Determining the Rydberg constant from the Balmer series of hydrogen

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Abstract

We aim to find a value for the Rydberg constant by measuring the wavelengths of the visible hydrogen emission spectral lines and using the Balmer series to fit those wavelengths to a linear plot.

1 Introduction

The Balmer series is a describes a subset of the spectral line emissions of a hydrogen atom. The wavelengths of the lines in this series are given by the formula

$$\frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right) [2] \tag{1.1}$$

where $n = 2$, $m = 3, 4, 5, \ldots$, and R is the Rydberg constant, given by

$$R = \frac{m_e e^4}{8\varepsilon^2 h^3 c} = (10973731.568160 \pm 0.000021)\,\mathrm{m}^{-1}[1] \tag{1.2}$$

Note that Equation 1.1 describes much more than just the Balmer series but only the Balmer series is needed for this experiment, and only 4 of the many spectral lines in the series at that. These 4 lines are called H-α (m=3), H-β (m=4), H-γ (m=5), and H-δ (m=6) and they will be used as they are the 4 that (formally) lie within the visible spectrum.

To determine a value for R, the wavelengths of H-α, H-β, H-γ, and H-δ will be measured and a linear fit will be made with $\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$ as the x values and $\frac{1}{\lambda}$ as the y values, thus R will be the gradient.

2 Apparatus

A Heath EU-700 Czerny-Turner monochromator with a photo-multiplier detector and pulse-counting electronics was used to measure the wavelengths of the spectral lines coming from a hydrogen spectral tube. The measurement system has three primary measurement parameters: slit width, dwell time, and step increment. The system counts how many times a photon was incident on the detector for the given wavelength, incrementing through a range of wavelengths measured in Angstroms.

The monochromator reports an incorrect wavelength, off by about 20 Angstroms, so the set-up needed to be calibrated. A HeNe laser of known wavelength (6328 A) was fired at the measurement system as show in Figure 2.1.

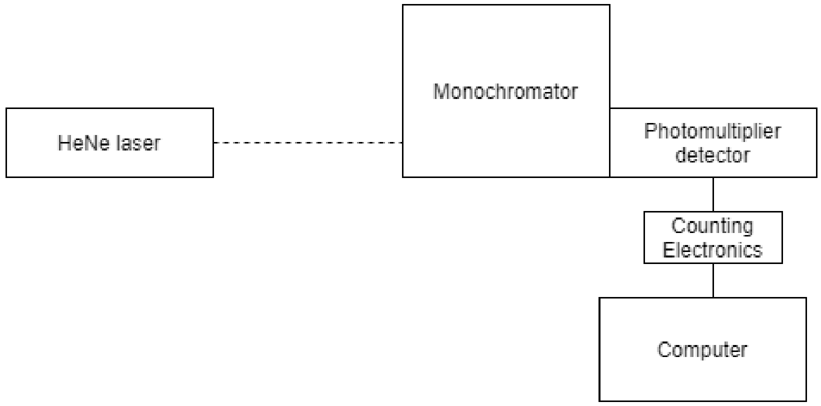

Figure 2.1: Calibration set-up

3 Method

3.1 Calibration

Using the wavelength of the HeNe laser as a known value, data was taken with a variety of parameters, the details of which can be found in section 4, around the expected value. Gaussians could be fitted to the data using `scipy.optimize.curve_fit` and a μ and σ extracted for each set. These values were combined in a mean weighted by their uncertainty σ using

$$\bar{\mu}_{wtd} = \frac{\sum\limits_{i=1}^{n} w_i \mu_i}{\sum\limits_{i=1}^{n} w_i} \tag{3.1}$$

$$\sigma_{wtd} = \sqrt{\frac{\frac{\sum_{i=1}^{n} w_i \mu_i^2}{\sum_{i=1}^{n} w_i} - (\bar{\mu}_{wtd})^2}{n-1}} \tag{3.2}$$

where $w_i = \frac{1}{\sigma^2}$ is the weighting [3].

The count data was assumed to be Poissonian and thus the uncertainty on a count of N is simply $\sqrt{N}$. This uncertainty was provided to `curve_fit`. Figure 3.1 shows an example of one calibration run.

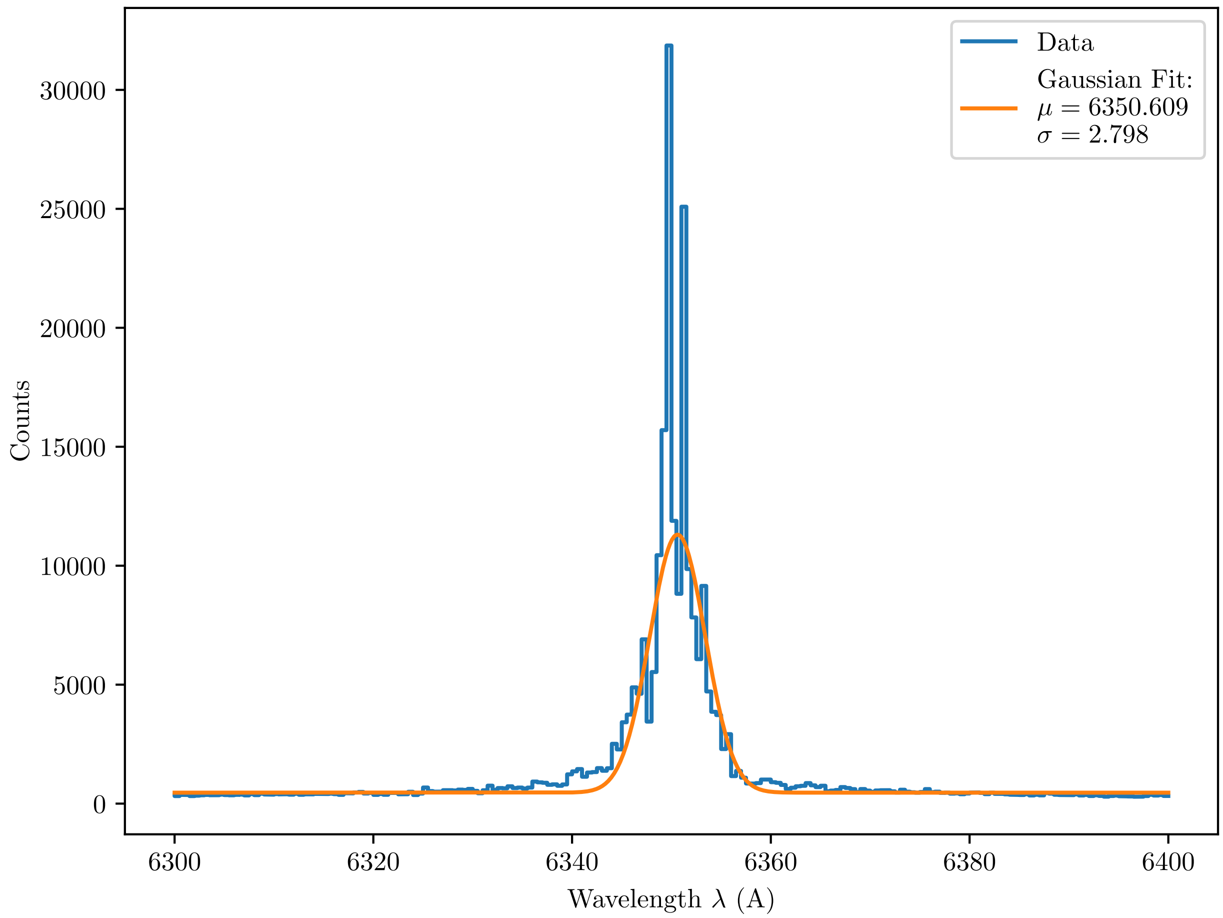

Figure 3.1: Calibration data with slit width 10 μm, dwell time 200 ms, increment 0.5 A, and on the range 6300-6400 A. Gaussian fit using `scipy.optimize.curve_fit` with the initial guess $\mu =$ the position of the maximum of the data and $\sigma = 1$. A vertical shift was included to account for background and an amplitude to account for Gaussians being a PDF.

The system scanned through an interval around the wavelength of the spectral lines with a variety of slit widths and wavelength increments

References

[1] 2018 CODATA list of the Fundamental Physical Constants, `https://physics.nist.gov/cuu/Constants/Table/allascii.txt`

[2] C. Foot, *Atomic Physics*, Oxford University Press, 2005

[3] Bevington, P. R., *Data Reduction and Error Analysis for the Physical Sciences*,McGraw-Hill, 1969

4 Appendix