Multi-room Heating System Modeled as a Dynamical System

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*Abstract*—Everyday cyber-physical systems can be represented as dynamical systems and modeled as such. In this paper, we describe the modeling of a multi-room (four) heating system as a dynamical system using MATLAB’s Simulink add-on. Given that all our assumptions for the model hold, it can regulate all room temperatures within a reasonable range. While the model is a largely simplified version of a realistic situation, it does well to simulate the basic interactions that take place to influence the dynamics of room temperatures with a heating system.

Keywords—Dynamical Systems, Heating System

# Introduction

For modeling a multi-room heating system, we cannot use an asynchronous-only model with fairness conditions to properly illustrate all the factors that contribute to the fluctuating temperatures of each room; there would not be enough synchrony among tasks to ensure that the temperatures are updated simultaneously. We instead focus our attention dynamical system modeling, as we can better describe the changes in the room temperatures as the dynamics of such a system with ordinary differential equations (ODEs). Additionally, the time synchrony between components is a better representation of the real-world factors that simultaneously drive the changes in temperature.

Consider now a heating system that regulates the temperature over four connected rooms with two heaters. The temperature of each room changes linearly with the difference with the outside temperature , the difference with each directly adjacent room’s temperature, and a heater’s current effect on the room. Then the dynamics of the system can be represented by the following equation:

where and , and , and , and . represents the influence that rooms have on each other. If the rooms are not directly adjacent, then . Additionally, if . represents the influence that the outside temperature has on room . represents the influence that a heater has on room . if there is no heater in room or if the heater is off and if a heater is on in the room. The system is parameterized by the constant matrix , the constant vector , the constant vector , and the initial room temperatures in the vector .

Additionally, there are constant vectors that are used to determine the placement of the heaters and whether they are on or off. If a heater is in room , then the heater is off if and on if . A heater is moved from room to room if: (1) room has no heater, (2) room has a heater, (3) , and (4) .

The inputs to the system are the vector for the initial placement of the heaters and . The outputs are the room temperatures .

The scenario we examined to build our model of the multi-room heating system was parameterized as:

With inputs:

For this parameterization, we aimed to keep for all .

# Assumptions

## are reasonable starting temperatures.

* Even though our model receives inputs, once these inputs are received, the system acts as a closed-loop system. The outside temperature, , remains constant once it is received as input and heater placement is maintained internally between time-steps.
* is chosen relative to the values of . For example, it would make no logical sense to have for all , but then have . (Do we need this?)
* The proportional temperature influence that the rooms have on each other, modeled by the matrix , is constant. The effects that the outside temperature and heaters, modeled by and respectively, have on each room are constant as well. This allows the dynamics to be modeled in a simplistic manner.

## Heaters can be moved instantaneously.

## A room cannot have more than one heater at a time.

* No uncontrolled inputs, such as people, other appliances, sunlight, etc. are considered.

## While modeled as a continuous time system, it must be run as a discrete time system. The model assumes a small step size for calculating dynamics.

# Related Work

We modeled a system with a relatively simple design. There were a limited number if heaters and the room temperatures were only affected by adjacent rooms and the outside temperature. We also simplified thermodynamic principles and airflow to be encompassed by a single temperature for each room. Further, the dynamic for our system were linear in order to make analysis of our model easier.

We did not take into account additional environmental factors like humidity or sunlight. Additionally, our system did not drive the room temperatures towards particular reference values as a PID controller might. Instead, we relied on logic that setting certain parameter values for would bound the temperatures within a certain range.

Most current houses and buildings use HVAC systems for temperature regulation rather than only a heating system. Thus, they are designed to perform temperature regulation in a wider range of temperatures and climates. Also, these modern systems have many more components and considerations to take into account than we looked at for our model.

Al-Rousan et al. [1] presented control and modeling of an HVAC system for a single room using dynamical system analysis. Whereas we abstracted away the heating method, they took into consideration the thermodynmaics of airflow for heat transfer. Their model works to regulate both temperature and humidity, and consisted of a humidifier, heater, cooler, fan, ductwork, and sensors for monitoring temperature and humidity. Their model has multiple components, each controlling one of these parts of the overall system. The dynamics over the state varaibles for each of these components are based on thermodynamic principles.

Inputs to their controller are temperature and humidity of the supplied air from the HVAC as well as the mass flow rate of the air. The outputs are the temperature and humidity of the room. The humidity and temperature were driven to set values using PID control. For added precision in converging to an ideal temperature and humidity level, the PID controller coefficients are modified based on whether the system is trying to heat vs. cool or humidify vs. de-humidify the room.

In addition to considering humidity as a factor for temperature, Al-Rousan et al. monitored temperature of the inner walls of the room as state variables with additional dynamics for these in the model. The temperature of the walls are also factored into the dynamics for the ambient room temperature and vise versa.

Compared to our model, which only looked at a high level version of romm temperatures, the model proposed by Al-Rousan et al. considered a much wider variety of environmental factors and a more complex temperature regulation system.

Peng and Passen [2] proposed a model for air conditioning (AC) that ours is more similar to as opposed to that proposed by Al-Rousan et al. Peng and Passen desinged a model more suited for user satisfaction. They separated the room into defined zones, with the idea being that the space near the air conditioner should be separate from the designated ‘working zone’ that inhabitants occupied in order to have better control over the room temperature of the ’working zone’.

Whereas we looked at the influence of multiple rooms on each other, they looked at the influence that nearby spaces, or zones, within a room have on each other. The dynamics for their system were based on Computational Fluid Dynamics theory, whereas our dynamics were a simplified version of heat flow. Despite this, the overall structure of the dynamics was quite similar to ours in that adjacent zone temperatures, or room temperatures for our model, directly influence the temperatures of adjacent zones. However, we considered the effects of outside temperature and heater status as influencing factors for room temperature dynamics whereas Peng and Passen considered the air flow from the AC as the only other factor influencing the state variables of zone temperature.

# Conclusions And Further Work

In this work, we designed a model for a multi-room heating system consisting of 4 rooms. We implemented and verified the model using MATLAB’s Simulink. The results show that our model is able to maintain room temperatures within a specified range.

In order to improve our model, we could make heater placement predictive rather than reactive. In order to do so, heater placement would need to take into account the values of the matrix and the vectors . While more computationally intensive, such a design could help narrow the temperature range the rooms are kept in.

Another alternative would be to model a more complex heater that can vary its heat output on a continuous range. That way, we could use a PID controller to drive the room temperatures to a more precise value rather than keep it within a particular range.

1. Table Type Styles

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