Estimating YY180726.2's η_{read} using 240 mK data

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Abstract

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DATA

We use the first 11 resonances in YY180726.2 temperature and power sweep data taken at 244–361 mK in yellow dewar by Fabien on 180803. VNA powers used were -20 dBm, -40 dBm, and -50 dBm, with a combined attenuation of $\sim$47 dB before entering the device check if the η_{read} plot is revised accordingly.

We use the Q_c fitted from the lowest power and temperature data, i.e. -50 dBm/244 mK, as a temperature and power-independent parameter for each of the 11 KIDs in this analysis. The reason is under this condition Q_c should be the largest relative to temperature and power-generated Q_i. We use the α and $Q_{i,0}$ fitted from $f_r(T)$ and $Q_i(T)$ by Fabien, T =244–361 mK, taken at the lowest (-50 dBm) VNA power, again as temperature and power-independent parameters for each of the 11 KIDs in this analysis. Note that instead of floating Δ_0 in the fit, the band gap was set to its BCS value, $\Delta_0 = 0.182$ meV, for aluminum.

For aluminum characteristic parameters, we use $N_0 = 17.2\ \mathrm{nm}^{-3}\mathrm{eV}^{-1}$ for single-spin density of state, $R = 9.64 \times 10^9\ \mathrm{nm}^3\mathrm{s}^{-1}$ for recombination constant, and since the thickness of our KID film is 30 nm, we assume it satisfies thin film, local limit, i.e. magnetic field penetrates the film with negligible attenuation while derivative "local" London equation applies. We also make the assumption that, since we are able to extract 75 resonances out of the design value of 80, and those chosen for this analysis are especially high in signal-to-noise ratio (SNR), the resonances we use for this analysis correspond exactly to the first 11 KIDs from the design, and from which we assign the corresponding film volumes.

THERMAL QUASI-PARTICLE GENERATION

In this analysis, we will mostly use

$$n_{qp} = \frac{1}{Q_{i,qp}\alpha\gamma_1\kappa_1} \tag{1}$$

as our standard method to calculate QP number density from observed $Q_{i,qp}$. The advantage of Eqn. (1) is it's independent of the mechanism of QP generation and should always apply once given the parameters needed by κ and γ *(will add equation later, assume everybody knows for now...)*, and α is fitted from temperature sweep data as explained previously. We calculate $Q_{i,qp}$ by

$$\frac{1}{Q_{i,qp}} = \frac{1}{Q_i} - \frac{1}{Q_{i,0}} \tag{2}$$

Q_i , where Q_i and $Q_{i,0}$ are from fitted data.

Fig. (1) shows the result of Eqn. (1) for the resonances at lowest VNA power. Also shown for comparison is the theoretical thermal QP density,

$$n_{qp,th} = 2N_0\sqrt{2\pi k_B T\Delta}e^{-\frac{\Delta}{k_B T}} \tag{3}$$

, which we calculate from the substrate temperature and assume $\Delta = \Delta_0$. One can see, when being excited by -50 dBm (∼-80 dBm at the device), the QP population is predominantly consistent with thermal QP within 10% error.

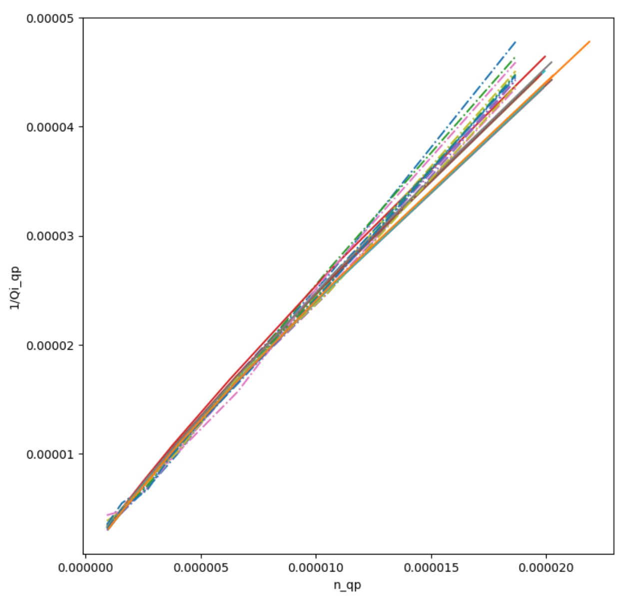


FIG. 1

READOUT POWER QUASI-PARTICLE GENERATION

To calculate the excessive QP generation by readout power, we aim for differentiating the QP population under appreciable external power with which dominated by thermal QP generation.

We begin from the general generation-recombination equilibrium condition,

$$\Gamma = Rn^2V \tag{4}$$

, where in our case R is for superconducting aluminum Cooper pair, and V is the volume of the inductor section of each KID *(Now V's are not accurately calculated and has a few % error...I'd need to do it soon)*. For low readout power, thermal QP-dominated case, this is simply

$$\Gamma_{th} = Rn_{qp,th}{}^2V_{sc} \tag{5}$$

For high readout power, we add readout power QP generation rate to the generation side of Eqn. (4),

$$\frac{\eta_{read}P_{read}}{\Delta} + \Gamma_{th} = Rn_{qp,tot}{}^2V_{sc} \tag{6}$$

, where again we assume $\Delta = \Delta_0$, and P_{read} is in fact only the portion of energy really interacts with the QP system,

$$P_{read} = \frac{2Q_r^2}{Q_cQ_{i,qp}}P_g \tag{7}$$

, where P_g is now the input "generator" power which can be read off from VNA and attenuators. Now we differentiate Eqn. (6) and Eqn. (5). With some rearrangement,

$$\eta_{read} = \frac{\Delta_0 Q_c Q_{i,qp}}{2P_gQ_r^2}RV_{sc}(n_{qp,tot}^2 - n_{qp,th}^2) \tag{8}$$

, where $n_{qp,tot}$ and $n_{qp,th}$ can be directly calculated by Eqn. (1) and Eqn. (2) from high and low readout power data, respectively. We choose the highest and lowest VNA power data, -20 dBm and -50 dBm (~-50 dBm and ~-80 dBm P_g) , for this calculation.

At the first try one we would attempt to plug in all the fitted numbers from data to calculate η_{read}, but there are many subtleties in choosing the correct inputs. First, while it's been verified previously, by subtracting $Q_{i,0}$'s contribution from Q_i, both fitted from low power data, thermal QP contribution dominates the remaining $Q_{i,x}$. However, one can't follow the same procedure to subtract fitted $Q_{i,0}$ using high power data and regards the remaining the only Q_{qp} and calculated n_{qp} from there, because as being constantly excited by external RF power, high power $Q_{i,0}$ contains readout power-generate QP contribution even at 0 K. So instead of subtracting $Q_{i,0}$ fitted from high power data, we assume the 0 K irreducible internal dissipation is independent of external power and use low power $Q_{i,0}$ in Eqn. (2) with high power Q_i to calculate high power $Q_{i,qp}$, which is then used to calculate $n_{qp,tot}$.

Secondly, to use Eqn. (1) with $Q_{i,qp}$ to calculate n_{qp}, one needs to input *QP system* temperature for the κ calculation. We assume it equals the *substrate* temperature, which in fact refers to the temperature of the thermometer on the cold plate, and we see the low power n_{qp} deviate by $\sim$10% from theoretical prediction, more obvious at higher temperature. Fig. (2) shows the κ-T relation for the 11 resonances. Since it's a (almost) linear decreasing relation in this range, if now the QP system has a temperature different from the substrate, the error made is linearly propagated to n_{qp}, while the percentile error may increase by $\sim$30% due to the $\sim$30% decrease in κ. It's known the QP system-substrate temperature difference can be increased by external excitation, for example, high readout power in our case. So for the high power analysis, we need to look for a method to determine the actual T_{qp}.

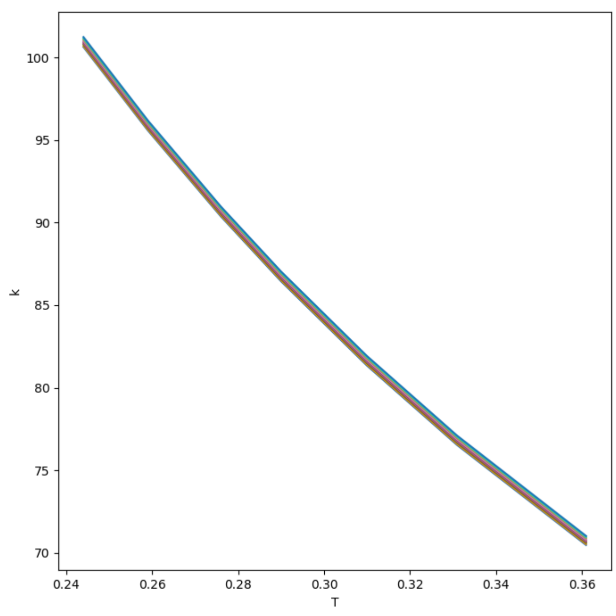


FIG. 2

We infer T_{qp} by isolating the information which only depends on QP thermal equilibrium – $Q_{qp,th}$ or $n_{qp,th}$. One way to obtain $Q_{qp,th}$ is using Eqn. (2), but this time we choose $Q_{i,0}$ fitted from high power data itself to be subtracted from high power Q_i. Assuming readout power QP generation rate is the same from 0 K to the actual operating temperature, the subtraction removes the contributions from the temperature/power-independent internal dissipation and the readout power together! Now Eqn. (2) gives the $Q_{i,th}$ only due to thermal QP at its own temperature. We then use the low power $Q_{i,qp}(T_{sub})$ relation as a lookup table to inverse-interpolate/extrapolate $T_{qp}(Q_{i,th})$ based on

$$\text{at low power, } Q_{i,qp}(T_{sub}) \sim Q_{i,th}(T_{qp,th})$$
$$\text{at high power, } T_{qp} = T_{qp,th}(Q_{qp,th})$$

The result is shown in Fig. (3). Due to uncertainty propagation, instead of having a stable $T_{qp}/T_{sub} > 1.0$, the number scatters around 1.0 by a few percent. *gonna continue anyway, at least we can get a sense of the range of η_{read} before finding a better method of calculation?*

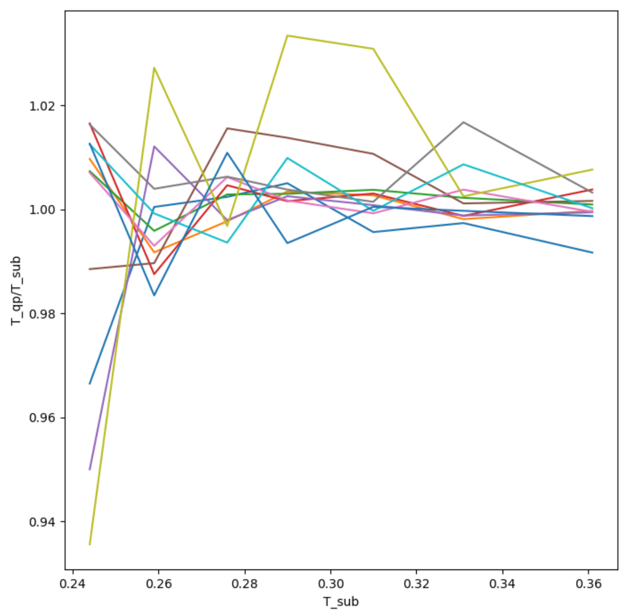


FIG. 3

Finally, we have had the correct Q_{qp} and T_{qp} to evaluate n_{qp} from Eqn. (1). To proceed to use Eqn. (8) to calculate η_{read}, there are a few remarks and reminders to be made. Since we would like to compute the energy coupled to the whole QP system, i.e. readout power and thermal generation alike, we use $Q_{i,qp}$ that combines both QP species, but not the $Q_{qp,th}$ for inferring T_{qp}, for the P_g-to-P_{read} conversion factor. We also choose to evaluate $n_{qp,th}$ at T_{sub} instead of T_{qp}, i.e. using n_{qp} from low temperature data, since we want extrac the excessing effect from readout power, and the QP system heating is part of effect and shouldn't be subtracted. The final result is in Fig. (4). As a comparison, the result assuming $T_{qp} = T_{sub}$ is shown in Fig. (5).

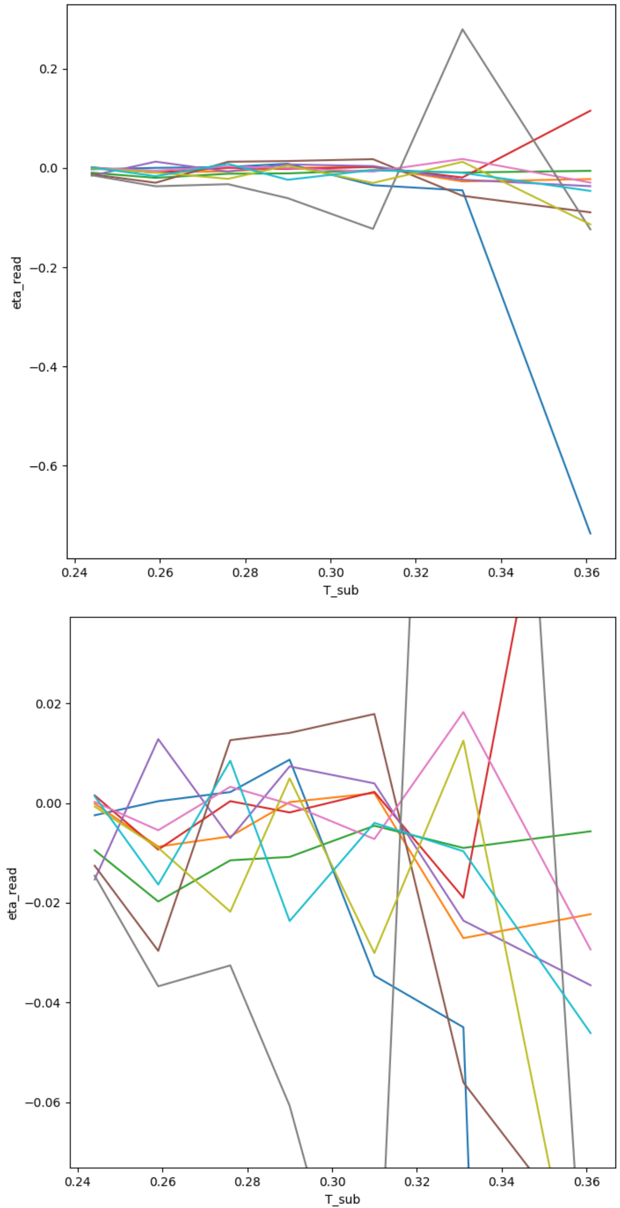


FIG. 4

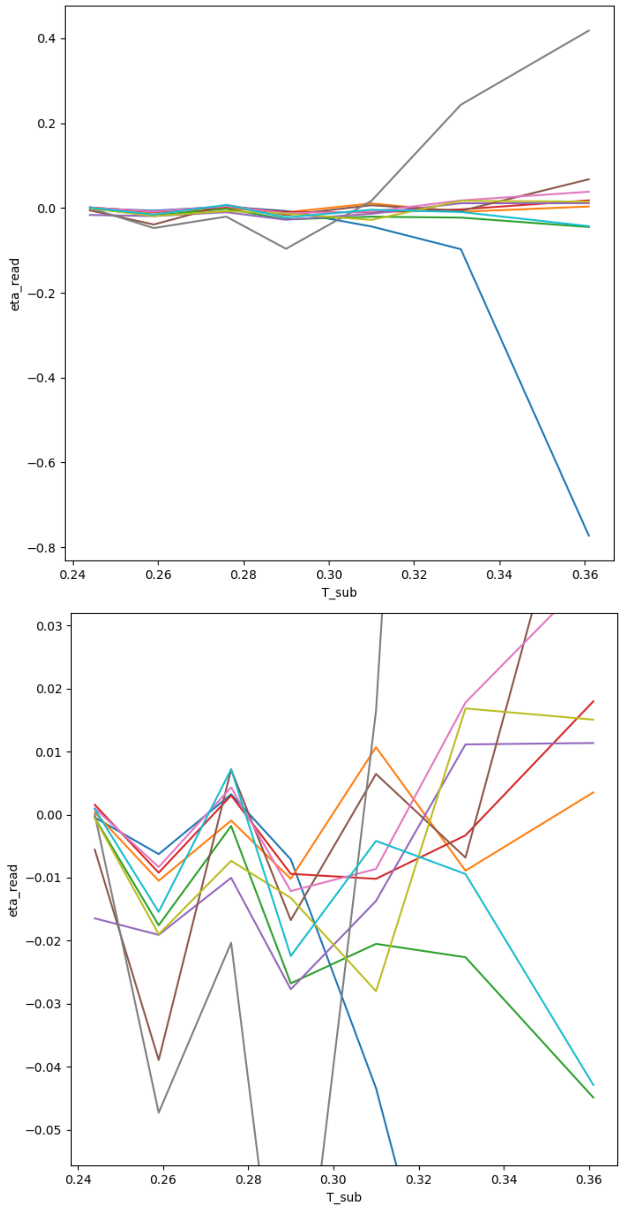

0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
eta_read
0.24
0.26
0.28
0.30
0.32
0.34
0.36
T_sub
0.03
0.02
0.01
0.00
−0.01
−0.02
−0.03
−0.04
−0.05
eta_read
0.24
0.26
0.28
0.30
0.32
0.34
0.36
T_sub

FIG. 5

POWER SWEEP ANALYSIS

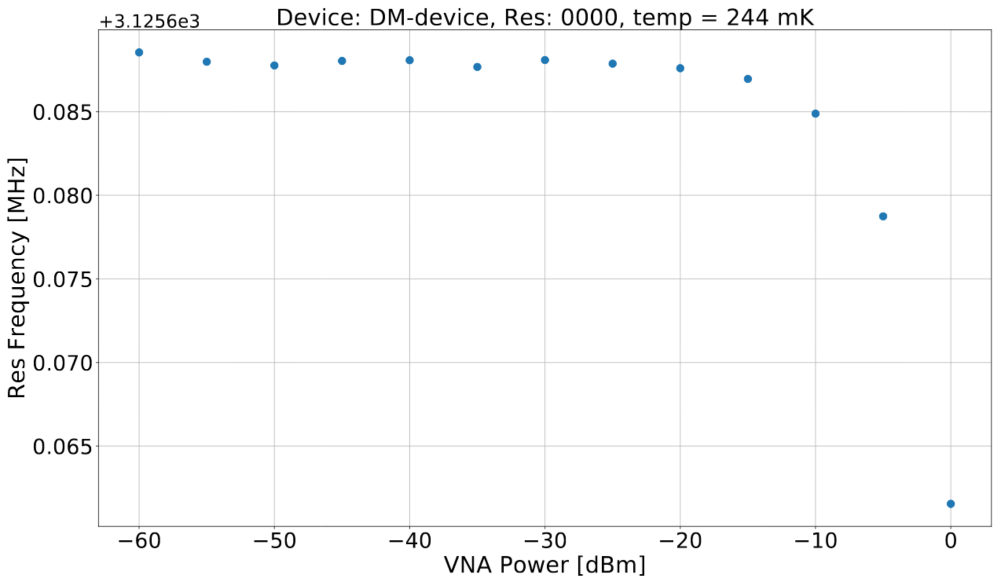


FIG. 6

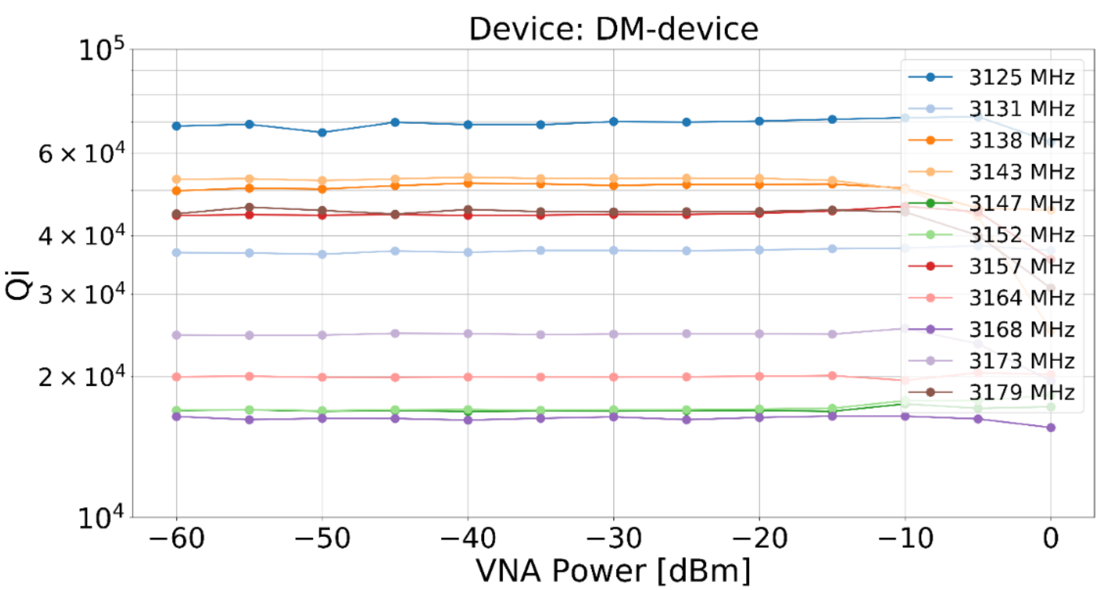


FIG. 7

Fig. (6) and Fig. (7) show the examples of resonant frequency and quality factor changes due to readout power, data from the first KID at 244 mK [?]. At around -20 dBm VNA power (∼-67 dBm at DMKID input), we see the readout power starts to generate an observable amount of QP that shifts the resonant frequencies and degrades the internal quality factors. Ideally, with the data taken at a power where a significant amount of QP is readout-generated relative to thermal QP and internal loss, e.g. -5 dBm or 0 dBm, one may follow the method described previously to calculate η_{read}. However, in order to fit for $Q_{i,0}$ at the specific power, which we have explained earlier is readout power dependent and is required in the calculation of T_{qp} and subsequently n_{qp} and η_{read}, the data must have a temperature sweep. Unfortunately, the highest power we used to take temperature sweep data is -20 dBm, and we have also used it to demonstrate the methodology and concluded the $n_{qp,read}$ is too small to arrive to a non-uncertainty-dominated η_{read}.

The next best thing we can do to estimate η_{read} is with our base temperature (244 mK) data taken at -50 dBm–0 dBm. To proceed, we make three assumptions:

1. Readout power-generated QP has negligible effect to $Q_{i,0}$ in -50 dBm data.
2. Internal loss quality factor, approximated by -50 dBm $Q_{i,0}$, is a constant for each KID.
3. Readout power-generated QP completely thermalizes with the whole QP system (Cooper pairs, thermal QP...).

We have demonstrated that once $Q_{i,0}$ is removed from the total Q_i, the remaining Q_i in low power data nicely follows the theoretical $Q_{qp,th}$, thus we think assumption 1. is valid. For assumption 2., we point out $Q_{i,0}$ varies $\sim 5\%$ for each resonance, but the data is from -50 dBm, -40 dBm, -20 dBm, all lower than where readout power is significant. Also the lowest temperature for the data is 244 mK, and it's known that Q_i is usually not $Q_{i,0}$-limited above $T_c/10$, which is 120 mK in our case [?]. We will further investigate assumption 2. in the coming dilution refrigerator test. There's a dedicated subsection in *Discussion* about the validity of assumption 3. We simply state here assuming infinitely fast QP thermalization overestimates n_{qp}, so it gives a conservative estimation to η_{read}. With assumption 3., we can deduce n_{qp} directly from $Q_{i,qp}$ using the thermal $Q_{qp,th}(n_{qp})$ relation without knowing T_{qp}.

We start by using Eqn. (2), where now Q_i comes from the fit of each readout power and $Q_{i,0}$ from the lowest power (-50 dBm). Note again all data use is at 244 mK. Fig. (8) shows the Q_i of different powers, with $Q_{i,0}$ also plotted for comparison. Immediately one sees the data contradicts our expectation in the way that the -50 dBm data has similar but not larger quality factors to all the high power curves except to 0 dBm. Also $Q_{i,0}$ is not as dominantly large compared to all the non-0 dBm Q_i. If $Q_{i,0}$ is indeed a constant independent of temperature as we have assumed, the low $Q_{i,0}$ implies we will be extracting relatively small dQ on top of a large $Q_i \sim Q_{i,0}$ as signal when we further cool the device to $\mathcal{O}(10)$ mK. The situation can be better visualized by plotting $Q_{i,0}$-removed Q_i (Fig. (9)). Only 0 dBm curve has comparable-or-smaller values to $Q_{i,0}$, while all other curves show much higher quality factors, meaning only with 0 dBm does energy dissipate comparably to QP generation and internal loss, but with less power, most of the energy dissipates to internal loss.

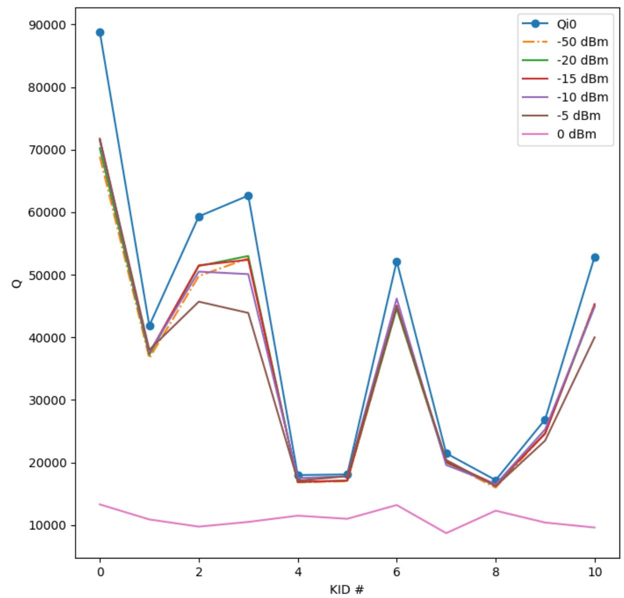


FIG. 8

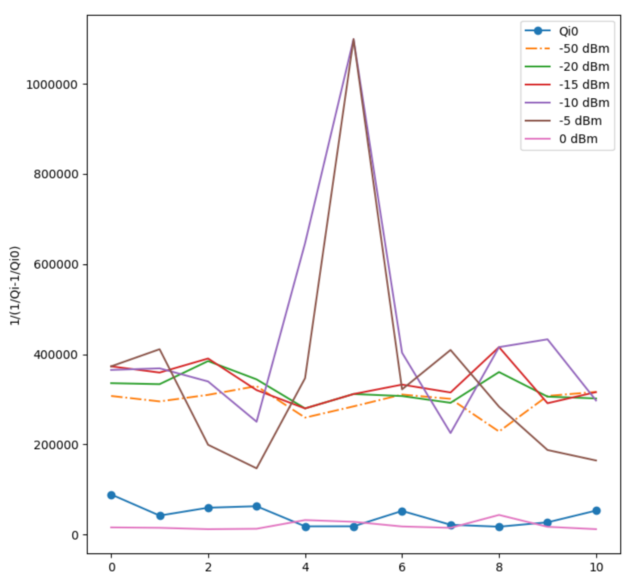


FIG. 9

We now interpolate/extrapolate the corresponding n_{qp} for these $Q_{i,0}$-removed Q_i, which we will sloppily call Q_{qp} until the end of this section, using -50 dBm "thermal" $Q_{qp,th}(n_{qp,th})$ relation as template. Since Q_{qp} is inverse proportional to n_{qp} (Eqn. (2)), instead of linearly interpolating Q_{qp} to n_{qp}, the interpolation is done between $1/Q_{qp}$ and n_{qp} to keep the error at linear order for small n_{qp}. The result is shown in Fig. (10)

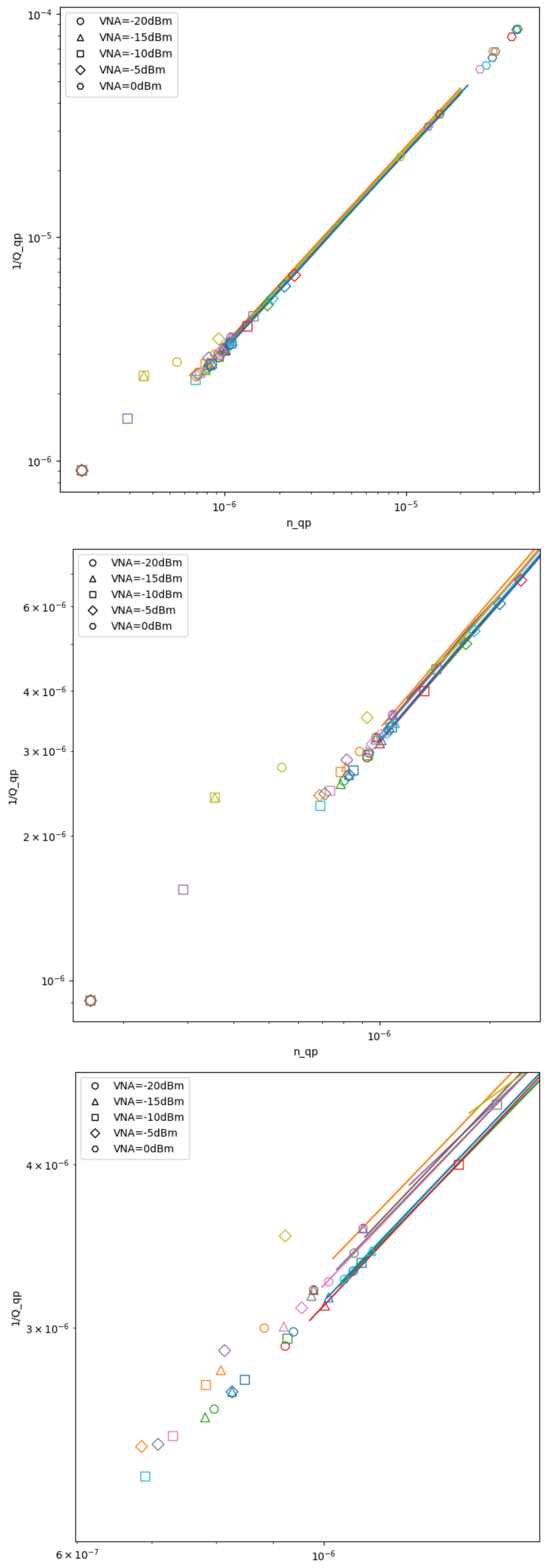

VNA=-20dBm
VNA=-15dBm
VNA=-10dBm
VNA=-5dBm
VNA=0dBm
1/Q_qp
n_qp
10^{-4}
10^{-5}
10^{-6}
10^{-6}
10^{-5}
VNA=-20dBm
VNA=-15dBm
VNA=-10dBm
VNA=-5dBm
VNA=0dBm
1/Q_qp
n_qp
6×10^{-6}
4×10^{-6}
3×10^{-6}
2×10^{-6}
10^{-6}
10^{-6}
VNA=-20dBm
VNA=-15dBm
VNA=-10dBm
VNA=-5dBm
VNA=0dBm
1/Q_qp
n_qp
4×10^{-6}
3×10^{-6}
6×10^{-7}
10^{-6}

FIG. 10

The 11 lines are the -50 dBm temperature sweep data of the chosen resonances, so the data points from left to right correspond to increasing temperatures and $n_{qp,th}$, and the left endpoints are of 244 mK base temperature. The hollow markers are the 244 mK power sweep data, whose n_{qp} are interpolated/extrapolated from the lines of the same color, denoting the same resonances. We see only the 0 dBm points lie in the high n_{qp}, low Q_{qp} region, while all other points are focused around the left endpoints of the thermal QP lines. It's equivalent to what we have explained in Fig. (8) of having only 0 dBm curve lies far from $Q_{i,0}$, while all other curves are close to it. We can also see in the zoom-in plots in Fig. (10) readout power is indeed positively correlated with $1/Q_{qp}$, i.e. readout power degradesQ_{qp}, as expected, although the effect is all very small compared to 0 dBm points. What's peculiar and somewhat unphysical is that there are many points sit at Q_{qp} larger than the left endpoints of the lines, i.e. on the left of the lines, which are basically the same base-temperature points but taken with the lowest power. It can also be seen in Fig. (9) at where the Q_{qp} exceeds the -50 dBm line.

We are ready to plug in the n_{qp} obtained from $Q_{qp,th}(n_{qp,th})$ to Eqn. (8) as $n_{qp,tot}$ and use $n_{qp,th}$ from low power data to calculate η_{read}, but first we need to sort out the factors for converting P_g to P_{read}. Fig. (11) shows the fitted Qc. It's mostly a constant independent of readout power as expected except for the divergent fit for KID#8, which we will discard. Unfortunately, the only Qc curve differs from the others is the 0 dBm one, and it's the only one we think we observe a significant generation of QP. We also need P_g specified *for each resonance*, meaning except the total attenuation to VNA power before DMKID, which is about -47 dB with a few dB uncertainty, we also need to know the off-resonance attenuation due to feedline for each KID. However, since we don't know the total amplification after DMKID, we can only read off the *relative* attenuation from S21. We decide to account for the feedline attenuation relative to the least attenuated resonance, KID#3, which is 9 dB higher in readout power than the most attenuated one, KID#10. Fig. (12) explains the relative attenuation with arbitrary zero. Note that this calibration only compensates the uncertainty by assuming all the KIDs accept the same readout power as the device input, but we still have an unknown attenuation, which now equals to the feedline attenuation to KID#3, and it's an unknown constant scaling to the power, and thus η_{read}. It's worth noting that this scaling can only reduce P_g, so it should only increase η_{read} once obtained in the future.

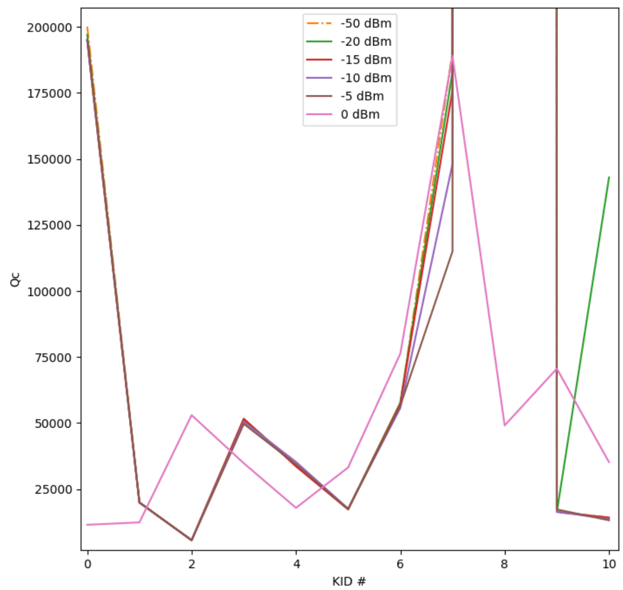


FIG. 11

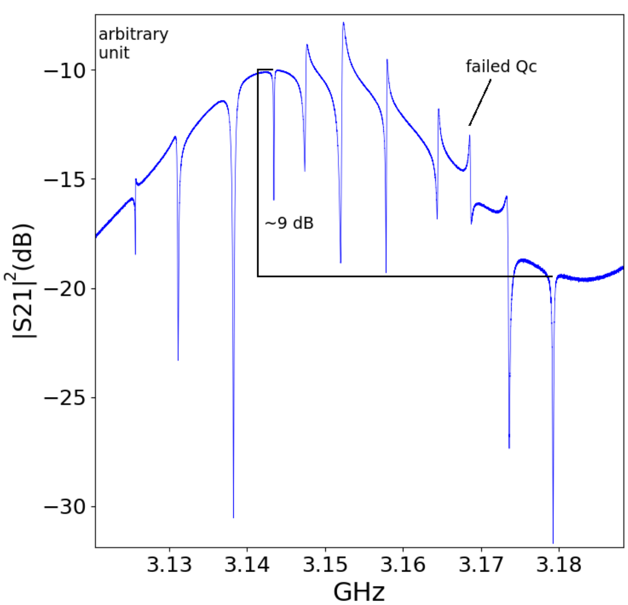
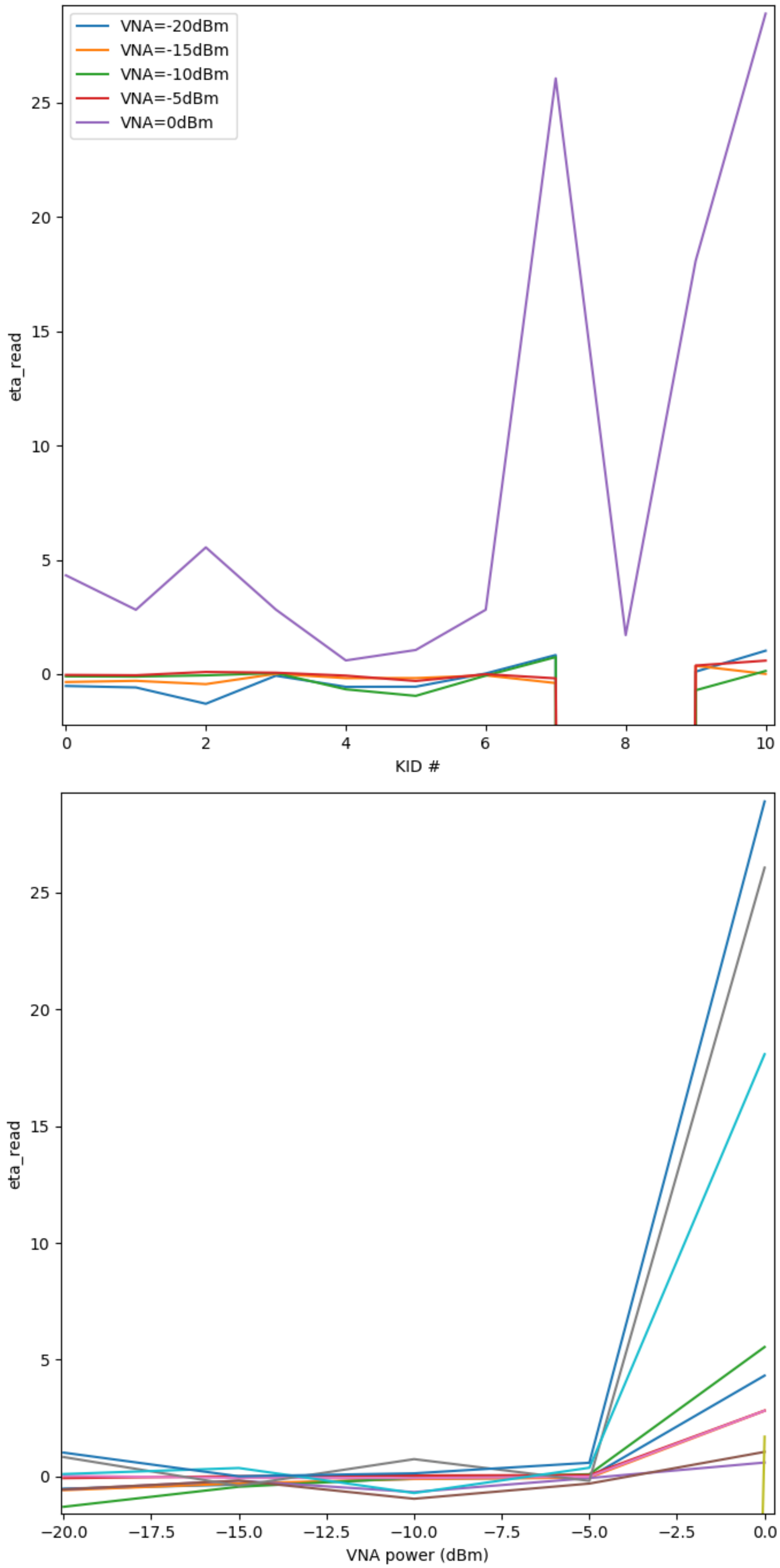


FIG. 12

Finally, the result of η_{read} is shown in Fig. (13).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VNA=-15dBm
VNA=-10dBm
VNA=-5dBm
VNA=0dBm
eta_read
KID #
0
2
4
6
8
10
5
15
20
25
eta_read
VNA power (dBm)
−20.0
−17.5
−15.0
−12.5
−10.0
−7.5
−5.0
−2.5
0.0

FIG. 13

DISCUSSION

T_{qp} (and $n_{qp,read}$) extraction

In the procedure described in last section, we first infer T_{qp} from a calculable $Q_{i,th}$ *by assuming the amount of QP being created by readout power is the same at 0 K and T_{qp}*, and then we can calculate κ at $T_{qp} = T_{qp,th}(Q_{i,th})$, which is then subsequently used for the $Q_{i,qp}$-based n_{qp} calculation. Another intuitive alternative is simply use an available $n_{qp}(Q_{qp})$ relation to convert Q_{qp} to its corresponding n_{qp}. In this case, different assumption made underlies the choices of the $n_{qp}(Q_{qp})$ "lookup table" and the Q_{qp} "input". One certain advantage is one skips the intermediate step of determining T_{qp}, which introduces calculation uncertainty from either the data of $Q(T)$ or, more likely, (linear) interpolation error from sparse temperature sweep. However, we still want to understand the physical assumptions for a better usage of the latter method. We begin by stating the two methods are mathematically equivalent, thus under the same $\Gamma_{read}(0) = \Gamma_{read}(T_{qp})$ assumption, if one chooses to use the Q_i with $Q_{i,0}(\textit{high power})$ being subtracted from. At this point the only available $n_{qp}(Q_{qp})$ tables would be either $n_{qp}(Q_i)$ or $n_{qp}(Q_{qp,th})$ from low temperature data, and we've already explained $n_{qp}(Q_{qp,th})$ matches theory well, so it's the lookup table to be use without introducing unnecessary uncertainty from incontrollable $Q_{i,0}(T = 0)$. On the other hand, one may also use the Q_i with $Q_{i,0}(\textit{low power})$, which is closer to the film's 0 K irreducible internal loss, being subtracted as the input to $n_{qp}(Q_{qp,th})$. It implies one treats the Q_i contributed by all sources except internal loss, thermal- and RF-generated QP alike, as one *thermalized* $Q_{qp,th}$ and looks for its corresponding $n_{qp,th}$. When external energy generates QPs from QP system, i.e. Cooper pair condensate, these QPs are in a completely out of equilibrium state with the Cooper pair-thermal QP subsystem, which by themselves are in thermal equilibrium. *Only when these "hot" QPs can immediately transfer the energy to the thermal QP subsystem and drive the whole system to maximal entropy state, are this $Q_{qp,``th"}$ and corresponding $T_{qp,``th"}$ and $n_{qp,``th"}$ representative to the system.* As $\kappa(T)$ is a decreasing function in temperature and is in the denominator of Eqn. (1), and this $T_{qp,``th"}$ is now an overestimation of T_{qp}, one ends up overestimating n_{qp} and also η_{read}. Note that, however, as we would like to be conservative on the estimation of η_{read}, this method serves as a reasonable upper limit estimation, while Fig. (4) and Fig. (5) may be viewed as recommended and lower limit estimates, respectively. (T_{qp}/T_{sub} and η_{read} plots generated this way look NO DIFFERENT from Fig. (4), implying $Q_{i,0}(high/low\ power)$ are indistinguishable, and we are not operating in readout-generated QP-dominated regime!)

CONCLUSION

Uh...no conclusion yet. According to Fabien's report, QP is not being generated predominately by readout power until VNA power is above -5 dBm, but here we are using -20 dBm for *high*. This explains the resulting η_{read} all scatters around 0, and no QP heating effect observed. We do have data taken at -5 dBm, but it's not a temperature sweep, which is necessary for determining α and $Q_{i,0}$. We need to either recool the device in yellow dewar and take data with high power, or cool it in dilution fridge to further suppress thermal QP. On a sidetrack, to my surprise $Q_{i,0}$ is non-negligible compared to $Q_{i,th}$ at this high temperature (240 mK). It needs to be subtracted in order to make $Q_{i,th}$ match theoretical thermal curve. Would it mean we will be dominated by $Q_{i,0}$ for internal loss going colder, thus unable to improve Q_i? It's sub-to-1 10^6, which won't change Q_{tot} but...uh...frustrating? The methodology is explained, and the code is made. Will try on higher power and see what happen.