# Capacitors

**11 August 2020**

**PHY2004W KDSMIL001**

## Contents

# 1 Introduction

In this report we will investigate capacitors, specifically in the context of a Resistor-Capacitor (RC) circuit. Firstly we will answer some questions regarding the theory of capacitors and RC circuits, then we will do some experiments and investigate the effects of different frequencies of AC voltage as well as different resistances on the behaviour of RC circuits.

# 2 Theory Questions and Answers

1. **Question**: How are $A$, $d$, and $\epsilon$ related, where $A$ is the area of one of the plates in a capacitor, $d$ is the distance between the two plates, and $\epsilon$ is the dielectric constant of the material between the plates.
   **Answer**: It's fairly easy to see that an increase in the area of the plates of a capacitor $A$ would result in an increase of the amount of charge each plate could hold. This in turn would increase the magnitude of the electric field between each plate and thus increase the potential difference between the plates when fully charged. This results in an increase in capacitance. Similarly the value of $\epsilon$, which is present in the equation for the electric field at a point, will have an effect on the capacitance. Increasing $\epsilon$ results in a decrease in $\vec{E}$ between the plates, allowing for more charge to build up on the plates. Finally, the value of $d$ will also have an inversely proportional effect on the electric field, meaning an increase in $d$ will lead to a decrease in $\vec{E}$ and thus a decrease in the amount of charge able to build up on the plates. This is all summed up in this equation for the capacitance of a capacitor:

$$C = \frac{A\epsilon}{d} \tag{2.1}$$

2. **Question**: By making use of dimensional analysis show that the units of the charging time of a capacitor $\tau$ as given by $\tau = RC$ is seconds (s).
   **Answer**: $R$ is given by $V/I$, the dimensions of which are

$$\begin{aligned} &\frac{M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}}{M^{\frac{1}{2}}L^{1\frac{1}{2}}T^{-2}} \\ \implies &L^{-1}T \end{aligned}$$

   We also know that $C$ is given by $Q/V$, the dimensions of which are

$$\begin{aligned} &\frac{M^{\frac{1}{2}}L^{1\frac{1}{2}}T^{-1}}{M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}} \\ \implies &L \\ \implies &[\tau] = [RC] = L^{-1}TL = T \end{aligned}$$

3. **Question**: Explain what effect an increase and decrease in frequency $\omega$ will have on the reactance and therefore the impedance of the $RC$ circuit.
**Answer**: The reactance is given by

$$X(\omega) = \frac{1}{\omega C}$$

meaning that reactance is inversely proportional to the frequency $\omega$. We also know that the impedance of an $RC$ circuit is given by $Z = R + iX(\omega)$. From this we can deduce that an increase in $\omega$ will result in a decrease in $X(\omega)$ which will in turn lead to the impedance being dominated more and more by $R$. As $\omega \to \infty, Z \to R$.

# 3 Practical Questions and Answers

4. **Question**: What is the correct way to insert a capacitor into a breadboard?
**Answer**: The correct way to insert a capacitor is Photo C, shown below:

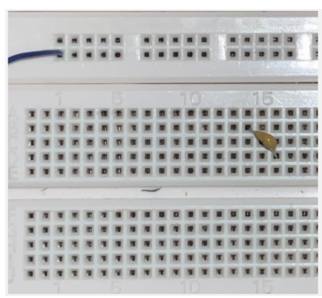


Figure 3.1: Photo C

The reason for this is the fact that in the other two pictures the capacitor was being short circuited, once by inserting both ends into the same power rail, the long connected pieces at the top and bottom, and once by inserting both ends into the same terminal strip, the shorter vertical pieces.

5. **Question**: Why does the charge on the capacitor (or the voltage across it) not reach the same minimum and maximum as the applied signal in the diagram below?

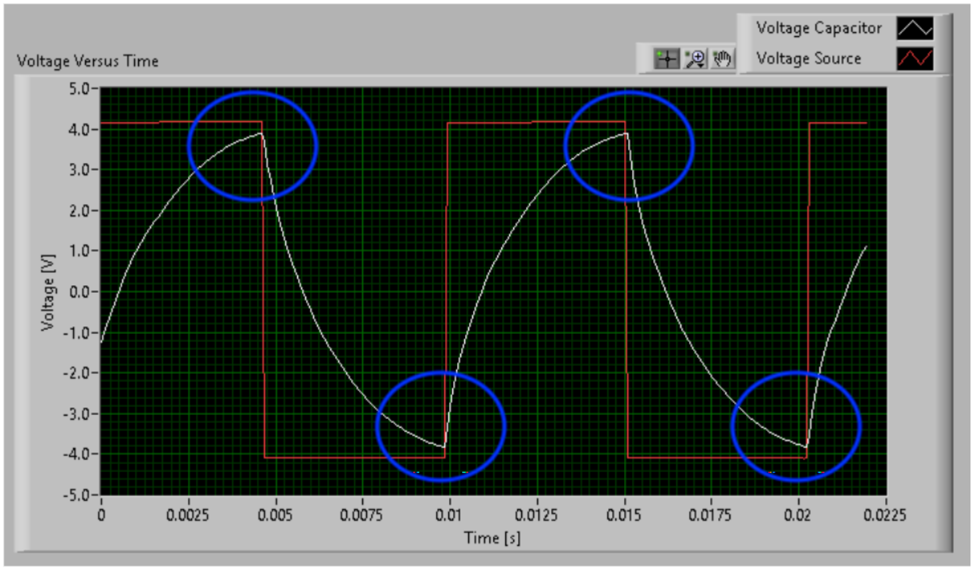


Figure 3.2: Voltage signal (white) across 100nF capacitor in series with 15 kΩ resistor

What would you change to ensure the capacitor reaches the same maximum and minimum as the applied signal?
**Answer**: The time it takes for a capacitor to reach its full capacity is governed by $\tau = RC$. This means that in order to decrease the time it takes to charge to its theoretical maximum we could decrease either the capacitance of the capacitor or the resistance of the resistor. This could be done by physically changing these components, such as decreasing the surface area of the plates of the capacitor, or simply by swapping out the components for those with a lower value.
We could also just reduce the frequency of the applied voltage, giving the capacitor more time to charge up before swapping the direction of the current, forcing it to discharge and charge up in the other direction.

# 4 Data Collecting

## 4.1 Resistance and Frequency Investigation

We were given data in the form of a text file that contained voltage measurements made by the myDAQ. The myDAQ measured the voltage across the 100nF capacitor in an RC circuit as well as across the function generator supplying a square wave voltage with peak-to-peak voltage of $V_{pp} = 8$. This circuit was set up with 5.6kΩ, 8.2kΩ, and 15kΩ resistors separately. Each circuit was recorded with 100Hz, 200Hz, 500Hz, and 1000Hz, giving us 12 datasets to work with. This data can be regarded as having a 2% uncertainty due to the accuracy of the equipment used. We used Python to interpret this data into the plots in subsection 5.1, the code for which is in Appendix 1

## 4.2 Decay Time from the Oscilloscope

The figure below is the screen of an oscilloscope measuring the voltage across the frequency generator and the capacitor in an RC circuit with capacitance C=100nF and resistance R=5.6kΩ. We want to determine the time it takes for the capacitor to fully discharge, that is the time difference between the capacitor being at full charge (in this case 4V) and it being at full charge in the "other direction", (having -4V across it).

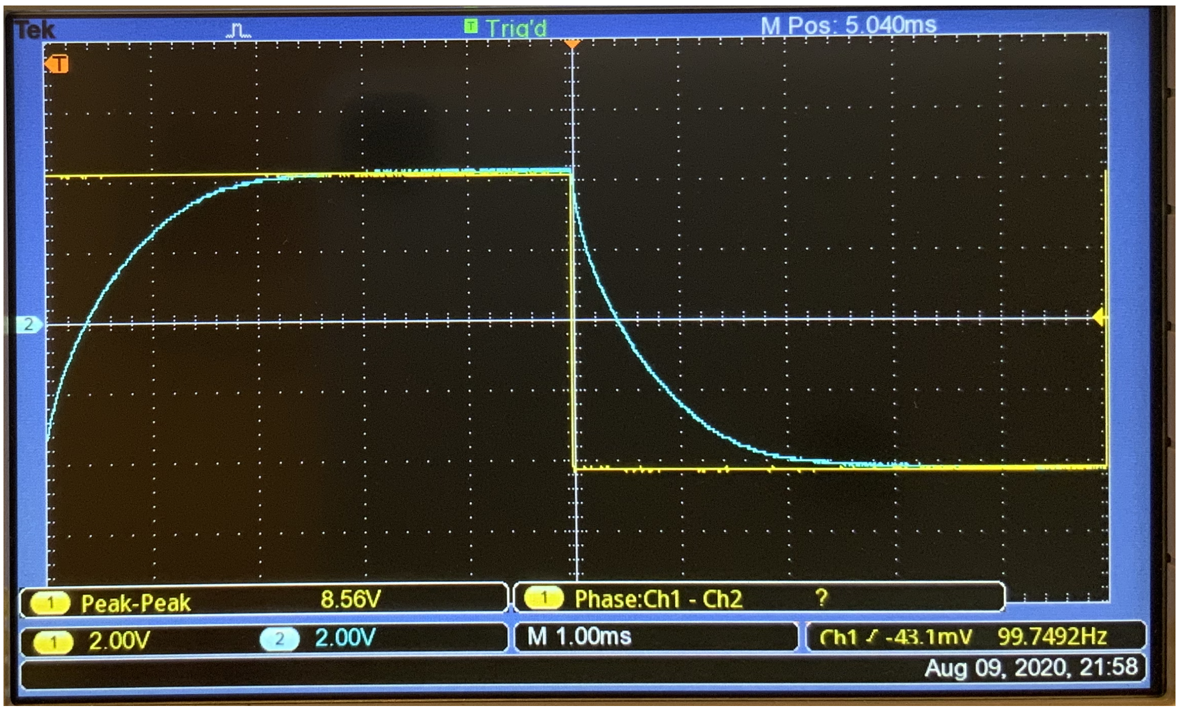



Figure 4.1: Oscilloscope Display

Looking at where the blue line meets the yellow line, it seems to properly come in contact after 17 subdivisions, which corresponds to 3.4ms. The is an analogue measurement so it has a triangular probability density function, so in order to calculate the uncertainty we use the following formula

$$u = \frac{a}{2\sqrt{6}} \tag{4.1}$$

where $a$ is the difference between our best upper bound and our best lower bound for the measurement. In our case our best guess could be between 3.2ms and 3.6 ms, so $a = 0.4$. This gives us $u = 0.081649$. We also need to consider that the oscilloscope has an uncertainty of 2%. Thus our uncertainty is $\sqrt{(0.081649)^2 + (2\% \cdot 3.4)^2} = 0.27325$ That gives us a final measurement of Decay Time $= 3.40 \pm 0.27$ms.

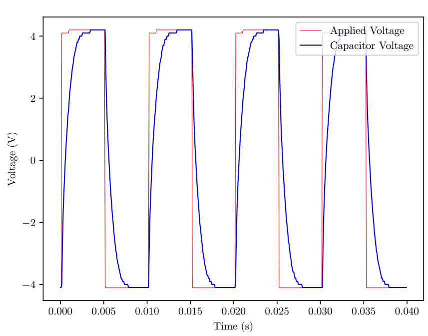
## 4.3 Current Measurement

In order to investigate the phase difference between current and source voltage in an RC circuit (subsection 5.3) we needed to collect some data. We did this by measuring the voltage supplied by the function generator, which was producing a sinusoidal signal with $V_{pp} = 8$, as well as the voltage across a 15k$\Omega$ resistor. This was done using the myDAQ as we did above. As before, the circuit was completed with a 100nF capacitor.

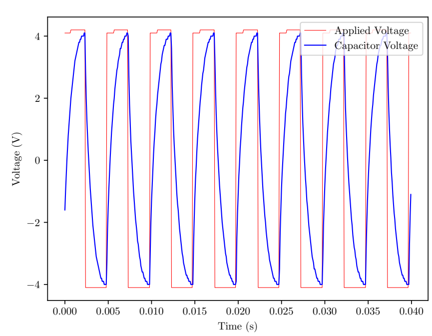
# 5 Analysis

## 5.1 Frequency and Resistor Response

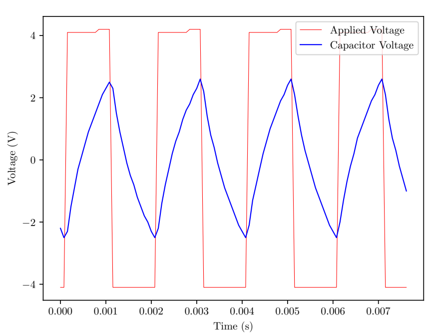
Apologies in advance for the small plot size but they are vectorised so you can zoom in as far as you like.



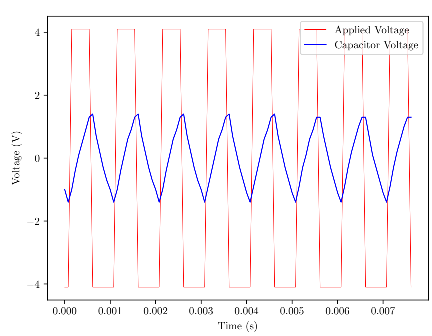

(a) 100Hz




(b) 200Hz



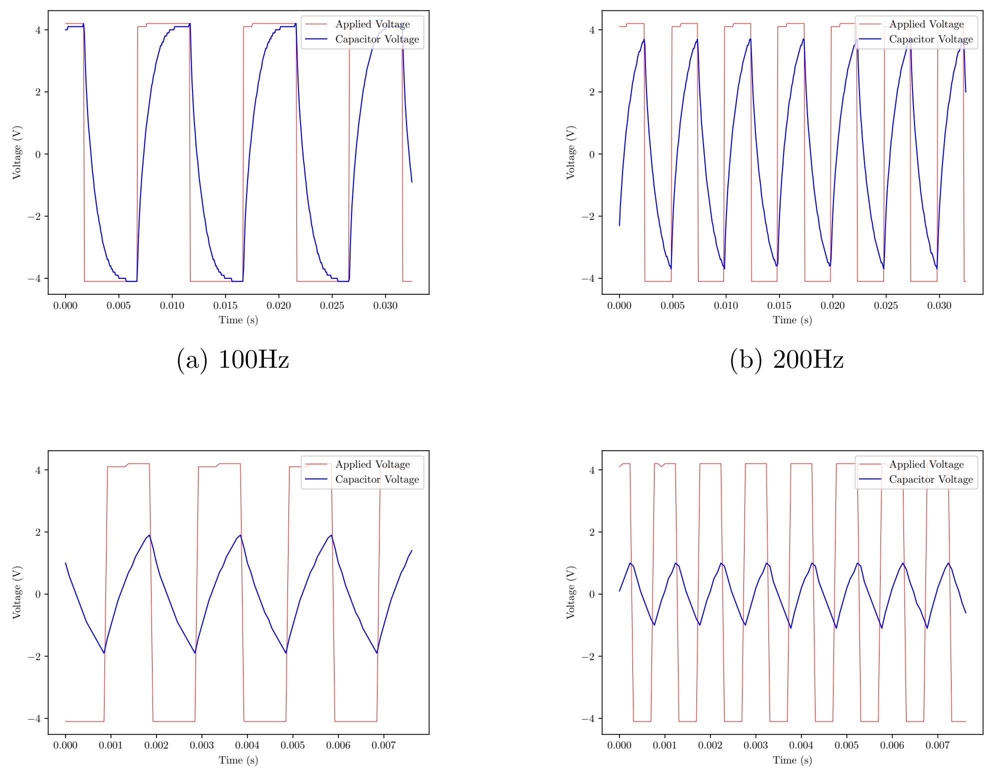

(c) 500Hz




(d) 1000Hz

Figure 5.1: Voltage readings for various frequencies of AC voltage across a 5.6k$\Omega$ resistor

From the figures above, it's clear to see that an increase in frequency leads to the capacitor not being able to charge up fully in the given time. This makes sense as nothing else in the circuit is changing so the time constant $\tau$ is remaining the same throughout as the time given to the capacitor for it to charge up decreases.




(a) 100Hz

(b) 200Hz

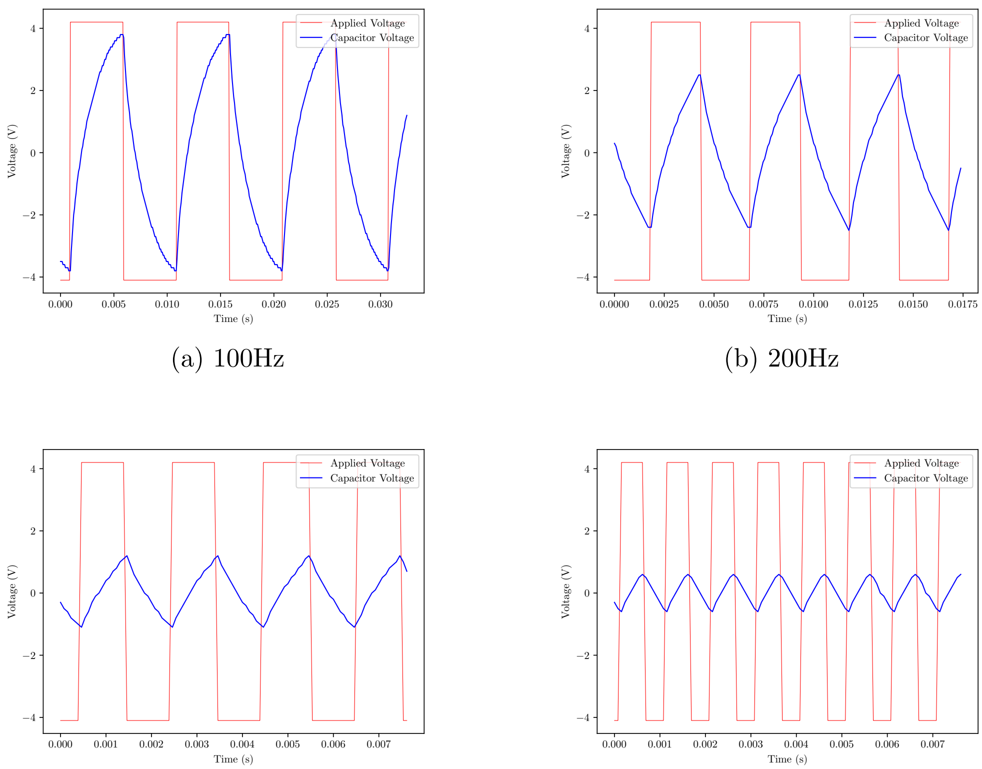
(c) 500Hz

(d) 1000Hz

Figure 5.2: Voltage readings for various frequencies of AC voltage across a 8.2k$\Omega$ resistor

The same is true for the plots above. The resistor in this circuit had a higher resistance and, as we expect from the equation $\tau = RC$, the capacitor couldn't even charge fully at 200Hz, where the circuit with less resistance could.

Below in Figure 5.3 we see exactly the same pattern. A higher resistance leads to the time constant being large and thus the capacitor cannot charge fully in the time given, with it charging less and less per cycle the higher the frequency of the applied voltage.




(a) 100Hz

(b) 200Hz

(c) 500Hz

(d) 1000Hz

Figure 5.3: Voltage readings for various frequencies of AC voltage across a 15kΩ resistor

All of these plots were created using the code in Appendix 1

## 5.2 Time Constant $\tau = RC$

In this section we will try to experimentally determine the value of the time constant for two RC circuits with C = 100nF and R = 15kΩ and 5.6kΩ respectively. To do this, we will look at the equations that describe the charging and discharging of a capacitor:

$$\text{Charging:} \quad V_C(t) = V_\epsilon[1 - e^{-\frac{t}{RC}}] \tag{5.1}$$

$$\text{Discharging:} \quad V_C(t) = V_\epsilon e^{-\frac{t}{RC}} \tag{5.2}$$

By linearising these equations we get the following

$$\text{Charging:} \quad \ln(V_\epsilon - V_C) = -\frac{t}{RC} + \ln(V\epsilon) \tag{5.3}$$

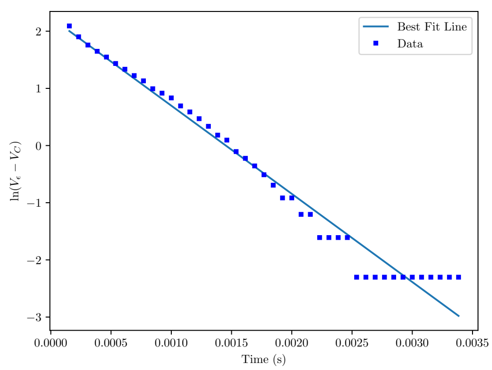
$$\text{Discharging:} \quad \ln(V_C) = -\frac{t}{RC} + \ln(V\epsilon) \tag{5.4}$$

Now we need to take the values from our collected data and plot, in the first case $\ln(V_\epsilon - V_C)$ against $t$ and in the second case $\ln(V_C)$ against $t$. We can then perform a linear least squares fit on this data to determine the slope, which will give us a value for $-\frac{1}{RC}$, from which we can extract the value of $\tau$. Below are the results.

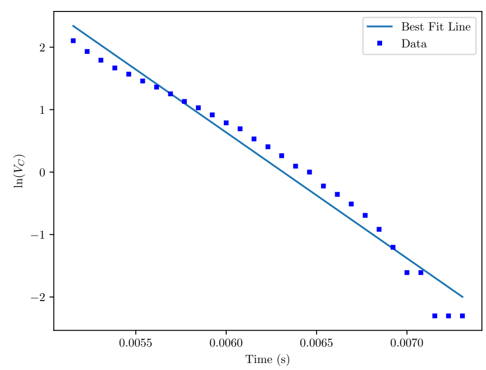
Starting with the 5.6kΩ resistor (Figure 5.4), our expected value is

$$\tau = RC = 5600 \times 1 \times 10^{-7} = 5.6 \times 10^{-4}$$

When we find $\tau$ for the charging and discharging plots and take their average we get a value of $\tau = 5.723\,022\,5 \times 10^{-4}$. The uncertainty on this measurement starts as a standard uncertainty for the line of best fit parameters and through propagation of uncertainties for the inversion and averaging out we get a final value of $\tau = 5.7230 \times 10^{-4} \pm 0.0309 \times 10^{-4}$. This is quite close to our expected value but does not agree within experimental uncertainty. The reason for this is likely only using one charging/discharging cycle to run the analysis on.




(a) Charging



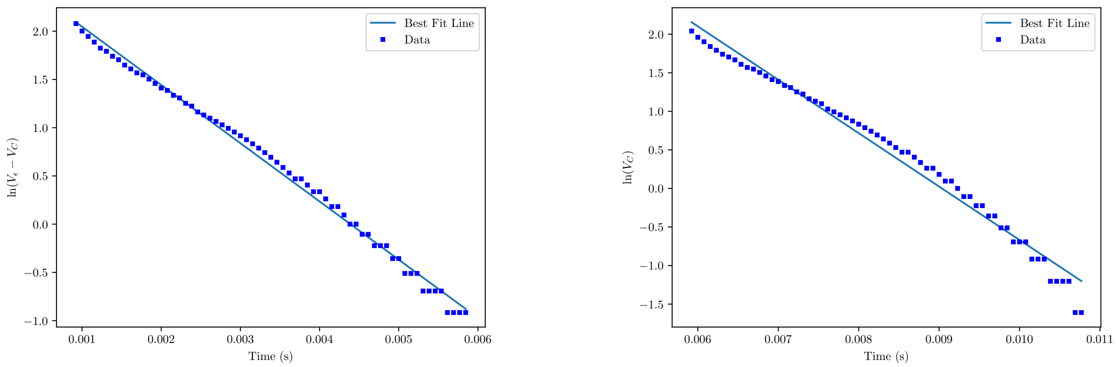

(b) Discharging

Figure 5.4: 5.6kΩ at 100Hz

For the 15kΩ resistor (Figure 5.5) our expected value is

$$\tau = RC = 15000 \times 1 \times 10^{-7} = 1.5 \times 10^{-3}$$

Performing exactly the same operations as above, we find $\tau = 1.5562 \times 10^{-3} \pm 0.0058 \times 10^{-3}$. Again this is fairly close but does not agree within experimental uncertainty. This is likely for the same reason: only using one charging/discharging cycle. All of the code for this section is in Appendix 2




(a) Charging (b) Discharging

Figure 5.5: 15kΩ at 100Hz

## 5.3 Phase Difference Between Current $I$ and Source Voltage $V_\epsilon$ in an RC Circuit

We will now investigate the phase difference between Current $I$ and Source Voltage $V_\epsilon$ in an RC circuit. To do this we will use the data collected in subsection 4.3. Below is the plot of the source voltage and the voltage across the resistor. We aren't displaying the current as it is minuscule compared to the source voltage, but the voltage will have the same period and phase difference.

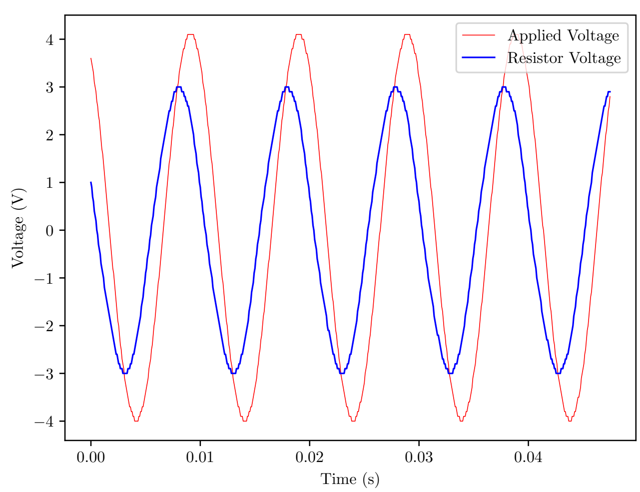



Figure 5.6: Voltage across a 15kΩ resistor in an RC circuit and the applied AC voltage

We used the function `scipy.signal.find_peaks` to find the positions of the maxima of both sets of data. We could then calculate the average time difference between peaks and found it to be exactly 0.001077. There was originally an uncertainty on this value but it turned out to be due to rounding errors in `float64` precision when calculating the standard deviation with `numpy.std`. However there *must* be some uncertainty to this measurement and since we didn't include the uncertainty due to the inaccuracy of the equipment used since we took an average, we'll use that for this. That gives us a difference in phase of $0.001077 \pm 0.000022$ seconds. Taking the wavelength into account ($0.00992267 \pm 0.00000019$), we can get a value in radians, namely $0.2170788\pi \pm 0.0000042$. All of the code for this section is in Appendix 3

## 5.4 Capacitance C

In this section we will verify the capacitance C of the capacitor used in this experiment. To do this we will derive an equation for C from the equation describing the discharging of a capacitor $V_C = V_\epsilon e^{-\frac{t}{RC}}$:

$$\begin{aligned}
V_{C1} &= V_\epsilon e^{-\frac{t_1}{RC}} \\
V_\epsilon &= V_{C2} e^{\frac{t_2}{RC}} \\
\therefore V_{C1} &= V_{C2} e^{\frac{t_2}{RC}} e^{-\frac{t_1}{RC}} \\
&= V_{C2} e^{\frac{t_2 - t_1}{RC}} \\
\implies \frac{V_{C1}}{V_{C2}} &= e^{\frac{t_2 - t_1}{RC}} \\
\implies \log_e\left(\frac{V_{C1}}{V_{C2}}\right) &= \frac{t_2 - t_1}{RC} \\
\implies C &= \frac{t_2 - t_1}{R \log_e\left(\frac{V_{C1}}{V_{C2}}\right)}
\end{aligned}$$

This would be valid if the discharging started at $V_C = 0$, but ours starts at $V_C = 4$ so we must adjust this equation, leaving us with

$$C = \frac{t_2 - t_1}{R \log_e\left(\frac{V_{C1}+4}{V_{C2}+4}\right)} \tag{5.5}$$

Calculating this using two reasonably chosen points we get

$$\begin{aligned}
t_1 &= 0.005692, V_{C1} = -0.5 \\
t_2 &= 0.007307, V_{C2} = -3.9 \\
\therefore C &= 8.111\,522\,479 \times 10^{-8}\text{F}
\end{aligned}$$

Now we need to consider the uncertainty of this measurement. It's a complicated one but if we break it down we see that uncertainties come from the measurement of the time and voltage for each point, which is 2% of the value as that's the accuracy of the equipment used in this experiment. Then the logarithm needs to be considered, as well as the final division. Note that there is no uncertainty on the value of the resistor as we know it to be 5.6kΩ. Below are each of the separate uncertainties as well as the final.

$$
\begin{aligned}
t_2 - t_1 &= 1.615 \times 10^{-3} \pm 0.185\,246\,984\,3 \times 10^{-3} && \text{: Type B, Addition Propagation} \\
\log_e\left(\frac{V_{C1}+4}{V_{C2}+4}\right) &= 3.555348061 \pm 0.7800052328 && \text{: Type B, Function Propagation} \\
R\log_e\left(\frac{V_{C1}+4}{V_{C2}+4}\right) &= 19909.94914 \pm 4369.029304 && \text{: Multiplication by a Constant} \\
C &= 8.111\,522\,479 \times 10^{-8} \pm 2.008\,132\,925 \times 10^{-8} && \text{: Division Propagation}
\end{aligned}
$$

and so our final result is $C = 8.112 \times 10^{-8} \pm 2.008 \times 10^{-8}$F, which agrees with our expected result of 100nF or $10 \times 10^{-8}$F.

# 6 Conclusion and Recommendations

To conclude, in many respects these experiments were successful. We investigated the relationship between resistor value in an RC circuit and the frequency of the AC voltage supplied to it and found that the resistance has a proportional effect on the time constant $\tau$ of the circuit, as we would expect from the equation $\tau = RC$.

We determined the time constant $\tau$ in its own right for two set-ups of RC circuit and found them to be close to, but not in agreement within experimental uncertainty with, the relevant expected values. This disagreement could likely be avoided in future experiments by taking data from multiple charging/discharging cycles to analyse.

We investigated the phase difference between current and source voltage in an RC circuit and found it to be around $\frac{\pi}{5}$.

Finally we verified that the capacitance in our circuit was in fact (around) 100nF by using equations that describe the discharging of a capacitor in an RC circuit. Our experimentally determined value agreed with the expected value but the uncertainty could likely be greatly improved by using more than one set of points and averaging the final value.

# 7 Appendix

Appendix 1: Code that interprets Resistance Frequency data

```
from matplotlib import pyplot as plt
import numpy as np
from numpy import cos, pi, sin, sqrt, exp, random
import matplotlib
# matplotlib.use('pgf')
matplotlib.rcParams.update({
    'pgf.texsystem': 'pdflatex',
    'font.family': 'serif',
    'text.usetex': True,
    'pgf.rcfonts': False,
})

file = open('Cap_100Hz_5k6.txt', 'r')
header = file.readline()
lines = file.readlines()
N = len(lines)
i=0
# data[0] is Time [s] - Voltage Capacitor
# data[1] is Voltage [V] - Voltage Capacitor
# data[2] is Time [s] - Voltage Source
# data[3] is Voltage [V] - Voltage Source
data = np.zeros((4, N))
# Reading the file, getting the data into the data array
for line in lines:
    line = line.strip()
    columns = line.split()
    data[0][i] = float(columns[0])
    data[1][i] = float(columns[1])
    data[2][i] = float(columns[2])
    data[3][i] = float(columns[3])
    i += 1
file.close()
# Plotting both the applied voltage and the voltage measured across
    the capacitor against time
plt.figure()
plt.plot(data[2], data[3], label='Applied Voltage', color='red', lw
    =0.5)
plt.plot(data[0], data[1], label='Capacitor Voltage', color='blue',
    lw=1)
# Making it all look better
plt.legend(loc=1)
plt.xlabel("Time (s)")
plt.ylabel("Voltage (V)")
plt.show()
```

Appendix 2: Code that determines $\tau$ from charging and discharging data

```
from matplotlib import pyplot as plt
import numpy as np
from numpy import cos, pi, sin, sqrt, exp, random
import matplotlib
# matplotlib.use('pgf')
matplotlib.rcParams.update({
    'pgf.texsystem': 'pdflatex',
    'font.family': 'serif',
    'text.usetex': True,
    'pgf.rcfonts': False,
})

file = open('Cap_100Hz_15k.txt', 'r')
header = file.readline()
lines = file.readlines()
N = len(lines)
i=0
# data[0] is Time [s] - Voltage Capacitor
# data[1] is Voltage [V] - Voltage Capacitor
# data[2] is Time [s] - Voltage Source
# data[3] is Voltage [V] - Voltage Source
data = np.zeros((4,N))
# Reading the file, getting the data into the data array
for line in lines:
    line = line.strip()
    columns = line.split()
    data[0][i] = float(columns[0])
    data[1][i] = float(columns[1])
    data[2][i] = float(columns[2])
    data[3][i] = float(columns[3])
    i += 1
file.close()
# 5k6 data for the first charging/discharging cycle
# chargingVE=data[3][2:45]
# chargingVC=data[1][2:45]
# dischargingVE=data[3][67:96]
# dischargingVC=data[1][67:96]+4
# chargingT=data[0][2:45]
# dischargingT=data[0][67:96]
# 15k data for the first charging/discharging cycle
chargingVE=data[3][12:77]
chargingVC=data[1][12:77]
dischargingVE=data[3][77:141]
dischargingVC=data[1][77:141]+4
chargingT=data[0][12:77]
dischargingT=data[0][77:141]
# Creating the arrays that will hold the final scatter plot data
chargingY=[]
dischargingY=[]
```

```python
# Calculating the y values for charging
for i in range(np.size(chargingT)):
    x=chargingVE[i]-chargingVC[i]
    chargingY.append(np.log(x))
# And discharging
for c in range(np.size(dischargingT)):
    v=dischargingVC[c]
    dischargingY.append(np.log(v))
# Fitting the linear least squares line for charging
def linearLeastSquares(x, y):
    Ni=np.size(x)
    m = ((Ni*sum(x*y)) - sum(x)*sum(y))/((Ni*sum(x**2))-(sum(x))**2)
    c = ((sum(x**2)*sum(y))-(sum(x*y)*sum(x)))/((Ni*sum(x**2))-(sum(x)**2))
    di=y-((m*x)+c)
    um = sqrt(((sum(di**2)/((Ni*sum(x**2))-(sum(x)**2)))*(Ni/(N-2))))
    uc = sqrt((((sum(di**2)*sum(x**2))/(Ni*((Ni*sum(x**2))-(sum(x)**2))))*(Ni/(Ni-2))))
    print('m=',m,'+/-',um)
    print('c=',c,'+/-',uc)
    return [m,c,um,uc]
chargingFit=linearLeastSquares(chargingT,chargingY)
dischargingFit=linearLeastSquares(dischargingT,dischargingY)
# Calculating tau and its uncertainty then printing
chargingTau=-1/chargingFit[0]
chargingTauUn=chargingTau*sqrt((chargingFit[2]/chargingFit[0])**2)
dischargingTau=-1/dischargingFit[0]
dischargingTauUn=(dischargingTau)*sqrt((dischargingFit[2]/dischargingFit[0])**2)
tau=(chargingTau+dischargingTau)/2
tauUn=sqrt(chargingTauUn**2 + dischargingTauUn**2)/2
print('tau =',tau,'+/-',tauUn)
# Plotting the charging scatterplot
plt.errorbar(chargingT, chargingY, fmt='bs', ecolor='black', label='Data', markersize=3)
plt.xlabel('Time (s)')
plt.ylabel('$\ln(V_\epsilon -V_C)$')
plt.plot(chargingT, chargingFit[0]*chargingT+chargingFit[1], label='Best Fit Line')
plt.legend()
# Plotting the discharging scatterplot
plt.figure()
plt.errorbar(dischargingT, dischargingY, fmt='bs', ecolor='black', label='Data', markersize=3)
plt.xlabel('Time (s)')
plt.ylabel('$\ln(V_C)$')
plt.plot(dischargingT, dischargingFit[0]*dischargingT+dischargingFit[1], label='Best Fit Line')
plt.legend()
plt.show()
```

## Appendix 3: Code that interprets Current data

```python
from matplotlib import pyplot as plt
import numpy as np
from numpy import cos, pi, sin, sqrt, exp, random
from scipy.signal import find_peaks
import matplotlib
# matplotlib.use('pgf')
matplotlib.rcParams.update({
    'pgf.texsystem': 'pdflatex',
    'font.family': 'serif',
    'text.usetex': True,
    'pgf.rcfonts': False,
})

file = open('Res_100Hz_15k.txt', 'r')
header = file.readline()
lines = file.readlines()
N = len(lines)
i=0
# data[0] is Time [s] - Resistor Voltage
# data[1] is Current [I] - Resistor Voltage
# data[2] is Time [s] - Voltage Source
# data[3] is Voltage [V] - Voltage Source
data = np.zeros((4, N))
# Reading the file, getting the data into the data array
for line in lines:
    line = line.strip()
    columns = line.split()
    data[0][i] = float(columns[0])
    data[1][i] = float(columns[1])
    data[2][i] = float(columns[2])
    data[3][i] = float(columns[3])
    i += 1
file.close()
# Finding the peaks of each
ResPeaks=find_peaks(data[1], distance=3)
AppPeaks=find_peaks(data[3], distance=3)
# Calculating the average wavelength from both sets
pi2Arr=np.concatenate((data[0][ResPeaks[0][1]]-data[0][ResPeaks[0][0]], data[0][ResPeaks[0][2]]-data[0][ResPeaks[0][1]], data[0][ResPeaks[0][3]]-data[0][ResPeaks[0][2]], data[0][AppPeaks[0][1]]-data[0][AppPeaks[0][0]], data[0][AppPeaks[0][2]]-data[0][AppPeaks[0][1]], data[0][AppPeaks[0][3]]-data[0][AppPeaks[0][2]]),axis=None)
pi2=np.mean(pi2Arr,dtype=np.float32)
# Calculating the average difference in seconds between relevant peaks from each set
diffs=np.concatenate((data[0][AppPeaks[0][0]]-data[0][ResPeaks[0][0]], data[0][AppPeaks[0][1]]-data[0][ResPeaks[0][1]], data[0][
```

```
AppPeaks[0][2]]-data[0][ResPeaks[0][2]], data[0][AppPeaks[0][3]]-data[0][ResPeaks[0][3]]), axis=None)
diff=np.mean(diffs,dtype=np.float32)
radiansDiff=2*diff/pi2
# Uncertainty calculations
pi2Un=np.std(pi2Arr,dtype=np.float32)/sqrt(np.size(pi2Arr))
diffUn=np.std(diffs,dtype=np.float16)/sqrt(np.size(diffs))
radiansDiffUn=radiansDiff*sqrt((diffUn/diff)**2 + (pi2Un/pi2)**2)
# Printing values and uncertainties
print('wavelength:', pi2, '+/-', pi2Un)
print('avg time difference:', diff, '+/-', diffUn)
print('phase diff in pi radians:', radiansDiff, '+/-', radiansDiffUn)
# Plotting both the applied voltage and the voltage measured across the resistor against time
plt.figure()
plt.plot(data[2], data[3], label='Applied Voltage', color='red', lw=0.5)
plt.plot(data[0], data[1], label='Resistor Voltage', color='blue', lw=1)
# Making it all look better
plt.legend(loc=1)
plt.xlabel("Time (s)")
plt.ylabel("Voltage (V)")
plt.show()
```