Determining the Rydberg constant from the Balmer series of hydrogen

KDSMIL001 — 18 August 2021

Abstract

We aim to find a value for the Rydberg constant by measuring the wavelengths of the visible hydrogen emission spectral lines and using the Balmer series to fit those wavelengths to a linear plot.

1 Introduction

The Balmer series is a describes a subset of the spectral line emissions of a hydrogen atom. The wavelengths of the lines in this series are given by the formula

$$\frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right)[2] \tag{1.1}$$

where $n = 2$, $m = 3, 4, 5, \ldots$, and R is the Rydberg constant, given by

$$R = \frac{m_e e^4}{8\varepsilon^2 h^3 c} = (10973731.568160 \pm 0.000021)\,\mathrm{m}^{-1}[1] \tag{1.2}$$

Note that Equation 1.1 describes much more than just the Balmer series but only the Balmer series is needed for this experiment, and only 4 of the many spectral lines in the series at that. These 4 lines are called H-α (m=3), H-β (m=4), H-γ (m=5), and H-δ (m=6) and they will be used as they are the 4 that (formally) lie within the visible spectrum.

To determine a value for R, the wavelengths of H-α, H-β, H-γ, and H-δ will be measured and a linear fit will be made with $\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$ as the x values and $\frac{1}{\lambda}$ as the y values, thus R will be the gradient.

2 Apparatus

A Heath EU-700 Czerny-Turner monochromator with a photo-multiplier detector and pulse-counting electronics was used to measure the wavelengths of the spectral lines coming from a hydrogen spectral tube. The measurement system has three primary measurement parameters: slit width, dwell time, and step increment. The system counts how many times a photon was incident on the detector for the given wavelength, incrementing through a range of wavelengths measured in Angstroms.

The monochromator reports an incorrect wavelength, off by about 20 Angstroms, so the set-up needed to be calibrated. A HeNe laser of known wavelength (6328 A) was fired at the measurement system as show in Figure 2.1.

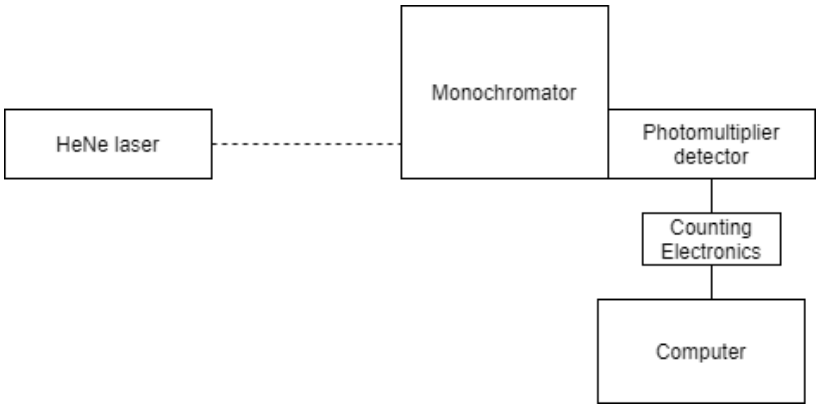

Figure 2.1: Calibration set-up

3 Method

3.1 Calibration

Using the wavelength of the HeNe laser (6328 A) as a known value, data was taken with a variety of parameters, the details of which can be found in section 4, around the expected value. Gaussians could be fitted to the data using `scipy.optimize.curve_fit` and a μ and σ extracted for each set. The difference between this μ and the known wavelength is the correction factor that needs to be applied to all measurements taken with this set-up. These corrections were combined in a mean weighted by their uncertainty σ using

$$\bar{\mu}_{wtd} = \frac{\sum_{i=1}^{n} w_i \mu_i}{\sum_{i=1}^{n} w_i} \tag{3.1}$$

$$\sigma_{wtd} = \sqrt{\frac{\frac{\sum_{i=1}^{n} w_i \mu_i^2}{\sum_{i=1}^{n} w_i} - (\bar{\mu}_{wtd})^2}{n-1}} \tag{3.2}$$

where $w_i = \frac{1}{\sigma^2}$ is the weighting [3].

The count data was assumed to be Poissonian and thus the uncertainty on a count of N is simply $\sqrt{N}$. This uncertainty was provided to `curve_fit`. Figure 3.1 shows an example of one calibration run. The final correction was found to be -22.46 ± 0.14.

3.2 Data

Data was taken for each of the spectral lines with a variety of parameters, the details of which can be found in section 4, and their wavelengths found as in subsection 3.1 by fitting a Gaussian. After subtracting the correction factor and combining the uncertainties of the wavelength and correction factor in quadrature, the wavelengths found for each spectral line were combined using Equation 3.1 and Equation 3.2 where again $w_i = \frac{1}{\sigma^2}$. Figure 3.2 shows an example for the fitting done.

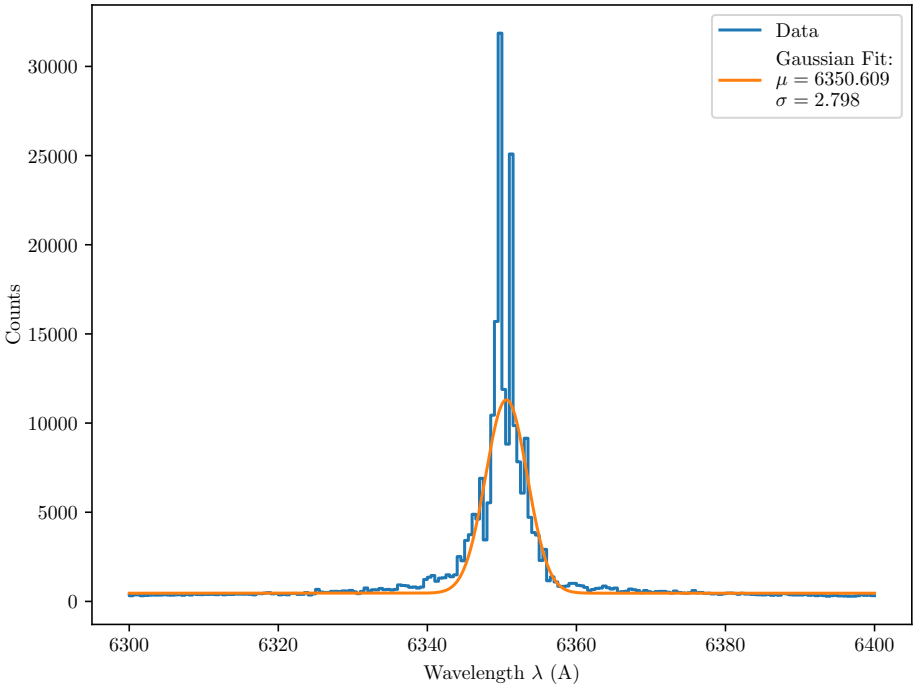

Figure 3.1: Calibration data with slit width 10 μm, dwell time 200 ms, increment 0.5 A, and on the range 6300-6400 A. The Gaussian was fit using `scipy.optimize.curve_fit` with the initial guess $\mu =$ the position of the maximum of the data and $\sigma = 1$. A vertical shift was included to account for background and an amplitude to account for Gaussians being a PDF.

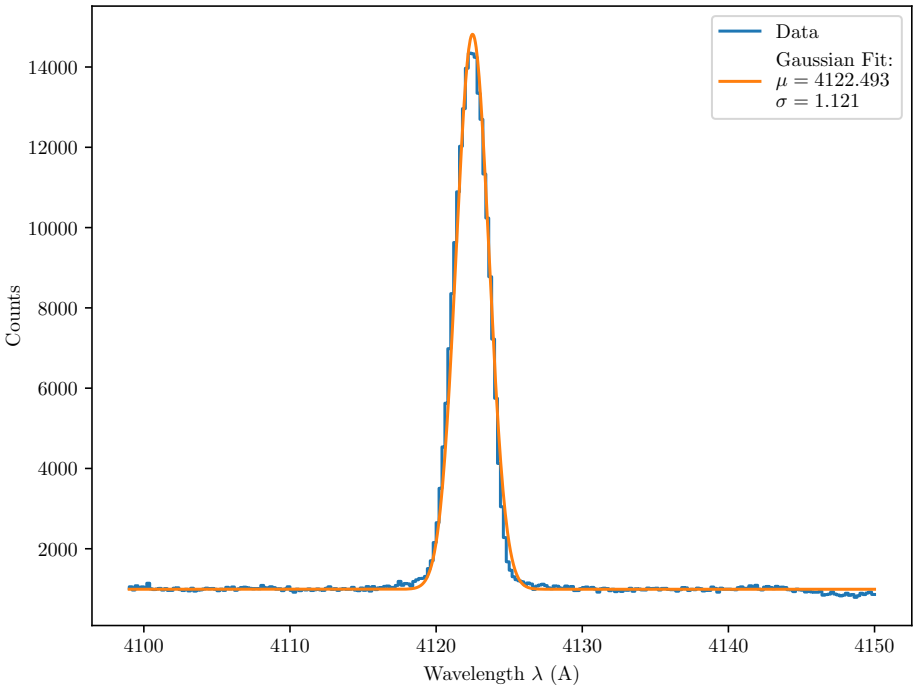

Figure 3.2: Data for spectral line H-δ with slit width 100 μm, dwell time 300 ms, increment 0.2 A, and on the range 4099-4150 A. The expected wavelength is 4101.734 ± 0.006[4] and after correction the measured wavelength is 4100.0 ± 1.1. The initial conditions for this fit were as in Figure 3.1

References

[1] 2018 CODATA list of the Fundamental Physical Constants, https://physics.nist.gov/cuu/Constants/Table/allascii.txt

[2] Foot, C., *Atomic Physics*, Oxford University Press, 2005

[3] Bevington, P. R., *Data Reduction and Error Analysis for the Physical Sciences*,McGraw-Hill, 1969

[4] Kramida, A., Ralchenko, Yu., Reader, J., and NIST ASD Team (2020). NIST Atomic Spectra Database (ver. 5.8), [Online]. Available: https://physics.nist.gov/asd [2021, August 21]. National Institute of Standards and Technology, Gaithersburg, MD. DOI: https://doi.org/10.18434/T4W

4 Appendix