How Much Higgs?

KDSMIL001 — September 2022

1 Introduction

This report aims to investigate the χ^2 and p-value test statistics using data from the ATLAS experiment, reconstructing the mass of the Higgs boson from the 4-lepton system. Our model gives us predictions for the background signal as well as the signal coming from the Higgs. To fit this model to our data we had two parameters to choose from: s_s multiplying the Higgs signal and s_b multiplying the background.

We first attempted fitting our model with just one parameter at a time, finding the χ^2/dof value for each fit. We could then try fitting with both parameters at the same time and look at that χ^2/dof value, comparing it to the one-parameter fits.

The p-value for the model before and after the fitting was the next thing in our sights, comparing the two to decide how likely the it is that the fitted model correctly describes the data we see.

Lastly we investigated the critical value needed to construct a confidence interval of 68% for a two-parameter fit

2 Exercises

2.1 Before fitting

Before fitting our model to the data, we first looked at the quality of the fit as it came from the data provided. Figure 2.1 shows the data, with $\chi^2/\text{dof} = 1.598$. This is a reasonable value as it's close to 1, but we can definitely do better. Note that for this report the χ^2/dof values will be approximate, rounding to 3 decimal places. This was chosen as any precision past 3 decimal places seemed to be insignificant in relation to the differences in χ^2/dof between fits.

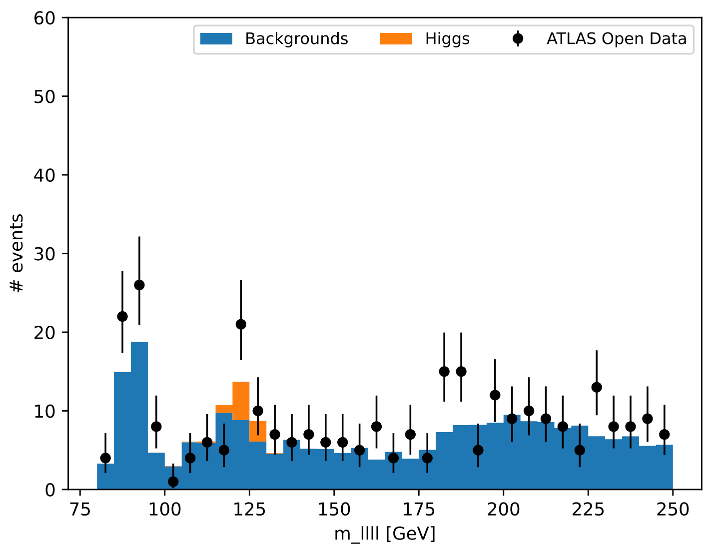


Figure 2.1: ATLAS open data of the mass of the 4-lepton system around the predicted mass of the Higgs boson. Also shown is the simulated prediction for the distribution of masses, made up of a background signal and a Higgs signal. For this model, $\chi^2/\text{dof} = 1.598$.

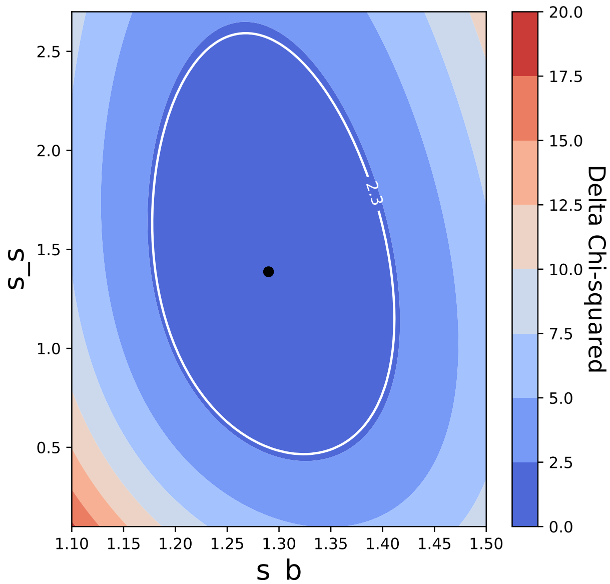
2.2 One-parameter fits

Our first attempts at fitting the model to our data were one-parameter fits, changing the multiplicative factor s_s for the Higgs signal first. At each new value we found a χ^2 value for the fit then subtracted the lowest value from the entire set, giving us a $\Delta\chi^2$ value for each s_s value in our interval. For a one-parameter fit, a confidence interval of 68% is defined as all values for which $\Delta\chi^2 \leq 1$ [1]. From this we could extract an uncertainty on the best-fit parameter. Our best-fit value was found to be $s_s = 2.01 \pm 0.76$ with $\chi^2/\mathrm{dof} = 1.558$.

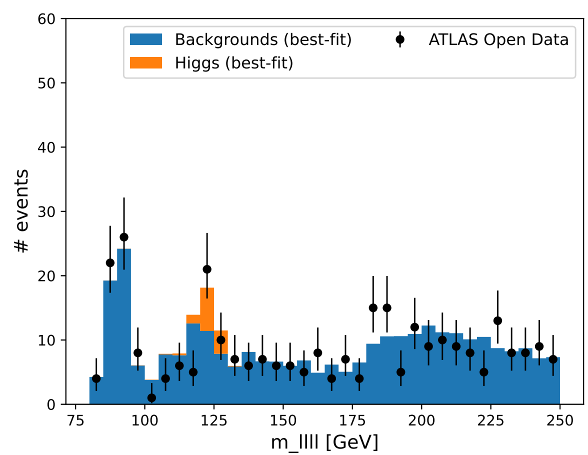
Doing the same but for s_b, multiplying the background signal, we found $s_b = 1.302 \pm 0.076$ with $\chi^2/\mathrm{dof} = 1.018$. This parameter clearly has more of an impact on the fit than s_s as can be seen from the lower χ^2/dof value. It also has a smaller uncertainty, which we can take as an indication that s_b is the right parameter to vary as our algorithm is quite sure that it should be around 1.3, not just 1.

2.3 A two-parameter fit

Of course, fitting just one parameter at a time is naive, so we then fit them both at the same time, looping over both and finding the minimum χ^2 point. This gave us best-fit values of $s_s = 1.39^{+1.16}_{-0.87}$ and $s_b = 1.29^{+0.12}_{-0.11}$ with $\chi^2/\mathrm{dof} = 1.039$. The estimation of these uncertainties came again from the 68% confidence interval, defined for a two-parameter fit as all pairs of values for which $\Delta\chi^2 \leq 2.3$ [1]. Figure 2.2a shows the contour plot of $\Delta\chi^2$ with respect to the $(s_s,\, s_b)$ pair as well as the contour of the critical value 2.3 defining the confidence interval. Fixing one parameter at its best-fit value, we were able to extract the upper and lower bound on the uncertainty for each parameter. This critical value of 2.3 will be investigated later on. Figure 2.2b shows the model fit to the data.



(a) Contour plot showing the $\Delta\chi^2$ value for each $(s_s,\, s_b)$ pair. The confidence interval on the best-fit pair (black dot) is shown as the area inside the contour at $\Delta\chi^2 = 2.3$. Upper and lower bounds on the confidence interval for each parameter were extracted to determine uncertainties.



(b) Histogram showing the data and the newly fitted model. $s_s = 1.39^{+1.16}_{-0.87}$, $s_b = 1.29^{+0.12}_{-0.11}$, and $\chi^2/\mathrm{dof} = 1.039$.

Figure 2.2: Contour plot and histogram for the two-parameter fit.

2.4 Comparing the one- and two-parameter fits

As seen before, the one-parameter fitting was much better when changing the scale of the background signal as opposed to the Higgs signal. This seems to make sense as the background signal is present over the entire range of interest, while the Higgs signal is only significant for a few of the bins in figure 2.1, so we might expect that changing the scale of the background has a larger impact on the overall fit of the model to our data. What is interesting about the two-parameter fit is that its χ^2/dof value is actually larger than that for the one-parameter background fit. Not only that, but the uncertainties on both s_s and s_b, for their respective one-parameter fits, are smaller than those determined from the two-parameter fit.

We can compare the best-fit values at a basic level, noting that for the two-parameter fit, s_s is a fair bit smaller than when fit on its own. This makes sense as in the two-parameter fit, the signal has already been lifted by the now larger background signal. It doesn't make sense, however, to compare the values with respect to their uncertainties, saying whether they agree or disagree. The reason for this is exactly what has been said: the signal sits on top of a different background, so it's been shifted by some amount. The fact that the best-fit s_b value doesn't change by much makes sense as well as its goal is to optimise the fit over the entire range of interest, only a small portion of which contains the Higgs signal. For this reason, it's likely to settle on the same best-fit value whether we include s_s or not.

We can also examine the shape of the $\Delta\chi^2 = 2.3$ contour in figure 2.2a, noting that it is slightly asymmetric. If it were a perfect ellipse with its axes aligned with the axes of the plot, we would be able to say there is no correlation between s_s and s_b, but there is clearly a non-zero amount of negative correlation, as well as an asymmetry as can be seen by the uneven shape at the bottom.

2.5 Goodness of fit

The p-value statistic is another useful measure of the goodness of fit of a model to some data. It is closely related to *chisq* but offers a different interpretation. Before fitting, so for figure 2.1, we found $p = 0.014877$. This means that if we assume our model to be correct, the chance of seeing the data we did, or anything more extreme, is around 1.5%. In other words, it's not that likely that this model is correct. After our two-parameter fit we found $p = 0.40642$, meaning that there's around a 41% chance to see data as or more extreme than ours, given our model is true. This is much better as, if the model perfectly described the process that created the data, we would expect an average p-value of 0.5 over many sets of data. Clearly our fitting process improved our model considerably.

2.6 Investigating the critical value for a two-parameter fit

Earlier, we simply quoted the critical value for a two-parameter fit as being 2.3, supported by Avni [1], but we would like to verify for ourselves that this is the correct value. To restate the problem, we need to find the critical value, which we will call d, for which 68% of all experiments have the "true value" within the region defined by that d. The true value in this case is the pair of best-fit values found before. To do this we generate a number of toy datasets, generating random bin values distributed according to gaussian distributions with means equal to the bin values of the fitted model and standard deviations equal to the square root of the means. We then check, for a range of d values, what fraction f of these toys have the best-fit pair within the confidence interval defined by d and the $\Delta\chi^2$ array for that toy. Figure 2.3 shows the results.

What can easily be seen in figure 2.3 is that a fraction of 0.68 apparently requires a critical value of around 3.3, quite far off the theoretical value of 2.3. This discrepancy could have a number of explanations.

Firstly, our search for the best-fit values was never going to return the exact right parameter values as we searched on a grid with a fixed spacing. It's more likely than not that the true value actually lies somewhere between the points that our method could test. Another method, perhaps using a gradient descent method or shrinking the grid spacing a few times to hone in on the value, would have yielded more accurate results.

Secondly, we made what we believe to be an incorrect assumption about the distribution of bin values. We needed to assume that they were gaussian distributed in order to generate the toys, but given that the best-fit model has bin values around 10 to 20, it's not safe to assume a gaussian distribution as there are

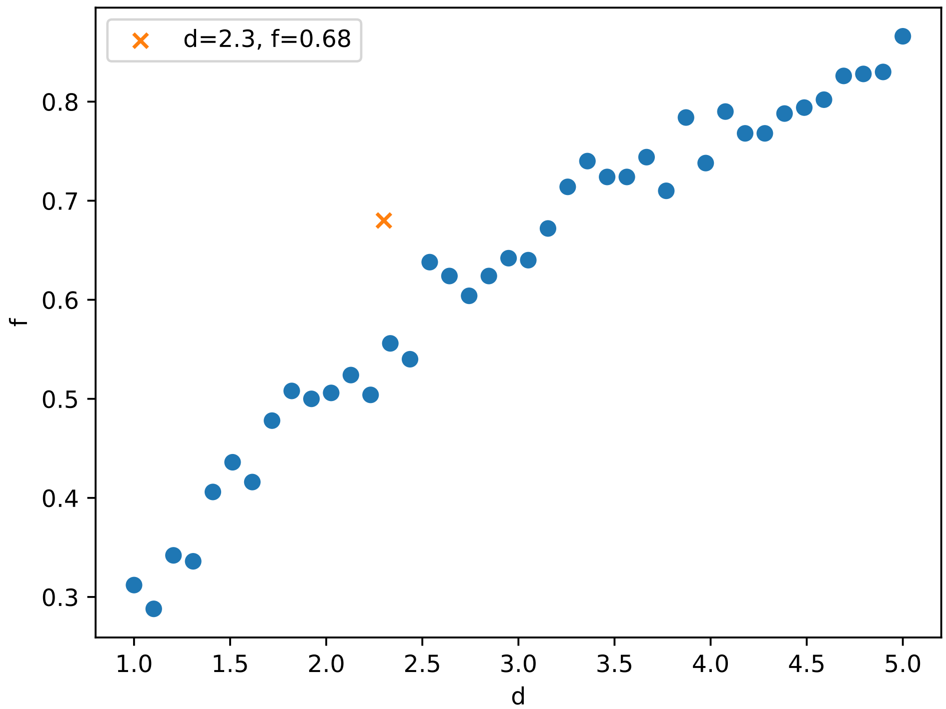


Figure 2.3: Fraction of toys f whose confidence interval for a given d contains the best-fit point as determined in section 2.3. Shown in orange is the expected d value for a fraction of 0.68 for a two-parameter fit. For each d value, 500 toys were used.

just too few events per bin. It is for this reason that we see f trending to a more shallow gradient as d increases and thus our critical value for a 68% confidence interval becomes about 3.3.

3 Conclusion

References

[1] Y. Avni. “Energy spectra of X-ray clusters of galaxies.” In: *Astrophysical Journal* 210 (Dec. 1976), pp. 642–646. DOI: 10.1086/154870.

A Code

First we import some crucial python modules that will allow us to analyse the data.

In [1]:
```
import uproot3
import uproot3_methods.classes.TLorentzVector as LVepm
import matplotlib.pyplot as plt
import time
import infofile
import numpy as np
import mplhep as hep
from scipy import stats
```

We need the following helper function to help use to calibrate the simualtion, we dont need to worry about what it actually does.

In [2]:
```
def get_xsec_weight(sample):
    info = infofile.infos[sample] # open infofile
    xsec_weight = (lumi*1000*info["xsec"])/(info["sumw"]*info["red_eff"]) #
*1000 to go from fb-1 to pb-1
    return xsec_weight # return cross-section weight
```

Here we are just defining the names of the files from where we will read the data and simulation. All data and MC samples should be downloaded to the local directory from https://vula.uct.ac.za/x/nF3H2b https://atlas-opendata.web.cern.ch/atlas-opendata/samples/2020/4lep/. The last file is the real collision data from ATLAS, the others are simulations of the physics processes we expect to have occured in the collisions.

In [3]:
```
files = [
"mc_361106.Zee.4lep.root",
"mc_361107.Zmumu.4lep.root",
"mc_410000.ttbar_lep.4lep.root",
"mc_363490.llll.4lep.root",
"mc_363492.llvv.4lep.root",
"mc_363356.ZqqZll.4lep.root",
"mc_345060.ggH125_ZZ4lep.4lep.root",
"mc_341964.WH125_ZZ4lep.4lep.root",
"mc_344235.VBFH125_ZZ4lep.4lep.root",
"mc_341947.ZH125_ZZ4lep.4lep.root",
"data.4lep.root"
]
```

We need to select out the most interesting collisions from the data and simulation so that we can as clear as possible signal of Higgs boson production. The exact details of the cuts are not very important. In summary, we select collision in which four leptons (electrons or muons) have been detected.

In [4]:

```
#------------DEFINING SOME VARIABLES AND OBJECTS FOR THE ANALYSIS-----------------------------------------------------

lumi = 10#fb^-1
nBins = 34

minMass = 80
maxMass = 250

bins_ar = np.linspace(minMass, maxMass, num=(nBins+1))
mc_hist_list = []
sample_names = []

f = plt.figure()

H_125 = np.zeros([nBins])
H_bkg = np.zeros([nBins])

btagWP77 = 0.6459

#-----------------------------------------------------------------------------------------------------------------------


for file in files: #looping over data and simulation files
    sample_name = file.split(".")[1]
    sample_names.append(sample_name)
    tree = uproot3.open("Data/"+file)["mini"]

    mcWeight, SumWeights, XSection, trigM, trigE, scaleFactor_PILEUP, scaleFactor_ELE, scaleFactor_MUON,scaleFactor_LepTRIGGER, lep_type, lep_pt, lep_eta, lep_phi, lep_E, lep_charge, lep_etcone20, lep_ptcone30, jet_n, jet_pt, jet_eta, jet_phi, jet_E, jet_MV2c10 = tree.arrays(["mcWeight", "SumWeights", "XSection","trigM", "trigE","scaleFactor_PILEUP", "scaleFactor_ELE", "scaleFactor_MUON","scaleFactor_LepTRIGGER","lep_type","lep_pt", "lep_eta","lep_phi", "lep_E", "lep_charge", "lep_etcone20", "lep_ptcone30", "jet_n", "jet_pt", "jet_eta", "jet_phi","jet_E", "jet_MV2c10"], outputtype=tuple)
    print("File has been successfully opened!")

    leplv = LVepm.TLorentzVectorArray.from_ptetaphi(lep_pt, lep_eta, lep_phi, lep_E)

    lep_reliso_pt = (lep_ptcone30 / lep_pt)
    lep_reliso_et = (lep_etcone20 / lep_pt)
    sum_lep_type = lep_type.sum()

    jetlv = LVepm.TLorentzVectorArray.from_ptetaphi(jet_pt, jet_eta, jet_phi, jet_E)
    jetlv = jetlv[jet_MV2c10.argsort()]
    tags = jet_pt[jet_MV2c10 > btagWP77]

    trig_cut = ( (trigM==1) | (trigE==1))
    lep_kinematics_cut  = ( (lep_pt.max() > 20000) & (lep_pt.min() > 7000) & (lep_eta.min() >-2.5) & (lep_eta.max() < 2.5))
    lep_type_cut  = ((sum_lep_type == 44) | (sum_lep_type == 48) | (sum_lep_type == 52))
    lep_iso_cut =  ((lep_reliso_pt.max() < 0.3) & (lep_reliso_pt.max() < 0.3))
    lept_count_cut = (leplv.counts ==4)
    lept_charge_cut = (lep_charge.sum()==0)

    # filtering events according to cuts above that select out interesting events
    event_cut = ( lep_kinematics_cut & lep_type_cut & lep_iso_cut  & lept_count_cut & lept_charge_cut)
```

```
    first_lep_p4 =  leplv[event_cut,0]
    second_lep_p4 = leplv[event_cut,1]
    third_lep_p4 =  leplv[event_cut,2]
    fourth_lep_p4 = leplv[event_cut,3]
    mcWeight = mcWeight[event_cut]

    scaleFactor_PILEUP = scaleFactor_PILEUP[event_cut]
    scaleFactor_ELE = scaleFactor_ELE[event_cut]
    scaleFactor_MUON = scaleFactor_MUON[event_cut]
    scaleFactor_LepTRIGGER = scaleFactor_LepTRIGGER[event_cut]

    #construct 4-lepton system by adding 4-vectors of the leptons vectorial
ly
    llll_p4 = first_lep_p4 + second_lep_p4 + third_lep_p4 + fourth_lep_p4


    # make histograms of the m_llll distribution for simulation and data
    if(file.split("_")[0] == "mc"):
        finalWeight = get_xsec_weight(sample_name)*(mcWeight)*(scaleFactor_
PILEUP)*(scaleFactor_ELE) *(scaleFactor_MUON)*(scaleFactor_LepTRIGGER)
        H, b = np.histogram(llll_p4.mass/1000.0, weights=finalWeight, bins=
bins_ar)
        mc_hist_list.append(H)
        if("H125" in file):
            H_125 = np.add(H, H_125)
        else:
            #print("Sample Name = " + str(file) + " exp. num. events = " +
 str(np.sum(finalWeight)) )
            H_bkg = np.add(H, H_bkg)
    else:
        finalWeight = np.ones(len(mcWeight))
        sample_name = "Data (10 fb^-1)"
        H_data, b = np.histogram(llll_p4.mass/1000.0, weights=finalWeight,
bins=bins_ar)
```

```
File has been successfully opened!
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<Figure size 640x480 with 0 Axes>
```
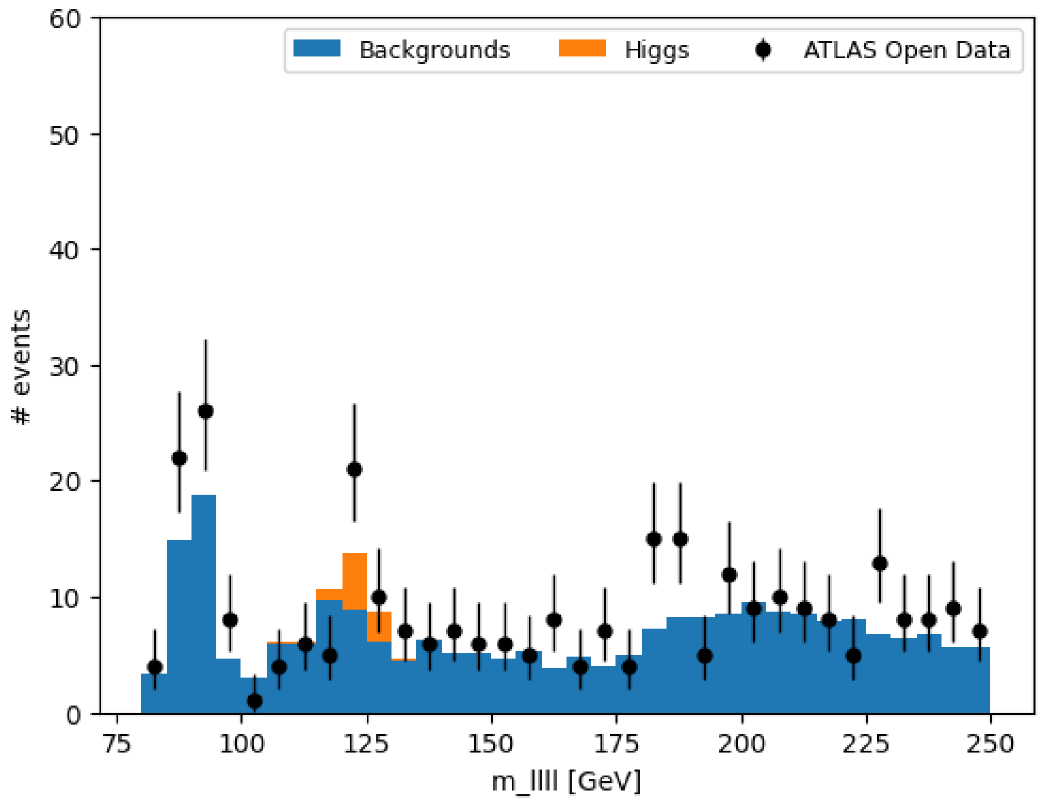

With our events selected, we can compare our data with our expectations from theory. Our best theory of particle physics is known as the Standard Model, so the predictions here represent the expectations from the Standard Model after a simulation of how the ATLAS detector detects particles has been applied.

Specifically, we plot a histogram of the number of collisions ("events") as a function of the mass of the four-lepton system. We use this mass because it distinguishes between the the Higgs signal, which has a peak at the higgs mass (125 GeV) and is zero elesehere, and the backgrounds which have a peak around 90 GeV and are flatter elsewhere.

In [5]:
```
hep.histplot([H_bkg, H_125], bins=bins_ar, stack=True, label=["Backgrounds"
, "Higgs"], histtype='fill')
hep.histplot([H_data], bins=bins_ar, stack=False, yerr=True, histtype="erro
rbar", color="black",label="ATLAS Open Data")

plt.legend(loc=1, ncol=3, fontsize=9)
plt.xlabel("m_llll [GeV]")
plt.ylabel("# events")
plt.ylim([0.0, 60])
# plt.savefig('../Report/Plots/no_fit_hist.pdf')
plt.show()
```



We start by fitting one parameter at a time and comparing the two fits using a chi-squared function.

Our chi-squared function is defined in the usual way, noting that the denominator is simply the predicted mean as we assume the data to be Poisson distributed.

In [6]:
```
def calcChiSq(obs, preds, numParams):
    #REPLACE THIS FUNCTION WITH A VALID CHI-SQUARED CALCULATION
    chiSq = np.sum( np.power( ( obs - preds ), 2) / ( preds ))
    ndf = len(obs) - numParams
    return chiSq, ndf
```

Here we find the chi-square/dof value for our model before any fitting happens

In [9]:
```
chi2_before_fit, dof_before_fit = calcChiSq(H_data, H_125 + H_bkg, 0)
print(f'Before fitting, chi-square/dof = {chi2_before_fit/dof_before_fit}')
```

```
Before fitting, chi-square/dof = 1.597845078902861
```

Fitting s_s

In [12]:

```
s_s_ar = np.linspace(1.0, 3.0, 100) # array of s_s values we will investigate

chi2_ar = np.empty( len(s_s_ar) ) # empty array of that will hold the chi2 values we will calculate

minChi2 = 1000000
mindof = 1
bestFit_s_s = 0.0 #starting values for min chi2 and best value of s_s

for s_s in range(0, len(s_s_ar)): #looping over out s_s values
    pred = (s_s_ar[s_s]*H_125) + (H_bkg) # generating a prediction according to this s_s
    chi2, dof = calcChiSq(H_data, pred, 1) # calculating chi2
    chi2_ar[s_s] = chi2 # adding chi2 value to chi2 array
    if(chi2 < minChi2): # check if this is the lowest chi2 we have seen so far
         minChi2 = chi2 # update lowest chi2 value seen
         mindof = dof   # update dof for lowest chi2
         bestFit_s_s = s_s_ar[s_s] # update value for best fit s_s

deltaChi2_ar = chi2_ar - minChi2 # make array of delta chi2 values

#we expect the chi2 vs. mZ curve to be quadratic, so let's fit that function to it.
z = np.polyfit(s_s_ar, deltaChi2_ar, 2) #"2" for a second-order polynomial
p = np.poly1d(z)

# we can display the estimated uncertianty on mZ via critical values of the delta chi-squared curve
y0 = 1.0
crit = (p - y0).roots # roots of the polynominal -1, i.e., the mz values where p = 1

#shading in the confience interval band
px=np.arange(crit[1],crit[0],0.001)

# fig, ax = plt.subplots()
# ax.plot(s_s_ar, deltaChi2_ar, 'k', linewidth=2, label="deltaChi2")
# ax.fill_between(px,p(px),alpha=0.5, color='g', label="uncertainty")

# plt.xlabel("s_s")
# plt.ylabel("delta chi-squared")
# plt.legend()
# plt.show()

print("Best fit value of s_s = " + str(round(bestFit_s_s, 3)) + " +/- " + str(round(np.abs(crit[0] - bestFit_s_s),2 )) )
print(f"minChi2/mindof: {minChi2/mindof}")
```

```
Best fit value of s_s = 2.01 +/- 0.76
minChi2/mindof: 1.5577434806307726
```

Fitting s_b

In [13]:
```
s_b_ar = np.linspace(1.15, 1.4, 100)

chi2_ar = np.empty( len(s_b_ar) )

minChi2 = 1000000
mindof = 1
bestFit_s_b = 0.0

for s_b in range(0, len(s_b_ar)):
    pred = (H_125) + (s_b_ar[s_b]*H_bkg)
    chi2, dof = calcChiSq(H_data, pred, 1)
    chi2_ar[s_b] = chi2
    if(chi2 < minChi2):
        minChi2 = chi2
        mindof = dof
        bestFit_s_b = s_b_ar[s_b]

deltaChi2_ar = chi2_ar - minChi2

z = np.polyfit(s_b_ar, deltaChi2_ar, 2)
p = np.poly1d(z)

y0 = 1.0 # this is the value of the delta chi-squared function that defines
the 68% CI for a one parameter fit.
         # we'll invetigate if this parmaeter makes sense in the final exer
cise.
crit = (p - y0).roots

px=np.arange(crit[1],crit[0],0.001)

# fig, ax = plt.subplots()
# ax.plot(s_b_ar, deltaChi2_ar, 'k', linewidth=2, label="deltaChi2")
# ax.fill_between(px,p(px),alpha=0.5, color='g', label="uncertainty")
# plt.xlabel("s_b")
# plt.ylabel("delta chi-squared")
# plt.legend()
# plt.show()

print("Best fit value of s_b = " + str(round(bestFit_s_b, 3)) + " +/- " + s
tr(round(np.abs(crit[0] - bestFit_s_b),3 )) )
print(f"minChi2/mindof: {minChi2/mindof}")
```

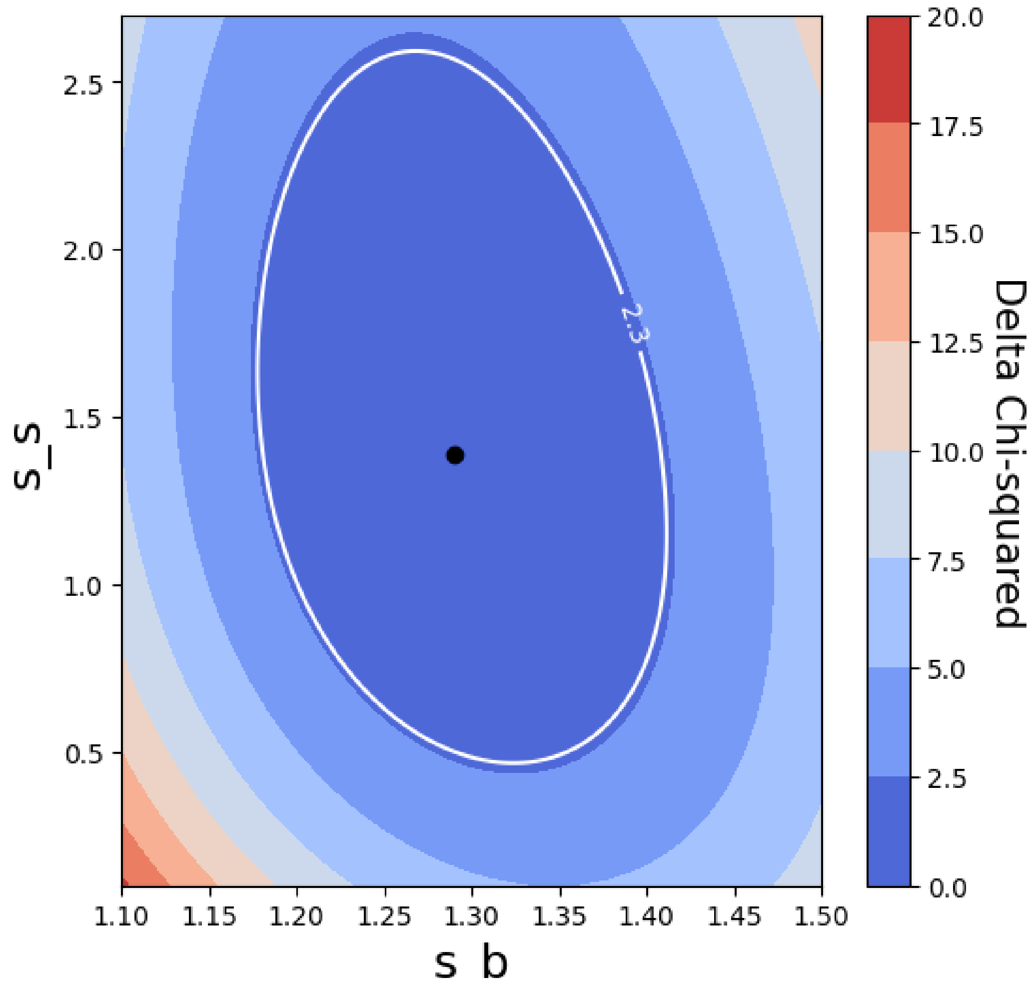
```
Best fit value of s_b = 1.302 +/- 0.076
minChi2/mindof: 1.0184456820283405
```

We can then move on to fitting them both at the same time.

Fitting s_s and s_b

In [14]:
```
s_s_ar = np.linspace(0.1, 2.7, 100)
s_b_ar = np.linspace(1.1, 1.5, 100)

chi2_ar = np.empty( (len(s_s_ar), len(s_b_ar) ))

bestFit_s_s = 0.0
bestFit_s_b = 0.0

minChi2 = 1000000
mindof = 1

for s_s in range(0, len(s_s_ar)):
    for s_b in range(0, len(s_b_ar)):
        pred = (s_s_ar[s_s]*H_125) + (s_b_ar[s_b]*H_bkg)
        chi2, dof = calcChiSq(H_data, pred, 2)
        chi2_ar[s_s, s_b] = chi2
        if(chi2 < minChi2):
            minChi2 = chi2
            mindof = dof
            bestFit_s_s = s_s_ar[s_s]
            bestFit_s_b = s_b_ar[s_b]

deltaChi2_ar = chi2_ar - minChi2
true_deltaChi2_ar = deltaChi2_ar    # For exercise 6

levels = [2.3] # this is the value of the delta chi-squared function that d
efines the 68% CI for a two parameter fit.
         # we'll invetigate if this parmaeter makes sense in the final exer
cise.
fig = plt.figure(figsize=(6,6))
ax = fig.gca()

cfset = plt.contourf(s_b_ar, s_s_ar, deltaChi2_ar,  cmap='coolwarm')
cset = plt.contour(s_b_ar, s_s_ar, deltaChi2_ar, levels=levels, colors=['wh
ite'])

bf_point = plt.scatter(bestFit_s_b, bestFit_s_s, color='black')
bf_point_index = [np.where(true_deltaChi2_ar == 0)[0][0], np.where(true_del
taChi2_ar == 0)[1][0]]


cbar = plt.colorbar(cfset)
cbar.set_label('Delta Chi-squared', fontsize=15, rotation=270, labelpad=15)
ax.clabel(cset, inline=1, fontsize=10)
ax.set_xlabel('s_b',fontsize=18)
ax.set_ylabel('s_s',fontsize=18)
# plt.savefig('../Report/Plots/two_fit_contour.pdf')
plt.show()

# Finding the bounds of the confidence interval for each parameter
s_s_bf_CI_left = s_s_ar[np.where(deltaChi2_ar[:,bf_point_index[1]] <= level
s[0])[0][0]]
s_s_bf_CI_right = s_s_ar[np.where(deltaChi2_ar[:,bf_point_index[1]] <= leve
ls[0])[0][-1]]
s_b_bf_CI_left = s_b_ar[np.where(deltaChi2_ar[bf_point_index[0],:] <= level
s[0])[0][0]]
s_b_bf_CI_right = s_b_ar[np.where(deltaChi2_ar[bf_point_index[0],:] <= leve
ls[0])[0][-1]]

s_s_bf_unc_left = np.abs(bestFit_s_s - s_s_bf_CI_left)
s_s_bf_unc_right = np.abs(bestFit_s_s - s_s_bf_CI_right)
s_b_bf_unc_left = np.abs(bestFit_s_b - s_b_bf_CI_left)
s_b_bf_unc_right = np.abs(bestFit_s_b - s_b_bf_CI_right)

#extract 2D result
```

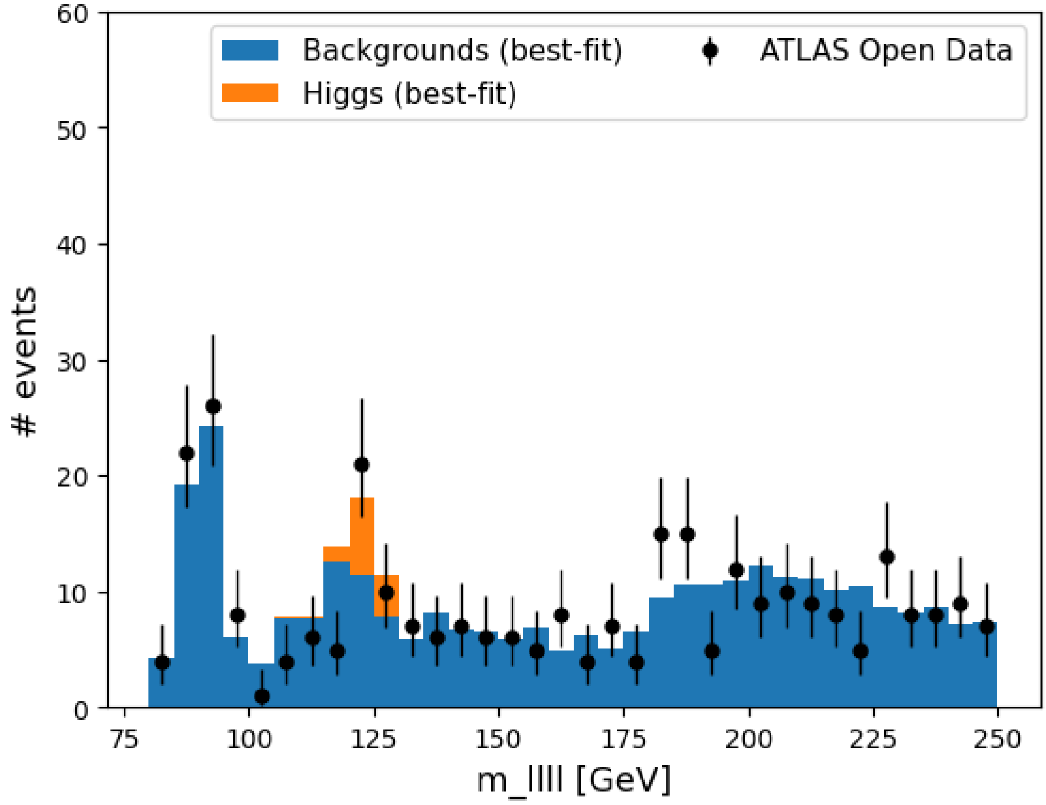
```
print(f'best-fit s_s = {bestFit_s_s:.5} (+ {s_s_bf_unc_right:.5} /- {s_s_bf
_unc_left:.5})')
print(f'best-fit s_b = {bestFit_s_b:.5} (+ {s_b_bf_unc_right:.5} /- {s_b_bf
_unc_left:.5})')
print(f'minChi2/mindof: {minChi2/mindof}')
```




```
best-fit s_s = 1.3869 (+ 1.1556 /- 0.86667)
best-fit s_b = 1.2899 (+ 0.11717 /- 0.10909)
minChi2/mindof: 1.038884905786871
```

With these s_s and s_b values, we can have a look at the model and how it compares to the data

In [15]:
```
f = plt.figure()

H_125_bf = bestFit_s_s*H_125
H_bkg_bf = bestFit_s_b*H_bkg

hep.histplot([H_bkg_bf, H_125_bf], bins=bins_ar, stack=True, label=["Backgr
ounds (best-fit)", "Higgs (best-fit)"], histtype='fill')
hep.histplot([H_data], bins=bins_ar, stack=False, yerr=True, histtype="erro
rbar", color="black",label="ATLAS Open Data")
plt.legend(loc=1, ncol=2, fontsize=11)
plt.xlabel("m_llll [GeV]", fontsize=13)
plt.ylabel("# events", fontsize=13)
plt.ylim([0.0, 60])
# plt.savefig('../Report/Plots/two_fit_hist.pdf')
plt.show()
```



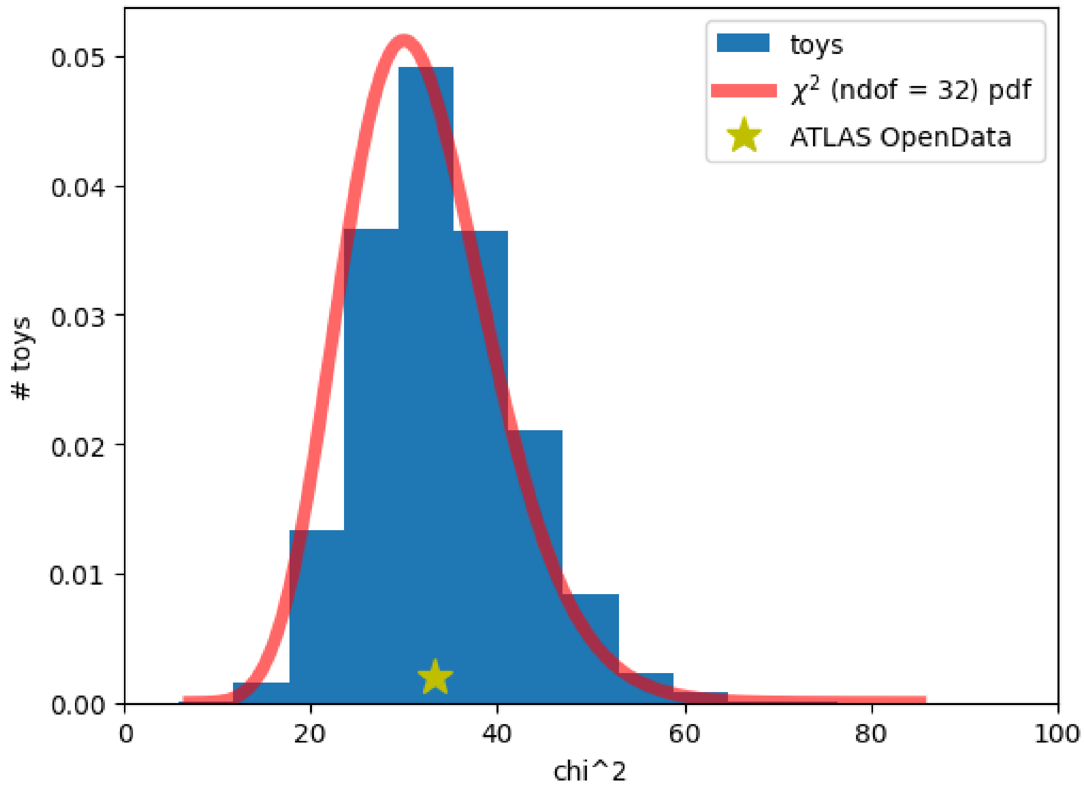
Goodness of fit

Here we can generate some toy distributions according to our model and see how the chi-square value for our data compares to that distribution

In [16]:
```
# generate the chi-2 distribution of N toy experiments based on the fitted
 model
# assume a gaussian-distributed bin height with mean = model pred. and sigm
a equal root model pred
# what then is the p-value of the data with respect to the distribution?

#function to generate a toy histogram given the mean for each bin and assum
ing the data is
# gaussian-distrubtu
def generate_toy(means):
        toy = np.empty(len(means))
        for i in range(0, len(means)):
            toy_bin = np.random.normal(means[i], np.sqrt(means[i]), 1)
            toy[i] = toy_bin
        #print("toy = " + str(toy))
        return toy

ntoys = 10000 # start with a small number, increase when you undertsand you
r results

means = H_bkg_bf + H_125_bf # take the results of the fit as the means of t
he fitted model

#print("means" + str(means))

chi2_toys = np.empty(ntoys)

for t in range(0, ntoys):
    toy = generate_toy(means)
    chi2_toys[t] = calcChiSq(toy, means, 0)[0]
    #print("chi2 " + str(chi2_toys[t]))

plt.figure()
fig, ax1 = plt.subplots()

#plot distribtion of chi-squared values from toys
bins_ar = np.linspace(0.0, 200, num=(nBins+1))
chi2Hist, chi2bins = np.histogram(chi2_toys, bins=bins_ar, density=True)
hep.histplot([chi2Hist], bins=chi2bins, histtype='fill', label="toys")

#plot expected distrutuon of chi-squared values from theory - chi-squared d
istribtuions with ndof = nbins -2
df = nBins - 2
x = np.linspace(stats.chi2.ppf(0.000001, df), stats.chi2.ppf(0.999999, df),
200)
label = '$\chi^{2}$ (ndof = ' + str(df) + ') pdf'
ax1.plot(x, stats.chi2.pdf(x, df), 'r-', lw=5, alpha=0.6, label=label)

# overlay the chi2 squared value from the ATLAS OpenData
chi2_data = calcChiSq(H_data, means, 0)[0]
plt.plot(chi2_data,0.002 , 'y*', ms=14, label='ATLAS OpenData')
plt.legend()
plt.xlabel("chi^2")
plt.ylabel("# toys")
plt.xlim(0,100)

print("chi2 value of ATLAS Open data is = " + str(chi2_data))
```

```
chi2 value of ATLAS Open data is = 33.24431698517987
```

```
<Figure size 640x480 with 0 Axes>
```



Related to that is the p-value. We can find it for the model before and after fitting, using the chi-square cumulative distribution function.

In [17]:
```
pVal_before_fit = 1 - stats.chi2.cdf(chi2_before_fit, dof_before_fit)    #
 No parameters so dof is just nBins

print(f'p value before fitting: {pVal_before_fit:.5}')

x = np.linspace(stats.chi2.ppf(0.000001, df), stats.chi2.ppf(0.999999, df),
200)
pVal = 1 - stats.chi2.cdf(chi2_data, df)

print(f'p value with fitting: {pVal:.5}')
```

```
p value before fitting: 0.014877
p value with fitting: 0.40642
```

Now we investigate the usage of a critical value of 2.3.

In [ ]:

```
d_arr = np.linspace(1, 5, 40)
N_toys = 500
model_means = H_125_bf + H_bkg_bf
min_chi_sq_arr = np.zeros( (len(d_arr), N_toys) )
min_dof_arr = np.zeros( (len(d_arr), N_toys) )

fracs = np.zeros(len(d_arr))

for d_index, d in enumerate(d_arr):
    num_within_d = 0
    for t in range(N_toys):
        chi2_ar_toy = np.empty( (len(s_s_ar), len(s_b_ar) ))

        H_toy = generate_toy(model_means)

        bestFit_s_s_toy = 0.0
        bestFit_s_b_toy = 0.0

        minChi2 = 1000000
        mindof = 1

        for s_s in range(0, len(s_s_ar)):
            for s_b in range(0, len(s_b_ar)):
                pred = (s_s_ar[s_s]*H_125) + (s_b_ar[s_b]*H_bkg)
                chi2, dof = calcChiSq(H_toy, pred, 2)
                chi2_ar_toy[s_s, s_b] = chi2
                if(chi2 < minChi2):
                    minChi2 = chi2
                    mindof = dof
                    bestFit_s_s_toy = s_s_ar[s_s]
                    bestFit_s_b_toy = s_b_ar[s_b]

        deltaChi2_ar_toy = chi2_ar_toy - minChi2

        if (deltaChi2_ar_toy[bf_point_index[0], bf_point_index[1]] <= d):
            num_within_d += 1


    fracs[d_index] = num_within_d / N_toys
```

In [24]:
```
frac_import = np.load('fracs.npy',)
plt.scatter(d_arr, frac_import)
plt.xlabel('d')
plt.ylabel('f')
plt.scatter([2.3], [0.68], label='d=2.3, f=0.68', marker='x')
plt.legend()
plt.savefig('../Report/Plots/d_f.pdf')
```