1. (a) We generate 20 000 random numbers according to the Gaussian distribution with form

$$P_{\text{Gaussian}}(N, \langle N \rangle) = \frac{1}{\sqrt{2\pi\langle N \rangle}} \exp\left(-\frac{(N - \langle N \rangle)^2}{2\langle N \rangle}\right) \tag{1}$$

i. The distribution of photon yields is shown in figure 1, along with its expected heights. For use in following questions, the first number generated was $N = 9\,758.529$, which rounds to $N = 9\,759$.

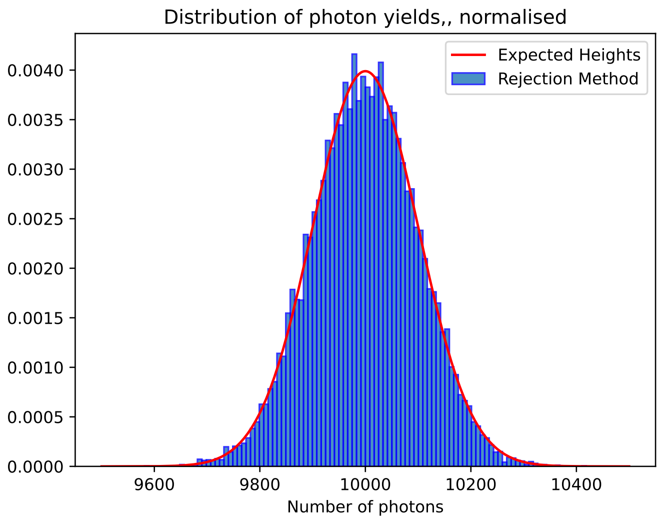

Figure 1: Normalised distribution of photon yields. Expected heights are given by the Gaussian distribution in equation (1).

ii. The average yield was calculated to be $9\,999.426\,6$, which is round about what we expect.

iii. We found the variance to be $10\,020.466$. The assumed distribution has a variance of 10 000, so this is close, but it's hard to put an uncertainty on it so we can't say whether it's reasonable or not.

(b) We plot the distribution that photon momentum magnitudes obey, namely the Boltzmann distribution:

$$P(p) \propto p^2 e^{-p/T} \tag{2}$$

Figure 2 shows the unnormalised distribution. Note that we plot it only on $p \in [0, 30]$ as we can't plot it all the way to infinity, and it drops off appreciably by $p = 30$. Integrating this distribution over $[0, \infty)$ we find a normalisation factor of 16, so we plot the normalised distribution in figure 3, given by

$$P(p) = \frac{p^2 e^{-p/T}}{16}. \tag{3}$$

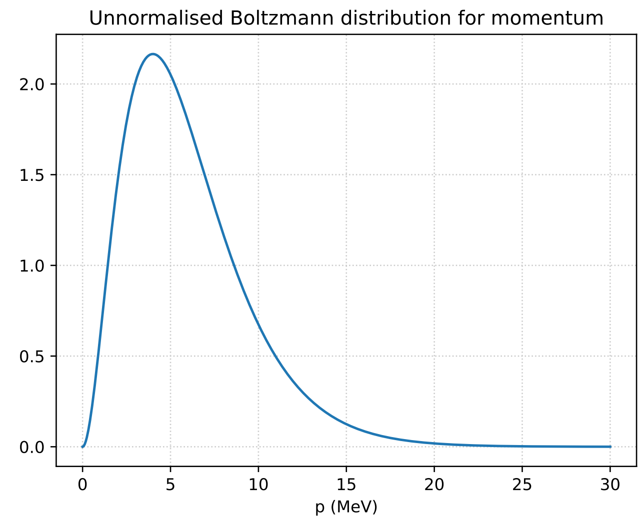

Figure 2: Unnormalised distribution of momentum magnitudes, following the Boltzmann distribution from equation (2) with $T = 2\,\mathrm{MeV}$.

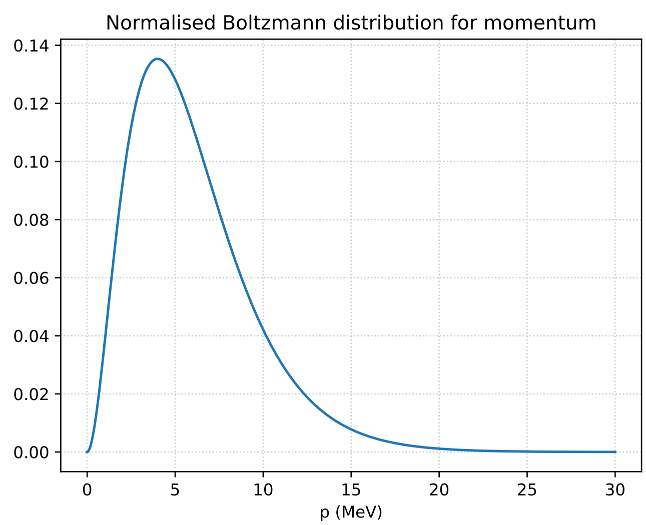

Figure 3: Normalised distribution of momentum magnitudes, once again following the distribution from equation (2) with $T = 2\,\mathrm{MeV}$, now divided by 16.

(c) We now generate a set of random photon momentum magnitudes according to the distribution found above, specifically the normalised version. The number of photons to generate for is given by the first generated number in 1. a).

  i. We plot the histogram in figure 4

  ii. The average p was found to be $5.9985\,\mathrm{MeV}$.

(d) don't forget to derive these boys

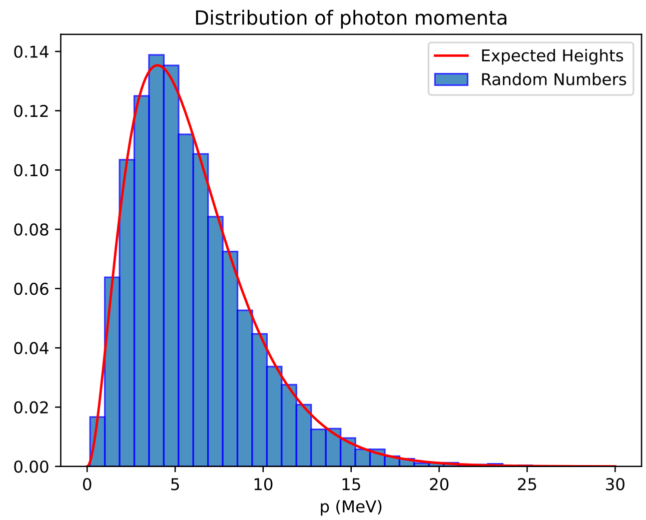

Figure 4: Distribution of photon momentum magnitudes p, generated according to equation (3), for 9 759 photons. The expected heights are of course given by equation (3) as well.

(e) We aim to determine the expressions needed for the inverse method of generating random numbers. We begin with θ.
We can start with the general form of the inverse transform method

$$x_i = \int_{-\infty}^{y_i} P(y')dy' \tag{4}$$

where x_i is a uniform number generated on $[0, 1]$. We know that θ starts at 0, so we can write

$$\begin{aligned} x_i &= \int_0^{\theta_i} \frac{1}{2} \sin\theta' d\theta' \\ &= \frac{1}{2}\left[-\cos\theta'\right]_0^{\theta_i} \\ &= \frac{1}{2}[-\cos\theta_i + 1] \\ \implies 2x_i - 1 &= -\cos\theta_i \\ \implies \theta_i &= \arccos(1 - 2x_i). \end{aligned}$$

Now we can tackle ϕ, once again starting equation (4). ϕ starts at $-\pi$, so we write

$$\begin{aligned} x_i &= \int_{-\pi}^{\phi_i} \frac{1}{2\pi} d\phi' \\ &= \frac{1}{2\pi}[\phi']_{-\pi}^{\phi_i} \\ &= \frac{1}{2\pi}[\phi_i + \pi] \\ \implies 2\pi x_i - \pi &= \phi_i \\ \implies \phi_i &= 2\pi\left(x_i - \frac{1}{2}\right). \end{aligned}$$

(f) We now use the inverse transform method to generate the distributions for θ and ϕ using the expressions found above. We will generate 9 759 values as for the momentum magnitudes.

i. We plot the histograms

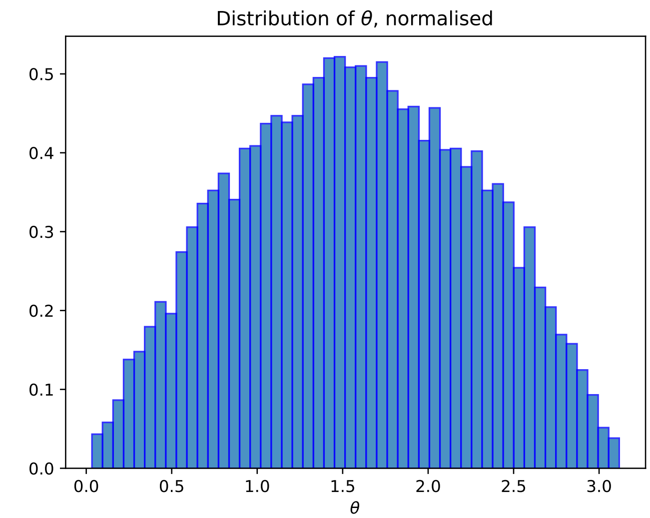

Figure 5: Distribution of 9 759 random θ values generated with the inverse transform method.

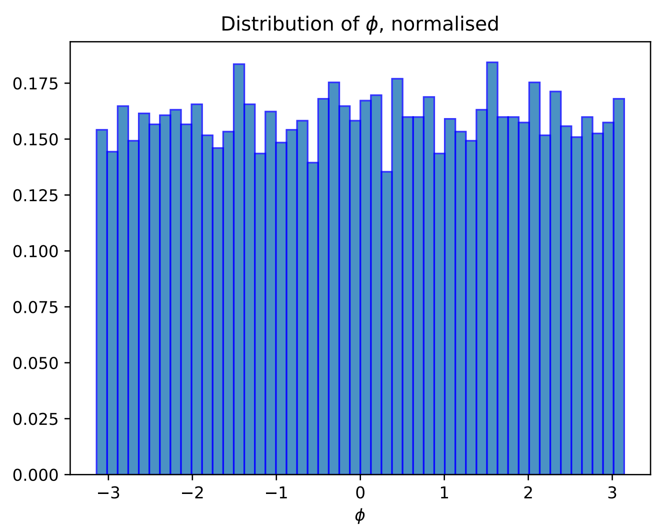

Figure 6: Distribution of 9 759 random ϕ values generated with the inverse transform method.

ii. We can find the average momentum in the z-direction for the first generated event. In spherical coordinates, the z coordinate is given by

$$z = \cos\theta$$

so simply finding this value for each photon in the event and taking the mean we find $\langle p_z \rangle = -0.000\,736\,8\,\mathrm{MeV}$.