Newborn stars are more likely to experience nearby SN explosions b/c they typically form in active star-forming regions, where there will be many high-mass stars forming around the same time. These stars have shorter lifetimes and thus will explode w/in the lifetimes of lower mass stars.

## G4

- Collisionless system = individual grav. interactions are unimportant in galaxy evolution, and grav. force can be described as arising from smooth  $\rho$  distribution, not pt sources.
- $t_{\text{relax}} = n_{\text{relax}} \cdot t_{\text{cross}}$ , time for individual encounters to change  $v$  by 100%.
- $n_{\text{relax}} = N/8\pi N$ ,  $t_{\text{cross}} = R/V$ ,  $t_{\text{relax}} \approx 10^{10} \text{ Myr}$

### • Galactic Stars as Collisionless Systems

• Collisionless systems are those in which direct collisions and near misses, ie grav. interactions b/w individual particles (aka stars), are unimportant for the evolution of the system (galaxy). This means the gravitational force can be described as arising from a smooth density distribution rather than a collection of individual stars.

• To see this, we consider the relaxation time of the galaxy, which is the time it takes for individual encounters to change a star's velocity by  $\sim 100\%$ :

$$t_{\text{relax}} = n_{\text{relax}} \cdot t_{\text{cross}}$$

-  $n_{\text{relax}}$  is the # of crossings necessary for the velocity to change by 100%, and is given by

$$n_{\text{relax}} \approx \frac{N}{8\pi N}$$

where  $N = \# \text{ of stars}$  (this equation takes a bit of work to derive so f\*\*\* it). For the MW,  $N \sim 10^{10}$  stars.

-  $t_{\text{cross}}$  is the time required for a typical star to cross the galaxy,

$$t_{\text{cross}} = R/V \approx 100 \text{ Myr}$$

$$\therefore t_{\text{relax}} = \frac{N t_{\text{cross}}}{8\pi N} \approx 10^{10} \text{ Myr} \gg t_{\text{universe}}$$

- When  $t_{\text{relax}} > t_{\text{universe}}$ , it means that it takes a very long time for individual gravitational interactions b/w stars to change a star's velocity by  $\sim 100\%$ . Individual stars are not important, and so the galaxy is a collisionless system where the gravitational force comes from a smooth density distribution rather than point sources.

• This argument breaks down for high stellar density regions, like globular clusters. In this case two-body relaxation (individual collisions) play an important role in globular cluster evolution.

## G5

- Dust is dominant cooling mechanism in later stages of MC collapse, after cloud becomes opaque to own radiation and must be re-radiated in IR
- Dust acts as catalyst for H<sub>2</sub> formation as H atoms get stuck on surface
- This H<sub>2</sub> can also act as cooling mechanism and lead to formation of more complex molecules

### • Importance of Dust in ISM and Star Formation

Dust is important for cooling in collapsing protostellar clouds, and as a catalyst for the formation of molecules in the ISM.

A star can only form if the collapsing cloud can get rid of its gravitational binding energy, which is usually done through radiation. Rotational transitions generally provide a cooling mechanism initially, but eventually the cloud becomes optically thick/opaque to its own radiation (probably b/c density is increasing, and  $T \sim K_p \sim n_0$ ). Dust then becomes the dominant cooling mechanism. Dust grains are heated to 60-100 K, at which point they begin re-radiating in the FIR. The collapsing cloud is transparent to radiation at these wavelengths, so it can continue collapsing and contracting as its gravitational energy is converted to radiation.

Dust grains also act as catalysts for the formation of H<sub>2</sub>. Neutral H atoms can become 'stuck' on the surface of a grain (trapped in the grain's potential well(?)), encounter other H atoms, and combine to form H<sub>2</sub>. This combination releases energy, which is enough to dislodge the H<sub>2</sub> from the grain surface, introducing new H<sub>2</sub> to the ISM. H<sub>2</sub> is another important cooling mechanism for star formation, and it can also lead to the formation of more complex molecules in the ISM.

- Examples of dust grain materials include PAHs, silicates, graphite, etc.
- Dust contributes significantly to extinction and reddening in our galaxy; see G9.

## G6

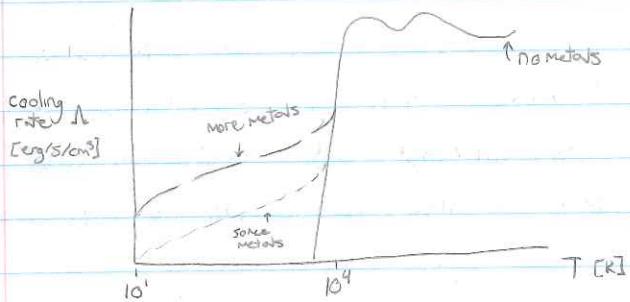
- H & He can't be collisionally excited to induce emission at low T of MCs
- Molecules dominate cooling at low densities through rotational transitions
- Dust dominates at high densities once cloud is too opaque, radiation must be absorbed and reemitted

### • H and He as Coolants

- H and He are poor coolants (in atomic form) because they can't be collisionally induced to emit photons at the low gas temperatures characteristic of molecular clouds ( $\sim 10^5$  K). The excitation temperature of H is  $\sim 10^5$  K ( $E_1 - E_2 = 13.6 - 3.4 = 10.2$  eV), so collisions will not be energetic enough to excite electrons and induce radiation by spontaneous emission.

- For this reason, other molecules dominate cooling in molecular cloud cores. At low densities (and temperatures) CO and O<sub>2</sub> are the dominant cooling mechanisms, while H<sub>2</sub>O can dominate at higher densities. These molecules are capable of collisional cooling which converts the colliding electron's KE to rotational energy, since the diatomic molecules have an additional degree of freedom compared to atomic H and He. This produces molecular lines in the microwave and mm wavelength range.

- As density increases and the optical depth gets worse (more opaque), dust grains dominate cooling (see G5). Thus molecular cloud cores can lose heat and collapse to form stars thanks to the presence of other compounds in the ISM, even if they only exist in small amounts.



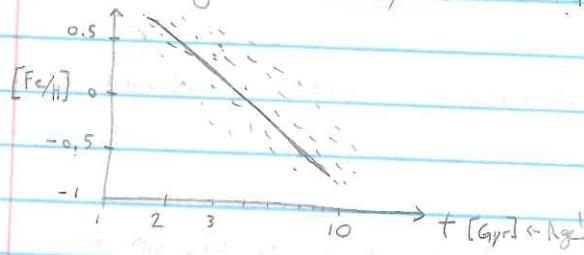
• Fine structure line emission of CI can also be a source of cooling in MCs.

# G7

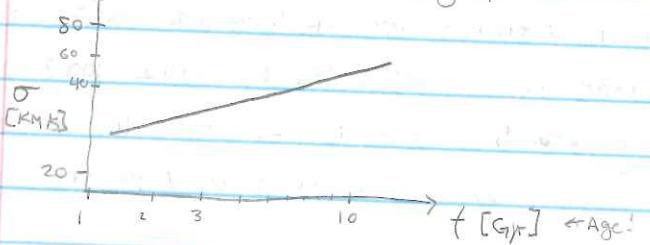
- 1st stellar pop'n is enriched w/ metals, later generations are higher  $Z$
- Old red stars have  $\sigma$  than young blue stars by scattering by MCs and transient spiral arms perturbs them and their grav. field
- Thus  $\sigma \propto Z$  and v.v.

## Properties of Stars in the Solar Neighborhood.

The properties of stars in the solar neighborhood are all related by their history of star formation and the evolution of the galaxy. As star formation progresses, the environment is enriched with metals following the deaths of massive stars. Later generations of stars are  $\therefore$  metal enriched, and there is a roughly inverse age-metalllicity relationship:



There is also a relationship b/w age and velocity dispersion. The  $\sigma$  of older cool red stars is generally higher than that of younger hot blue stars, implying that stars are born on nearly circular orbits, but as the stellar population ages,  $\sigma$  increases as roughly  $\sigma \propto t^{1/2}$ .



This increase is likely not due to the accumulation of small velocity kicks from passing stars, since galaxies are collisionless systems (see G4). Instead it's thought that scattering by <sup>(vertically)</sup> Molecular Clouds and <sup>(radially)</sup> transient spiral arms, which produce irregularities in the galaxy's gravitational field, affects  $\sigma$ .

Combining these relations, we can say that older stars have higher  $\sigma$  and lower metallicity, and v.v.  $\therefore$  we have a relationship b/w  $\sigma$  and  $Z$  as well: as metallicity increases,  $\sigma$  decreases, since these two parameters are both dependent on the age of the stellar population.

# G 8

- photoionization, ionized  $e^-$  contributes to thermal E through collisions, ionization of metals by star light and He by x-ray rays from compact objects
- photodissociation of  $H_2$ , excited  $e^-$  falls to vibrational continuum and dissociates molecule
- cosmic rays, non thermal particles ionize H
- dust photoelectric emission, photon absorbed on surface excites  $e^-$ , which escapes

## • Heat Sources in the ISM

- 3 photoionization, 1 photodissociation, 1 photoelectric emission

Less Important

### • Photoionization

- when a bound  $e^-$  absorbs a photon it can gain enough energy to be liberated from the nucleus. The free  $e^-$  then contributes to the thermal energy of the gas through collisions w/ ions and other free  $e^-$ s. Contact w/ ions  $\rightarrow$  recombination, contact w/ bound  $e^-$ s  $\rightarrow$  excitation and radiation of excitation energy, contact w/ free  $e^-$ s  $\rightarrow$  Bremsstrahlung radiation. Thus the ISM is heated via photoionization of C, Si, Fe, and  $\sim 2.1$  eV is deposited into the ISM per ionization. Source: starlight.

### • Photodissociation of $H_2$

- photons w/  $E = 11.2 - 13.6$  eV can dissociate  $H_2$  by exciting an  $e^-$  to an excited state which can then fall down to the vibrational continuum, and dissociate. This releases  $\sim 0.4$  eV per dissociation. Source: star light.

### • Cosmic Rays

- high energy non-thermal ( $\sim$  MeV) protons and electrons can ionize H, and the freed  $e^-$ s can then contribute to heating the ISM by further ionizing atoms and scattering events w/ other free  $e^-$ s (ie elastic collisions, which allow  $e^-$ s to share energy w/ the gas). Source: obviously cosmic rays from high E sources

### • X-Ray Photoionization

- X-rays can ionize H, and the energetic  $e^-$  released can further ionize atoms. Since the H cross section for X-ray absorption is smaller than the, X-ray ionization of Fe (and other metals) contributes more to heating at such wavelengths. Source: X-rays from compact objects or the hot ISM.

More Important

### • Dust Photoelectric Emission

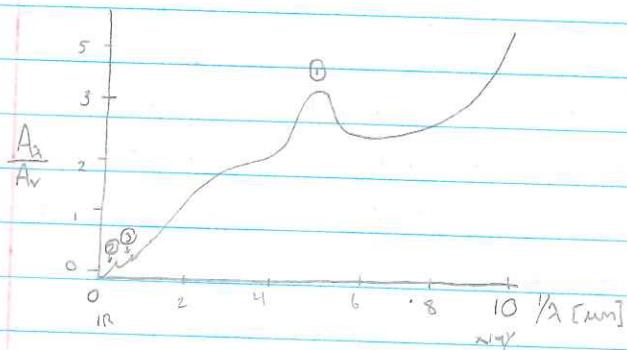
- when an energetic photon is absorbed onto the surface of a dust grain, an  $e^-$  can be excited and escape from the surface. If this emission occurs efficiently it can be a major heat source in diffuse neutral clouds in the ISM. Source: PAHs Mostly.

- Also: stellar superwinds, SNe, shock waves, etc. can all heat their surroundings.

# G 9

- Extinction is  $\lambda$ -dep. process by which light is scattered and absorbed in ISM
- Depends on Mie Theory:  $\sigma \sim S^6/24, S^3/\lambda, S^2$ , assume  $S \ll 1 \mu\text{m}$
- Deviations from Mie Theory produce features; 2175 Å bump = PAH resonance scattering, 10 μm bump = Si-O bond stretching, 3.4 μm bump = C-H bond stretching in hydrocarbons

## Interstellar Extinction Curve



$$A_\lambda = (M_{\text{obs}} - M_{\text{expected}}) \text{ at } \lambda$$

$A_V$  = extinction at  $\sim 0.5 \mu\text{m}$

- Interstellar extinction is  $\lambda^2$ -dependent process by which light is scattered (and absorbed) by dust in the ISM. The features in the extinction curve thus depend on the dust's scattering cross section, which depends on both  $\lambda$  and the size of the dust grain (Mie theory):

$$\begin{aligned} \sigma_\lambda &\sim S^6/24, \quad \lambda \gg S \\ &\sim S^3/\lambda, \quad \lambda \approx S \\ &\sim S^2, \quad \lambda \ll S \end{aligned}$$

- At  $\lambda \approx 0.55 \mu\text{m}$ , grains are  $\sim 0.1 \mu\text{m}$ , nonspherical, aligned
- At  $\lambda \lesssim 0.3 \mu\text{m}$ , grains are  $\lesssim 0.05 \mu\text{m}$ , minimally aligned
- But overall grains ~~are~~  $\approx 0.015 \mu\text{m}$  contribute substantially.

- however, there are deviations from this dependency. The main ones are:

### ① 2175 Å Bump

- this deviation is due to resonance scattering of PAHs or possibly graphite

### ② 10 μm Bump

- absorption bands due to the stretching of Si-O bonds; difficult to see

### ③ 3.4 μm Bump

- due to C-H bond stretching in hydrocarbons; also very difficult to see

## G 10

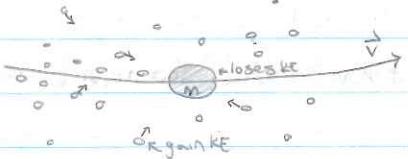
- grav. attraction causes over density of particles behind moving mass
- $a \sim -mv^2/2$ , explain dimensionally
- Friction b/w satellite and DM halo = lose KE/AM, spiral in, tidally disrupted, stars added as coherent streams b/c satellites have  $\downarrow \sigma$ , large galaxy unaffected.

### Dynamical Friction

When a massive particle moves through a distribution of massive particles, the 'background' particles are gravitationally attracted to the moving particle and accelerate toward its path. This creates an over-density along the path which is larger on the tail end, and the gravitational force of this over-density decelerates the moving particle due to dynamical friction:

$$\frac{dv}{dt} \sim -\frac{mv^2}{1v^3}$$

thus the more massive the particle is, and the more dense the particle distribution is, the more it decelerates, while the faster it travels, the less it decelerates.

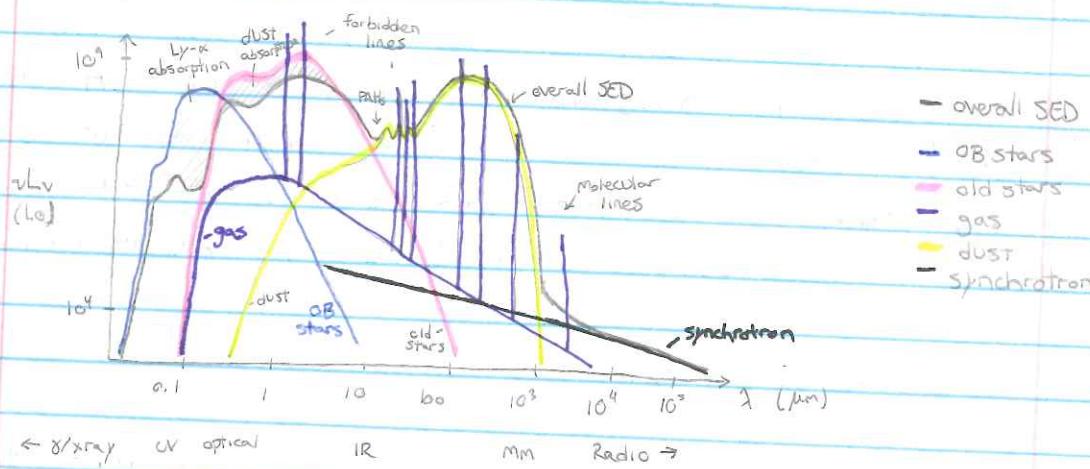


During galaxy mergers, dynamical friction between the satellite/smaller galaxy and the DM halo of the larger galaxy causes the smaller one to lose KE and AM as it orbits. This causes the orbit to decay and inspiral, transferring its KE/AM to the particles in the DM halo. As the small galaxy approaches the center it gets tidally disrupted, and since small galaxies generally have lower velocity dispersions (Tully-Fisher relation;  $\downarrow L = \downarrow \sigma$ ), the galaxy's constituents will be added to the larger as coherent streams. The larger galaxy will remain largely unaffected.

# G II

- Draw components: OB stars, old stars, dust + PAH feature, gas + forbidden/molecular lines, synchrotron, absorption at Ly- $\alpha$
- Explain SED of galaxy is combination of SEDs of emitting bodies within it

## Galaxy SED Features



- The SED of a galaxy is composed of the combined SEDs of the emitting bodies within it. Most of the emission in the optical to IR is related to stars, either directly or indirectly, while at other wavelengths other contributions affect the SED.
- Gamma/Xray - due to cosmic ray + ISM interactions, accretion onto compact objects, hot shocked gas. This doesn't contribute much in the MW.
- UV - blue/OB star<sup>BB</sup> emission, absorption of Ly- $\alpha$  photons
- Optical - older star blackbody emission, absorption of light (re-emitted in IR), forbidden line emission by metals
- IR - dust blackbody emission, PAH and complex organic molecule emission produces complex forest of lines
- MM - Molecular (rotational) line emission from cold molecular gas
- Radio - synchrotron emission from relativistic e<sup>-</sup> + ISM B-field interaction, free-free emission (Bremsstrahlung) from HII regions

## G 12

- Assume N & size of MW;  $N_{100} = N_{\text{MW}} \cdot \frac{V_{100}}{V_{\text{MW}}} = 10^6 \text{ stars}$
- Assume SFR, assume avg mass  $\rightarrow$  SFR in #/yr. Assume B-age, get # stars that formed in B-lifetime. Assume IMF, integrate  $\rightarrow$  frac of stars B-type or less. Scale by vol. ratio = 20 stars
- From SFR in #/yr,  $\times$  by age, scale by vol. ratio = 5000 \*

- Stars within 100pc of the Sun.

- To determine the # of stars w/in 100pc of the Sun, we start with a couple assumptions about the # of stars in the MW and the size of the MW:

$$N_{*,100} = N_{*,\text{MW}} \cdot \frac{V_{100}}{V_{\text{MW}}} = (10^{10} *) \cdot \frac{\frac{4}{3}\pi R^3}{\pi R^2 h} = (10^{10} *) \cdot \frac{\frac{4}{3}\pi (100\text{pc})^3}{\pi (10^4\text{pc})^2 / (10^3\text{pc})} = 10^6 *$$

- To determine the # of stars here that are B-type or earlier, we'll make a bunch of assumptions:

- the SFR of the MW is  $\sim M_{\text{luminous matter}} / t_{\text{MW}} = 10^0 M_{\odot} / 10^{10} \text{ yr} = 1 M_{\odot}/\text{yr}$
- the average star has  $M = 0.2 M_{\odot}$ , since M-stars are most common. So the SFR in terms of # of stars formed is  $SFR / (0.2 M_{\odot}/*) = 5 * / \text{yr}$ .
- a typical B-star has  $M_B \sim 8 M_{\odot}$  and  $t_B \sim 10^7 \text{ yr}$ . The upper mass limit for earlier stars is  $\sim 100 M_{\odot}$
- the # of stars formed will depend on the IMF, which we'll assume is described by  $\zeta_{\text{Salpeter}}(M) \sim M^{-2.35}$
- $\therefore$  fraction of stars  $\leq B$  in MW =  $\int_{8 M_{\odot}}^{100 M_{\odot}} dM M^{-2.35} \approx 0.04$
- $\hookrightarrow$  # stars formed w/in lifetime of B-star  $= SFR \# \cdot t_B = (5*/\text{yr}) (10^7 \text{ yr}) = 5 \times 10^7 *$
- $\hookrightarrow$  # stars  $\leq B$  formed w/in this time in MW =  $(5 \times 10^7 *) (0.04) = 2 \times 10^6 *$
- $\hookrightarrow$  # stars  $\leq B$  formed w/in this time w/in 100pc =  $(2 \times 10^6 *) \cdot \frac{V_{100}}{V_{\text{MW}}} \approx 20 *$

- To determine the # of stars younger than  $10^8 \text{ yr}$  in our 100pc sphere, we consider the # of stars that could have formed w/in  $10^8 \text{ yr}$  in the entire MW, and then scale down to our sphere:

- # stars  $< 10^8 \text{ yr}$  in MW =  $SFR \# \cdot t_{10^8} = (5*/\text{yr}) (10^8 \text{ yr}) = 5 \times 10^8 *$
- # stars  $< 10^8 \text{ yr}$  w/in 100pc =  $(5 \times 10^8 *) \cdot \frac{V_{100}}{V_{\text{MW}}} \approx 5000 *$

# G13

- Collapse begins when cloud exceeds Jeans length; is perturbed by another cloud or shock wave.
- Collapse goes on  $t_{\text{dyn}} \sim \sqrt{G\rho}$
- As  $\rho \uparrow$  core becomes optically thick, radiation can't remove E, T increases until star approaches HSE and contracts on KH timescale,  $t_{\text{KH}} = \frac{GM^2}{RL}$
- Contraction ends when fusion can support star against gravity. Endpoint depends on M b/c diff. processes begin to dominate energy production depending on  $T_{\text{core}}$ .

- Cloud Collapse and Contraction

- A molecular cloud core will begin to collapse due to interaction w/ another cloud, shock waves from SNe/HII regions, or entering a higher density region of a spiral arm. The MCC ( $T \approx 10\text{ K}$ ,  $n \approx 10^3 \text{ H}_2/\text{cm}^3$ ) will begin to collapse on a free fall/dynamical timescale once the cloud exceeds its Jeans mass/length:

$$t_{\text{dyn}} \sim \sqrt{G\rho}, \quad R_J \sim v_{\text{sound}}/t_{\text{dyn}}$$

- As the  $\rho$  increases in the interior as  $\rho \sim 1/r^2$  (assume constant T; see P8 and P20), the core becomes optically thick, meaning it's too opaque to radiate away the energy produced by gravitational contraction as fast as it's produced.

- The  $\rho$  and T continue to increase until the protostar approaches HSE, and from here it contracts on a Kelvin-Helmholtz timescale (quasi-static contraction):

$$t_{\text{KH}} \sim \frac{GM^2}{RL} \quad (\sim 10^7 \text{ yr})$$

- this is b/c a protostar's luminosity is produced by gravitational contraction, and using the virial theorem,  $L = E_f = \frac{1}{2}U_f = \frac{1}{2}\frac{GM^2}{R_f}$ .

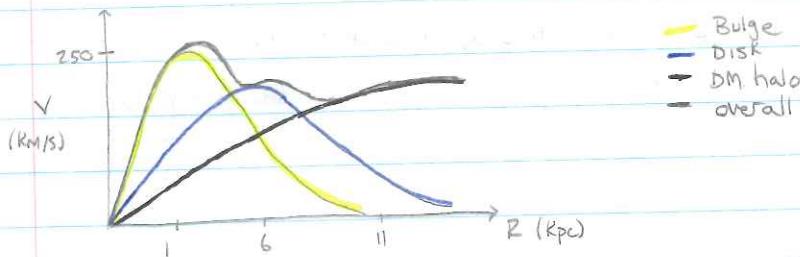
- Eventually the core T rises to the point where nuclear fusion ignites, and contraction ends when this fusion can produce enough energy to support the star on its own. At this point, the new star moves onto the MS and evolves on a nuclear timescale,  $t_{\text{nuc}} \sim Emc^2/L$ .

- The endpoint of contraction depends on mass because different processes will begin to dominate the energy production depending on the core temperature. High mass stars can achieve higher T and ignite the CNO cycle rather than the P-P chain, so the point at which nuclear fusion takes over from grav. contraction will vary.

# G 14

- Draw rotation curve components: bulge, disk, halo
- $F_{\text{cent}} = F_{\text{grav}} \rightarrow M \propto R \rightarrow \rho \propto M/R^3 \sim 1/R^2$
- high  $v$  at large  $R$  tells us normalization of  $\rho$  distribution is high, so lots of DM

## Galactic Rotation Curve



If the rotation curve is flat at large radii, that means  $v(r) = \text{constant}$ . We can determine the relationship b/w mass and radius then by equating the centripetal force and gravitational force:

$$\frac{MV^2}{r} = \frac{GMm}{r^2}$$

$$v = \sqrt{\frac{GM}{r}} = \text{constant}$$

$$\therefore M \sim r$$

- by mass conservation we have  $dM = 4\pi r^2 \rho dr$ , or  $M = \frac{4}{3}\pi r^3 \rho$

$$\therefore \rho \sim M r^{-3}$$

$$\sim r \cdot r^{-3}$$

$$\therefore \rho \sim r^{-2}$$

Thus the shape of the rotation curve informs us about the density distribution. The fact that  $v_c$  is very high at large radii means that the normalization of this distribution ( $\rho_0$ ) must be high - there is a lot of mass following this  $1/r^2$  density distribution.

To determine the circular/rotational velocity in other galaxies, you can measure the velocity (usually of the gas in a galaxy) along your line of sight on the longest axis of the galaxy, and scale that by its inclination:

$$V_c = \frac{V(0, y)}{\sin i}$$



# G15

• Table of phase, composition, cooling

• HIM = HI/ions, X-ray emission. WIM = HII, free-free, optical, fine structure.

WNM = HI, optical, fine structure (21 cm). CNM = HI, fine structure (21 cm).

H<sub>2</sub>, Diffuse/Dense = H<sub>2</sub>, fine structure (CO, 21 cm, CII)

## • Thermal phases in the ISM

• The baryons in the ISM have a wide range of  $T$  and  $\rho$ , but most baryons have  $T$ 's that fall close to characteristic states or phases. Six distinct phases account for most of the M and V of the ISM:

### ① • Coronal Gas / Hot Ionized Medium

-  $T \gtrsim 10^{5.5} K$ ,  $n_H \sim 0.004 \text{ cm}^{-3}$ ,  $f_V$  (volume fraction)  $\sim 0.5$

- gas that's been shock heated by blast waves from SNe. The gas is collisionally ionized (ions like O II exist here), and cools through adiabatic expansion and X-ray emission.

### ② • HII Gas / Warm Ionized Medium

-  $T \sim 10^4 K$ ,  $n_H \sim 0.3 - 10^4 \text{ cm}^{-3}$ ,  $f_V \sim 0.1$

- gas that's been photoionized by UV radiation from hot massive OB stars. The gas cools via optical emission lines, fine-structure line emission, and free-free emission.

### ③ • Warm HI / Warm Neutral Medium

-  $T \sim 5000 K$ ,  $n_H \sim 0.6 \text{ cm}^{-3}$ ,  $f_V \sim 0.4$

- Mostly atomic gas heated by photoelectrons from dust, ionization by starlight and cosmic rays. Cooling is also done by optical line and fine-structure line emission here, like 21 cm.

### ④ • Cool HI / Cold Neutral Medium

-  $T \sim 100 K$ ,  $n_H \sim 30 \text{ cm}^{-3}$ ,  $f_V \sim 0.01$

- Mostly atomic gas heated by the same as above but cooled pretty much exclusively by fine structure line emission, like 21 cm.

### ⑤ • Diffuse Molecular Gas (H<sub>2</sub>)

-  $T \sim 50 K$ ,  $n_H \sim 100 \text{ cm}^{-3}$ ,  $f_V \sim 0.001$

- Similar to the cool HI clouds / CNM, but the higher density allows H<sub>2</sub> to form and stay in the cloud interior. Cooling also occurs via fine structure lines, like 21 cm + CO.

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## ⑥ Dense Molecular Gas ( $H_2$ )

- $T \sim 10-50\text{ K}$ ,  $n_H \sim 10^{3-6}\text{ cm}^{-3}$ ,  $f_V \sim 10^{-4}$
- these are gravitationally bound clouds that are dense and thus 'dark', as their visual extinction is  $\gtrsim 3$  mag through their central regions. This is due to the presence of dust grains. Star formation occurs here, and cooling is achieved by CO line emission and CI fine structure line emission.

Phase	$n_H [\text{cm}^{-3}]$	$T [\text{K}]$	$f_V$	Composition	Cooling
HIM	0.004	$10^{5-6}$	0.5	HII, ions	X-ray emission, adiabatic expansion
WIM	$0.3-10^4$	$10^4$	0.1	HII	Optical emission lines, fine-structure "", free-free emission
WNM	0.6	5000	0.4	HI	Optical lines, fine structure (21 cm)
CNM	30	100	0.01	HI	Fine structure
Diffuse $H_2$	100	50	0.001	$H_2$	Fine structure; 21 cm, CO lines
Dense $H_2$	$10^{3-6}$	$10-50$	$10^{-4}$	$H_2$	Fine structure, CO line, CI line

- Stupid Table w/ Region, star type, age, Z, σ
- Bulge = old K-M giants, 0.2-10 Gyr, solar Z, ~10-100 σ.
- Disk = M-dwarf/giants, <10 Gyr, low Z, ~10-100 σ
- OCS = young stars w/ single SF burst, 1 Gyr, solar Z
- GCS = popn II w/ MS turnoff w/ F star, 10-13 Gyr, low Z, ~100 σ
- Halo = old stars from in-situ / mergers, 10-13 Gyr, low Z, ~100 σ
- Elliptical = no SF so pop II stars, 10-13 Gyr

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## Stellar Populations in the Galaxy

Region	Star Description/Comments	Age (Gyr)	Z [Fe/H]	σ (km/s)	Radius (kpc)	M (M <sub>⊙</sub> )
i) Bulge	<ul style="list-style-type: none"> <li>old stars; K-M giants, RR Lyrac</li> <li>range of Z suggest early enrichment</li> </ul>	0.2-10	-2 → 0.5 (solar according to Isb)	50-150	4	$10^{10}$
ii) Disk	<ul style="list-style-type: none"> <li>A-M dwarfs, giants, sub-giants</li> <li>Thick disk has older (K-M) stars and lower Z (Isb doesn't believe in thick v. thin disk fyi)</li> </ul>	< 10	-2 → -0.5	10-50	25	$10^{9-10}$
iii) Open Clusters	<ul style="list-style-type: none"> <li>Young stars formed in single SF burst (if young, will include OBs)</li> <li>close encounters w/ GMCs can gravitationally disrupt them</li> </ul>	~1	0 (Z <sub>0</sub> )			$10^{2-3} \text{ star's}$
iv) Globular Clusters	<ul style="list-style-type: none"> <li>Population II Stars</li> <li>MS turn-off ~ A-F stars</li> </ul>	10-13	< -0.8		~40	$10^5 \text{ star's}$
v) Halo	<ul style="list-style-type: none"> <li>mostly older stars, some form in situ, some disrupted from satellite galaxies</li> </ul>	10-13	< -5 → -0.5	100	> 100	$10^9$
vi) Ellipticals	<ul style="list-style-type: none"> <li>Starved of gas so low/no SF</li> <li>pop'n II stars, most formed rapidly around Z ~ 2-3.</li> </ul>	10-13			1-200	$10^{8-13}$

• The degeneracy of age and metallicity, as probed by color, makes it tough to characterize the stellar pop'n of elliptical galaxies.

Hilary

# G17

- Nebular line diagnostics = line ratio  $\sim$  density ratio for 2 excited levels in an atom; use Boltzmann eqn to get T
- Balmer Jump = recombination produces continuum due to  $e^-$  KE, w/ discontinuities at  $KE_e = 0$ , which depends on  $e^-$  distribution and thus T
- Dielectronic Recomb. = compare collisional excitation rate to DR rate b/c have diff T-dep.

## • Temperature of H II Regions

• There are 3 main ways to determine the temperature of an H II region:

### ① Nebular Line Diagnostics

- this makes use of ions that have 2 excited levels that are 'energetically accessible' with  $\Delta E \sim K_B T_{\text{gas}}$ , such that the level populations are sensitive to  $T_{\text{gas}}$ . By measuring the ratio of the line emission strengths, you can relate that to the relative number density of each level, and use the Boltzmann equation to get T:

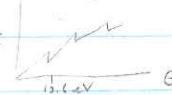
$$\frac{L_\alpha}{L_\beta} = \frac{h\nu_\alpha \text{Volume} N_\alpha A_\alpha}{h\nu_\beta \text{Volume} N_\beta A_\beta} \sim \frac{N_\alpha}{N_\beta} = \frac{g_\alpha}{g_\beta} \exp\left(\frac{-\Delta E_{\alpha\beta}}{K_B T_{\text{gas}}}\right)$$

- generally the temperature of the gas is  $\sim 10^4$  K, so a good probe will be when  $\Delta E \sim 10$  eV ( $1\text{eV} \approx 10^3$  K)

- if  $E_\alpha \approx E_\beta$ , then the relative rates of populating the levels via collisions won't depend on T. The ratio will have a  $\uparrow^\rho$  and  $\downarrow^\rho$  limit, so measuring the line ratio for such states can inform you about the density of the gas.

### ② Balmer Jump in the Recombination Continuum

-  $e^-$ 's will have a continuum of KE, so when they recombine w/ ions, the radiated photon will have  $E = KE + \text{binding } E$ . This will result in a continuum of recombination radiation with sharp discontinuities at the binding energies of the atom (when  $KE_e = 0$ ). The strength of the jump/discontinuity depends on the distribution of  $e^-$ 's w/  $KE=0$ , which then depends on T. By comparing the strength of the jump to the strength of recombination lines, a measure of T can be made. A common choice is the Balmer jump (365 nm), compared to various recombination lines of hydrogen.



### ③ Dielectronic Recombination Lines

- when an  $e^-$  collides w/ an atom/ion, it will release some energy which can be used to excite another  $e^-$  in the atom. For instance, an  $e^-$  can collisionally ionize excited levels of C II, but can also produce excited levels of C III by dielectronic recombination. The rate coefficients for collisional excitation and D.R. will have different T dependences, making their line coefficients a useful diagnostic tool.

## G 18

- Modeling the chemical evolution of the MW predicts that  $\approx 1/2$  the solar neighborhood stars will have  $Z \leq 1/4 Z_{\odot}$ , but we see  $\approx 2\%$  w/  $Z$ .
- Model assumes no initial metals in disk; closed box, IMF/SFR:  $M(z) = M(0)[1 - e^{-z/p}]$
- Solar include initial metals, not closed box (influx of  $\approx 2$  gas), diff IMF w/ high M stars in early galaxy

### The G-Dwarf Problem

- Using the IMF and SFR to model the chemical evolution of the disk predicts too many low metallicity stars at low masses compared to what we observe today.

#### The Model goes as follows:

- Assume the newly formed galaxy is all gas and no stars or metals.
- Assume the galaxy is a closed box - no gas/dust is allowed to leave or enter.
- Assume an IMF and SFR. For each generation of stars you'll get a yield  $p'$  of metals, which will describe the amount of ISM enrichment you get as time passes and stars form and explode.
- Working out a bunch of math, you find that the mass of stars below some metallicity will be  $M(z) = M(0)[1 - \exp(-z/p)]$

- This closed box model implies  $\approx 1/2$  of the solar neighborhood stars should have  $Z \leq 1/4 Z_{\odot}$  ( $Z_0 \approx 0.02$ ). In reality, we see that only  $\approx 2\%$  of F and G dwarfs have such low  $Z$ , implying that this simple model is incorrect.

#### Solutions:

- the disk had some initial metals
  - ↳ this could occur if rapidly evolving massive stars enriched the ISM before the gas and dust settled into a disk.
- the closed box model assumption is invalid
  - ↳ If a lot of metal-poor gas has accreted onto the disk since its formation, this would result in dilution of the ISM and a lower mass density, leading to less SF early on. We'd thus see fewer metal-poor old stars today.
- the IMF was different in the early galaxy
  - ↳ If a larger fraction of more massive stars formed early on, we would see fewer low mass and low metallicity stars still existing today, and more metal enrichment for later-forming stars, leading to fewer metal-poor stars overall.

# G 19

- Spiral arms are stationary density waves. Can be explained by elliptical orbits whose long axes vary smoothly w/ radius in form of their directions. Not material waves b/c otherwise get winding.
- Structure related to large scale galaxy properties: tightly wound, smoothness, ~ Lbulge/Ldisk and  $M_{gas}/M_{stars}$ . Also show dust lanes, blue knots, HII regions, high SFR
- Classified by orientation relative to direction of rotation; trailing v. leading

## Spiral Structure Characteristics

- This question is far too ambiguous so I'm taking an 'observational characteristics' approach!
- The central idea of spiral structure theory is that spiral arms will form if stars and gas clouds move on elliptical orbits whose major axis positions vary smoothly as a function of radius. As the long axis of any orbit rotates with radius, the orbits become condensed in some places compared to others. If each orbit was uniformly distributed with gas clouds, the surface density would be higher wherever the orbits more closely approach each other.
- The Lin-Shu hypothesis states this succinctly: spiral structure is a stationary density wave.
- Spiral structure is closely related to the large scale properties of galaxies. For example, the Hubble sequence points out that how tightly wound spiral arms are and how smooth they are are related to  $L_{bulge}/L_{disk}$  and  $M_{gas}/M_{stars}$ .
- Spiral arms have characteristic dust lanes caused by absorption in dense clads of gas and dust, as well as blue knots of star formation which are often accompanied by HII regions, indicative of rapid SF.
  - The SFR in spiral arms is thus much higher than the rest of the disk. Interstellar gas is also essential for persistent spiral structure.
- Spiral arms can be classified by their orientation relative to the direction of the galaxy's rotation. Trailing arms are significantly more common than leading arms, but this is actually tough to determine observationally because you need to determine the galaxy's inclination.
- We know that the arms must be density waves rather than material waves b/c otherwise the differential rotation of the galaxy would wind up the arms on a shorter timescale than the age of the galaxy. A typical analogy for this stationary density wave is cars moving through a traffic jam.