# LRC Circuit

KDSMIL001 PHY2004W

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## Contents

# 1 Introduction

In this report we will investigate the LRC circuit in two different forms; the parallel resonant circuit and the series resonant circuit. More on what those are a bit later. For now we just need to know that an LRC circuit is comprised of an inductor (L), a resistor (R), and a capacitor(C), as well as some kind of applied voltage. In this experiment we will be using an alternating voltage as it results in some interesting behaviours. We will get into these behaviours more in later sections but for now all we need to know is that any kind of LRC circuit will have a resonant driving frequency at which the current (and thus the voltage) as well as the impedance in the circuit will be at an extremum. In order to see how each configuration will behave, let's investigate them analytically first.

# 2 The Parallel Resonance LRC Circuit

The parallel resonance set-up for an LRC circuit is one in which the inductor and capacitor are wired in parallel with each other and then that inductor-capacitor module is wired in series with the resistor, as shown below.

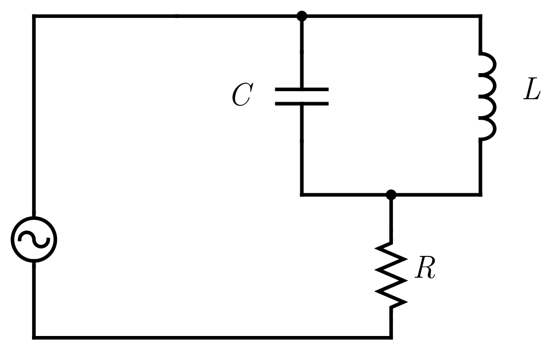


Figure 2.1: Parallel Resonance Circuit

In order to mathematically describe these circuits we use what we call a Transfer Function, in this case denoted $H(\omega)$, which is the magnitude of the ratio of output voltage across the resistor $V_{out}$ over the applied voltage $V_{in}$. Finding these voltages in terms of inductance and so on leads us to

$$H(\omega) = \frac{R(1-\omega^2 LC)}{\sqrt{R^2(1-\omega^2 LC)^2 + \omega^2 L^2}} \tag{2.1}$$

which, when differentiated, has a minimum at $\omega_0 = \frac{1}{\sqrt{LC}}$. Note that this $\omega$ is $2\pi f$ where $f$ is the driving frequency. If we plot this function with some values for $L, C, R$, we get something like Figure 2.2.

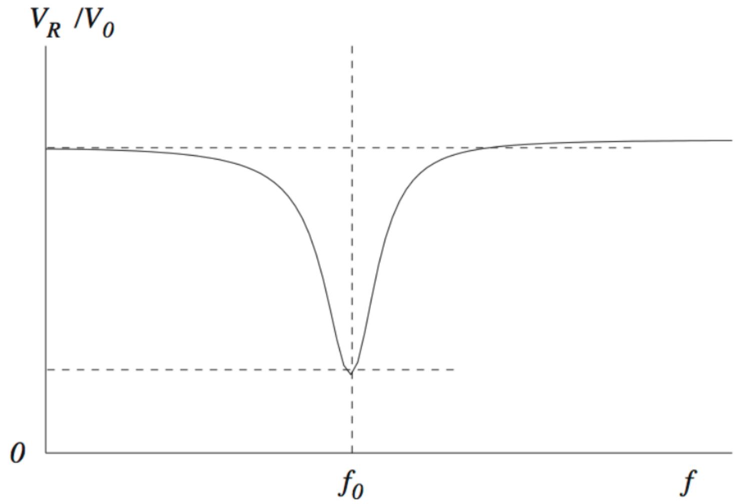


Figure 2.2: Parallel Resonance Curve

We'll discuss this more in the following section.

# 3 The Series Resonance LRC Circuit

In this set-up we have all three components, the inductor, capacitor, and resistor, in series, as below.

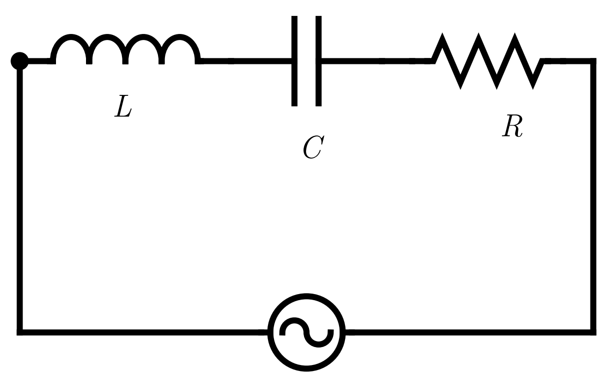


Figure 3.1: Series Resonance Circuit

As you might imagine, the transfer function for this configuration is slightly different to Equation 2.1. In fact we will derive it now.

We begin with the definition of the transfer function:

$$H(\omega) = \frac{V_{out}}{V_{in}}$$

where $V_{in}$ is the voltage supplied by our AC source and $V_{out}$ is the voltage drop across the resistor, which we know will be

$$V_{out} = iR$$

but as this is a series circuit we know that the current in all the components will always be the same value

$$i = \frac{V_{in}}{|Z(\omega)|}$$

where $Z(\omega)$ is the impedance of this circuit. Again this is a series circuit so the total impedance is merely the sum of the impedance of the respective components.

$$\begin{aligned}
Z(\omega) &= Z_L + Z_C + Z_R \\
&= (0 + jL\omega) + (0 - j\frac{1}{C\omega}) + (R + 0j) \\
&= R + j(L\omega - \frac{1}{C\omega})
\end{aligned}$$

where we're using $j$ as the imaginary unit. Current is not complex, which is why we have $|Z|$ previously. We can calculate this value

$$\begin{aligned}
|Z(\omega)| &= \sqrt{Z \cdot Z*} \\
&= \sqrt{(R + j(L\omega - \frac{1}{C\omega}))(R - j(L\omega - \frac{1}{C\omega}))} \\
&= \sqrt{R^2 + (L\omega - \frac{1}{C\omega})^2} \\
&= \sqrt{R^2 + L^2\omega^2 - 2\frac{L}{C} + \frac{1}{C^2\omega^2}} \\
&= \frac{1}{C\omega}\sqrt{C^2\omega^2R^2 + L^2C^2\omega^4 - 2LC\omega^2 + 1} \\
&= \frac{1}{C\omega}\sqrt{(C\omega R)^2 + (1 - LC\omega^2)^2}
\end{aligned}$$

Which means that we have

$$\begin{aligned}
i &= \frac{V_{in}}{\frac{1}{C\omega}\sqrt{(C\omega R)^2 + (1 - LC\omega^2)^2}} \\
&= \frac{V_{in}C\omega}{\sqrt{(C\omega R)^2 + (1 - LC\omega^2)^2}} \\
\implies V_{out} &= \frac{RV_{in}C\omega}{\sqrt{(C\omega R)^2 + (1 - LC\omega^2)^2}} \\
\implies H(\omega) &= \frac{RV_{in}C\omega}{V_{in}\sqrt{(C\omega R)^2 + (1 - LC\omega^2)^2}}
\end{aligned}$$

So we have our transfer function for the series resonance circuit:

$$H(\omega) = \frac{RC\omega}{\sqrt{(C\omega R)^2 + (1 - LC\omega^2)^2}} \tag{3.1}$$

This equation has the same extremum as the parallel circuit, except it has a maximum at $\omega_0 = \frac{1}{\sqrt{LC}}$ rather than a minimum. Plotting this function we did in Figure 2.2 we get

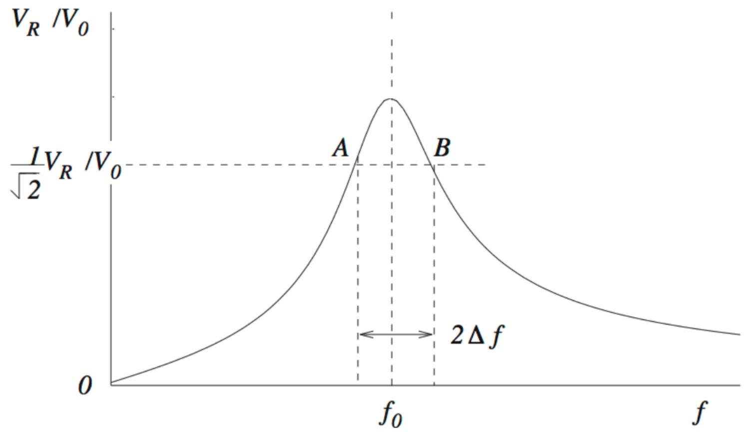


Figure 3.2: Series Resonance Curve

These plots have $f$ along the horizontal axis as that's the input frequency but that is no problem as $f = \frac{\omega}{2\pi}$. From the values of $2\Delta f$ and $f_0$ we can define a parameter we call the quality factor

$$Q = \frac{f_0}{2\Delta f} = \frac{\omega_0 L}{R}$$

that describes the "sharpness" of this curve, the larger $Q$ is, the sharper the curve. From Figure 3 we see that the resistance and inductance in the circuit has an impact on $Q$ and thus on the shape of the resonance curve. Note that this quality factor and its associated usefulness is not unique to the series resonance circuit, it applies to the parallel circuit too and we will use this in later sections to analyse specific circuits.

# 4 Experiment

In the experimental section of this report we will be investigating the behaviour of both the parallel and series resonance circuits by way of setting them up on a breadboard to see how they behave in reality, as well as using a simulation of each.

## 4.1 Apparatus

We used the following items, all with standard measurement uncertainty of 2%:

- 1 1 k$\Omega$ resistor (actual measured value 984.4 $\Omega$).
- 2 100 $\Omega$ resistors (actual measured value 98.0 and 98.6 $\Omega$).
- 1 96.51 nF capacitor (actual measured value 93.94 nF).
- 1 70 mH inductor (actual measured value 73.62 mH).
- 1 myDAQ.
- 1 screwdriver.
- 1 function generator.
- 1 digital oscilloscope.
- 1 breadboard.
- Wire (with negligible resistance).

Our parallel circuit was set up with the 1 k$\Omega$ resistor and the series circuit was set up with the two 100 $\Omega$ resistors in series to achieve a total resistance of 200 $\Omega$. They were set up in the same configuration as in Figure 2.1 and Figure 3.1.

## 4.2 Method

To collect our data (thanks Prof Blumenthal) we started off using the function generator to apply a sinusoidal alternating voltage across the parallel and series circuits with the oscilloscope measuring the voltage across the resistor. By adjusting the frequency of the function generator we were able to find the point at which the voltage across the resistor was minimised or maximised, depending on the type of circuit. The oscilloscope displays for each frequency are pictured in Figure 4.1 and Figure 4.2

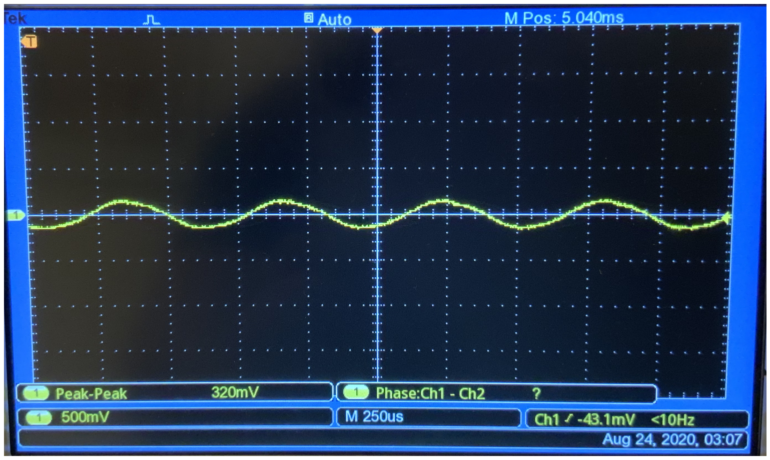


Figure 4.1: Parallel Resonance Circuit at resonance

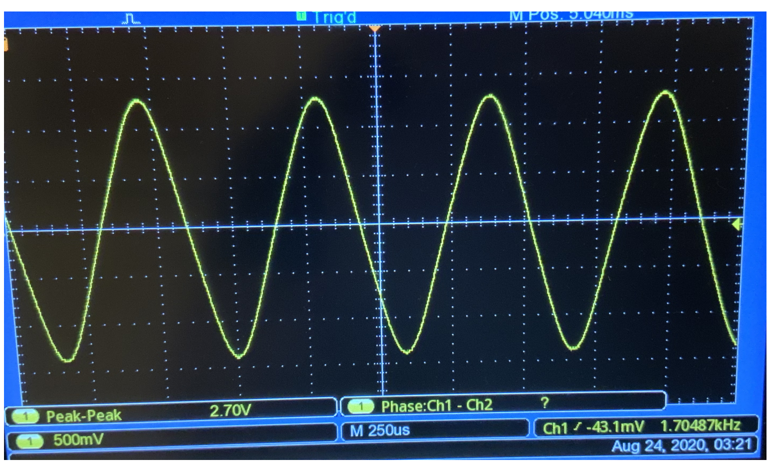


Figure 4.2: Series Resonance Circuit at resonance