

1. To use importance sampling with Monte Carlo integration, we identified that the integrand of

$$I = \int_{-1}^6 dx \int_0^5 dy \int_0^5 dz e^{-x/3} \left(1 + 0.1 \ln(\sqrt{x^2 + y^2 + z^2 + 1})\right) \quad (1)$$

is mostly flat over the interval, in terms of  $y$  and  $z$  at least, so we can sample those values just by a uniform distribution over the interval. The  $x$  values need to be sampled according to  $e^{-x/3}$ , using the inverse transform method. To do this we consider  $w_i$ 's distributed uniformly on  $[0, 1]$ . Then

$$\begin{aligned} w_i &= \int_{-\infty}^{x_i} e^{-x'/3} dx' \\ &= \int_{-1}^{x_i} e^{-x'/3} dx' \\ &= -3(e^{-x_i/3} - e^{-1/3}) \\ \implies x_i &= -3 \ln \left( -\frac{w_i}{3} + e^{1/3} \right) \end{aligned}$$

gives us the distribution of  $x$  values.

Now we want to calculate  $I$  using

$$I = \frac{1}{N} \sum_{i=1}^N \frac{f(x_i, y_i, z_i)}{w(x_i, y_i, z_i)}. \quad (2)$$

In this case  $f(x, y, z)$  is the original integrand.  $w(x, y, z)$  needs to be normalised, so can't just be  $e^{-x/3}$ , it needs to be divided by the product of the integral of it over the integration interval, including the  $y$  and  $z$  intervals, which turns out to be  $-3(e^{-2} - e^{1/3}) \times 25$ .

Generating  $x_i$  values using the inverse transform method, and  $y_i$  and  $z_i$  uniformly, we used equation (2) and generated 1 000 000 points per coordinate. Doing this 100 times and taking the mean, we found

$$I = 107.120\,26 \pm 0.000\,35 \quad (3)$$

where the uncertainty is simply the standard deviation divided by  $\sqrt{\text{numRuns}} = 10$ .

2. In order to estimate the integral

$$\langle x \rangle = \frac{\int_0^\infty x e^{-x} dx}{\int_0^\infty e^{-x} dx} \quad (4)$$

using the Metropolis method, we chose to generate  $x_i$  values according to  $e^{-x}$  over the interval  $[0, \infty)$ . This need not be normalised as we are dividing by the integral of it anyway.

We generated around 1 000 000 points using this method. After skipping the first 100 to avoid the transient stage, and taking every 58 points to avoid correlation, we had around 30 000 points. The number of initial values to skip was chosen somewhat arbitrarily, as we could not find a way to determine the equilibration of the method. The number to skip between points was chosen as a result of the application of the autocorrelation function.  $\Delta = 2$  was chosen as it gave us an acceptance ratio of around 43%, and the starting point was chosen to

be  $x = 0$ , as  $e^{-x}$  has its maximum there over the interval.  
The integral could then be found using

$$I = \frac{\sum_{i=1}^N x_i}{N} \tag{5}$$

where  $N$  is the number of points generated.  
Performing this method 100 times and taking the mean, we found

$$I = 1.001\,16 \pm 0.000\,50 \tag{6}$$

where the uncertainty is simply the standard deviation divided by  $\sqrt{\text{numRuns}} = 10$ .