universität wien

Algorithms and data structures 1

Theoretical exam

2nd night appointment

02/10/2023

Surname:	
Student number:	

The information is printed on both sides!

20	24	31	27	24	32	21	26
+	+	+	+	+	+	+	+
1	2	3	4	5	6	7	8th

Task 1 [2]

In the table above, enter the digits of your student number in the empty boxes in front of which there is a plus sign. Do the additions and find the numbers 1 to 8.

Task 2 [18]

a) [10] In C++-like pseudocode, create a recursive function *f* with an integer parameter n to which the Master theorem is applicable and its running time is in *ÿ(n2 log(n))* .

b) [3] Using the master theorem, show that *f* has the desired running time.

c) [5] In C++-like pseudocode, create a function *g* with an integer parameter n that calls *f* and whose running time is in *ÿ(n2(log(n)) 3)* .

Task 3 [20]

The values up to . (from task 1) are stored in an array in this order from left to right. Sort the values in ascending order

a) [10] Quicksort (Always use the last - rightmost - value as the pivot element). b) [4] Merge sort
c) [6] Heap sort

Task 4 [20]

a) [9] Insert the values up to from task 1 (in this order) into an initially empty hash table of length 7.
Use ***h(k) = k%7*** as the hash function and double hashing for collision handling.
The second hash function is ***g(k) = k%3 +*** 2.
Sketch the state of the hash table after each insert step.

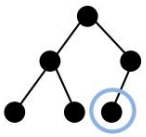
b) [1] Delete the value from the table and sketch the state of the hash table. c) [4] Enter **(after** deleting according to task b) the collision path (index positions visited) when searching for the value.

d) [2] Why is the “recoverable” mark used in double hashing? e) [2] Why is it recommended to use a table size that is a prime number for double hashing? f) [2] Name 2 dynamic hashing methods. (Names will do.)

Task 5 [20]

a) [5] In C++-like pseudocode, give the definition of an efficient data structure for a max-heap that stores integer values.

b) [5] In C++-like pseudocode, provide a method that finds the node in the heap that will be swapped with the root during deletion and prints its value. In the usual graphical representation, this node is in the lowest level on the far right (see figure).



The “replacement node” when deleted

c) [5] In C++-like pseudocode, give an efficient method to get the maximum of the stored in the max heap output values.

d) [5] What runtime orders do your methods from point b) and point c) have with respect to the number n stored in the heap of the elements?

Task 6 [20]

The following adjacency matrix with path costs for a directed graph is given
(the values up to . are to be taken from task 1):

$$\begin{pmatrix} 0 & 64 & 058 \\ 2 & 00 & \\ & 0407^{3} & \\ 20 & 100 & \\ 422 & & 00^{7} \end{pmatrix}$$

a) [2] Sketch the directed graph.

b) [10] Use Dijkstra's algorithm to determine the shortest paths from node 1 (first row, first column of Matrix) to all other nodes of the graph.

c) [8] Use Kruskal's algorithm to determine a minimally spanning tree of the shadow of the graph. (She obtain the shadow of the graph by neglecting the directions of the edges. If two nodes are then connected by two edges, these edges are combined into one. In other words: Two nodes x and y in the shadow are connected by an undirected edge if and only if at least one of the edges from x to y or from y to x exists in the originally directed graph. They choose the minimum of all as the weight of the undirected edge
directed edges represented by them.)