Cosmology Study Notes

Miranda Herman
25 April 2018

Disclaimer: This is not meant to be a thorough all-encompassing study guide for the cosmology section of the quals. It's just a thing I started writing to understand how all of the random stuff in cosmology is connected, because I didn't really like the way most textbooks approach this topic, and I didn't take the cosmology course! These notes have tons of gaps in them, skip a lot of details, and are definitely not-quite-right sometimes, but I tried to make my uncertainties apparent while writing.

References:
Carroll & Ostlie, 2nd ed.
Schneider, 2nd ed.
Dodelson 2003

History of the Universe (the abridged version though, obviously)

In the beginning, for absolutely no reason at all as far as we can tell, the universe came into existence via a **Big Bang**, and it had some weirdly specific cosmological parameters that initially made me super paranoid. We can't see that far back in time to actually observe this Big Bang, but we do have some supporting evidence for the predictions this model makes about our observable universe. For example, the recessional velocity of galaxies, the CMB temperature and fluctuations, and BBN abundances are pretty conclusive. On the other hand, there are a few issues with the standard model of the Big Bang that we can't really explain without invoking this crazy concept of inflation, like the horizon problem, the flatness problem, and the magnetic monopole problem. Before we dive into all the observations and stuff, let's just compile a timeline of events, starting with the earliest possible time we can logically discuss: the **Planck time**.

$$t_P = \sqrt{\frac{\hbar G}{c^5}} \approx 10^{-44}\ s$$

At times earlier than this, we don't have the physics necessary to describe the state of the universe, because before this space and time can't really be thought of as separate. Things were crazy af, man. We think that the four fundamental forces (gravity, electromagnetism, and the strong and weak nuclear forces) were all united into one giant **Theory of Everything** force via some crazy af mathematical symmetries that kept them all together. At this point the universe was just some weird plasma and we have no idea what it was really composed of, just that there were entirely **random quantum fluctuations** that caused slight over- and under-densities throughout. The universe was about 10^{32} K or 10^{19} GeV. For reference, particle physicists really have no idea how particles behave above a few hundred GeV.

Now right around the Planck time, the universe underwent some insane **spontaneous symmetry breaking** for no apparent reason, and gravity decided it had had enough with the other 3 forces. So gravity broke off as a separate force, and the remaining 3 stayed joined and are described by various **Grand Unified Theories**, none of which have really been worked out fully. This triforce unification would've lasted until the universe was about 10^{-34} s old, corresponding to a temperature of 10^{29} K or 10^{15} GeV. We still have no idea what the universe was composed of at this point, but some more spontaneous symmetry breaking caused the Grand Unified Theories force to split into the strong nuclear force and the **electroweak force** (the unification of the electromagnetic and weak nuclear force). This spontaneous symmetry breaking caused a choice in the 'direction' or discrete quantum numbers describing the quantum (Higgs) field to be made. In theory, any place in the field where a discontinuity in this choice existed, you'd get a defect in the field. And anywhere you get a pointlike discontinuity, you'd get a **magnetic monopole**. Now I don't really understand this next link, but apparently the universe was so ridiculously hot/high energy at the end of this GUT era that it made a lot of magnetic monopoles. Like, a lot a lot. Plus, they're theorized to be insanely massive -- like 10^{16} times the mass of a proton -- but the problem is, we've never seen one. So where they at?

To solve this problem (and some others which I will get to eventually), cosmologists introduced a mostly-crazy-but-not-enough-to-reject-it-because-it-actually-fixes-a-lot-of-issues idea of **inflation**, a period right around the GUT era but *before* the GUT force split apart, where the universe did some crazy stuff. The idea is, at this point the universe entered a weird state called a **false vacuum** where the universe was not actually in the state with the lowest possible energy density, which is the most favorable state to be in. Instead, it had supercooled to far below 10^{28} K while still remaining in a high energy-density state with unbroken symmetry. This false vacuum is crazy, dude. Like, the energy density of the false vacuum is constant, just like it is for dark energy, and according to the fluid equation it would have to have a negative pressure. Wat. Now I know what you're thinking: does that mean the universe was composed of dark energy? Observations of the Hubble flow almost definitely say no, but we have literally nothing else to work with. So until someone comes up with a better idea for the energy density of this false vacuum, we're just going to assume it's dark energy.

Ok now remember all those random quantum fluctuations that were just chillin up until now? Well get ready because they are about to mess things up for us during this inflation period. These fluctuations were basically due to the uncertainty principle, which tells us that if we're sure about the time, we know far less about the energy. So while most of the universe was described by this weird false vacuum, there must've been a tiny region that actually entered a true vacuum state, and the pressure inside this tiny bubble would've been basically zero. Now, since the false vacuum has negative pressure like we said, that means this tiny bubble had higher pressure relative to the false vacuum, causing it to expand like there was no tomorrow. So at about 10^{-36} s, this constant energy density of the false vacuum dominated the acceleration equation and the scale factor of the universe grew exponentially. This was the period of inflation.

Inflation came to an end around 10^{-34} s when the universe underwent some more spontaneous symmetry breaking for no apparent reason. This caused the GUT force to split into the strong and electroweak forces like we mentioned above, and the elevated energy density of the false vacuum was released, reheating the universe back to its pre-inflation temperature of like 10^{28} K. This burst of energy produced a bunch of **particle-antiparticle pairs** and from here the Big Bang theory carries on as expected.

Before we discuss what happened after this period in the super early universe, we should talk about all the problems inflation solves for us that the OG version of the Big Bang theory can't account for. First let's talk about the **horizon problem**. We'll touch more on this later when we get to recombination, but suffice to say that before inflation our bubble was in thermodynamic equilibrium, and so everything within the bubble was in causal contact. Inflation caused most of the volume of our bubble to be carried far beyond our currently observable universe, so our observable universe is really just an insignificantly tiny fraction of that larger bubble which we can no longer see. But, since every part of our observable universe was in thermodynamic equilibrium prior to inflation, it makes sense that the CMB on large scales (larger than 1° across) is smooth. Without the exponential expansion due to inflation, there would be no way for these distant regions of the universe to be in causal contact with each other, because their light cones would never intersect in the time since the Big Bang.

Second, we can discuss the **flatness problem**, which we'll also come back to later, probably. At first glance the universe is just weirdly, suspiciously flat -- i.e. the density of the universe (which includes photons and baryons and dark energy) is almost exactly equal to the critical density between having positive curvature (closed universe) and negative curvature (open universe). In other words, the density parameter is effectively 1. The OG Big Bang model just assumes 'that's the way it is, kiddos,' because if the universe wasn't very very flat initially, we literally just wouldn't have been able to exist to observe it this way. During inflation (at t_{infl}), however, the density parameter varied with scale factor as:

$$\Omega(t) = 1 + \left(\frac{k\,c\,t_{infl}}{a(t)}\right)^2$$

Since the scale factor *a(t)* increased exponentially during inflation, this means the density parameter became effectively 1, which is the same as saying k = 0 in this equation (i.e. the inflationary universe has no curvature and is flat af). So the flatness problem is solved by inflation too.

Finally, the **magnetic monopole problem** we mentioned way before. With inflation, defects in the quantum field (aka monopoles) are really just remnants of the false vacuum, still with unbroken symmetry and a higher energy density, and thus a lot of mass. These defects likely arose around the boundaries of all the tiny true-vacuum bubbles that formed and caused inflation. So when inflation ended and our tiny bubble had expanded by like more than 30 orders of magnitude, these defects/monopoles became really really sparse. If only a few monopoles existed around every single bubble, and our observable universe only takes up an insignificant fraction of that initial bubble, then the chances of us observing a monopole are slim to none. This is effectively true for all the particles that existed prior to inflation -- the rapid expansion would've diluted their numbers to insignificant amounts, so any particles in appreciable quantities that we see in the universe today must have popped into existence after inflation, due to the burst of particle-antiparticle formation that was fueled by the energy released by the false vacuum.

Alright now we're ready to move on to explaining the rest of the history of the universe. The expansion of the universe was no longer dominated by dark energy at this point, but by radiation instead, so the scale factor went as $t^{1/2}$. So like we said, after inflation the strong and electroweak forces separated and we were left with a hot mess of a universe. A bunch of particles and antiparticles were created, like quarks and leptons and photons and gluons and some weird hypothetical particles and their antiparticles that could transform into quarks and antiquarks. According to GUT, these weird particles changed to quarks slightly faster than the antiparticles changed to antiquarks as the universe cooled, which left the universe with ever so slightly fewer antiquarks than quarks. However, right after inflation it was way too hot for the quarks and gluons to stick together to form hadrons, so the universe was just a hot dense **quark-gluon soup**.

The quarks and antiquarks survived until about 10^{-11} s (post-Big Bang) when the universe cooled to about 10^{12} K or 100 MeV, and another spontaneous symmetry breaking occurred. This one we've actually confirmed experimentally, and we know that this symmetry breaking separated the electroweak force into the **electromagnetic and weak nuclear forces**. This gave mass to the W- and Z-bosons (carriers of the weak nuclear force), differentiating them from photons (carriers of the electromagnetic force). The quarks were finally able to combine to form hadrons (baryons and mesons, but for the love of god don't ask me wtf a meson is), with a slightly higher number of baryons forming over anti-baryons. This event is called **baryogenesis**. There was then a ton of particle-antiparticle annihilation, which destroyed basically all of the antimatter. The remaining matter is what we see in the universe today, and the insane amount of photons produced in this annihilation event is the initial source of the CMB. All of the particles (baryons, leptons, photons) were in equilibrium at this point via processes like Compton scattering, Bremsstrahlung, electron-positron pair production/annihilation, neutrino-electron scattering, etc.

When the universe was about 1 s old, the temperature was about 10^{10} K or 1 MeV and **neutrinos became decoupled** from the rest of the universe's constituents, meaning the temperature was too low for the neutrino reactions to keep the neutrinos in thermal equilibrium with the other particle species. The neutrinos froze out with a characteristic thermal distribution corresponding to 10^{10} K, which should have produced a **Cosmic Neutrino Background**, but we are not great at detecting neutrinos and this CNB would have a temperature of like 1.9 K today, so we haven't seen evidence of this yet.

When the universe was about 10 s old with a temperature of about 10^9 K or 100 keV, electron-positron pair production couldn't be maintained because the temperature of the photons had decreased with the expansion of the universe, so **annihilated electron-positron pairs** couldn't be replaced by the formation of new pairs. Almost all of the electrons and positrons were annihilated, but a slightly higher number of electrons persisted to make up for the overall positive charge density resulting from the protons. This annihilation process added a crazy amount of additional photons to the universe, all still in thermodynamic equilibrium with the surrounding matter (except neutrinos, of course). We should also talk about **neutrons** here, which can be produced via reactions involving protons and electron/positrons and neutrinos/antineutrinos, and which have a decay time of about 10 minutes. With a decreased number of electrons and positrons available for such reactions, plus the whole neutrino freeze-out event, the production of neutrons basically came to a halt, and the number density of neutrons vs. protons was frozen in. Neutrons continued to decay into protons until the temperature dropped sufficiently below 10^9 K.

After these first 10 s and for the next 3 minutes, as the temperature slowly dropped below 10^9 K, **primordial nucleosynthesis (BBN)** took place where protons and neutrons were able to fuse into as many deuterium nuclei as they possibly could, until basically all the neutrons were bound up in deuterium. The temperature of the universe was still high enough that photons could photo-dissociate the nuclei, but the production of deuterium relies on strong interactions and is therefore super efficient. After this, there were a bunch of possible reactions to get to ^{4}He so the formation of helium proceeds very quickly after the deuterium bottleneck. ^{4}He has a really high binding energy, so once it's formed, it can't be destroyed by photo-dissociation. However, the reactions that produced this helium were different from the ones that take place in stars via the P-P chain (but I'm not about to memorize reaction equations so don't ask me about that). Forming appreciable quantities of heavier nuclei was difficult because there are no stable nuclei with A = 5, so only a small amount of ^{6}Li and ^{7}Li and ^{8}Be was formed after ^{4}He. The reactions that formed these nuclei were hindered by the fact that there was very little D or ^{3}H or ^{3}He around, and any further nucleosynthesis was hindered by the fact that there are no stable nuclei with A = 8. After 3 minutes the expansion of the universe caused the temperature of the matter to drop until BBN beyond this wasn't possible.

So after 3 minutes, 75% of the baryonic mass of universe was in the form of protons (hydrogen), 25% was in the form of ^{4}He, and there were trace amounts of other nuclei. All of this material was in thermodynamic equilibrium with the photons and electrons, with the temperature too high to allow electrons to fuse to nuclei without being immediate ionized by the photons. But before we move on to the next period of evolution in the universe, we should talk about the observational evidence that supports this piece of the Big Bang model: measurements of the **primordial abundances of D and ^{4}He**.

So to start, the density of D and ^{4}He in the universe depends on the baryon density for a few reasons. First, if the baryon density were higher, the baryon-photon ratio would be higher, and so D could start to form earlier. Extending this argument, the amount of ^{4}He that formed would also have depended on the initial density of baryons. Basically the higher the baryon density, the less D you're left with at the end of BBN and the more ^{4}He you get. But how are we supposed to measure the abundance of an element throughout the entire observable universe? QSO absorption lines. We can measure the relative strengths of the Ly-alpha absorption lines of H and D, which have slightly different transition frequencies due to the difference in their masses. The relative line strength tells us about number density , so we can measure the **D/H ratio** and compare it to what the Big Bang model predicts for the baryon density of the universe. And it does a decent job! But wait. Where do these absorption lines come from? The absorption lines are produced when the energetic photons emitted by QSOs encounter cool metal-poor intergalactic gas, which absorbs these photons. The intergalactic gas probably hasn't seen any star formation that would increase its metallicity, so the D/H ratio of the gas should be very close to its primordial value. Measuring this fraction of D tells us that the baryon fraction of the universe is really really small, and so the rest of the matter must be in some form other than baryons: dark matter. And who tf knows what that is.

Ok so we've been able to provide observation support for BBN, which is cool. Let's discuss what happened next in the evolution of the universe: **recombination**. This name is actually stupid af because at no point in the entire history of the baby universe had nuclei and electrons ever been combined before. But whatever. Basically, the universe was super boring for a few hundred thousand years. After about 50,000 years the energy density of the universe shifted towards being dominated by matter (including baryons and dark matter) rather than radiation. This means the scale factor no longer evolved as $t^{1/2}$ but rather $t^{2/3}$.

Despite this shift, it was still too hot for electrons and nuclei to combine, so the universe was just a plasma of nuclei and electrons and photons all kept in thermodynamic equilibrium by scattering events involving free electrons. All the particles were just patiently waiting for the universe to expand enough so that the temperature could go down and allow neutral atoms to form. Essentially, as the universe expanded and reduced the number density of free electrons, the average time between scatterings of a photon by an electron started to approach the timescale of the expansion. At this point the **photons started to become decoupled** from the electrons. Furthermore, as the temperature dropped to about 3000 K (less than 1 eV) nuclei and electrons started to combine (and stay combined) into neutral atoms. This occurred at a lower temperature than the ionization temperature of H (13.7 eV or 10^5 K) because the Wien's tail of the Planck distribution still had a ton of high energy photons capable of ionizing any newly formed atoms. So the abundance of neutral atoms wasn't appreciably greater than that of ionized atoms until the temperature fell well below this value. This occurred when the universe was about 400,000 years old.

This recombination event caused a huge drop in opacity for the photons. With all the electrons bound up in atoms, there wasn't much to scatter the photons and redirect their path. Radiation and matter completely decoupled at this point, and radiation streamed freely through the newly transparent universe. The photons were described by a blackbody distribution since they were all in thermodynamic equilibrium before their last scattering event, and so as the universe has expanded the characteristic temperature of this radiation has decreased with the increasing scale of the universe. The free-streaming radiation is seen as the **Cosmic Microwave Background** today, because the photons have been cosmologically redshifted from 3000 K to just 2.73 K on their journey from the '**surface of last scattering,**' at a redshift of about 1100, to us. The agreement between theory and observation of this background radiation is one of the main pillars of the Big Bang model, alongside BBN. The third pillar (galaxy recessional velocity) will take a little more work to build up to because we need to create some galaxies first.

Following recombination, there was a whole lot of nothing for like 100 Myr. The universe entered the **Dark Ages**, where there was no real structure and no visible light, since the CMB photons pretty quickly shifted from visible wavelengths to infrared as the universe expanded. But eventually we had to form some structure, otherwise we wouldn't be here. If the universe was so incredibly homogeneous and isotropic, how could that happen? Well surprise surprise, it wasn't actually perfectly homogeneous -- at least not on small scales. Remember those quantum fluctuations from pre-inflation? Well it turns out the **density perturbations** cause by those guys actually propagated all the way through from the very early phases of cosmic evolution (I'm not 100% sure about this crap). These inhomogeneities must have been really really tiny during recombination, but they still left their mark on the CMB in the form of tiny **temperature anisotropies**. Basically the angular distribution of the CMB temperature tells us about the matter inhomogeneities that existed during recombination. If the temperature is a little lower than average, it's because photons were **gravitationally redshifted (Sachs-Wolfe effect)** -- they lost energy while climbing out of the gravitational potential well that existed in regions of over-densities. If the temperature is a little higher, that indicates that photons didn't lose as much energy as average because they came from places with a shallower gravitational well than average. BUT WAIT, THERE'S MORE.

These density fluctuations would have only become more pronounced with time. As the universe expanded, over-dense regions would have expanded more slowly due to their self-gravity, and eventually this would have allowed **large-scale structure** to form. But before we get into that, let's discuss exactly how these density fluctuations developed, and how they fit into this picture of recombination, the CMB, and the later development of large-scale structure in the universe. This is going to take literally forever because the CMB is intricate af, so buckle your seatbelt.

To start us off on this super confusing explanation of density fluctuations, let's remind ourselves that right around recombination the universe was made up of four main components: photons, baryons, neutrinos, and dark matter. Now, density fluctuations can occur for each of these components *independently*. If the fluctuations are proportional to each other, so that the components don't exchange any energy with each other, we have **adiabatic or curvature fluctuations**. In this case, the sum of all the density fluctuations is non-zero, which affects the local curvature of spacetime. On the other hand, if the sum of the density fluctuations for our four components is zero, we have **isothermal or isocurvature fluctuations**. These involve an increase in the potential density increase that is suppressed or frozen in, whatever that means. For instance you might have a spatial variation in the types of particles present, which would create a pressure difference, but couldn't produce an actual density variation since the particles' motions would be limited by the frequent interaction between matter and radiation. I really have no idea wtf this actually means, but apparently anytime density fluctuations are present, you can describe them as the sum of both adiabatic and isothermal fluctuations, kind of like a sum of kinetic and potential energy when discussing the energy of a system, I guess.

One of the main things to consider with these density fluctuations is how they produced **baryonic acoustic oscillations (BAOs)** in the early universe. The initial quantum fluctuations produced gravitational instabilities such that over- and under-densities developed as matter (baryons and dark matter and everything else) collapsed inward. As they fell and the density of a region increased, the compression causes a tiny bit of heating until radiation pressure reversed the motion of the baryons. This created sound waves that oscillated due to the interaction between this pressure gradient and gravity. The dark matter, however, didn't really care about this pressure because it has no way to interact with the baryon-photon plasma other than gravitationally. So the dark matter just continued to coalesce at the centers of these gravitational wells, with the radiation and baryonic matter oscillating in a type of shell around it. Of course, this all happened on a variety of scales, which makes it really hard to think about.

The adiabatic and isothermal fluctuations remained distinct until radiation and matter decoupled during recombination, after which the particles in isothermal fluctuations (which until this point were basically frozen in place) were no longer bound by the radiation field. This meant the particles could actually do something about the pressure differences that arose and *move* to produce real density perturbations rather than just potential ones. At this point, the two types of fluctuations became indistinguishable.

The cool thing about the temperature fluctuations (arising from the density fluctuations) of the CMB is that, for a given angular position on the celestial sphere, they can be expressed as a sum of spherical harmonics:

$$\frac{\delta T(\theta,\phi)}{T} = \sum a_{l,m} Y^{l}{}_{m}(\theta,\ \phi)$$

By measuring the temperature variations in all directions, we can determine the value for the coefficients. Then, since we have no non-arbitrary way of choosing the zero-point for our angles, we can take an angular average to determine the average amplitude of the coefficients for these spherical harmonics. This gives us the **CMB power spectrum** as a function of the multipole moment *l*, which tells you about the amplitude on different angular scales:

$$C_l = \frac{1}{2l+1} \sum |a_{l,m}|^2$$

The form of this power spectrum tells us about the fundamental mode and harmonic modes of the acoustic oscillations that were frozen in at the time of last scattering. The first and largest peak corresponds to an angular size of about 1°, or l = 200, and is due to compression of a large region that reached its maximum compression at the time of decoupling. The first trough is produced by a smaller region that started oscillating earlier, and since it was smaller, it was able to oscillate more quickly, arriving at dT = 0 by the time of decoupling. The next peak arises from an even smaller region that was able to oscillate through its maximum compression and reach maximum rarefaction (expansion, I guess… opposite of compression) at decoupling. This second peak is smaller than the first because the temperature variation caused by compression is greater in magnitude than that of rarefaction. This is due to the biasing effect of the gravitational force exerted by dark matter on the baryons. By comparing the heights of the first and second peaks we can learn about the density of baryons in the universe, since the relative suppression of the second peak gets more pronounced the more baryonic matter you have.

The third peak is due to an oscillation that reached its second maximum compression at decoupling. This peak is sensitive to the density of dark matter in the universe (no idea why tho), and so the comparable heights of the third and second peak indicate that most of the matter is in the form of dark matter. This means the CMB power spectrum can be used to determine **cosmological parameters** such as the density parameters describing the universe, which tells us about the curvature. At smaller angular scales, the rest of the peaks start to die off around l > 1000. This is because random-walking photons can diffuse between the compressions and rarefactions of short-wavelength sound waves, decreasing their amplitudes and thus the magnitude of the temperature variation. At large angular scales (l < 50), the power spectrum is pretty low and flat. This is because regions with angular sizes this large never experienced acoustic oscillation. The density fluctuation on this scale was so vast that it remained outside of the particle horizon until recombination, so CMB photons from this region were just affected by the primordial temperature fluctuation and the Sachs-Wolfe effect (gravitational redshift as they climbed out of gravitational potential wells). The cool thing about all of these harmonics (if you're into that sort of thing, idk) is that they support the idea of inflation. The oscillations of the density fluctuations of a given size (and thus a given oscillation frequency) had to have reached their maximum compressions and rarefactions simultaneously, and thus started their oscillations simultaneously, in order to produce the regularity in the harmonic structure we see today.

I hope your seatbelt is still buckled because there's another aspect of the CMB that we need to talk about and it is just as horrifically confusing as BAOs are: **CMB polarization**. Measuring the polarization of the CMB is really hard for two reasons. One, the magnitude is really small -- only 1% the size of the temperature fluctuations -- and two, there's a ton of polarized **foreground emission** that you have to account for, like galactic synchrotron emission, Bremsstrahlung (free-free) emission, thermal dust emission, and the SZ effect, all of which mess with CMB photons between the time when they decoupled from matter and when they finally reach us. But wtf makes them polarized in the first place? Thomson scattering. The electric field of incident unpolarized radiation causes free electrons in the photon-baryon soup to vibrate. This vibration produces scattered light that is **linearly polarized**. But wait. If the radiation field coming at the electrons is isotropic, there will be no net polarization. So we ask again, wtf makes CMB photons polarized?

Ok don't stress because the answer is still Thomson scattering, just slightly-more-complicated Thomson scattering. To get a net polarization the radiation field must be more intense along one axis than another, which means it has a quadrupole moment. This quadrupole moment can't be sustained if Thomson scattering occurs super often, since the photon directions get randomized really quickly. This tells us that the observed polarization of the CMB must have been set in place at the time of decoupling, when free electrons combined with nuclei and Thomson scattering was no longer happening, allowing the quadrupole to stay and polarize the photons. Now, where does this quadrupole moment come from?

There are two ways to get a local quadrupole moment. The first is based on the flow of the photon-baryon fluid, which had a velocity gradient as implied by the presence of BAOs. As an electron falls radially inward toward a concentration of dark matter, from its frame of reference the neighboring fluid elements are all moving toward it. The fluid in the radial direction is approaching it faster than the fluid in the transverse direction, so the radiation in the radial direction is more intense. This produces a quadrupole moment and polarizes photons as they undergo scattering by the electron. The polarization pattern (that is, the direction of the CMB's observed electric field) will be directed radially away from cooler temperature fluctuations, forming tangential loops around warmer fluctuations. This polarization pattern is known as an **E-mode**. Since E-modes are due to density fluctuations, they're known as **scalars**. This would be a great time to talk more about how such modes depend on the **Stokes parameters *Q* and *U***, but I really don't understand them and I really don't care to because I'll literally never deal with them again after I pass this insane exam.

The second way to get a local quadrupole moment is through gravitational waves produced by primordial density fluctuations (idk man, just roll with it). When a gravitational wave passes through the photon-baryon soup, it stretches the space containing the photons along one axis and compresses it along a perpendicular axis. This produces a quadrupole moment, and the resulting polarization pattern spirals outward from the temperature fluctuations with the direction of the spiral dependent on the sign of the temperature fluctuation. This polarization pattern is known as a **B-mode**. Since B-modes are due to gravitational waves, they're known as **tensors**, and are expected to be most prominent at angular scales larger than a degree. So overall, the polarization pattern of the CMB is a combination of E- and B-modes. Since the two modes are closely tied to the temperature fluctuations of the CMB, the CMB power spectrum and the polarization power spectrum (is that what's it's called? Who knows? Who cares?) are correlated in terms of the multipole moments associated with their peaks and troughs.

One more question we should bring up in reference to CMB polarization. How do we measure the polarization? I honestly have no idea because the papers I've googled are way too technical, but here's what I've gleaned. You need something like a polarization-sensitive bolometer, generally a receiver that operates in the few-hundred GHz range and is sensitive to the incoming electric field. Not sure sure what to do with that information, but I know that in general to measure the polarization of light you just slap on a polarizer and rotate it until you find the angle with the highest intensity of radiation, and that's how you determine the polarization.

Ok I am sick of talking about the CMB so we're going to move tf on, even though it's still going to show up and bite us in the butt in like 3 paragraphs. After recombination the baryons and photons no longer interacted, so while the photons were free to stream through the universe, the baryons were left in a shell of fixed radius around each of the dark matter clumps. The characteristic size of these shells depends on the sound speed of the baryon-photon fluid traveling with the pressure wave -- faster than about half the speed of light. This corresponds to about 150 Mpc in comoving coordinates, and can tell us a lot about the distribution of galaxy clusters, which we'll discuss more below. But basically the point is, BAOs not only leave an imprint on the CMB in the form of temperature anisotropies, but they also leave an imprint on the clustering of galaxies and matter in the universe today.

So the baryons were gravitationally attracted to the clumps of dark matter that had formed, and so gradually fell inward and increased the over-density in the region. When the density fluctuation dρ/ρ had grown to about 1, these regions of dark matter and ordinary matter detached from the Hubble flow and began to collapse into the early structures we can observe today. Since we know how large the baryon shells around each dark matter clump would have been, we can determine the expected separation between galaxy clusters that would have formed from the initial density perturbations (as we said above, about 150 Mpc). If we know the separation of these structures, and we can measure the angle subtended by them on the sky, then we can determine their distances from us. BAOs can therefore be used as a **standard ruler** because of the effect they've had on the structure formed in the universe. I guess this would be as good a time as ever to talk about the **2-point correlation function**.

Galaxies are not randomly distributed in space, and instead form in groups and clusters and even larger structures. This means the probability of finding a galaxy in some volume is not the same everywhere, and instead, the probability of finding a galaxy at point X depends on whether there's a neighboring galaxy at point Y. The probability is thus:

$$dP = n[1+\xi(r)]\, dV$$

where *n* is the average number density of galaxies and the horrific squiggle (xi) is the two-point correlation function, which depends on the separation of two galaxies. The TPCF describes whether galaxies are more concentrated (> 0) or more dispersed (< 0) than average and has the general form:

$$\xi(r) = \left(\frac{r}{r_o}\right)^{-\gamma}$$

where r_o is the correlation length, which generally has a value around 5 h^{-1} Mpc (but this varies with luminosity; brighter galaxies have longer correlation lengths) and gamma is usually about 1.7. We can measure the correlations between galaxy positions and thus arrive at this equation by performing spectroscopic redshift surveys of galaxies. The fun thing about the TPCF is that, since it's related to the BAOs that were responsible for galaxy/structure formation, you can relate the TPCF to a **power spectrum** via a Fourier transform (in case you hadn't had enough of power spectra yet):

$$P(k) = 2\pi \int_0^\infty dx\, x^2\, \frac{\sin(kx)}{kx}\xi(x)$$

Roughly, the power spectrum describes the level of structure as a function of length scale, *L = 2pi/k*. So not only is this power spectrum related to the TPCF, it's also obviously related to the CMB power spectrum with the C_l's and crap, and you can use one to get the other.

Now that we have the beginnings of structure formation figured out (galaxy formation from the bottom-up method with cold dark matter), we can talk about what happened next in the history of the universe. It took about 200 Myr for the first stars and galaxies to form. The UV radiation from the first generation of stars (Population III stars) and AGN began to reionize the universe, and this was conveniently called the epoch of **reionization**.

Reionization took a while -- between a redshift of z ~ 15 to 6 -- and was due pretty much exclusively to photoionization. Collisional ionization can be ruled out because for it to be efficient the IGM would need to be really hot, which we know isn't the case based on the near-perfect black body of the CMB. So like we said, it must have been hot stars (mainly) and AGN (less so) rather than the hot IGM that produced all the energetic photons capable of re-ionizing the universe. So how do we get these stars? If we assume that gas/baryonic matter had to fall into the dark matter halos in order to form galaxies, then the gas must have needed to cool and condense into clouds from which the first stars could form. What sort of cooling mechanism would have been capable of chilling the gas? There was no metal in this primordial gas, so cooling via metal lines is out of the question. Atomic H and He are both pretty terrible coolants, since their excitation temperatures are pretty high. Molecular H, on the other hand, does a great job and dominates the cooling rate of primordial gas below ~ 10^4 K. This would have allowed for the first stars to form, but these stars would have been considerably different from the ones forming today. They're known as **Population III stars**. Without any metals, their opacity would have been much lower. This means they would have been a lot hotter and brighter (and thus had shorter lifetimes), and this high temperature made them more efficient sources for ionizing photons.

Ironically, the stars that formed with the help of H_2 cooling actually destroyed any H_2 in their vicinity, since H_2 is pretty easy to photo-dissociate with photon energies above ~ 11 eV. Without this H_2 no further cooling and thus no further star formation could take place in these regions. So the stars existed with a small bubble of ionized material around them, while the rest of the universe's baryons remained neutral. Of course, since these stars were massive they very quickly exploded as supernovae, ejecting all of the metals they produced into the IGM. This metal enrichment of the IGM can be observed in the spectra of high redshift ($z \sim 6$) QSOs, as well as the Ly-alpha forest which is produced by the IGM. Both show non-vanishing metallicities.

In order for further ionization of the universe to take place, we need more stars. Without molecular H we're left only with atomic H (and maybe He) to cool things down, and since cooling works best when $kT \sim h\nu$, only in more massive halos ($\sim 10^7$ M_{sun}, which are hotter) are able to cool via atomic H at a slightly later epoch (confusing, I know. The hotter it is the better it cools, because fml). Thus massive halos experience efficient cooling and star formation, forming the first proto-galaxies. These are then capable of ionizing the surrounding IGM in the form of **HII regions** (ionized H), which gradually expand as more ionizing photons are produced. Eventually these expand and overlap with each other until all of the IGM is ionized. At this point reionization is complete and the temperature of the IGM is about 10^4 K. This is because the energy of an ionizing photon is generally a little higher than 13.6 eV, so the energy difference is transferred to the electron during photoionization, and the electron is coupled to the other gas particles via Coulomb interaction. This surplus of energy results in heating of the IGM.

A quick note on **He reionization**. Since He requires about 4 times as much energy to ionize as H, and even the most massive stars don't produce photons with energies this high, it's thought that quasars had to have been responsible for He reionization around $z \sim 3$. This is based on a statistical analysis of the Ly-alpha forest, plus He absorption lines and the He Gunn-Peterson effect in high-redshift QSOs.

So now that we know what reionization was, we can discuss the observational constraints we have on it in more detail. There are actually a bunch: the Gunn-Peterson trough and the Lyman-alpha forest, secondary anisotropies in the CMB, and 21 cm emission from neutral hydrogen in the IGM. First let's talk about the **Lyman-alpha forest** first, since that's popped up a few times without much explanation. The Ly-a forest shows up as a series of *absorption* lines in the spectra of QSOs (and other things probably but idgaf) at wavelengths bluer than the Ly-a *emission* line at **1216 A** (UV). As a quick refresher, this emission is due to a bound electron in the first excited state of a hydrogen atom dropping to the ground state (n=2 to 1). Ly-a absorption therefore occurs when a ground-state electron absorbs a photon of 1216 A and jumps to the first excited state. Don't confuse this with 912 A, which is the wavelength of light an electron in the ground state needs to absorb to become fully ionized (n=1 to infinity). Also don't confuse it with 6563 A, which is the H-alpha line (n=3 to 2, or Balmer-alpha), and also don't confuse it with 21 cm emission (hyperfine transition of neutral H where the spin of the electron flips). Confusing, right? Anyway, the Ly-a forest is a series of absorption lines blue-ward of 1216 A that are caused by the diffuse distribution of gas in the IGM. The sources of these absorbed photons are distant, highly redshifted quasars, and as the photons travel toward us they pass through absorbing clouds of *neutral* H in the IGM that are closer to us than the original source. So the absorption lines are produced at lower redshifts, meaning they're shifted blue-ward relative to the original quasar (i.e. the thing producing the emission in the first place). So all of the absorption lines in the Ly-a forest are due to the same Ly-a transition happening at different redshifts.

I know what you're thinking -- neutral H? I thought we wanted to know about reionization? We do, and the Ly-a forest tells us about the process of reionization, since it wasn't an instantaneous event. Think of it this way: if you have a low-z quasar (maybe $z \sim 0.2$), the light it's emitting is travelling toward us through IGM that's basically entirely re-ionized already. There won't be many pockets of neutral H left for the photons to travel through and be absorbed, so the spectrum of the quasar won't show many absorption lines and will mostly be a continuum blue-ward of its Ly-a emission line. Now consider the spectra of a high-z quasar (maybe $z > 6$). The photons emitted by this quasar will be travelling through IGM that isn't completely ionized yet (at least at the beginning of the photons' journey), so there will be a lot of neutral H to bump into on their way toward us. This will produce far deeper and broader absorption lines. As the photons continues their journey, the universe is becoming more ionized and the fraction of neutral H is decreasing, so the absorption lines will become less pronounced. By measuring the line widths and strengths, you can determine the column density of neutral H undergoing this Ly-a absorption; the stronger the line appears, the more neutral hydrogen you must have, and therefore the less ionized the universe must have been at that time.

So how does all of that help us constrain the epoch of reionization? Through the **Gunn-Peterson trough**. The Gunn-Peterson trough is basically the extreme version of the Ly-a forest. At very high redshifts the universe was hardly re-ionized at all yet (but like post recombination, obviously, so not too too high a redshift). Photons emitted by quasars at these distances would initially be travelling through mostly neutral H, meaning almost all of the photons could be absorbed by the gas to produce absorption lines that are so broad and deep that basically no photons from this era reach us today. The redshift at which this trough ends (i.e. when the neutral hydrogen is nearly depleted and the universe is almost entirely re-ionized) is roughly $z \sim 6$. At this point we start to see more of the Ly-a forest, since only pockets of neutral H remain to absorb the photons. So the Gunn-Peterson trough allows us to place constraints on the *end* of the epoch of reionization.

Alright so that's one constraining method down. Next let's discuss the **secondary anisotropies of the CMB**. Basically, reionization resulted in a lot of newly freed electrons available for Thomson scattering in the IGM, so the scattering optical depth went up (i.e. a photon's mean free path was shorter; it couldn't travel as far without being scattered). The main effect of this scattering is to change the observed amplitude of the temperature fluctuations and polarization of the CMB. So the CMB provides two ways to constraint the epoch of reionization. The first is through the **reionization bump** that's observed at low multipoles (large angular scales) in the polarization power spectrum, since there was increased Thomson scattering and thus increased polarization at the horizon scale during this epoch. The reionization bump therefore tells you about when reionization *began* in the universe.

The second way is consider the **kinetic Sunyaev-Zeldovich effect**. I guess now is as good a time as ever to sort out the difference between the kinetic and thermal SZ effect. Let's talk about thermal SZ first. When CMB photons pass through hot intracluster gas in galaxy clusters, they experience inverse Compton scattering whereby the photons gain energy through their interaction with relativistic electrons. The CMB will no longer look like a blackbody, but will be skewed slightly, since the number of lower energy photons goes down and the number of higher energy photons goes up as lower energy photons are shifted to higher energies. The kinetic SZ effect occurs when the cluster has a peculiar velocity, so the intracluster gas has some bulk motion that produces a Doppler shift on the CMB photons. This k-SZ effect can take place with the ionized bubbles in the epoch of reionization as well, since the bubbles (Stromgren spheres) can have a peculiar velocity. This results in a change in the amplitude of the temperature fluctuations seen in the CMB power spectrum. The strength of this effect scales with the number of bubbles and therefore informs us about the *duration* of reionization.

Alright so one more reionization probe is left to discuss: **21 cm emission**, or the hyperfine transition of neutral hydrogen. In more detail, the 1s ground state of H is split into two levels due to the magnetic moment interaction between the proton and electron. When the electron spin is aligned with the proton, it has slightly higher energy, and vice versa. When the electron spin flips from aligned to anti-aligned, a photon is emitted. This spin flip can happen spontaneously but is very rare, so generally collisions between gas particles and photons are required to induce this emission (I could go into a lot more detail and try to figure out population levels and introduce the Boltzmann equation and spin temperature, but no thank you). I think the basic idea is that, prior to reionization, we didn't see a lot of 21 cm emission or absorption because there wasn't really a mechanism for producing the 21 cm line other than spontaneous emission. When the first stars turned on, the neutral H could absorb their photons and become excited, and when the electrons fell back to the ground state they could go into either of the two spin states. If their post-interaction spin state differed from their pre-interaction spin state, then you would see 21 cm line absorption (i have no clue why it was absorption instead of emission, and at this point I really do not care). As the epoch of reionization comes to an end and the amount of HI available to undergo this spin flip decreases, the amount of 21 cm absorption observed also decreases. The 21 cm line is therefore a probe of the duration of the reionization epoch, I think.

At this point we've reionized the universe and we've started to create stars and galaxies from the larger structure that began with density fluctuations and dark matter clumps and infalling baryonic material. For the rest of the history of the universe (i.e. 1 Gyr after the Big Bang), more galaxies form, we get metal-enriched stars and ISM and enter the matter-dominated era and later the dark energy-dominated era, until we find ourselves here, capable of observing the universe and trying to remember all the intricate details that lead to our own formation. Who knows how it'll all turn out in the end.

There's a lot of stuff we've skipped, like the exact equations describing the geometry of our universe, how the scale factor changes over time, the development of the CDM spectrum, the tired light hypothesis (stupid and unnecessary to learn about since it's wrong), and the steady state universe (also dumb and not worth remembering, in my opinion). We've also neglected the recessional velocity of galaxies as observation evidence for a cosmic Big Bang (since that's sort of extragalactic territory, kind of), explanations of the various CMB foregrounds (thermal SZ effect, integrated Sachs-Wolfe effect, synchrotron radiation, dust), descriptions of the many different kinds of QSOs and how they're all related (again, sort of extragalactic), and the many methods we have for measuring cosmological parameters, like Type Ia supernovae and even gravitational lensing. We didn't really touch on all the dark matter candidates either. I suppose that'll be for another time. Good luck!