PHY2111

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Contents

Chapter 2

Kinematics in One Dimension

2.1 Lab 1

Make sure to always review the instructions for all lab reports. Everything must be typed, including calculations. Answer all questions in your own words including definitions; write complete sentences and show all of your work, including steps in your calculations. Follow the instructions in the assignment folder to submit your worksheet.

2.1.1 Part A - Experimental Errors and Uncertainty

Pre–Lab Questions

1. Explain the difference between accuracy and precision.

   Accuracy is how close a measured or average value is to the true value, whereas precision is how close the individual measured values are to one another (repeatability)

2. Name a statistical quantity that can be used to indicate an accuracy level.

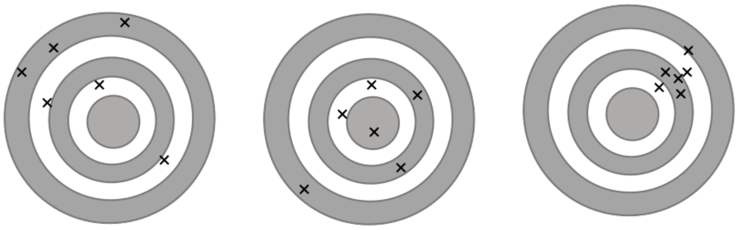
   Percent Error can be used to indicate an accuracy level.

3. Name a statistical quantity that can be used to indicate a precision level.

   Standard Deviation can be used to indicate a precision level.

4. To what level of accuracy (1 cm, 0.1 cm, 0.01 cm, 0.001 cm) can the ruler used to measure the lengths of the pencil in Table 1 of the Lab 1 Instructions Sheet be read?

   It can be read to $0.01\,\mathrm{cm}$**, because the measurements are accurate to this degree.**



In the above cartoon, three bullseyes have been drawn representing dart boards. Each has had six darts thrown at it by three different throwers, A, B, and C, respectively.

5. Which thrower is the most accurate? Explain your choice.

   Thrower B is the most accurate because, on average, their darts cluster closest to the center.

6. Which thrower is the most precise? Explain your choice.

   Thrower C is the most precise because the grouping is closest together, regardless of the distance from the center.

7. Which would you want on your team? Explain your choice.

   Ideally, a thrower who is both accurate and precise, but I would choose thrower B because they have a possibility of actually hitting the center, rather than thrower C, who would almost certainly never hit the center.

Post–Lab Questions

Part 1

A student measures the value of the acceleration due to gravity several times and obtains the following results (all in $\mathrm{m/s^2}$ units):

$$9.95,\ 10.10,\ 9.97,\ 9.99,\ \text{and}\ 10.02$$

Given: The accepted value for the acceleration due to gravity on Earth is $9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$.

Remark.

1. Mean

$$\overline{x} = \frac{9.95 + 10.10 + 9.97 + 9.99 + 10.02}{5} = 10.006\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

2. Standard Deviation

$$\begin{aligned}(9.95 - 10.006)^2 &= 0.003136\\(10.10 - 10.006)^2 &= 0.008836\\(9.97 - 10.006)^2 &= 0.001296\\(9.99 - 10.006)^2 &= 0.000256\\(10.02 - 10.006)^2 &= 0.000196.\end{aligned}$$

Sum: 0.01372

$$\sigma = \sqrt{\frac{0.01372}{5-1}} = 0.0585662.$$

8. Determine the uncertainty range for this set of data, using any calculation tool at your disposal. Please record the uncertainty range here with appropriate significant figures and units as discussed in the above pages.

$$g = 10.01 \pm 0.06\,\frac{\mathrm{m}}{\mathrm{s}^2}.$$

9. Determine both the accuracy and precision levels in percentage form. Show your work here.

$$\begin{aligned}\text{Percent Error} &= \frac{|10.01 - 9.81|}{9.81} \times 100\,\% \approx 2.0\,\%\\ \text{Precision Level} &= \frac{0.06}{10.01} \times 100\,\% \approx 0.6\,\%.\end{aligned}$$

10. Do random errors alone seem to account for any inaccuracies from the accepted value, or does a systematic error appear to be present? Explain your reasoning.

$$\begin{aligned}10.01\,\frac{\mathrm{m}}{\mathrm{s}^2} - 9.81\,\frac{\mathrm{m}}{\mathrm{s}^2} &= 0.2\,\frac{\mathrm{m}}{\mathrm{s}^2}\\ \frac{0.2\,\frac{\mathrm{m}}{\mathrm{s}^2}}{\sigma\;\mathrm{or}\;0.06} &= 3.33.\end{aligned}$$

Since the measured mean differs from the expected value ($9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}$) by about 3 times the standard deviation, random errors alone do not seem to account for the total inaccuracy of the measurements.

Part 2

We may not always be able to determine an accuracy level using a percent error calculation when an accepted value for a quantity is unknown. However, if we have multiple ways of determining a quantity, such as independent measuring methods or theories, we can compare two different results for the same quantity using a percent difference. Comparing the uncertainty range between the two methods is very useful as well.

Later on in Module 2, we will study frictional forces along with coefficients of friction which are unitless numbers that describe the amount of friction between various surfaces. Let us compare a set of measurements for a friction coefficient between a wooden and rubber surface using two different methods of measurement: a simple pulling technique versus an incline sliding technique. Here's the data:

11. Complete the values in the table for the uncertainty ranges and precision levels for each set and find the percent difference between the two sets. Show your worked out calculations below.

Table 2.1: Coefficients of Friction between wood and rubber surfaces

Direct Pulling Method	Incline Plane Method
0.72	0.71
0.70	0.91
0.79	0.66
0.74	0.79
0.75	0.87

Remark.

1. Direct Pulling Method

$$\overline{x}_{\text{pull}} = \frac{0.72 + 0.70 + 0.79 + 0.74 + 0.75}{5} = 0.74$$
$$\sigma_{\text{pull}} \approx 0.034 \longleftarrow \text{Calculated using Google Sheets.}$$

Uncertainty Range $= 0.74 \pm 0.03$

Precision Level $= \frac{0.034}{0.74} \times 100\,\% \approx 4.6\,\%$

2. Incline Plane Method

$$\overline{x}_{\text{incline}} = \frac{0.71 + 0.91 + 0.66 + 0.79 + 0.87}{5} = 0.788$$
$$\sigma_{\text{incline}} \approx 0.11 \longleftarrow \text{Calculated using Google Sheets.}$$

Uncertainty Range $= 0.79 \pm 0.11$

Precision Level $= \frac{0.11}{0.79} \times 100\,\% \approx 14\,\%$

The percent difference between the two average values is

$$\frac{|0.79 - 0.74|}{(0.74 + 0.79) \div 2} \times 100\,\% = 6.5\,\%.$$

Table 2.2: Coefficients of Friction between wood and rubber surfaces

	Direct Pulling Method	Incline Plane Method
	0.72	0.71
	0.70	0.91
	0.79	0.66
	0.74	0.79
	0.75	0.87
Uncertainty Range	0.74 ± 0.03	0.79 ± 0.11
Precision Level (%)	4.6 %	14 %
Percent Difference	6.5 %	

12. Which data set seems to be more reliable and why? Does this set have less random or systematic error?

    The Direct Pulling Method has a significantly smaller standard deviation, so it is more precise and probably more reliable in terms of random error. There isn't really a value to compare against, so there doesn't seem to be a large systematic offset.

13. Given the percent difference between the two sets, do you think the two sets provide a statistically different value for those coefficients, or do you think they are statistically the same? Use the uncertainty ranges to help explain this.

    The difference between the means $(0.788 - 0.74 \approx 0.05)$ **is small, and the uncertainty ranges overlap. This means that the sets are not statistically much different.**

14. Comparing the two uncertainty ranges, does any of the data appear to be suffering from any systematic effects? Why or why not?

    There is no strong indication of a systematic effect in one method vs. the other. In other words, no single shift is making all values bigger or smaller in a way that is outside the other method's range.

2.1.2 Part B - Graphical Analysis and Techniques

Pre-Lab Questions

15. State two things that graphs are able to provide when comparing two variables.

    Graphs provide visual trends and relationships, which are hard to visualize without a graphical image. They also represent quantitative parameters, which can let you extract important information about the dataset, such as a slope or intercept.

16. What is the recommended type of plot to use for these graphs?

    A scatter plot is recommended, which shows each point individually.

Procedure

The goal of this exercise is for you to determine the relationship and constant of proportionality between the radius and area of a circle. You may already know what this relationship is, but here you will attempt to "prove" it to yourself. You will be provided the diameters of several circles, from which you can find the respective radii. The areas of the circles will be found by an independent method. If we then plot a graph of area vs. radius for these circles, hopefully the shape of the curve generated will suggest what the relationship is and allow you to "zero in on it" just like in the example from the instructions (separate instructions document).

We will be using some data collected from circles of varying size cut out from rigid sheets of paper. If we first determine the area of the rectangular sheets of paper and measure their mass, we can compute the density of the paper. Thus, the area of the cut-out circles can be determined by measuring their mass and using the same density value.

Let us define the two-dimensional (or surface) density as:

$$D = \frac{m}{A}$$

where m is the mass, and A is the area it covers. Since a cut out of this same paper will have the same density as the entire sheet, we can solve for the area by using the same density and measured mass. Thus, we have

$$A = \frac{m}{D}.$$

Below is a set of data collected for two sheets of paper used to generate the circles we will use. That is followed by the dimensions of the cut-out circles.

Table 2.3: Measurements of Paper Sheets

	Sheet 1	Sheet 2
Mass (g)	9.198	9.104
Length (cm)	27.93	28.01
Width (cm)	21.63	21.62
Area (cm^2)	604.1	605.6
Density ($\frac{\mathrm{g}}{\mathrm{cm}^2}$)	0.01522	0.01503

Remark. The average density D_{avg} is

$$D_{\text{avg}} = \frac{0.01522 + 0.01503}{2} \approx 0.015\,13\ \frac{\mathrm{g}}{\mathrm{cm}^2}.$$

Table 2.4: Measurements of Paper Circles

Diameter (cm)	Mass (g)	Area cm^2	Radius (cm)	Radius^2 (cm^2)
4.88	0.31	20.36	2.44	5.95
6.19	0.48	31.80	3.10	9.58
7.09	0.62	41.26	3.55	12.57
7.89	0.77	50.78	3.95	15.56
9.15	1.01	66.91	4.58	20.93
10.35	1.27	84.03	5.18	26.78
11.75	1.67	110.21	5.88	34.52
15.63	2.89	191.01	7.82	61.07

17. Complete the area and density values in Table 2.3. Be sure to provide one sample calculation of each here and remember to limit the digits appropriately. Recall that the area of a rectangle is length times width. Also, fill in the average density at the bottom of the table.

Remark (Sample Calculation).

a. Given

$$\begin{aligned} \text{Mass} &= 9.198\,\text{g} \\ \text{Length} &= 27.93\,\text{cm} \\ \text{Width} &= 21.63\,\text{cm}. \end{aligned}$$

b. Calculations

$$\begin{aligned} A &= 27.93\,\text{cm} \times 21.63\,\text{cm} \approx 604.1\,\text{cm}^2 \\ D &= \frac{9.198\,\text{g}}{604.1\,\text{cm}^2} \approx 0.015\,23\,\frac{\text{g}}{\text{cm}^2}. \end{aligned}$$

18. Using the average density found for Table 2.3, use the masses of the circles in Table 2.4 to determine their respective areas (using the equation from earlier, $A = \frac{m}{D}$). Please provide one sample calculation here. Also compute the radii values from the diameters in Table 2.4.

Remark (Sample Calculation).

a. Given

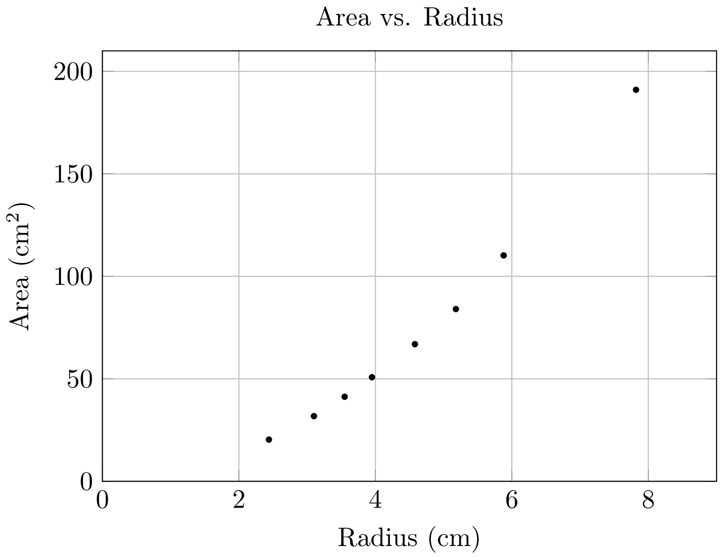
$$\begin{aligned} D_{\text{avg}} &= 0.015\,13\,\frac{\text{g}}{\text{cm}^2} \\ \text{Diameter} &= 4.88\,\text{cm} \\ \text{Mass} &= 0.31\,\text{g}. \end{aligned}$$

(The table has been rounded)

b. Calculations

$$\begin{aligned} A &= \frac{0.308\,\text{g}}{0.015\,13\,\frac{\text{g}}{\text{cm}^2}} \approx 20.36\,\text{cm}^2 \\ r &= \frac{4.88\,\text{cm}}{2} = 2.44\,\text{cm} \\ r^2 &= 2.44^2 \approx 5.95\,\text{cm}^2. \end{aligned}$$

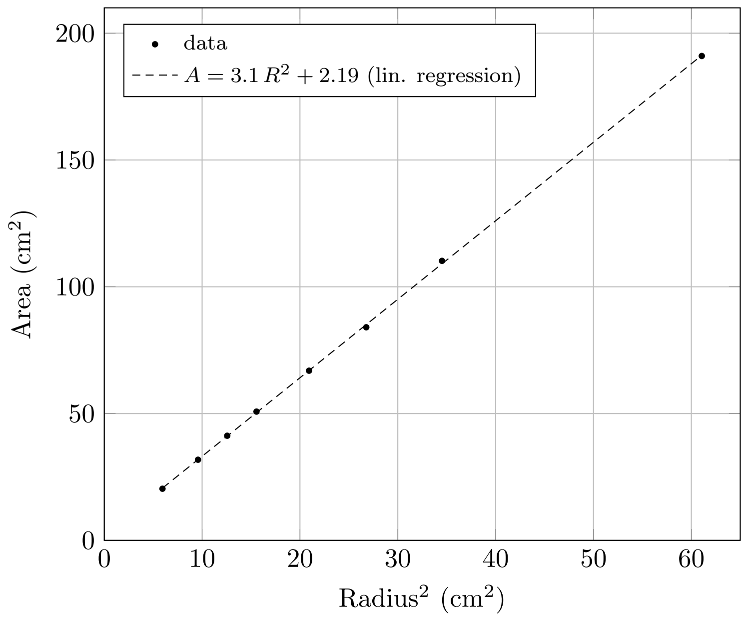
19. Prepare a graph of Area vs. Radius and insert that here (copy and paste functions should work between Excel and Word). Double check to be sure you have the area data along the vertical axis and the radii along the horizontal axes. If not, you will want to reverse the columns in your spreadsheet before generating the graph. Remember to provide a title and label both the axes in a similar fashion as the above examples.



20. Assuming the data points suggest some mathematical relationship, can you tell what it is? It should be parabolic. We can now draw the conclusion that the area of the circle is proportional to the square of the radius. If this is true, then a plot of Area vs. Radius^2 should yield a straight line. Go back to Table 2.4, square each radius value and complete the final column on the right. Again, keep appropriate significant figures.

    The mathematical relationship between a circle's area and its radius is, of course, $A = \pi r^2$.

21. Prepare a plot of Area vs Radius^2. Again, double check the data ranges along the axes so you have the area data along the vertical axis, and label all the axes appropriately. The points should now suggest a straight line. Click on a data point and select Trendlines under the Chart menu. Insert the linear function and be sure to check mark the option to display the equation on the graph. Place this graph here.



Post–Lab Questions

22. If we ignore the y-intercept (hopefully it is close to zero on your graph), the equation of a line is $y = mx$. Substituting in the variables from the graph, the equation becomes $A = cr^2$, where A is the area, r is the radius and c represents the constant of proportionality. Judging by the value of the slope you got from the second graph and your geometry knowledge for the area of a circle, what common name do we have for this constant of proportionality?

    The common name is pi $(\pi) = 3.14159265\ldots$

23. Given the true value of this constant, find the percent error of the slope. Show your work here.

$$\begin{aligned}\text{Percent Error} &= \left|\frac{c_{\text{exp}} - c_{\text{true}}}{c_{\text{true}}}\right| \times 100\,\% \\ &= \left|\frac{3.1 - \pi}{\pi}\right| \times 100\,\% \approx 1.32\,\%.\end{aligned}$$

24. Obviously, there are random errors in the mass and length measurements, as well as minor variations across a sheet of paper which will generate uncertainty in our slope. A quick way to estimate this uncertainty is to find the percent difference between the two density values found in Table 2.3. Compute this percent difference and use that percentage to find the uncertainty in the slope value. Summarize the uncertainty range for the slope here. Show all your work.

Remark.

Sheet 1: $D_1 = 0.015\,22\,\frac{\mathrm{g}}{\mathrm{cm}^2}$

Sheet 2: $D_2 = 0.015\,03\,\frac{\mathrm{g}}{\mathrm{cm}^2}$

$$\begin{aligned} \Delta D &= |D_1 - D_2| = |0.01522 - 0.01503| = 0.000\,19\,\frac{\mathrm{g}}{\mathrm{cm}^2} \\ \overline{D} &= \frac{D_1 + D_2}{2} = \frac{0.01522 + 0.01503}{2} = 0.015\,125\,\frac{\mathrm{g}}{\mathrm{cm}^2} \\ \text{Percent Diff.} &= \frac{\Delta D}{\overline{D}} \times 100\,\% \approx 1.26\,\% \\ \Delta c &= c_{\mathrm{exp}} \times 0.0126 \approx 3.1 \times 0.0126 \approx 0.04. \end{aligned}$$

So, the slope is

$$c = 3.1 \pm 0.04.$$

25. Does the accepted value for this known constant fall within the uncertainty range just determined above? What does this imply about the existence of any systematic errors in the data? In other words, do random errors alone explain the accuracy level attained in question.

The accepted value is pi ($3.14159265\ldots$), and the experimental result is 3.1 ± 0.04. This gives an uncertainty range from 3.06 to 3.14, meaning that the accepted value of π does not fall within the estimated uncertainty range. There may be systematic errors in the experimental procedure that shift the experimental value away from the accepted value of pi, or more data points would shift the estimate higher so that it includes pi.