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Problem 7

Find the area of a triangle with height 555 m and base 260 m, with uncertainties of 12 m and 15 m, respectively. Find the area, A_m .

$$\begin{aligned} A &= \frac{1}{2}bh \\ A &= \frac{1}{2} \times 260 \text{ m} \times 555 \text{ m} \\ &= 72\,150 \text{ m}^2. \end{aligned}$$

Using the function rule, the uncertainty in the area is given by

$$\Delta A = \sqrt{\left(\frac{\partial A}{\partial b} \Delta b\right)^2 + \left(\frac{\partial A}{\partial h} \Delta h\right)^2}.$$

Since

$$\frac{\partial A}{\partial b} = \frac{1}{2}h \quad \text{and} \quad \frac{\partial A}{\partial h} = \frac{1}{2}b,$$

we have

$$\frac{\partial A}{\partial b} = \frac{1}{2} \times 555 \text{ m} = 277.5 \text{ m} \quad \text{and} \quad \frac{\partial A}{\partial h} = \frac{1}{2} \times 260 \text{ m} = 130 \text{ m}.$$

Now, substitute the uncertainties $\Delta b = 15 \text{ m}$ and $\Delta h = 12 \text{ m}$:

$$\begin{aligned} \Delta A &= \sqrt{(277.5 \text{ m} \times 15 \text{ m})^2 + (130 \text{ m} \times 12 \text{ m})^2} \\ &\approx 4445.2 \text{ m}^2. \end{aligned}$$

$$A_m = 72150 \pm 4445.2 \text{ m}^2.$$