PHY2111

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Contents

Chapter 1

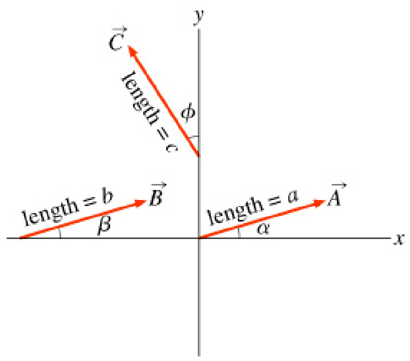
Vectors and Coordinate Systems

1.1 3.1-3.4

1.1.1 Problem Set

1.1.1.1 (Problem 1) ± Resolving Vector Components with Trigonometry

Part A Find the components of the vector $\vec{A}$ with length $a = 1.00$ and angle $\alpha = 15.0°$ with respect to the x-axis as shown in the figure below.



↪ **Solution.**

$$A_x = 1.00 \cos 15.0 = 0.965925826289$$
$$A_y = 1.00 \sin 15.0 = 0.258819045103.$$

□

Part B Find the components of the vector $\vec{B}$ with length $b = 1.00$ and angle $\beta = 10.0°$ with respect to the x axis as shown in the figure.

↪ **Solution.**

$$B_x = 1.00 \cos 10.0 = 0.984807753012$$
$$B_y = 1.00 \sin 10.0 = 0.173648177667.$$

□

Part C Find the components of the vector $\vec{C}$ with length $c = 1.00$ and angle $\phi = 35.0°$ as shown in the figure.

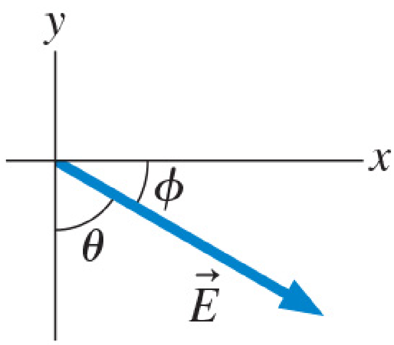
↪ **Solution.** The angle with respect to the x axis would be

$$90 + 35 = 125.$$

$$C_x = 1.00 \cos 125 = -0.573576436351$$
$$C_y = 1.00 \sin 125 = 0.819152044289.$$

□

1.1.1.2 (Problem 2) Problem 3.3 – Enhanced – with Hints and Feedback



Parts A-B What are the x and y components of the vector $\vec{E}$ in terms of the angle θ and the magnitude E shown in the figure above? Express your answers in terms of some or all of the variables θ, E.

↪ **Solution.**

$$E_x = E\cos\left(\frac{3\pi}{2}+\theta\right)$$
$$E_y = E\sin\left(\frac{3\pi}{2}+\theta\right).$$

□

Parts C-D For the same vector, what are the x and y components in terms of the angle ϕ and the magnitude E? Express your answers in terms of some or all of the variables ϕ, E.

↪ **Solution.**

$$E_x = E\cos(2\pi-\phi)$$
$$E_y = E\sin(2\pi-\phi).$$

□

1.1.1.3 (Problem 3) Problem 3.7 – Enhanced – with Hints and Feedback

Parts A-B Find the x and y components of $\vec{v} = 9.5\,\frac{\mathrm{m}}{\mathrm{s}}$, 30° clockwise from the positive y axis.

↪ **Solution.**

$$v_x = 9.5\,\frac{\mathrm{m}}{\mathrm{s}}\cos\left(90^\circ - 30^\circ\right) = 4.75\,\frac{\mathrm{m}}{\mathrm{s}}$$
$$v_y = 9.5\,\frac{\mathrm{m}}{\mathrm{s}}\sin\left(90^\circ - 30^\circ\right) = 8.23\,\frac{\mathrm{m}}{\mathrm{s}}.$$

□

Parts C-D Find the x and y components of $\vec{a} = 1.7\,\frac{\mathrm{m}}{\mathrm{s}^2}$, 30° above the negative x-axis.

↪ **Solution.**

$$a_x = 1.7\,\frac{\mathrm{m}}{\mathrm{s}^2}\cos\left(180^\circ - 30^\circ\right) = -1.47\,\frac{\mathrm{m}}{\mathrm{s}^2}$$
$$a_y = 1.7\,\frac{\mathrm{m}}{\mathrm{s}^2}\sin\left(180^\circ - 30^\circ\right) = 0.85\,\frac{\mathrm{m}}{\mathrm{s}^2}.$$

□

Parts E-F Find the x and y components of $\vec{F} = 60.0\,\mathrm{N}$, 36.9° counterclockwise from the positive y-axis.

↪ **Solution.**

$$F_x = 60.0\,\mathrm{N}\cos\left(90^\circ + 36.9^\circ\right) = -36.03\,\mathrm{N}$$
$$F_y = 60.0\,\mathrm{N}\sin\left(90^\circ + 36.9^\circ\right) = 47.98\,\mathrm{N}.$$

□

1.1.1.4 (Problem 4) $\pm$ Vector Addition and Subtraction

Let vectors

$$\begin{aligned}\vec{A} &= (1, 0, -3)\\ \vec{B} &= (-2, 5, 1)\\ \vec{C} &= (3, 1, 1)\,.\end{aligned}$$

Calculate the following, and express your answers as ordered triplets of values separated by commas.

Parts A-F

$\hookrightarrow$ **Solution.**

A. $\vec{A} - \vec{B}$

We can simply subtract the x, y, and z components (vector subtraction):

$$\begin{aligned}\vec{A} - \vec{B} &= (A_x - B_x, A_y - B_y, A_z - B_z)\\ &= (3, -5, -4)\,.\end{aligned}$$

B. $\vec{B} - \vec{C}$

$$(-5, 4, 0)\,.$$

C. $-\vec{A} + \vec{B} - \vec{C}$

$$(-6, 4, 3)\,.$$

D. $3\vec{A} - 2\vec{C}$

$$(-3, -2, -11)\,.$$

E. $-2\vec{A} + 3\vec{B} - \vec{C}$

$$(-11, 14, 8)\,.$$

F. $2\vec{A} - 3\left(\vec{B} - \vec{C}\right)$

$$(17, -12, -6)\,.$$

$\square$

1.1.1.5 (Problem 5) Problem 3.10

Part A Part A was just graphing the vector.

Part B Find the magnitude of $\vec{A} = 3.0\hat{\imath} + 7.0\hat{\jmath}$.

↪ **Solution.** $\hat{\imath}$ and $\hat{\jmath}$ are unit components, i.e. $(1, 1)$. Therefore,

$$\begin{aligned} A &= \sqrt{3.0^2 + 7.0^2} \\ &= \sqrt{58}. \end{aligned}$$

□

Part C Find the direction angle of $\vec{A} = 3.0\hat{\imath} + 7.0\hat{\jmath}$ measured clockwise from the positive x axis.

↪ **Solution.**

$$\theta = \arctan\left(\frac{7.0}{3.0}\right) = 66.8^\circ.$$

□

Remark. Parts E-L were solved in the exact same way.

1.1.1.6 (Problem 6) Problem 3.12 – Enhanced – with Expanded Hints

Let

$$\vec{A} = 6\hat{\imath} - 2\hat{\jmath}$$
$$\vec{B} = -2\hat{\imath} + 6\hat{\jmath}$$
$$\vec{C} = \vec{A} + \vec{B}.$$

Parts A-C

↪ **Solution.**

A. What is the component form of vector $\vec{C}$?

$$C_x = A_x + B_x = 6\hat{\imath} + \left(-2\hat{\imath}\right) = 4\hat{\imath}$$
$$C_y = A_y + B_y = -2\hat{\jmath} + 7\hat{\jmath} = 5\hat{\jmath}$$
$$\vec{C} = 4\hat{\imath} + 5\hat{\jmath}.$$

B. What is the magnitude of vector $\vec{C}$

$$C = \sqrt{4^2 + 5^2} = \sqrt{41}.$$

C. What is the direction of vector $\vec{C}$?

$$\theta_C = \arctan\left(\frac{5}{4}\right) = 51.3°.$$

□

1.1.1.7 (Problem 7) Problem 3.14

Let

$$\vec{A} = 4\hat{\imath} - 2\hat{\jmath}$$
$$\vec{B} = -3\hat{\imath} + 5\hat{\jmath}$$
$$\vec{C} = 2\vec{A} + 3\vec{B}.$$

$\hookrightarrow$ **Solution.** This was solved in a similar way to problem 6. $\square$

1.1.1.8 (Problem 8) Problem 3.14

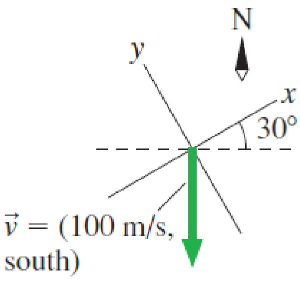
Let

$$\vec{E} = \hat{\imath} + 2\hat{\jmath}$$
$$\vec{F} = 2\hat{\imath} - \hat{\jmath}.$$

$\hookrightarrow$ **Solution.** This was solved in a similar way to problem 7. $\square$

1.1.1.9 (Problem 9) Problem 3.20 – Enhanced – with Hints and Feedback

A velocity vector is shown below.



Parts A-B What are the x and y components of the velocity vector?

↪ **Solution.** First, we can make it relative to the axis by

$$\vec{v}_{rel} = 90^\circ - 30^\circ = 60^\circ.$$

Then, we can figure out the angle from the x axis by

$$180^\circ + 60^\circ = 240^\circ.$$

Compute the x component by

$$v_x = 100 \cos 240 = -50^\circ.$$

Compute the y component by

$$v_y = 100 \sin 240 = -86\,6^\circ.$$

□

1.1.1.10 (Problem 10) Problem 3.27

Part A Find a vector that points in the same direction of the vector $\left(\hat{\imath}+\hat{\jmath}\right)$ and whose magnitude is 1.

↪ **Solution.** The magnitude of $\hat{\imath}$ is 1, and similar with $\hat{\jmath}$.

We can simply find the normalized vector v_n by taking the sum of the magnitudes of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$ (which are both 1) and dividing each by the root of the sum M:

$$
\begin{aligned}
M &= \sqrt{1+1} \\
v_n &= \frac{\hat{\imath}}{M} + \frac{\hat{\jmath}}{M} \\
&= \frac{\hat{\imath}}{\sqrt{2}} + \frac{\hat{\jmath}}{\sqrt{2}}.
\end{aligned}
$$

□

1.1.1.11 (Problem 11) Problem 3.34

You are fixing the roof of your house when a hammer breaks loose and slides down. The roof makes an angle of 25° with the horizontal, and the hammer is moving at 7.5 m/s when it reaches the edge. Assume that the hammer is moving from the top of the roof to its right edge.

Parts A-B What are the horizontal and vertical components of the hammer's velocity just as it leaves the roof?

↪ **Solution.** Given the angle and the magnitude of the hammer's velocity, this is a simple problem.

First, the hammer is sliding downwards, so

$$\theta = 360° - 26° = 335°.$$

Since $\cos\theta = \frac{\text{adj}}{\text{hyp}}$ where the hypotenuse is the magnitude of the velocity and the adjacent side is the desired horizontal component, we can simply solve for the adjacent:

$$\begin{aligned} \cos 335° &= \frac{v_x}{7.5\,\frac{\text{m}}{\text{s}}} \\ v_x &= 7.5\,\frac{\text{m}}{\text{s}} \cos 335° \\ &= 6.8\,\frac{\text{m}}{\text{s}}. \end{aligned}$$

Similarly,

$$\begin{aligned} v_y &= 7.5\,\frac{\text{m}}{\text{s}} \sin 335° \\ &= -3.2\,\frac{\text{m}}{\text{s}}. \end{aligned}$$

□

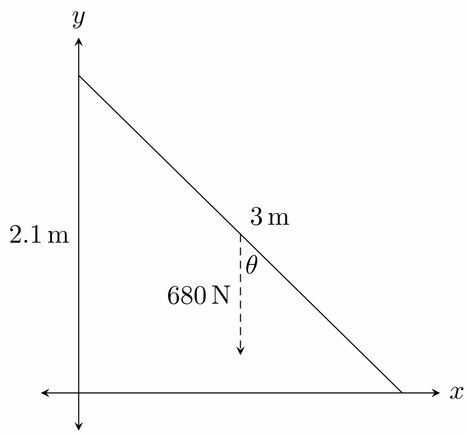
1.1.1.12 (Problem 12) Problem 3.40

Tom is climbing a 3.0 m long ladder that leans against a vertical wall, contacting the wall 2.1 m above the ground. His weight of 680 N is a vector pointing vertically downward. (Weight is measured in newtons, abbreviated N.)

Part A What is the magnitude of the component of Tom's weight parallel to the ladder?

↪ **Solution.**

We can draw a picture of the triangle formed by the ladder and the wall:



We can find theta by using $\cos\theta = \frac{\text{adj}}{\text{hyp}}$:

$$\cos\theta = \frac{2.1}{3}$$
$$\theta = \arccos\left(\frac{2.1}{3}\right) = 45.572\,995\,999\,2^\circ.$$

Then we can figure out the angle relative to the ladder:

$$a = -45.572\,995\,999\,2^\circ.$$

Therefore, the magnitude of his weight parallel to the ladder is

$$|680\,\text{N}\cos\left(-45.572\,995\,999\,2^\circ\right)| = 476\,\text{N}.$$

□

Part B What is the magnitude of the component of Tom's weight perpendicular to the ladder?

↪ **Solution.** Similarly, his perpendicular component is simply the y com-

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. . .

ponent relative to the ladder:

$$|680\,\mathrm{N}\sin\left(-45.572\,995\,999\,2^\circ\right)| = 485.61\,\mathrm{N}.$$

$\square$

1.1.1.13 (Problem 13) Problem 3.23

The position of a particle as a function of time is given by $\vec{r} = \left(6.6\hat{\imath} + 4.0\hat{\jmath}\right) t^2\mathrm{m}$, where t is in seconds.

Parts A-F

↪ **Solution.** We can find the magnitude m and scale by the time:

$$\begin{aligned} m &= \sqrt{6.6^2 + 4.0^2} = 7.717\,512\,552\,63\,\mathrm{m} \\ d(t) &= mt^2. \end{aligned}$$

A. $d(0) = 0\,\mathrm{m}$

B. $d(2.5) = 48.234\,453\,453\,9\,\mathrm{m}$

C. $d(5.7) = 48.234\,453\,453\,9\,\mathrm{m}$

The particle's speed at t is given by

$$s(t) = |d'(t)| = |2mt|.$$

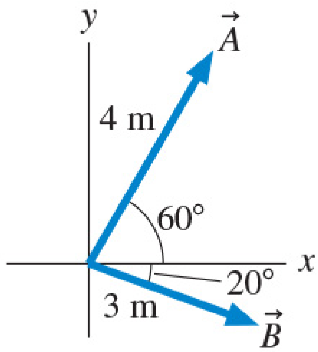
D. $s(0) = 0\,\frac{\mathrm{m}}{\mathrm{s}}$

E. $s(2.5) = 38.587\,562\,763\,1\,\frac{\mathrm{m}}{\mathrm{s}}$

F. $s(0) = 87.979\,643\,1\,\frac{\mathrm{m}}{\mathrm{s}}$

□

1.1.1.14 (Problem 14) Problem 3.25 – Enhanced – with Expanded Hints



Part A Find vector $\vec{C}$ such that $\vec{A} + \vec{B} + \vec{C} = \vec{0}$. Write your answer in component form.

↪ **Solution.** First, the components of vectors $\vec{A}$ and $\vec{B}$:

$$\begin{aligned}
A_x &= 4\cos 60 = 2 \\
A_y &= 4\cos 60 = 3.46410161514 \\
B_x &= 3\cos -20 = 2.81907786236 \\
B_y &= 3\cos -20 = -1.02606042998.
\end{aligned}$$

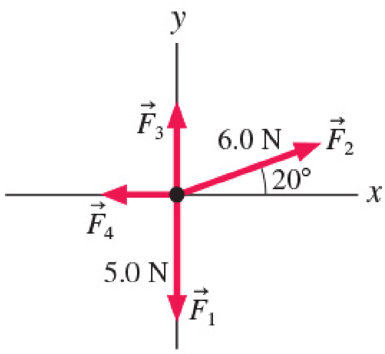
Now, we can solve for $\vec{C}$:

$$\begin{aligned}
\vec{C} &= -\vec{A} - \vec{B} \\
&= -(2, 3.46410161514) - (2.81907786236, -1.02606042998) \\
&= (-4.81907786236, -2.43804118516) \\
&= -4.81907786236\hat{\imath} - 2.43804118516\hat{\jmath}.
\end{aligned}$$

□

1.1.1.15 (Problem 15) Problem 3.45 – Enhanced – with Expanded Hints

Four forces are exerted on the object shown in the figure below. The net force on the object is $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 4.0\hat{\imath}\text{N}$.



Parts A-B What are the components of $\vec{F}_3$ and $\vec{F}_4$?

↪ **Solution.**

First we will write out the components of the known vectors.

$$\begin{aligned}\vec{F}_1 &= (0, -5)\\ \vec{F}_2 &= (6\cos 20, 6\sin 20)\,.\end{aligned}$$

Since the horizontal component of the net force is zero, we can see that the vertical component of $\vec{F}_3$ must be equal and opposite to the sum of the vertical components of $\vec{F}_1$ and $\vec{F}_2$.

$$\begin{aligned}0 &= \vec{F}_{3y} + \vec{F}_{1y} + \vec{F}_{2y}\\ \vec{F}_{3y} &= -\vec{F}_{1y} - \vec{F}_{2y}\\ &= -(-5) - (6\sin 20) = 2.94787914005.\end{aligned}$$

Therefore, $\vec{F}_3 = (0, 2.94787914005)$.

We can do something similar for $\vec{F}_4$.

$$\begin{aligned}4 &= \vec{F}_{4x} + \vec{F}_{2x}\\ \vec{F}_{4x} &= 4 - \vec{F}_{2x}\\ &= -1.63815572472.\end{aligned}$$

Therefore, $\vec{F}_4 = (-1.63815572472, 0)$. □