Max Kiene

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Problem 9

$$a = 12.0 \pm 0.5\,\text{cm}, \quad b = 8.5 \pm 0.5\,\text{cm}, \quad c = 9.8 \pm 0.5\,\text{cm},$$

the volume of an ellipsoid is

$$V = \frac{4}{3}\pi\,a\,b\,c.$$

$$V = \frac{4}{3}\pi(12.0)(8.5)(9.8) \approx 4188\,\text{cm}^3.$$

Using the function rule,

$$\Delta V = V\sqrt{\left(\frac{\Delta a}{a}\right)^2 + \left(\frac{\Delta b}{b}\right)^2 + \left(\frac{\Delta c}{c}\right)^2}.$$

Calculate the relative uncertainties:

$$\frac{\Delta a}{a} = \frac{0.5}{12.0} \approx 0.04167, \quad \frac{\Delta b}{b} = \frac{0.5}{8.5} \approx 0.05882, \quad \frac{\Delta c}{c} = \frac{0.5}{9.8} \approx 0.05102.$$

$$\begin{aligned}
\Delta V &\approx 4188\sqrt{(0.04167)^2 + (0.05882)^2 + (0.05102)^2} \\
&\approx 4188\sqrt{0.001736 + 0.003460 + 0.002602} \\
&\approx 4188\sqrt{0.007798} \\
&\approx 4188 \times 0.0883 \\
&\approx 370\,\text{cm}^3.
\end{aligned}$$

$$V_\text{m} = (4188 \pm 370)\,\text{cm}^3.$$