PHY2111

Max Kiene

Spring 2025

Contents

Chapter 1

Vectors and Coordinate Systems

1.1 Quiz

1.1.1 Problem 1

Parts A-C Are the following statements true or false?

A. The magnitude of a vector can be different in different coordinate systems. **False**

B. The direction of a vector can be different in different coordinate systems. **False**

C. The components of a vector can be different in different coordinate systems. **True**

1.1.2 Problem 2

A velocity vector $43°$ below the positive x-axis has a y-component of $-32\,\mathrm{m/s}$.

1.1.2.1 Part A

What is the value of its x-component? Express your answer with the appropriate units.

↪ Solution.

$$\begin{aligned}\tan(-43°) &= \frac{-32°}{x} \\ x &= \frac{-32°}{\tan(-43°)} \\ &\approx 34.3\,\frac{\mathrm{m}}{\mathrm{s}}.\end{aligned}$$

□

1.1.3 Problem 3

Let

$$\begin{aligned}\vec{A} &= 4\hat{i} - 2\hat{j} \\ \vec{B} &= -3\hat{i} + 4\hat{j} \\ \vec{D} &= \vec{A} - \vec{B}.\end{aligned}$$

Part A What is the component form of the vector $\vec{D}$?

↪ Solution.

$$\begin{aligned}\vec{D} &= \left[4\hat{i} - \left(-3\hat{i}\right)\right] + \left[-2\hat{j} - 4\hat{j}\right] \\ \vec{D} &= 7\hat{i} - 6\hat{j}.\end{aligned}$$

□

Part B What is the magnitude of vector $\vec{D}$?

↪ Solution.

$$\begin{aligned}\|\vec{D}\| &= \sqrt{7^2 + (-6)^2} \\ &= \sqrt{85} \\ &\approx 9.23.\end{aligned}$$

□

Part C What is the direction of the vector $\vec{D}$? Express your answer in degrees.

↪ Solution.

$$\begin{aligned}\theta &= \arctan\left(-\frac{6}{7}\right) \\ &\approx 40.6^\circ \text{ below the positive } x \text{ axis.}\end{aligned}$$

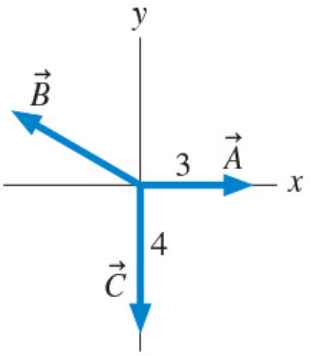
□

1.1.4 Problem 4

For the three vectors shown in the figure below,

$$\vec{A} + \vec{B} + \vec{C} = 1\hat{j}.$$

What is vector $\vec{B}$?



Part A Write $\vec{B}$ in component form. Express your answer in terms of the unit vectors $\hat{i}$ and $\hat{j}$.

↪ **Solution.** We know that

$$\begin{aligned}\vec{A} &= 3\hat{i} = (3, 0)\\ \vec{C} &= -4\hat{j} = (0, -4)\\ \vec{V}_{\text{sum}} &= \hat{j} = (0, 1)\,.\end{aligned}$$

Therefore,

$$\begin{aligned}\vec{B} &= \vec{V}_{\text{sum}} - \vec{A} - \vec{C}\\ &= (-3, 5) = -3\hat{i} + 5\hat{j}.\end{aligned}$$

□

Part B What is the magnitude of $\vec{B}$?

↪ **Solution.**

$$\begin{aligned}\|\vec{B}\| &= \sqrt{(-3)^2 + 5^2}\\ &= \sqrt{34}\\ &\approx 5.83.\end{aligned}$$

□

Part C What is the direction angle of $\vec{B}$? Express your answer in degrees.

...

...

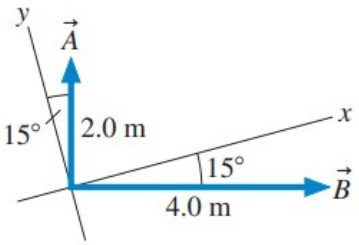
↪ **Solution.**

$$\begin{aligned}\theta &= \arctan\left(\frac{5}{-3}\right) \\ &\approx 59^\circ \text{ above the negative } x \text{ axis.}\end{aligned}$$

□

1.1.5 Problem 5

The figure below shows vectors $\vec{A}$ and $\vec{B}$.



Part A Find $\vec{D} = 4.70\vec{A} + \vec{B}$. Express components of $\vec{D}$ in meters and separated by a comma.

↪ **Solution.** First, we will find the components of $\vec{A}$ and $\vec{B}$ relative to the Cartesian plane.

$$\begin{aligned}\vec{A}_\theta &= 90° - 15° = 75° \\ \vec{A}_x &= 2.0\,\text{m}\cos(75°) \\ \vec{A}_y &= 2.0\,\text{m}\sin(75°)\,.\end{aligned}$$

$$\begin{aligned}\vec{B}_x &= 4.0\,\text{m}\cos(-15°) \\ \vec{B}_y &= 4.0\,\text{m}\sin(-15°)\,.\end{aligned}$$

Then,

$$\begin{aligned}4.70\vec{A} &= 4.70\,(2.0\,\text{m}\cos(75°)\,, 2.0\,\text{m}\sin(75°)) \\ &= (2.43\,\text{m}, 9.08\,\text{m})\,.\end{aligned}$$

$$\begin{aligned}&(2.43\,\text{m}, 9.08\,\text{m}) + (4.0\,\text{m}\cos(-15°)\,, 4.0\,\text{m}\sin(-15°)) \\ &= (6.297\,\text{m}, 8.044\,\text{m})\,.\end{aligned}$$

□

1.1.6 Problem 6

The minute hand on a watch is 3.00 cm in length. Use a coordinate system in which the y-axis points towards the 12 on the watch face.

Parts A-B

A. What is the displacement vector of the tip of the minute hand from 8:00 to 8:20 a.m.? Enter the x and y components of the displacement vector in centimeters separated by a comma.

B. What is the displacement vector of the tip of the minute hand from 8:00 to 9:00 a.m.? Enter the x and y components of the displacement vector in centimeters separated by a comma.

↪ **Solution.** At 8:00 a.m., the minute hand would be pointing directly up (positive y-axis).

At 8:20 a.m., the minute hand would be pointing 30° below the positive x-axis (or $-30°$). This is because there are 12 hours on the clock, meaning there are 3 segments per quadrant, where each segment is $\frac{90°}{3} = 30°$. Since there are 60 minutes in an hour with $\frac{60}{12} = 5$ minutes per segment, 20 minutes past any hour means that the minute hand would be $\frac{20}{5} = 4$ segments clockwise from the positive y-axis.

Now, we can find the x and y components of the "vectors" that are the minute hand at 8:00 ($\vec{A}$) and 8:20 ($\vec{B}$) and subtract $\vec{A}$ from $\vec{B}$.

$$\begin{aligned} \vec{A} &= 3.00\,\text{cm}\,(0, 1) \\ \vec{B} &= 3.00\,\text{cm}\,(\cos(-30), \sin(-30)) \\ \text{Displacement of Tip} &= \vec{B} - \vec{A} \\ &\approx (2.60, -4.50)\,. \end{aligned}$$

For part B, the minute had at any hour is pointing vertically up. Therefore, the displacement over an hour (even though the hand travels in a circular path) would be 0. □

1.1.7 Problem 7

Trevon drives with velocity

$$\vec{v}_1 = \left(50\hat{i} - 10\hat{j}\right) \text{ mph}$$

for 1.0 h, then

$$\vec{v}_2 = \left(20\hat{i} + 30\hat{j}\right)$$

for 2.0 h.

Part A What is Trevon's displacement vector? Enter the x and y components of the displacement vector in miles separated by a comma.

↪ **Solution.** We can simply add the vectors.

$$\begin{aligned}
\vec{D}_x &= 50\hat{i} + 2.0\,\text{hr} \cdot 20\hat{i} = 90\hat{i} \\
\vec{D}_y &= -10\hat{j} + 2.0\,\text{hr} \cdot 30\hat{j} = 50\hat{i} \\
\vec{D} &= 90\hat{i} + 50\hat{i}.
\end{aligned}$$

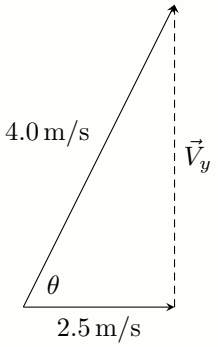
□

1.1.8 Problem 8

Jack and Jill ran up the hill at 4.0 m/s. The horizontal component of Jill's velocity vector was 2.5 m/s.

Parts A-B What was the angle of the hill? Express your answer in degrees.

↪ **Solution.** I will draw a diagram for clarity.



$$\begin{aligned}\cos(\theta) &= \frac{2.5}{4.0} \\ \theta &= \arccos\left(\frac{2.5}{4.0}\right) \\ &\approx 51.32^\circ.\end{aligned}$$

$$\begin{aligned}\vec{V}_y &= \sin 4.0\,(51.32) \\ &\approx 3.12\,\frac{\text{m}}{\text{s}}.\end{aligned}$$

□

1.1.9 Problem 9

A jet plane taking off from an aircraft carrier has acceleration

$$\vec{a} = \left(14\,\mathrm{m/s^2},\ 22^\circ \text{ above horizontal}\right).$$

Parts A-B

A. What is the horizontal component of the jet's acceleration? Express your answer with the appropriate units.

B. What is the vertical component of the jet's acceleration? Express your answer with the appropriate units.

↪ **Solution.**

$$a_x \approx 14 \times 0.9272 \approx 12.98\,\mathrm{m/s^2}$$
$$a_y \approx 14 \times 0.3746 \approx 5.24\,\mathrm{m/s^2}.$$

□